AIP | Physics of Plasmas

Large amplitude dust acoustic solitary wave with positively charged dust grain

Samiran Ghosh and M. R. Gupta

Citation: Physics of Plasmas (1994-present) **13**, 044503 (2006); doi: 10.1063/1.2193913
View online: http://dx.doi.org/10.1063/1.2193913
View Table of Contents: http://scitation.aip.org/content/aip/journal/pop/13/4?ver=pdfcov
Published by the AIP Publishing

Large amplitude dust acoustic solitary wave with positively charged dust grain

Samiran Ghosh[a)]
Government College of Engineering and Textile Technology, 4, Cantonment Road, Berhampore 742101, Murshidabad, West Bengal, India

M. R. Gupta
Centre for Plasma Studies, Faculty of Science, Jadavpur University, India

(Received 20 December 2005; accepted 17 March 2006; published online 18 April 2006)

Large amplitude solitary waves are investigated in a dusty plasma containing electrons, positive ions, negative ions, and positively charged dust grains [N. D'Angelo, J. Phys. D **37**, 860 (2004)] by the Sagdeev potential. Numerical investigations related to Q machine dusty plasma with a positive charge reveal that the nonlinear dust acoustic wave possesses only a supersonic compressive soliton. The range of Mach numbers where such solitary waves exist is also investigated. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2193913]

In a dusty plasma, the dust grains may be charged negatively by plasma electron and ion currents or positively by secondary electron emission, UV radiation, or thermionic emission etc.[1] Due to the higher thermal velocity of electron than ion, the dust grains usually acquire a negative charge in low temperature laboratory dusty plasma.[2] However, in a laboratory Q machine plasma, positively charged dust grains may be produced by replacing the plasma electrons with negative ions whose thermal velocity is smaller than that of ions (positive ions).[3]

Recently, the excitation of both linear "dust acoustic wave" (DAW)[4] and "dust ion acoustic wave"[5] due to a zero order electric field in the presence of collisions is investigated theoretically in a multicomponent electronegative dusty plasma with positive charged dust grains.[3] The nonlinear aspects (such as solitons, shocks, etc.) of multicomponent dusty plasma with negatively charged dust grains have been investigated, both experimentally[6,7] and theoretically.[8–10] But, the physics of electrostatic nonlinear structure in multicomponent electronegative dusty plasma with positively charged dust grains has not yet been investigated. Hence it is instructive to examine the possibility of formation of nonlinear potential structures in such a multicomponent electronegative dusty plasma. In this brief communication (BC), we investigate the large amplitude solitary potential structure of DAW in such multicomponent electronegative dusty plasma by the Sagdeev potential.

We consider a four component unmagnetized dusty plasma consisting of Boltzmann distributed electrons, positive ions, negative ions, and positively charged cold dust grains. At $x\to-\infty$, charge on the dust grain surface is ze where the plasma is assumed to be an undisturbed uniform state [$\phi=0$ and $n_j=n_{j0}$ ($j=e,\ +,\ -,$ and d)] so that

$$n_{e0}+n_{-0}=n_{+0}+zn_{d0} \qquad n_{+0}/n_{-0}=(1-\epsilon z)+\alpha, \tag{1}$$

where $\epsilon=n_{d0}/n_{-0}$ and $\alpha=n_{e0}/n_{-0}$.

The one-dimensional behavior of the DAW is described by the following normalized fluid equations:

$$\partial_T N+\partial_X(NV_d)=0, \tag{2}$$

$$\partial_T V_d+V_d\partial_X V_d=-\partial_X\Phi, \tag{3}$$

$$\epsilon z\partial_X^2\Phi=\alpha e^{\Phi}+e^{\Phi/\sigma_-}-((1-\epsilon z)+\alpha)e^{-\Phi/\sigma_+}-\epsilon zN, \tag{4}$$

where $\sigma_\pm=T_\pm/T_e$. The time (T) and space (X) scales are normalized in units of the dust plasma frequency $\omega_{pd}(=\sqrt{z^2e^2n_{d0}/\epsilon_0 m_d})$ and dust plasma Debye length $\lambda_d(=\sqrt{\epsilon_0 T_e/n_{d0}z^2e^2})$. The velocity V_d is normalized by $\omega_{pd}\lambda_d$ whereas electrostatic potential ϕ and dust density N are normalized by T_e/e and n_{d0}, respectively; $\Phi=e\phi/T_e$ and $N=n_d/n_{d0}$.

The usual linear analysis shows that in terms of the aforementioned normalizing units the linear DAW velocity in the long wavelength limit is

$$\lambda=\sqrt{\frac{\epsilon z}{\alpha+1/\sigma_-+((1-\epsilon z)+\alpha)/\sigma_+}}. \tag{5}$$

We study the nonlinear dynamics of large amplitude DAW in the wave frame defined by $\xi=X-VT$, where V is the velocity of such waves. The Mach number for such waves is given by

$$M=V/\lambda. \tag{6}$$

The fluid equations (2) and (3) can then be integrated exactly with the boundary conditions $\Phi,\ \partial_\xi\Phi\to0;\ N\to1;\ V_d\to0$ as $\xi\to\pm\infty$. After elimination of velocity, we find the following expression for dust fluid density:

$$N=(1-2\Phi/V^2)^{-1/2}. \tag{7}$$

Transforming Poisson's equation (4) to the wave frame ξ and using (7) and then integrating we get

a)Electronic mail: sran_g@yahoo.com

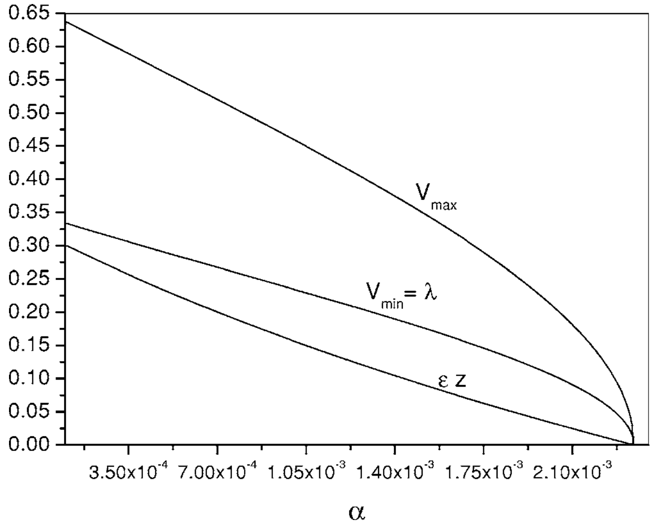

FIG. 1. Variations of ϵz, $V_{\min}=\lambda$ [Eq. (5)] and $V_{\max}$ as a function of α.

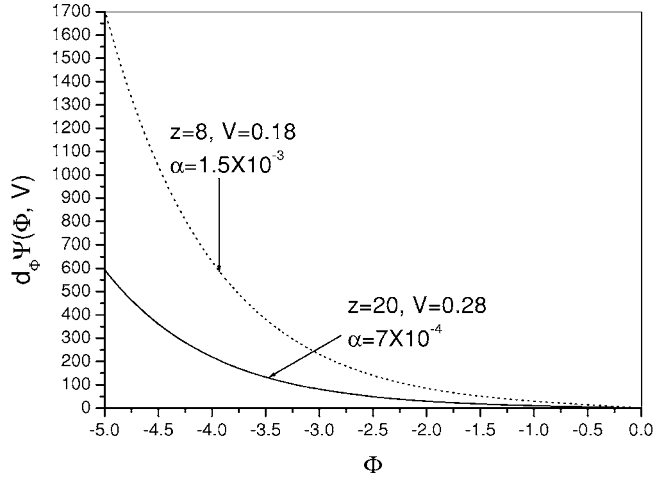

FIG. 2. Variations of $d_\Phi \Psi(\Phi,V)$ [Eq. (13)] as a function of $\Phi(<0)$.

$$\frac{1}{2}\left(\frac{d\Phi}{d\xi}\right)^2 + \Psi(\Phi,V) = 0, \tag{8}$$

where $\Psi(\Phi,V)$ is the Sagdeev potential given by

$$\begin{aligned}\Psi(\Phi,V) = (\epsilon z)^{-1}[&\alpha(1-e^{\Phi}) + \sigma_-(1-e^{\Phi/\sigma_-}) \\ &+ \sigma_+((1-\epsilon z)+\alpha)(1-e^{-\Phi/\sigma_+}) \\ &- \epsilon z V^2(\sqrt{1-2\Phi/V^2}-1)].\end{aligned} \tag{9}$$

Equation (8) represents the energy integral for a classical particle of unit mass moving with velocity $d\Phi/d\xi$ in a potential $\Psi(\Phi,V)$, where V appears as a parameter. For the existence of a solitary wave, one requires that the potential satisfy the following conditions[11]:

$$\begin{aligned}&\text{(i)} \quad \Psi(\Phi,V)|_{\Phi=0} = 0 = \partial_\Phi \Psi(\Phi,V)|_{\Phi=0}, \\ &\text{(ii)} \quad \Psi(\Phi,V)|_{\Phi=\Phi_m} = 0, \\ &\text{(iii)} \quad \Psi(\Phi,V) < 0; \quad 0 < |\Phi| < |\Phi_m|,\end{aligned} \tag{10}$$

where Φ_m is the amplitude of the solitary wave. The first part of condition (i) is obvious and the second part is also satisfied by the charge neutrality condition [Eq. (1)]. Thus condition (i) implies that for the solitary wave, the Sagdeev potential $\Psi(\Phi,V)$ possesses a double root at $\Phi=0$. The third condition in (10) is analogous to the analysis of a particle in the potential well, which leads to

$$\partial^2_\Phi \Psi(\Phi,V)\Big|_{\Phi=0} < 0 \qquad V^2 > V^2_{\min} = \frac{\epsilon z}{\alpha + 1/\sigma_- + ((1-\epsilon z)+\alpha)/\sigma_+} = \lambda^2, \tag{11}$$

where $V_{\min}$ is the lower limit of the large amplitude wave velocity V. Hence, the existence of a potential well, necessary for a solitary wave propagation, one must have $V > V_{\min}$ or equivalently

$$M > M_{\min} = V_{\min}/\lambda = 1. \tag{12}$$

Thus subsonic solitary waves do not exist. The variation of $V_{\min}$ with α is shown in Fig. 1. The nature of the solitary wave (compressive or rarefactive) is determined by the following conditions[11]:

$$d_\Phi \Psi(\Phi,V)\Big|_{\Phi=\Phi_m>0} > 0; \quad d_\Phi \Psi(\Phi,V)\Big|_{\Phi=\Phi_m<0} < 0. \tag{13}$$

The variations of $d_\Phi \Psi(\Phi,V)$ as a function of $\Phi(<0)$ for different $V > V_{\min}=\lambda$ is shown in Fig. 2. Figure 2 shows that $d_\Phi \Psi(\Phi,V)(>0)$ increases with the decrease of $\Phi(<0)$ and thereby ruling out the possibility of rarefactive $(\Phi<0)$ dust acoustic solitary wave. On the other hand, for compressive soliton $(\Phi>0)$, we see that N is real provided $V^2 > 2\Phi_m$ [Eq. (7)]. Also from condition (ii) in (10), we obtain the following relation between soliton amplitude Φ_m and solitary wave velocity: V,

$$\begin{aligned}&\Phi_m = -\beta(\Phi_m) - \frac{\beta(\Phi_m)^2}{2V^2}, \\ &\beta(\Phi_m) = \frac{(\alpha(1-e^{\Phi_m}) + \sigma_-(1-e^{\frac{\Phi_m}{\sigma_-}}) + \sigma_+((1-\epsilon z)+\alpha)(1-e^{-\frac{\Phi_m}{\sigma_+}}))}{\epsilon z}.\end{aligned} \tag{14}$$

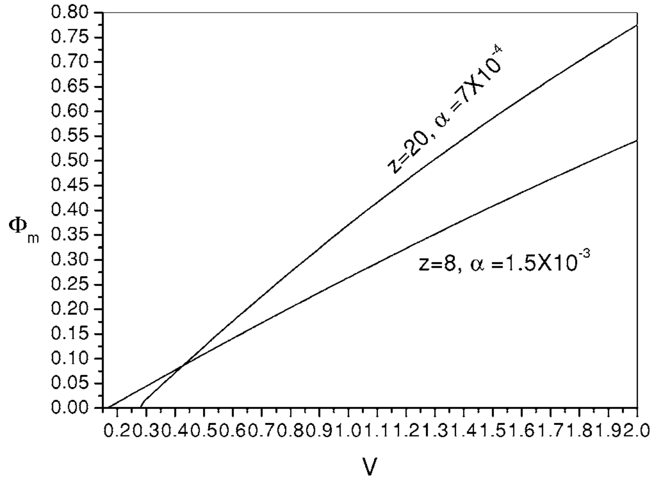

FIG. 3. Variation of soliton amplitude Φ_m with solitary wave velocity V [Eq. (14)]. Corresponding Mach numbers are given by $M=V/\lambda$ [Eq. (6)].

We numerically solve this equation for Φ_m subject to the condition $V>V_{\min}$ for different α and the variations are shown in Fig. 3.

For numerical computation, we consider the set of dusty plasma parameter related to Q machine with positive charge[3]: K^+ as positive ion (mass $m_+ \sim 6.68\times 10^{-26}$ kg), SF_6^- as negative ions (mass $m_- \sim 2.44\times 10^{-25}$ kg), $T_e=T_+ \sim 0.2$ eV, $T_- \sim 0.1$ eV, dust grain radius $r_0=1$ μm, dust mass density $\sim 10^3$ kg m^{-3} and $\epsilon=0.01$. On the basis of these plasma parameters, the charge on dust grains z is determined as a function of α from the following equilibrium current balance equation[3]:

$$((1-\epsilon z)+\alpha)C_+ e^{-ze^2/4\pi\epsilon_0 r_0 T_+} - C_-\left(1+\frac{ze^2}{4\pi\epsilon_0 r_0 T_-}\right) - \alpha C_e\left(1+\frac{ze^2}{4\pi\epsilon_0 r_0 T_e}\right)=0, \qquad (15)$$

where $C_j=\sqrt{8T_j/\pi m_j}$.

The results of this BC can be summarized as follows.

(1) Equation (15) shows that the maximum value of α for which dust grains are positively charged is $\alpha_{\max}\sim 2.1\times 10^{-3}$ (Fig. 1: ϵz vs α), but if $\alpha>\alpha_{\max}$, the dust grains are negatively charged. Figure 1 shows that the value of $\epsilon z(\epsilon=0.01)$ i.e., dust charge z, $V_{\min}=\lambda$ and also $V_{\max}$ decreases with the increase of α.

(2) The variations of $d_\Phi \Psi(\Phi,V)$ as a function $\Phi(<0)$ are shown in Fig. 2. Figure 2 shows that $\Psi(\Phi,V)$ is a monotonic increasing function of $\Phi(<0)$ and thus there does not exist any $\Phi_m(<0)$ for which $\Psi(\Phi_m,V)=0$ provided $V>V_{\min}=\lambda$. Hence for the above-mentioned plasma parameters, there does not exist any rarefactive solitary wave. Thus in a Q machine multicomponent dusty plasma, the DAW possesses only supersonic $(M>1)$ compressive solitary waves.

(3) The variations of Sagdeev potential $\Psi(\Phi,V)$ [Eq. (9)] with Φ for different α, z, V are depicted in Fig. 4. The dotted and solid curves of Fig. 4 show that for lower values of α, i.e., higher values of z and V the depth as well as width of the potential well increases.

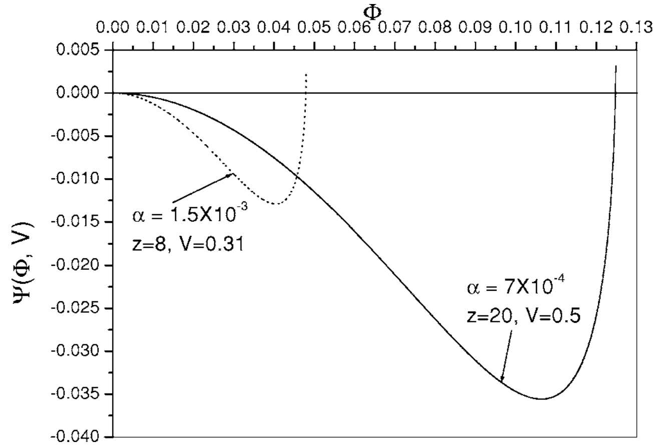

FIG. 4. Variation of Sagdeev potential $\Psi(\Phi,V)$ [Eq. (9)] with potential Φ.

(4) The peak amplitude of the solitary wave increases with the increase of solitary wave velocity V and hence with an increase in the Mach number $M=V/\lambda$ with a decrease in α which are shown in Fig. 3.

(5) The potential profiles of the compressive soliton are depicted in Fig. 5, which are finite width solitons. On the other hand, for the same plasma parameters the dust number density N [Eq. (7)] profiles of compressive solitons are drawn in Fig. 6, which are more spiky than the potential profiles. This is due to the fact that the dust number density N is related to potential Φ by the relation (7) which shows that for a given value of V, the dust number density (N) increases rapidly with the increase in potential (Φ).

(6) Numerical analysis of $\Psi(\Phi,V)$ [Eq. (9)] reveals that there exists a maximum value $V=V_{\max}$ [for $z=8$, $\alpha=1.5\times 10^{-3}$, $V_{\max}=0.32$; $z=20$, $\alpha=7\times 10^{-4}$, $V_{max}=0.52$] for which $\Psi(\Phi,V_{\max})=0$ has the only solution $\Phi=0$; for $V\geqslant V_{\max}$ there does not exist any nonzero solution of $\Psi(\Phi,V_{\max})=0$. This happens due to the fact that for $V\geqslant V_{\max}$, Φ_m as given by Eq. (14) becomes so large that it exceeds the value $\Phi=V^2/2$. The variations of $V_{\max}$ with α is

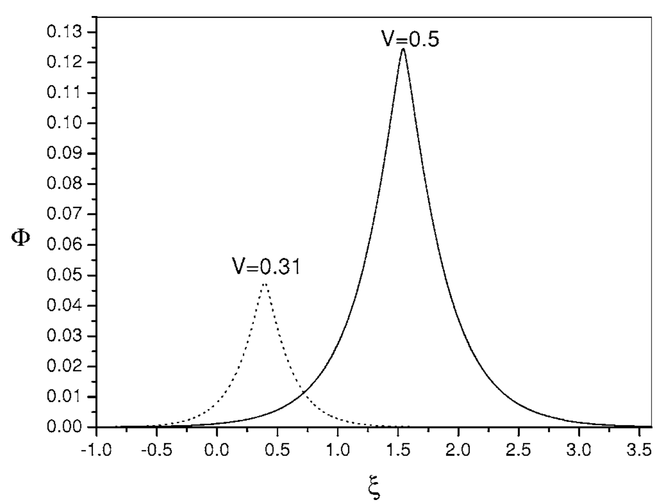

FIG. 5. Variation of solitary potential Φ with ξ for different V. The corresponding Mach numbers are $M=5.96$ (dotted curve) and $M=6.1$ (solid curve).

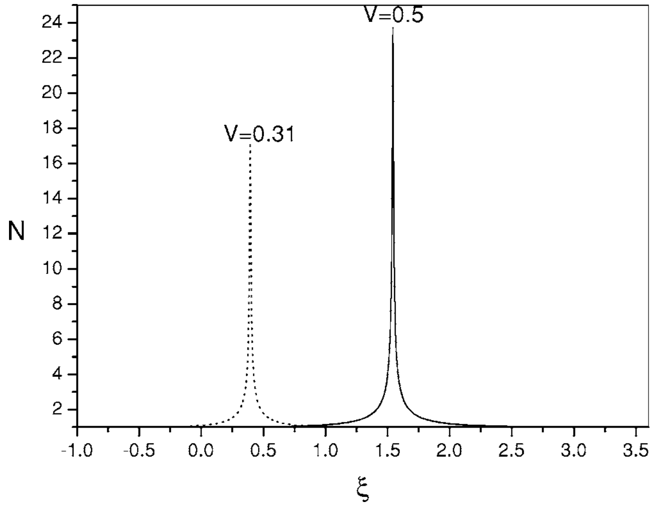

FIG. 6. Variation of dust number density N [Eq. (7)] with ξ.

drawn in Fig. 1. To find $V_{\max}$, we solve $\Psi(\Phi, V) \geqslant 0$ at $\Phi = V^2/2$ for different α. The solution of this equation yields the maximum value $V_{\max}$ and correspondingly the maximum Mach number $M_{\max} = V_{\max}/\lambda$. For the given parameters $M_{\max} \approx 6.2(z=8)$ and $6.4(z=20)$. For any value of $V > V_{\max}(M > M_{\max})$ the dust grains cannot get across the traveling wave potential barrier $\Phi = V^2/2$ and are reflected upstream by this potential barrier. This prevents the formation of a solitary wave and leads to the formation of double layer (monotonic shock like structure).

(7) In this BC the charge on the dust grains are assumed to be constant. But, the charge fluctuations may cause interesting physical phenomena like shock wave formation in dusty plasma (due to delayed charging[10]). The effects of dust charge fluctuations on nonlinear properties of large amplitude DAW using this model is under investigation. Our preliminary results predict that the dust charge fluctuations may produce DA shock. The results of the investigation will be communicated in due course.

The authors would like to thank Professor N. D'Angelo of University of Iowa, Iowa City, Iowa, for helpful discussions during the course of this work.

[1] E. C. Whipple, Rep. Prog. Phys. **44**, 1197 (1981); M. Hornayi, Annu. Rev. Astron. Astrophys. **34**, 383 (1996).
[2] A. Barkan, N. D'Angelo, and R. L. Merlino, Phys. Rev. Lett. **73**, 3093 (1994).
[3] N. D'Angelo, J. Phys. D **37**, 860 (2004).
[4] N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. **38**, 453 (1990); A. Barkan, R. L. Merlino, and N. D'Angelo, Phys. Plasmas **2**, 3563 (1995).
[5] N. D'Angelo, Planet. Space Sci. **38**, 1143 (1990); P. K. Shukla and V. P. Silin, Phys. Scr. **45**, 508 (1992).
[6] Y. Nakamura, H. Bailung, and P. K. Shukla, Phys. Rev. Lett. **83**, 1602 (1999); Y. Nakamura and A. Sharma, Phys. Plasmas **8**, 3921 (2001).
[7] Q. Z. Luo, N. D'Angelo, and R. L. Merlino, Phys. Plasmas **6**, 3345 (1999).
[8] J. X. Ma and J. Liu, Phys. Plasmas **4**, 253 (1997); Samiran Ghosh, S. Sarkar, M. Khan, M. R. Gupta, and K. Avinash, Phys. Lett. A **298**, 49 (2002).
[9] Samiran Ghosh, Phys. Plasmas **12**, 094504 (2005).
[10] M. R. Gupta, S. Sarkar, Samiran Ghosh, M. Debnath, and M. Khan, Phys. Rev. E **63**, 046406 (2001); Samiran Ghosh, S. Sarkar, M. Khan, K. Avinash, and M. R. Gupta, Phys. Plasmas **10**, 977 (2003).
[11] R. Z. Sagdeev, *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, 1966), Vol. IV, p. 23.