KAON MASS IN DENSE MATTER

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Abstract

The variation of kaon mass in dense, charge-neutral baryonic matter at beta-equilibrium has been investigated. The baryon interaction has been included by means of nonlinear Walecka model, with and without hyperons and the interaction of kaons with the baryons has been incorporated through the Nelson-Kaplan model. A self-consistant, one-loop level calculation has been carried out. We find that at the mean field level, the presence of the hyperons makes the density-dependence of the kaon mass softer. Thus, the kaon condensation threshold is pushed up in the baryon density. The loop diagrams tend to lower the kaon condensation point for lower values of $a_3 m_s$. We also find that the S-wave kaon-nucleon interaction plays the dominant role in determining the on-set of kaon condensation and the contribution of the P-wave interaction is insignificant.

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The study of hadronic matter at large temperature and densities is a field of immense current interest because of the possibility of novel phases, such as pion condensation, kaon condensation, quark-matter formation etc, appearing at higher baryonic densities and temperatures, which may be achieved in relativistic heavy ion collisions and /or dense stars. One of the possible interesting features of the dense hadronic matter, investigated by several authors [1, 2, 3, 4, 5, 6, 7] of late, is that a charge-neutral, dense hadronic matter may undergo kaon condensation at sufficiently high densities (baryon densities $\sim$ 3-5 times nuclear matter density). If this is true, then one would expect that the cores of neutron stars may consist of kaon-condensed baryonic matter. The nuclear equation of state in the presence of kaon condensation being softer, this would affect the limits on neutron star masses and radii. A good understanding of the density at which the kaon condensation may occur is also of interest in the context of heavy ion collisions aimed at looking for quark matter as this density is quite close to the density of the expected hadron to quark matter phase transition. Thus, one would like to know whether one passes through a kaon-condensed phase between the baryonic and the quark phases as the nuclear density is increased. The kaon-condensation threshold, $i.e.$, the density at which kaon-condensation begins, is given by the density at which the effective kaon mass in the nuclear medium equals the kaon chemical potential. For a system having zero strangeness, this threshold is equivalent to having zero kaon mass. Thus the investigation of the behaviour of the kaon mass as a function of baryon density is of importance.

The possibility of kaon condensation in the nuclear matter was first investigated by Nelson and Kaplan[1]. In their calculation these authors used a $SU(3) \times SU(3)$ Lagrangian (see later for the details). In their model the kaon condensation is predominantly driven by an attractive S-wave kaon-nucleon interaction. This should be contrasted with the pion condensation which is driven by the P-wave pion-nucleon interaction. The calculation of Nelson and Kaplan was done in the mean field approximation and the hyperons, which are likely to appear in the dense hadronic matter, were not included. The calculation of Nelson and Kaplan has been extended by a number of authors [3, 4, 5, 6, 7]. In some of the extensions the nonlinear Walecka model [7] has been used to incorporate the baryonic interactions and to employ a somewhat more realistic nuclear equation of state. The hyperons which are

expected to appear at higher nuclear density are also included. However, most of these calculations are mean field calculations and do not include higher loop corrections.

In the present work we have studied the properties of kaons in the nuclear medium. In particular, we have computed the kaon propagator in the nuclear medium and determined the effective kaon mass in the nuclear medium from the pole position of the kaon propagator. We go beyond the mean field approximation (essentially Fig. 1(a)) and include the oyster diagrams (Fig. 1(b)) in our calculations. We also investigate the effect of P-wave kaon-baryon interaction on the kaon propagator (fig. 1(c)).

Before going on to describing our calculation, let us first consider the two relevant Lagrangians referred to in the above. The Nelson-Kaplan $SU(3) \times SU(3)$ Lagrangian is [1, 6]

$$
\begin{aligned}
\mathcal{L}_{NK} &= \frac{1}{4}\text{Tr}\ \partial_\mu U \partial^\mu U^\dagger + c\text{Tr}\ (m_q(U + U^\dagger - 2)) + \text{Tr}\ \bar{B}\gamma^\mu i\partial_\mu B + i\ \text{Tr}\ \bar{B}\gamma_\mu[V_\mu, B] \\
&+ D\ \text{Tr}\ \bar{B}\gamma_\mu\gamma_5\{A_\mu, B\} + F\ \text{Tr}\ \bar{B}\gamma_\mu\gamma_5[A_\mu, B] + a_1\ \text{Tr}\ \bar{B}(\zeta m_q\zeta + h.c.)B \\
&+ a_2\ \text{Tr}\ \bar{B}B(\zeta m_q\zeta + h.c.) + a_3\ \text{Tr}\ (m_q U + h.c.)\ \text{Tr}\ \bar{B}B,
\end{aligned}
\tag{1}
$$

where B is the baryon octet, M is the pseudoscalar meson octet, m_q is quark mass matrix, f is the pion decay constant, $U = \exp(\sqrt{2}iM/f)$ and $\zeta^2 = U$. The mesonic vector and axial vector currents (V^μ and A^μ respectively) are given by,

$$
V_\mu = \frac{1}{2}(\zeta^\dagger\partial_\mu\zeta + \zeta\partial_\mu\zeta^\dagger)
\tag{2}
$$

$$
A_\mu = \frac{1}{2}(\zeta^\dagger\partial_\mu\zeta - \zeta\partial_\mu\zeta^\dagger)
\tag{3}
$$

In our calculation, the quark masses are chosen to be $m_u = m_d = 0$ and $m_s = 150 MeV$ and the pion decay constant is chosen to be 93 MeV. Apart from these, the other parameters of the Lagrangian are the weak interaction constants $D = 0.81$, $F = 0.44$ (with $g_A = F + D$) and a_1, a_2 and a_3. The parameters a_1 and a_2 are determined by fitting the nucleon, lambda and cascade masses. In terms of the strange quark mass (m_s), $a_1 m_s = -64 MeV$ and $a_2 m_s = -134 MeV$. The parameter a_3 is related to the strangeness content of the nucleon and kaon-nucleon sigma term Σ^{kn} [6]. It must be noted that Σ^{kn} is not well-determined experimentally and therefore a_3 is not known particularly well. In our calculation, $a_3 m_s$ has been varied from -134 MeV to -310 MeV. These correspond to zero and 20% strangeness content of the nucleon, respectively.

The nonlinear Walecka model Lagrangian is [9]

$$
\begin{aligned}
\mathcal{L}_W &= \sum_i \bar{B}_i(i\gamma^\mu\partial_\mu - m_i + g_{\sigma i}\sigma + g_{\omega i}\omega_\mu\gamma^\mu - g_{\rho i}\rho_\mu^a\gamma^\mu T_a)B_i - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} \\
&+ \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}\rho_{\mu\nu}^a\rho_a^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu^a\rho_a^\mu \\
&- \frac{1}{3}bm_N(g_{\sigma N}\sigma)^3 - \frac{1}{4}c(g_{\sigma N}\sigma)^4 + \sum_l \bar{\psi}_l(i\gamma^\mu\partial_\mu - m_l)\psi_l.
\end{aligned}
\tag{4}
$$

Here B, σ, ω_μ, ρ_μ^a, and ψ_l are the baryon, sigma, omega, rho and lepton fields respectively, g's are the baryon -meson coupling constants and b and c are the coefficients of the cubic and quartic σ meson self interaction terms. The strangeness 0 and -1 baryons and electrons and muons are included in the present calculation. The masses of rho and omega mesons are chosen to be their experimental values and the sigma mass and meson-baryon coupling constants are chosen by fitting the nuclear saturation properties[8]. The coupling constants of the mesons with the strange baryons are not fixed by the nuclear saturation properties. We therefore use the quark model estimates for these[9].

The procedure adopted in the present calculataion is as follows. The equation of state of the hadronic matter is calculated from the Walecka model (4) in the mean field approximation. The calculation yields the densities of different baryon species and their effective masses as a function of baryon density. This information is then used in the calculation of the kaon propagator $D(\vec{k}, \omega)$

$$
D(\vec{k}, \omega) = \frac{1}{\omega^2 - k^2 - m_k^2}(1 + \Pi(\vec{k}, \omega)D(\vec{k}, \omega))
\tag{5}
$$

in the hadronic medium. The kaon self energy $\Pi(\vec{k}, \omega)$ is calculated, from the Lagrangian (1), by including the diagrams shown in Fig. 1. Thus, our calculation includes the diagrams upto one-loop level and upto fourth power in $1/f$. The loop diagrams, which are divergent, have been properly regularised so as to yield the physical mass of Kaon at zero baryon density using standard dimensional regularisation techniques [10]. In Fig. 1, (a) and (b) are due to the S-wave interaction, whereas (c) is due to the P-wave interaction. For the sake of brevity we have not given the full expression of self energy here. This will be reported in a full paper elsewhere.

The pole of the kaon propagator, as a function $\vec{k}$ gives the relation between the energy and momentum of kaons in the medium. In particular, the value of the real part of ω which

satisfies the equation [11]

$$D^{-1}(\vec{k} \rightarrow \vec{0}, \omega) = \omega^2 - m_k^2 - Re[\Pi(|\vec{k}| \rightarrow 0, \omega)]$$
$$= 0 \qquad (6)$$

defines the kaon mass in the medium.

Here we would like to state that, ideally, one should use the same model to compute the properties of the hadronic matter as well as those of the kaon. In absence of a comprehensive model which would describe the nuclear matter properties as well as the kaon-baryon interaction, one is forced to employ two different models as we have done. However, we must stress that it would be incorrect to add the two Lagrangians ($\mathcal{L}_{NK}$ and $\mathcal{L}_W$) and solve the problem since these two model Lagrangians are developed for two different purposes. $\mathcal{L}_{NK}$ has a $SU(3) \times SU(3)$ symmetry whereas $\mathcal{L}_W$ does not. Furthermore, at a conceptual level, such an approach will amount to serious overcounting. For example, the σ meson of the Walecka model Lagrangian essentially arises from the interacting two-pion system. On the other hand, the interactions of pseudoscalar mesons are explicitly included in the Nelson-Kaplan model.

The behaviour of the kaon mass in the nuclear medium is displayed in Fig. 2. In this figure, the mean field results as well as the results of the full calculation are plotted. The results are for $a_3 m_s = -134 MeV$. We have also shown the results when the hyperons are excluded. The calculation shows that the loop diagrams tend to increase the kaon mass for $a_3 m_s = -134 MeV$. In fact, we find that the contribution of P-wave interaction (Fig. 1(c)) is negligible. One reason for this is that the contribution of this diagram is proportional to the square of the kaon momentum; in the static limit, therefore, this diagram contributes very little. We have used different parameter sets of the Walecka model in our calculations. In particular, the nuclear compressibility has been varied from 250 MeV to 350 MeV. We find that the kaon mass does not depend much on the compressibility.

The effect of the inclusion of hyperons on the kaon mass is also displayed in Fig. 2. We find that the reduction in the kaon mass is somewhat smaller when the hyperons are included. Thus, the density at which the kaon mass vanishes is increased when the hyperons are included and therefore the threshold for kaon condensation goes up with the inclusion of

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the hyperons. Of course, the hyperons (primarily Σ^-'s and Λ's) appear when the nuclear density is about 0.4 fm^{-3}. Hence the curves for with and without hyperons are identical below this density. Qualitatively, this behaviour of the kaon mass can be explained as follows. At higher densities, the energy would be minimised when the strangeness fraction is about unity. For example, the quark matter would have strangeness fraction of unity if the difference between strange and nonstrange quark masses is small in comparison with the quark chemical potential. In the hadronic matter, the strangeness fraction is increased by introducing hyperons or by having kaon condensate. Thus the presence of hyperons means that the system already has some strangeness fraction and therefore the kaon condensation is inhibited. We further find that the variation in the meson-baryon coupling strength does not appreciably affect the effective kaon mass. This result is somewhat at variance with the observation of earlier authors [7]. The reason for this difference is most probably the naive addition of $\mathcal{L}_{NK}$ and $\mathcal{L}_{\mathcal{W}}$ in ref. [7].

The dependence of the kaon mass on $a_3 m_s$,*i.e.*, the strangeness content of the nucleon, is shown in Fig. 3 at the mean field level. We find that the kaon mass decreases more rapidly when $a_3 m_s$ decreases from -134 MeV to -310 MeV. Thus, with the decrease in $a_3 m_s$ the threshold for kaon condensation decreases. A similar trend has been observed by other authors[7]. However, we believe the smaller values of $a_3 m_s$ are probably not meaningful, although there is some justification for these values in terms of strangeness content of the nucleon and pion-nucleon Σ term[12]. The reason is that at the lower values of $a_3 m_s$, the kaon mass is as small as 350 MeV when the nuclear density is 0.1 fm^{-3}, which is about two-thirds of nuclear density. Clearly, this would have a significant impact on the behaviour of kaons in nuclei. We are not aware of any experimental evidence which indicates such small K^- masses.

Fig. 4 is plotted to study the effect of $a_3 m_s$ on the higher order diagram. We find that higher order diagram disfavours the kaon condensation for higher values of $a_3 m_s$ as effective kaon mass vanishes at higher densities. But for lower values of $a_3 m_s$ oyster diagram favours the condensation. In other words, the higher order diagrams in the kaon-baryon interaction favours the condensation for higher strangeness fraction of the nucleon.

In conclusion, we have shown that in the presence of hyperons kaon mass vanishes at

higher density compared to the case where there is no hyperon in the medium. The effective kaon mass is more sensitive to the value of a_3m_s than the different parameter sets in the Walecka model which we have used here. The higher order diagrams in S-wave K-N interaction causes the kaon mass to vanish at lower densities for lower values of a_3m_s. Here we would like to point out that a lower value of a_3m_s corresponds to higher strangeness fraction in nucleon. If the strangeness fraction is higher than 20% [12] then the kaon mass may be zero even at normal nuclear matter densities. This implies that kaon condensation may occur at normal nuclear matter or even at lower densities which, needless to say, is ruled out by physical arguments. One possible way out may be to incorporate the effects of finite size of mesons and baryons which may increase the critical density for baryons substantially. Studies in such direction are in progress.

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