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Instability of dust acoustic wave due to nonthermal ions in a charge varying dusty plasma

Samiran Ghosh[a)]
College of Textile Technology, Berhampore 742101, Murshidabad, West Bengal, India

R. Bharuthram
University of Natal, Durban-4041, South Africa

Manoranjan Khan[b)] and M. R. Gupta
Centre for Plasma Studies, Faculty of Science, Jadavpur University, Kolkata-700 032, India

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The effects of nonthermal ions with excess of fast (energetic) ions on linear dust acoustic (DA) wave propagation has been investigated incorporating the dust charge variation and the isothermal dust pressure variation. It is seen that due to the dust charge variations in the presence of nonthermal ions, instead of the usual damping, there is a growth of the DA wave if the ion nonthermality parameter $a > 15(1+\sigma_i)/(8-72\sigma_i)$, $\sigma_i(=T_i/T_e \ll 1)$, $T_i(T_e)$ is the ion (electron) temperature, and there may occur, under certain conditions, exponentially growing mode with zero real frequency. It is also seen that in the absence of dust charge variations there also occurs a zero real frequency, exponentially growing mode if the ion nonthermality parameter $a > 1$. In absence of dust charge variations or in the presence of adiabatic dust charge variations, finite dust temperature T_d can stabilize the instability. However, in the presence of nonadiabatic dust charge variations T_d cannot stabilize the instability. © *2004 American Institute of Physics.* [DOI: 10.1063/1.1760584]

I. INTRODUCTION

The dust grains immersed in a plasma become highly charged due to collisions with electrons and ions. For low frequency and very low frequency oscillations, the presence of such highly charged heavy mass dust grains introduce some new eigen modes, called the "dust ion acoustic" (DIA) wave[1] and the "dust acoustic" (DA) wave.[2] For the DA wave[2] the highly charged heavy dust grains mass provides the inertia and the inertialess Boltzmann distributed thermal electrons and ions provide the restoring force, whereas the DIA wave is the usual ion acoustic wave mode in a dusty plasma with phase velocity $\propto n_{i0}/n_{e0}$[1] [$n_{i0}(n_{e0})$ is equilibrium number density of ion (electron)]. These modes have been observed in laboratory experiments with dust grains of fixed charge.[3,4] The electrostatic charging of the dust grains immersed in a plasma is the main feature of dust plasma interactions in a dusty plasma. In a truly dusty plasma, the charge on the dust grains q_d is an extra dynamical variable which controls the grain motion and it fluctuates according to the fluctuating electrostatic force in the plasma. This extra dynamical variable q_d is to be determined from the dust charging equation $\omega_{pd}/\nu_{ch} dq_d/d(\omega_{pd})t = I(q_d,\phi)/\nu_{ch}$, where $I(q_d,\phi)$ is the total current, ν_{ch} is the charging frequency and ω_{pd} is the dust plasma frequency. In the theory of *nonadiabatic* dust charge variations, it is assumed that the ratio ω_{pd}/ν_{ch} is small but finite, i.e., $\omega_{pd}/\nu_{ch} \neq 0$, whereas in the theory of *adiabatic* charge variations, it is assumed that ω_{pd}/ν_{ch} is very small, so that $\omega_{pd}/\nu_{ch} \approx 0$. In the linear theory, it is seen that the nonadiabatic charge variations lead to the damping of the linear wave modes,[5–8] whereas in the nonlinear analysis, the nonadiabatic dust charge variations cause an anamolous dissipation in the plasma. This dissipation leads to the generation of shock waves in a dusty plasma.[9–11]

Theoretically the wave propagation characteristics of linear, as well as, nonlinear DA waves in a dusty plasma, consisting of negatively charged dust grains, Maxwellian electrons and nonthermal ions have been studied.[12,13] In all these works, it is assumed that the charge on the dust grains are fixed. The effects of both nonadiabatic and adiabatic dust charge variations in presence of nonthermal ions (which were not considered in these earlier investigations), drastically modify the electrostatic mode. The linear and nonlinear wave propagation charactristics of the DA wave in the presence of nonthermal ions in a charge varying dusty plasma have not yet been studied.

In this paper we have investigated the linear DA wave propagation characteristics incorporating the effects of nonthermal ions, the isothermal pressure variations of dust grains and both nonadiabatic and adiabatic dust charge variations. In collisionless plasmas, the velocity distribution most commonly used is the Maxwellian distribution. However, observations of space plasmas indicate the presence of electron and ion populations that are far from thermodynamic equilibrium. Thus, the occurrence of nonthermal ions and electrons are a common feature in such an environment. Nonthermal ions from the earth's bow-shock have been observed by the Vela satellite,[15] as well as in and around the earth's foreshock.[16] The ASPERA experiment on the Phobos satel-

a)Electronic mail: sran_g@yahoo.com
b)Bengal Engineering College (Deemed University), Shibpur, Howrah, India.

lite has detected nonthermal ion fluxes from the upper ionosphere of Mars.[17] Closer to earth, fast nonthermal ions have been recently observed by the Nozomi satellite in the vicinity of the Moon.[18] It appeared from the observations that the nonthermal ions have a ring structure in the velocity phase space. The mechanism suggested for the formation of a ring structure for nonthermal ions is that some of the solar wind ions are deflected in the close vicinity of the Moon. Such deflected ions have large velocities (the bulk velocity of the solar wind) and they move by the force of the electric field and gyrate about the magnetic lines. The ring structure in velocity phase space is suggested to arise in this manner; a possible one such velocity distribution is given by Eq. (2) of Sec. II.

The presence of nonthermal ions modifies the ion current and also the dust charging frequency. The modified expression of ion current in the presence of nonthermal ions has been derived using orbit motion limited (OML) approach[14] and the variations of dust charging frequency with the ion nonthermality parameter $a>0$ have been shown graphically. Numerical investigations reveal that, due to the dust charge variations in the presence of nonthermal ions, the DA wave becomes damped for $a<15(1+\sigma_i)/(16-24\sigma_i)$ and it will grow exponentially for $a>15(1+\sigma_i)/(8-72\sigma_i)$, whereas when $15(1+\sigma_i)/(16-24\sigma_i)<a<15(1+\sigma_i)/(8-72\sigma_i)$, the DA wave will grow in a low dust density region and will get damped in a high dust density region. It is seen that due to the presence of nonthermal ions the DA wave disappears under certain conditions and in its place there occurs a zero frequency purely growing mode. It is also seen that the dust temperature plays a stabilizing role.

The paper is organized in the following manner. Section II contains the formulation of the problem. The dispersion relation of DA wave and the roots of dispersion relation are derived in Sec. III. The numerical results are given in Sec. IV. Finally, in Sec. V, we present a summary of our results.

II. FORMULATION OF THE PROBLEM

A plasma model consisting of fully ionized collisionless, unmagnetized, nonrelativistic dusty plasma consisting of Boltzmann distributed electrons, nonthermal ions and negatively charged warm adiabatic dust grains is considered. The dust charge variations are also considered. Thus at equilibrium we have

$$n_{i0}=n_{e0}+z_d n_{d0}, \tag{1}$$

where n_{j0} $(j=e,i,d)$ are the equilibrium number densities of jth (e=electron, i=ion, d=dust) species and $-z_d e$ is the equilibrium charge of the dust surface.

As the ions are assumed to be nonthermal distributed, a possible three-dimensional equilibrium state ion velocity distribution function satisfying collisionless Vlasov equation with a population of fast (energetic) particle is given by

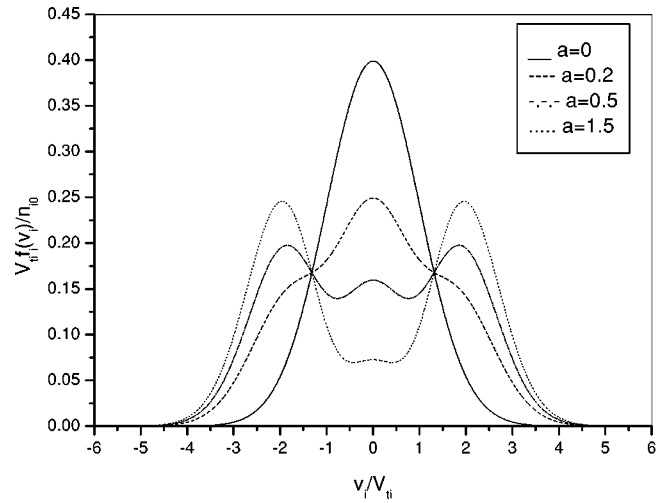

FIG. 1. Steady state ion distribution function as given by Eq. (3) for different values of nonthermal parameter a. Solid line for $a=0$, dashed line for $a=0.2$, dash dotted line for $a=0.5$ and dotted line for $a=1.5$.

$$F_i(v_i)=F_i(v_x,v_y,v_z)=\frac{n_{i0}}{(1+3a)}\left(\frac{1}{2\pi V_{ti}^2}\right)^{3/2}\times\left[1+4a\left(\frac{1}{2}\frac{v_x^2}{V_{ti}^2}+\frac{\Phi}{\sigma_i}\right)^2\right]\times\exp\left(-\frac{v_x^2+v_y^2+v_z^2}{2V_{ti}^2}-\frac{\Phi}{\sigma_i}\right), \tag{2}$$

where a is the ion nonthermal parameter which determines the number of fast(energetic) ions, v_x,v_y and v_z are the x,y, and z components of the ion velocity v_i, $V_{ti}=\sqrt{T_i/m_i}$ is the ion thermal velocity, $T_i(T_e)$ is the ion(electron) temperature, m_i is the ion mass and $\Phi=e\phi/T_e$ and $\sigma_i=T_i/T_e$, where ϕ is the electrostatic potential. Integrating the ion distribution function (2) over the velocities v_y and v_z, the steady state one-dimensional (with $v_x=v_i$) ion velocity distribution with a population of fast ion is given by[19,20] ($\phi=0$)

$$f_i(v_i)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F_i(v_x,v_y,v_z)dv_y dv_z =\frac{n_{i0}}{(1+3a)\sqrt{2\pi V_{ti}^2}}\left(1+a\frac{v_i^4}{V_{ti}^4}\right)\exp\left(-\frac{v_i^2}{2V_{ti}^2}\right). \tag{3}$$

A direct comparison between the steady state Maxwellian ion distribution function [$a=0$ in (3)] and the steady state ion distribution function with a population of fast particles is shown in Fig. 1 for a number of values of the nonthermal parameter a.

In the presence of a nonzero potential $\Phi=e\phi/T_e$ integrating the distribution function (2), we get the following ion number density:

$$n_i=\int_R F_i(v_i)d^3v_i=n_{i0}\left[1+\frac{4a}{1+3a}\left(\frac{\Phi}{\sigma_i}+\frac{\Phi^2}{\sigma_i^2}\right)\right]\times\exp\left(-\frac{\Phi}{\sigma_i}\right), \tag{4}$$

where R is the domain of integration in the ion velocity space and $\sigma_i=T_i/T_e$ and T_e is the electron temperature.

The velocity distribution function of electrons are assumed to Maxwellian so that the electron number density is given by

$$n_e=n_{e0}\exp(\Phi). \tag{5}$$

A. Dust grain charging

In this paper we use the orbital motion limited (OML) approach[14] to describe the charging of the dust grains.

The electron and ion currents to the dust grains are given by

$$I_j=q_j\int_{R_j}\sigma_{sj}(v_j,\phi_d)v_jF_j(v_j)d^3v_j, \tag{6}$$

where R_j is the domain of integration in the jth species velocity space, $\sigma_{sj}=\pi r_0^2(1-2q_j\phi_d/m_jv_j^2)$ is the effective dust grain collisional cross section, $\phi_d=q_d/4\pi\epsilon_0r_0$ is the dust surface potential relative to the plasma potential ϕ, q_j is the electric charge of the $j(e=\text{electron},\ i=\text{ion})$th species of the plasma particles, q_d is the dust charge and $4\pi\epsilon_0r_0$ is the capacitance of the spherical dust grain of average radius r_0.

As it is assumed that the electrons are Maxwellian, substituting the Maxwellian velocity distribution of electrons in (6) and then integrating (6) we get the following expression for electron current:

$$I_e=-\pi r_0^2e\sqrt{\frac{8T_e}{\pi m_e}}n_{e0}\exp(\Phi+zQ_d). \tag{7}$$

Similarly substituting (2) into (6) and then integrating (6) we get the following expression of ion current:

$$I_i=\pi r_0^2e\sqrt{\frac{8T_i}{\pi m_i}}\frac{n_{i0}}{(1+3a)}\left[\left(\left(1+\frac{24a}{5}\right)+\frac{16a}{3}\frac{\Phi}{\sigma_i}+4a\frac{\Phi^2}{\sigma_i^2}\right)-z\frac{Q_d}{\sigma_i}\left(\left(1+\frac{8a}{5}\right)+\frac{8a}{3}\frac{\Phi}{\sigma_i}+4a\frac{\Phi^2}{\sigma_i^2}\right)\right]\exp\left(-\frac{\Phi}{\sigma_i}\right), \tag{8}$$

where $Q_d=q_d/z_de$, z_de is the magnitude of the equilibrium dust charge and $z=z_de^2/4\pi\epsilon_0r_0T_e$ is the nondimensional dusty plasma parameter. In the above expression the parameter a arises due to the effects of nonthermal ions (put $a=0$, get the usual expression of ion current for Maxwellian ions). Hence the nonthermal ions also modifies the ion current.

Considering only electron and ion currents due to collisions with plasma particles, the dust grain charging equation is given by

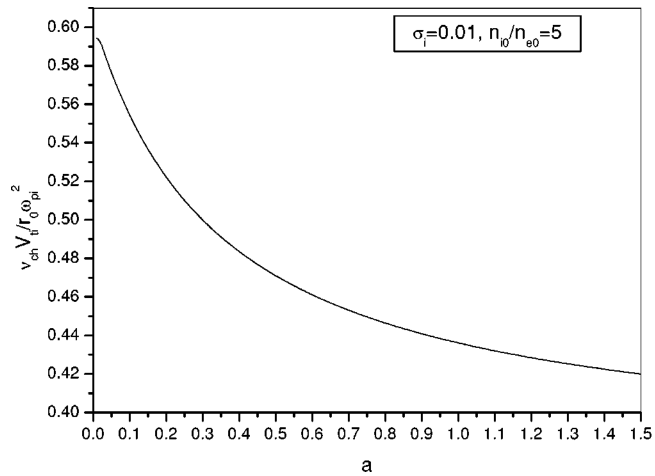

FIG. 2. Variation of normalized dust charging frequency (ν_{ch}) [Eq. (10)] with ion nonthermality parameter a for $\sigma_i=0.01$ and $n_{i0}/n_{e0}=5$.

$$\frac{dQ_d}{dt}=\frac{I_e+I_i}{z_de}. \tag{9}$$

The dust charging frequency ν_{ch} is given by

$$\nu_{ch}=-\left.\frac{\partial(I_e+I_i)}{z_de\,\partial Q_d}\right|_{Q_d=-1,\Phi=0}=\frac{r_0}{\sqrt{2\pi}}\frac{\omega_{pi}^2}{V_{ti}}\left(\frac{5+8a}{5+15a}\right)\left[1+z+\left(\frac{5+24a}{5+8a}\right)\sigma_i\right] \tag{10}$$

and the electron ion number density ratio is given by

$$\frac{n_{i0}}{n_{e0}}=\sqrt{\frac{m_i}{\sigma_im_e}}\frac{\left(\frac{5+15a}{5+8a}\right)}{\left(\frac{z}{\sigma_i}+\frac{5+24a}{5+8a}\right)}e^{-z}. \tag{11}$$

To derive the above relation we use the current balance equation $I_e+I_i|_{Q_d=-1,\Phi=0}=0$. The effects of nonthermal ions on the dust charging frequency has been shown graphically in Fig. 2.

B. The basic equations

The dynamics of the negatively charged dust grains is governed by the following continuity and momentum equations for the dust fluid together with Poisson's equation:

$$\frac{\partial N_d}{\partial t}+\frac{\partial(N_dv_d)}{\partial x}=0, \tag{12}$$

$$\frac{\partial v_d}{\partial t}+v_d\frac{\partial v_d}{\partial x}=-c_{da}^2Q_d\frac{\partial\Phi}{\partial x}-V_{td}^2N_d^{-1}\frac{\partial N_d}{\partial x}, \tag{13}$$

$$\frac{\partial^2\Phi}{\partial x^2}=-\frac{1}{\lambda_{De}^2}\left[\left(\frac{n_{i0}}{n_{e0}}\right)N_i-N_e+\left(\frac{z_dn_{d0}}{n_{e0}}\right)N_dQ_d\right], \tag{14}$$

where $N_d=n_d/n_{d0}$, $\lambda_{De}=\sqrt{\epsilon_0T_e/n_{e0}e^2}$ is the electron Debye length, $c_{da}=\sqrt{z_dT_e/m_d}$ is the usual DA speed, $V_{td}=\sqrt{T_d/m_d}$ is the dust thermal velocity, T_d is the dust temperature, m_d is the dust mass, n_{d0} is the equilibrium dust number density, and v_d is the dust fluid velocity.

The normalized charge variable Q_d is determined by Eq. (9), which after some algebra can be rewritten as

$$\frac{\omega_{pd}}{\nu_{ch}}\left[\frac{\partial Q_d}{\partial t}+v_d\frac{\partial Q_d}{\partial x}\right]=\beta_d\omega_{pd}\left[\frac{\left(\left(\frac{5+24a}{5+8a}-\frac{zQ_d}{\sigma_i}\right)+\frac{40a}{15+24a}\left(2-\frac{zQ_d}{\sigma_i}\right)\frac{\Phi}{\sigma_i}+\frac{20a}{5+8a}\left(1-\frac{zQ_d}{\sigma_i}\right)\frac{\Phi^2}{\sigma_i^2}\right)\exp\left(-\frac{\Phi}{\sigma_i}\right)}{\frac{5+24a}{5+8a}+\frac{z}{\sigma_i}} - \exp(\Phi+z(1+Q_d))\right], \tag{15}$$

where

$$\beta_d=\frac{z+\left(\frac{5+24a}{5+8a}\right)\sigma_i}{z\left[1+z+\left(\frac{5+24a}{5+8a}\right)\sigma_i\right]}, \tag{16}$$

and $\omega_{pd}=\sqrt{z_d^2e^2n_{d0}/\epsilon_0 m_d}$ is the dust plasma frequency.

III. DISPERSION RELATION

To study the low frequency electrostatic modes, we perturbed all the dependent variables about their equilibrium values as follows:

$$N_d=1+\delta N_d;\quad v_d=0+\delta v_d,$$
$$\Phi=0+\delta\Phi;\quad Q_d=-1+\delta Q_d. \tag{17}$$

Substituting $N_e=n_e/n_{e0}$ and $N_i=n_i/n_{i0}$ from (4), and (5) and then using the above perturbations, the linearized basic equations become

$$\frac{\partial\delta N_d}{\partial t}+\frac{\partial\delta v_d}{\partial x}=0, \tag{18}$$

$$\frac{\partial\delta v_d}{\partial t}=c_{da}^2\frac{\partial\delta\Phi}{\partial x}-V_{td}^2\frac{\partial\delta N_d}{\partial x}, \tag{19}$$

$$\frac{\partial^2\delta\Phi}{\partial x^2}=-\frac{1}{\lambda_{De}^2}\left[\left(\frac{n_{i0}}{n_{e0}}\right)\left(\frac{a-1}{1+3a}\right)\frac{\delta\Phi}{\sigma_i}-\delta\Phi-\left(\frac{z_dn_{d0}}{n_{e0}}\right)\delta N_d+\left(\frac{z_dn_{d0}}{n_{e0}}\right)\delta Q_d\right], \tag{20}$$

$$\frac{\omega_{pd}}{\nu_{ch}}\frac{\partial\delta Q_d}{\partial t}=\beta_d\omega_{pd}\left[\frac{\alpha+\frac{z}{\sigma_i}\beta}{z+\left(\frac{5+24a}{5+8a}\right)\sigma_i}\delta\Phi-\frac{1}{\beta_d}\delta Q_d\right], \tag{21}$$

where α and β are given by

$$\alpha=\frac{8a-15-\sigma_i(15+72a)}{15+24a};$$
$$\beta=\frac{16a-15-\sigma_i(15+24a)}{15+24a}. \tag{22}$$

A. Nonadiabatic charge variation

In this case we assume that ratio $\omega_{pd}/\nu_{ch}\neq 0$, i.e., the ratio is a finite but small quantity. Therefore, to study the propagation of a low frequency small amplitude DA wave, we consider that all the above perturbed quantities are propagating as $\exp[i(kx-\omega t)]$. Thus the linearized equations (18)–(21) in the long wave length limit $k^2\lambda_{De}^2\ll 1$, yields the following dispersion relation for the DA wave:

$$D(\omega,k)\equiv\left(\frac{\omega^2}{k^2}-V_{td}^2\right)\left[1-\left(\frac{n_{i0}}{n_{e0}}\right)\frac{a-1}{\sigma_i(1+3a)}-\left(\frac{n_{i0}}{n_{e0}}-1\right)\frac{\beta_d}{\left(1-i\frac{\omega}{\nu_{ch}}\right)}\left(\frac{\alpha+\frac{z}{\sigma_i}\beta}{z+\left(\frac{5+24a}{5+8a}\right)\sigma_i}\right)\right]-\left(\frac{n_{i0}}{n_{e0}}-1\right)c_{da}^2=0, \tag{23}$$

where $D(\omega,k)$ is the plasma dispersion function. In deriving this plasma dispersion function the equilibrium state charge neutrality condition (1) has been used. Now we turn to the determination of roots of the dispersion relation.

With $\omega=\omega_r+i\omega_i$, the real part of the frequency is given by $\mathrm{Re}\,D(\omega,k)=0$ yielding the frequency of the DA wave:

$$\frac{\omega_r^2}{k^2c_{da}^2}=\sigma_d+\frac{\left(\frac{n_{i0}}{n_{e0}}-1\right)}{1-\left(\frac{n_{i0}}{n_{e0}}\right)\frac{a-1}{\sigma_i(1+3a)}-\beta_d\left(\frac{n_{i0}}{n_{e0}}-1\right)\left(\frac{\alpha+\frac{z}{\sigma_i}\beta}{z+\left(\frac{5+24a}{5+8a}\right)\sigma_i}\right)}, \tag{24}$$

where $\sigma_d=T_d/z_dT_e=V_{td}^2/c_{da}^2$ arises due to the presence of warm dust grains.

The damping rate, i.e., the imaginary part of the root of the dispersion relation of the DA wave is given by

$$\omega_i=-\frac{\operatorname{Im}D(\omega_r,k)}{\frac{\partial \operatorname{Re}D(\omega_r,k)}{\partial\omega_r}}\Rightarrow\frac{k^2c_{da}^2}{2\nu_{ch}}\frac{\beta_d\left(\frac{n_{i0}}{n_{e0}}-1\right)^2\left(\frac{\alpha+\frac{z}{\sigma_i}\beta}{z+\left(\frac{5+24a}{5+8a}\right)\sigma_i}\right)}{\left[1-\left(\frac{n_{i0}}{n_{e0}}\right)\frac{a-1}{\sigma_i(1+3a)}-\beta_d\left(\frac{n_{i0}}{n_{e0}}-1\right)\left(\frac{\alpha+\frac{z}{\sigma_i}\beta}{z+\left(\frac{5+24a}{5+8a}\right)\sigma_i}\right)\right]^2}. \tag{25}$$

The above expression (25) shows that the damping arises here due to the dust charge variation as the damping decrement $\omega_i\propto\beta_d$ (the other plasma parameter remains constant), the term arising due to dust charge variation. Both the expressions (24) and (25) also show that the presence of nonthermal ions modify the propagation characteristics of linear DA wave which are shown graphically in Figs. 3–6 for different plasma parameters.

Several feature of physical interest arise depending on the values of ω_r and ω_i. One of them is that the DA wave may grow or may be damped depending on the algebraic sign of ω_i provided $\omega_r^2>0$. The condition for growth $\omega_i>0$ is

$$\alpha+\frac{z}{\sigma_i}\beta>0\Rightarrow\frac{8a-15-\sigma_i(15+72a)}{15+24a}+\frac{z}{\sigma_i}\frac{(16a-15-\sigma_i(15+24a))}{(15+24a)}>0. \tag{26}$$

This above inequality always holds good if the ion nonthermality parameter

$$a>\frac{15(1+\sigma_i)}{8-72\sigma_i}, \tag{27}$$

provided $\sigma_i<1/9$ as the ion nonthermality parameter $a>0$, whereas the condition for damping $\omega_i<0$ is

$$\frac{8a-15-\sigma_i(15+72a)}{15+24a}+\frac{z}{\sigma_i}\frac{(16a-15-\sigma_i(15+24a))}{(15+24a)}<0. \tag{28}$$

This inequality always holds good if the ion nonthermality parameter

$$a<\frac{15(1+\sigma_i)}{16-24\sigma_i}. \tag{29}$$

But if $15(1+\sigma_i)/(16-24\sigma_i)<a<15(1+\sigma_i)/(8-72\sigma_i)$, then there will be growth or damping depending on the values of z/σ_i. Actually $z/\sigma_i=z_de^2/4\pi\epsilon_0r_0T_i$ is the solution of the equilibrium current balance equation $I_e+I_i|_{\Phi=0,Q_d=-1}=0$. The numerical solution of (11) shows that for lower values of n_{i0}/n_{e0}, i.e., for low dust density $z/\sigma_i>1$ and for higher values of n_{i0}/n_{e0}, i.e., for high dust density $z/\sigma_i<1$ as it is expected because $dQ_d/dn_d<0$. Hence for low dust density, there will be growth of DA wave and for high dust density there will be damping of DA wave (see Figs. 3 and 4).

Another feature of interest is that for $\alpha+(z/\sigma_i)\beta>0$, the right-hand side (RHS) of (25) is positive but if the RHS of (24) becomes negative, then there is no DA mode. In its place there appears a purely growing mode or nonoscillating growing mode about a zero real frequency. The condition for this instability to arise is

$$\sigma_d+\frac{\left(\frac{n_{i0}}{n_{e0}}-1\right)}{1-\left(\frac{n_{i0}}{n_{e0}}\right)\frac{a-1}{\sigma_i(1+3a)}-\beta_d\left(\frac{n_{i0}}{n_{e0}}-1\right)\left(\frac{\alpha+\frac{z}{\sigma_i}\beta}{z+\left(\frac{5+24a}{5+8a}\right)\sigma_i}\right)}<0 \tag{30}$$

together with condition (26) or (27). On the other hand, if a satisfies condition (29) together with the condition (30), then the instability occurs only when $|\omega_r^2|>|\omega_i\nu_{ch}|$. If the dust temperature T_d is low or dust grains are cold, i.e., $T_d=0\Rightarrow\sigma_d=0$, then the condition (30) holds depending on the values of n_{i0}/n_{e0} and we get non oscillating growing mode about a zero real frequency. But, if the dust temperature T_d is sufficiently high so that the RHS of (24) becomes positive, then instead of nonoscillating growing mode we get oscillating growing mode about a nonzero real frequency. Here the growth of DA arises through the imaginary part of the solution of the dispersion relation (23) as $\omega_i>0$ for $\alpha+(z/\sigma_i)\beta>0$. Thus the finite dust temperature can not stabilize the instability.

B. Adiabatic charge variation

In this case we assume that the dust plasma frequency is much smaller than the dust charging frequency so that $\omega_{pd}/\nu_{ch}\approx 0$ and hence the left-hand side (LHS) of Eq. (21) $=0\Rightarrow\delta Q_d=\beta_d(\alpha+z/\sigma_i\,\beta/z+(5+24a/5+8a)\sigma_i)\delta\Phi$. Hence due to this assumption, the dispersion relation of DA wave becomes

$$D(\omega,k)\equiv\left(\frac{\omega^2}{k^2}-V_{td}^2\right)\left[1-\left(\frac{n_{i0}}{n_{e0}}\right)\frac{a-1}{\sigma_i(1+3a)}-\beta_d\left(\frac{n_{i0}}{n_{e0}}-1\right)\left(\frac{\alpha+\frac{z}{\sigma_i}\beta}{z+\left(\frac{5+24a}{5+8a}\right)\sigma_i}\right)\right]-\left(\frac{n_{i0}}{n_{e0}}-1\right)c_{da}^2=0\Rightarrow\frac{\omega^2}{k^2c_{da}^2}=\sigma_d$$
$$+\frac{\left(\frac{n_{i0}}{n_{e0}}-1\right)}{1-\left(\frac{n_{i0}}{n_{e0}}\right)\frac{a-1}{\sigma_i(1+3a)}-\beta_d\left(\frac{n_{i0}}{n_{e0}}-1\right)\left(\frac{\alpha+\frac{z}{\sigma_i}\beta}{z+\left(\frac{5+24a}{5+8a}\right)\sigma_i}\right)}. \tag{31}$$

This dispersion relation shows that a zero frequency nonoscillating growing mode is obtained, if the inequality (30) holds. For low dust temperature or cold dust grains $(T_d=0)$, the inequality (27) holds, whereas for hot dust grain the above inequality does not hold and we get the usual DA wave mode. Thus, the finite dust temperature can stabilize the instability in the case of adiabatic dust charge variations.

C. No charge variation

In this case we assume the charges on the dust grains are fixed, i.e., $\delta Q_d=0$. Therefore, the dispersion relation of DA wave in the presence of nonthermal ions becomes

$$\frac{\omega^2}{k^2c_{da}^2}=\sigma_d+\frac{\left(\frac{n_{i0}}{n_{e0}}-1\right)}{1-\left(\frac{n_{i0}}{n_{e0}}\right)\frac{a-1}{\sigma_i(1+3a)}}. \tag{32}$$

This dispersion relation shows that for $a>1$, depending on the values of n_{i0}/n_{e0} and σ_i, instead of DA mode, we get a zero frequency growing mode if

$$\frac{\left(\frac{n_{i0}}{n_{e0}}-1\right)}{\frac{n_{i0}}{n_{e0}}\frac{a-1}{\sigma_i(1+3a)}-1}>\sigma_d. \tag{33}$$

This inequality holds good for low dust temperature or cold dust grains. But for sufficiently high dust temperature the above inequality does not hold and instead of a zero frequency growing mode, we get the usual DA wave mode. Thus the finite dust temperature can can also stabilize the instability in absence of dust charge variations.

It should be noted that with $a=0$ one recovers the standard results for DA waves in presence of Boltzmann distributed ions.

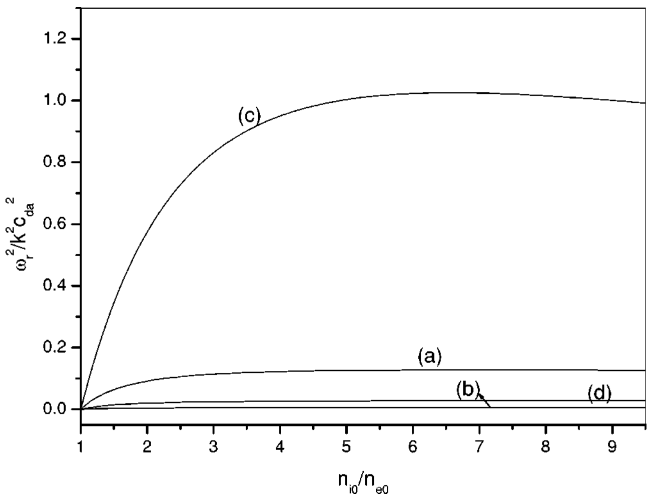

FIG. 3. Plot of normalized real frequency (ω_r) with n_{i0}/n_{e0} The curves are (a) $\sigma_i=0.01$, $a=15(\sigma_i+1)/(16-24\sigma_i)-0.1$; (b) $\sigma_i=0.01$, $a=0$; (c) $\sigma_i=0.05$, $a=15(\sigma_i+1)/(16-24\sigma_i)-0.1$; (d) $\sigma_i=0.05$, $a=0$ (Boltzmann distributed ions).

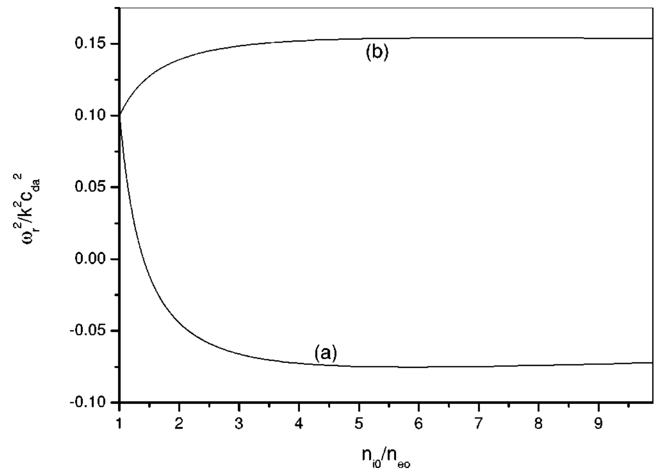

FIG. 5. Plot of normalized real frequency (ω_r) with n_{i0}/n_{e0}. The curves are (a) $\sigma_d=0.1$, $\sigma_i=0.1$ and $a=[15(1+\sigma_i)/(8-72\sigma_i)]+0.01$; (b) $\sigma_d=0.1$, $\sigma_i=0.1$, $a=0$.

IV. NUMERICAL RESULTS

Figure 2 shows the variation of dust charging frequency (ν_{ch}) with nonthermality parameter a for $\sigma_i(=T_i/T_e)=0.01$, $n_{i0}/n_{e0}=5$. This figure shows that the dust charging frequency decreases with the increase of ion nonthermality parameter $a(>0)$.

The variations of real frequency (ω_r) and imaginary frequency (ω_i) with n_{i0}/n_{e0} for different plasma parameters are shown in Figs. 3–6. The four curves in Figs. 3 and 4 refer to the following choices of σ_i and nonthermal parameter a: (a) $\sigma_i=0.01$, $a=[15(1+\sigma_i)/(16-24\sigma_i)]-0.1$ [satisfies condition (29)]; (b) $\sigma_i=0.01$, $a=0$; (c) $\sigma_i=0.05$, $a=[15(1+\sigma_i)/(16-24\sigma_i)]-0.1$ [satisfies condition (29); (d) $\sigma_i=0.05$, $a=0$. These two figures show that with the increase of n_{i0}/n_{e0} and σ_i, the real frequency (Fig. 3) and also the damping decrement (Fig. 4) of the DA wave increases. These two figures also show that the presence of nonthermal ions increases the magnitude of both $|\omega_r|$ and $|\omega_i|$.

The two curves in Figs. 5 and 6 refer to the following choices of σ_i, σ_d and nonthermal parameter a: (a) $\sigma_d=0.1$, $\sigma_i=0.1$ and $a=[15(1+\sigma_i)/(8-72\sigma_i)]+0.01$ [satisfies condition (27)]; (b) $\sigma_d=0.1$, $\sigma_i=0.1$, $a=0$. The curve (a) in Fig. 5 shows that due to the presence of nonthermal ions, the real frequency $\omega_r^2<0\Rightarrow\omega_r$ is imaginary, whereas the imaginary frequency $\omega_i>0$ [Curve (a) in Fig. 6]. This figure also shows that, due to the presence of finite dust temperature $\sigma_d=0.1$, for lower values of n_{i0}/n_{e0}, the real frequency is positive. The curve (b) of Fig. 5 shows that, in the absence of nonthermal ions ($a=0$), i.e., for Boltzmann distributed ions, the real frequency is always positive and the imaginary frequency ω_i is always negative [Curve (b) in Fig. 6, shows the usual damping arising due to nonadiabatic dust charge variations]. Hence, for these plasma parameters, the DA mode disappears [Curve (a) in both the Figs. 5 and 6 for higher values of n_{i0}/n_{e0}] and we get a nonoscillating or zero frequency purely exponentially growing mode. On the other

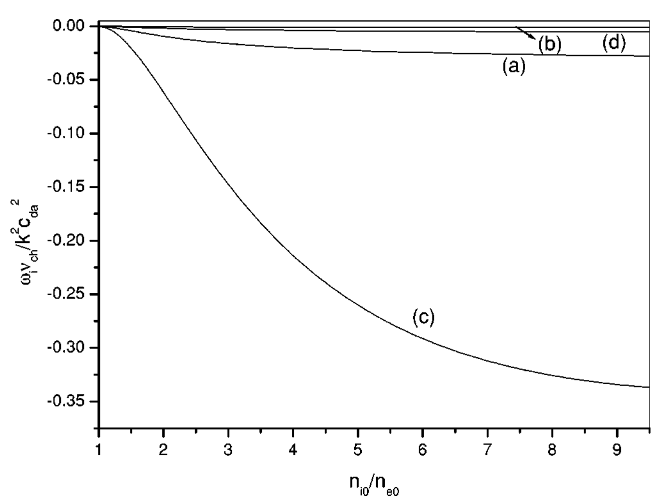

FIG. 4. Plot of normalized imaginary frequency (ω_i) with n_{i0}/n_{e0}. The curves and the other plasma parameters are same as in Fig. 3.

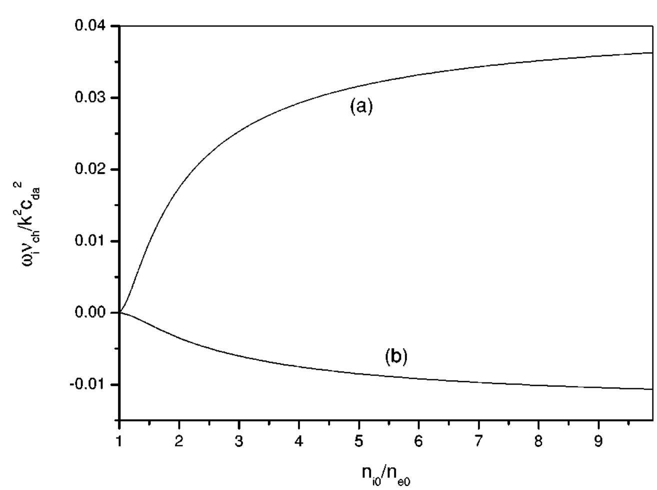

FIG. 6. Plot of normalized imaginary frequency (ω_i) with n_{i0}/n_{e0}. The curves and the other plasma parameters are same as in Fig. 5.

hand, for nonzero finite dust temperature, for lower values of n_{i0}/n_{e0}, $\omega_r^2>0$ and we get an oscillating exponentially growing DA mode.

V. DISCUSSIONS

The following is the summary of our investigation of the linear DA wave propagations in the presence of nonthermal ions, isothermal pressure variation of dust grains and dust charge variations.

(1) Presence of nonthermal ions modifies the ion current [Eq. (9)] and also the dust charging frequency [Eq. (10) and Fig. (2)].

(2) Due to the presence of nonthermal ions for $a>15(1+\sigma_i)/(8-72\sigma_i)$ there exists a oscillating exponentially growing DA wave, whereas instead of the DA wave mode there exists a nonoscillating or zero frequency growing mode if condition (30) is satisfied. On the other hand, for $a<15(1+\sigma_i)/(16-24\sigma_i)$ there occurs propagation of DA wave with typical damping decrement.

(3) For dusty plasma with adiabatic dust charge variation or with fixed charge dust grains, finite dust temperature can stabilize the DA instability, whereas it cannot stabilize the DA instability for dusty plasma with nonadiabatic dust charge variations.

(4) Our results can be useful in understanding the behavior of DA waves in space and astrophysical plasma environments and also in understanding physical phenomena like condensation of dust grains, where nonthermal ion are found to occur.[15–18]

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