

Infrared behaviour of massless QED in space-time dimensions $2 < d < 4$

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Abstract

We show that the logarithmic infrared divergences in electron self-energy and vertex function of massless QED in 2+1 dimensions can be removed at all orders of $1/N$ by an appropriate choice of a non-local gauge. Thus the infrared behaviour given by the leading order in $1/N$ is not modified by higher order corrections. Our analysis gives a computational scheme for the Amati-Testa model, resulting in a non-trivial conformal invariant field theory for all space-time dimensions $2 < d < 4$.

Keywords: infrared divergence, $1/N$, logarithmic corrections

PACS numbers: 11.15.Bt, 11.15.Pg

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Massless QED in 2+1 dimensions is of interest for various reasons. It provides a theoretical laboratory for studying the infrared (iR) divergences of perturbation theory [1, 2, 3, 4, 5, 6, 7, 8, 9] and chiral symmetry breaking [6, 7, 8, 9, 10, 11, 12]. It also arises naturally in several theories of high temperature superconductivity [13, 14, 15]. Also remarkably the theory is not simply super-renormalizable, it is ultraviolet (uV) finite. For a Green function with F (B) number of external fermion (boson) lines, the superficial degree of divergence is $\delta(F, B) = 4 - (3/2)F - B - L$, where L is the number of loops. Consider the possible uV divergent diagrams:

1. One-loop fermion self-energy $\Sigma(p)$ has $\delta = 0$. But the mass renormalization is absent as a consequence of chiral symmetry. Therefore, the contribution is uV finite.
2. One-loop vacuum polarization $\Pi_{\mu\nu}(q)$ has $\delta = 1$. But gauge invariance requires that it has the form

$$\Pi_{\mu\nu}(q) = (q^2\delta_{\mu\nu} - q_\mu q_\nu)\Pi(q^2). \quad (1)$$

(We consider Euclidean Green functions throughout this paper.) As two powers of the photon momenta are pulled out, $\Pi(q^2)$ effectively has $\delta = -1$ and therefore it is uV finite.

3. Two-loop vacuum polarization has $\delta = 0$. It is also uV finite due to gauge invariance.

The same power counting shows that the iR divergences become increasingly worse with the number of loops. The iR superficial degree of divergence is given by $\Delta = -\delta$. In fact, there is a more severe type of iR divergence in perturbation theory. Self-energy insertions on any internal line of a loop give rise to infrared divergent contributions even for hard external momenta [16]. For the example shown in Fig. 1, let us perform the d^3l integration first. Clearly, the integrand will contain more and more factors of $1/l^2$ with increasing number of self-energy insertions on the photon line. There is a similar problem with self-energy insertions on any internal fermion line of a loop. As a result, perturbation theory does not exist.

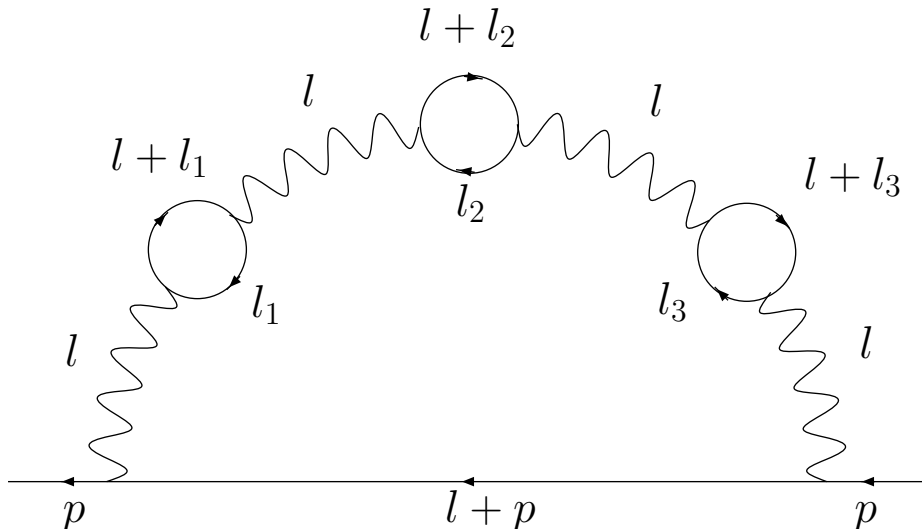


Figure 1: If the d^3l integration is performed first, the Feynman integral diverges even for hard external momentum.

A resummation of perturbation theory using $1/N$ expansion dramatically alters the situation [2]. The Lagrangian is

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \sum_i \bar{\psi}^i(i\not{\partial} - e\not{A})\psi^i \quad (2)$$

where ψ^i ($i = 1, \dots, N$) are N species of charged four-component spinors, all massless. Enforcing $U(2N)$ invariance [6] ensures that fermion mass is not generated to any order in perturbation theory. The charge e has an engineering dimension $1/2$. We take the large N limit with Ne^2 fixed. Then to the leading order in $1/N$, the one-loop vacuum polarization $\Pi^{(1)}(q^2)$ has to be included with the free photon propagator. Due to the masslessness of the fermion, $\Pi^{(1)}(q^2)$ is singular at $q = 0$ [1, 2]:

$$\Pi^{(1)}(q^2) = \frac{\mu}{q}, \quad \mu = \frac{Ne^2}{8}. \quad (3)$$

With the conventional gauge choice,

$$D_{\mu\nu}(q) = \frac{\delta_{\mu\nu} - q_\mu q_\nu / q^2}{q^2 + \mu q} - (\alpha - 1) \frac{q_\mu q_\nu}{q^4}. \quad (4)$$

This changes the infrared behaviour of the photon propagator from being inversely quadratic to inversely linear in momentum.

We consider a rearranged perturbation theory, with this as the “free” photon propagator, but otherwise the usual fermion propagator and vertex. The only difference with the usual perturbation theory is that the *one-loop* vacuum polarization contributions are not to be included in the new diagrams. Now the ultraviolet divergences are absent in any order as before, as the free photon propagator is as usual inversely quadratic for large momenta. On the other hand the infrared behaviour is now very different. The iR superficial degree of divergence is now

$$\Delta(F, B) = B + F - 3. \quad (5)$$

It is to be observed that this is independent of the number of loops and depends only on the number of external lines. This is analogous to the uV degree of divergence of a renormalizable theory. For non-exceptional Euclidean momenta $\{q\}$ which go to zero uniformly like

$$q = \rho Q, \quad (6)$$

the Green function is singular as $\rho^{-\Delta}$. In effect the scale dimension of photon has changed from the canonical $1/2$ to 1 , while that of the fermion remains at the canonical value 1 . This infrared limit corresponds to an infrared stable fixed point [6].

Subintegrations can spoil the elegant picture of the iR behaviour described above [1, 2, 3, 4]. The danger is from subdiagrams with $\Delta = 0$ which can generate a logarithmic singularity in momenta external to this subdiagram. Thus powers of logarithms arise from various subdiagrams. These logs can shift the infrared behaviour away from that given by the naive fixed point described earlier.

The fermion self-energy naively has $\Delta = -1$. However, the absence of self-mass correction makes the effective Δ zero. Indeed an explicit calculation of $\Sigma(p)$ in one loop gives a log correction [1, 3, 8]. See Ref. [15] for computation of the anomalous dimension of the fermion, suggesting that the canonical value is wrong. This further casts doubt on the relevance of the naive iR fixed point. The vertex correction also has $\Delta = 0$, and gives rise to logs in an arbitrary gauge.

However, the fermion self-energy depends on the gauge chosen, and we have to address the gauge invariant Green functions to unambiguously describe the iR behaviour. The simplest such objects are the Green functions involving only photons. As a consequence of Ward identities, the photon fields appear only in the field strength combination, and the Green functions are gauge invariant. Now there are sufficient indications [4, 6, 8, 17] in the two-loop order that the logs from fermion self-energy and vertex corrections cancel as a consequence of gauge invariance. It is also conjectured that such cancellations take place at all orders [4, 6]. But an explicit demonstration is lacking.

The situation is very similar to the uV logs in QED in 3+1 dimensions. There are log divergences in one-loop fermion self-energy and vertex corrections. When these are plugged into a two-loop calculation of the vacuum polarization, we expect two powers of log, as the vacuum polarization itself has effective $\delta = 0$. But an explicit calculation [18] shows a cancellation of the squares of logarithms between the fermion self-energy and vertex correction diagrams, so that only one power of log results. Johnson, Willey and Baker [19] have shown such a cancellation to all orders. Their strategy is to prove that a gauge choice exists in which the fermion self-energy and vertex corrections are free of log divergence. We will adopt this approach here. The demonstration is, however, much simpler in 2+1 dimensions.

We may expect that (with a specific choice of the gauge), if log corrections in fermion self-energy and vertex corrections are absent, such insertions into other Green functions do not lead to log corrections. Then the simple picture of iR behaviour is true.

We should not expect the absence of log corrections in higher orders even for a particular choice of the gauge parameter α in Eq. (4). The reason is that the α -dependent part is inversely quadratic in momentum and cannot cancel the iR logs coming from the inversely linear part. However we can choose the non-local Kondo-Nakatani gauge [8, 9] in which the photon propagator is

$$D_{\mu\nu}(q) = \frac{\delta_{\mu\nu} - \xi q_\mu q_\nu / q^2}{q^2 + \mu q}. \quad (7)$$

Now the part dependent on the gauge parameter ξ contributes to the infrared divergences in the same way as the rest, and there is hope that the log corrections vanish with a particular choice of the parameter. Indeed this has been checked in the lowest order (see Ref. [8] for the case of the fermion self-energy and Appendix A for the vertex correction).

We demonstrate here that with an appropriate choice of ξ in each order of $1/N$ expansion, the fermion self-energy and the vertex corrections have no logarithmic infrared divergences, and there are no log corrections to the leading $1/N$ infrared behaviour of any Green function. Our proof is iterative. We presume this to hold to $O(N^{-n})$, and show that this is then true to $O(N^{-n-1})$. (By a connected Green function of $O(N^{-n})$, we mean the following. Powers of e in the corresponding tree diagram, if any, are to be disregarded.

Each remaining e^2 is to be replaced by N^{-1} . Finally, each fermion loop contributes a factor of N .)

The Feynman integrals of a Green function $g(\{q\})$ have the following components depending on the external momenta $\{q\}$ and the loop momenta $\{l\}$:

(i) The photon propagators as in Eq. (7) with q replaced with $\Sigma l + \Sigma q$, where Σl (Σq) denote appropriate linear combinations of the internal (external) momenta.

(ii) The fermion propagators $1/(\Sigma \not{l} + \Sigma \not{q})$

(iii) Integration $\int d^3l/(2\pi)^3$ over each loop momentum.

The vertex factors do not depend on the momenta. We now choose the Euclidean external momenta $\{q\}$ going to zero uniformly as in Eq. (6), where Q are of $O(1)$. We also take $\{Q\}$ to be non-exceptional, i.e., no proper subset of $\{Q\}$ sums to zero. Let us make a change in the variables of loop integrations: $l \rightarrow \rho L$. Pulling out *one* factor of ρ from the denominator of each photon propagator and each fermion propagator, we get $g(\{q\}) = \rho^{-\Delta} G(\{Q\}, \rho)$. Here Δ is simply the naive infrared degree of divergence of the Green function, as given in Eq. (5). $G(\{Q\}, \rho)$ has the same expression as $g(\{Q\})$, apart from a modified photon propagator with a denominator $\rho(\Sigma L + \Sigma Q)^2 + \mu|\Sigma L + \Sigma Q|$.

Therefore, setting $\rho = 0$ formally, we get an expression for $G(\{Q\}, \rho = 0)$ which is exactly that of QED, apart from a photon propagator with a denominator $\mu|\Sigma L + \Sigma Q|$, i.e., inverse linear in momentum. The other rules are unchanged (except that the one-loop vacuum polarization corrections are ignored).

First note that as Q are all of $O(1)$ and non-exceptional, $G(\{Q\}, \rho = 0)$ is iR finite (Appendix B). However, $G(\{Q\}, \rho \rightarrow 0)$ can have uV divergence, with the uV superficial degree of divergence just the negative of that given in Eq. (5). Thus the situation is as in a renormalizable theory, with uV divergences only in self-energy and vertex parts at all orders. (As a consequence of gauge invariance, the photon self-energy corrections are anyway free of uV divergences even now. Also, overlapping divergences within the photon self-energy corrections can be handled in the usual way.) In effect, we have mapped the iR divergence of $g(\{\rho Q\})$ for $\rho \rightarrow 0$ to the uV divergence of $G(\{Q\}, \rho \rightarrow 0)$. The cut-off for the uV divergence is provided by $1/\rho$. We have presumed that by a choice of the gauge parameter, the fermion self-energy and vertex corrections of lower orders contained in our diagrams have been rendered finite. Then $G(\{Q\}, \rho \rightarrow 0)$ is finite except when it is a fermion self-energy or vertex correction.

Now we concentrate on the case of the vertex correction. We enclose the self-energy and vertex parts within boxes as in Fig. 2. Since these (lower order) contributions are presumed iR finite by appropriate choice of the gauge parameter, we may set $\rho = 0$ for such boxes. We may shrink the boxes to points and get the skeleton diagram, which has uV superficial degree of divergence $\delta = 0$. These points are assigned ρ independent factors. Since all proper subdiagrams of the skeleton diagram have $\delta < 0$, there is just a $\ln \rho$ divergence and not a higher power of logarithm.

We now show how to calculate the coefficient of the part which diverges as $\ln \rho$. Let us apply the operation $[\rho(d/d\rho)]_{\rho=0}$ on the skeleton diagram. The operation $d/d\rho$ modifies the denominator of each photon propagator of the skeleton, one at a time, to $(\rho|\Sigma L + \Sigma Q| + \mu)^2$. We denote this modified photon propagator by a cut (see Fig. 2). In order to be able to take the limit $\rho \rightarrow 0$, we scale the loop variables back to l , i.e., replace L with l/ρ . As $\delta = 0$, the net effect of $[\rho(d/d\rho)]_{\rho=0}$ is the original expression for the skeleton evaluated

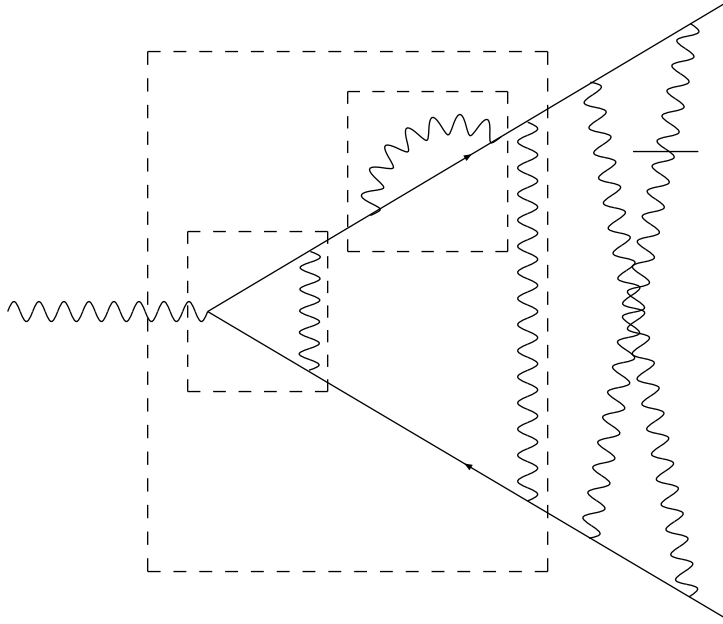


Figure 2: A contribution to the coefficient of $\ln \rho$.

at zero external momenta, except that the denominator of the cut photon propagator is modified to $(|\Sigma l| + \mu)^2$. (The denominators of the other photon propagators are $(\Sigma l)^2 + \mu|\Sigma l|$.) First, note that the photon propagators are inverse quadratic in momentum for large momenta just like the usual propagator, and hence our expression is uV finite. Secondly, note that μ provides an iR cutoff to the cut photon propagator, resulting in $\Delta = -1$. This again confirms that there is just a $\ln \rho$ divergence, and explicitly determines the numerical coefficient of N^{-n-1} . In Appendix A, this procedure for the extraction of the $\ln \rho$ term is illustrated by working out the case of the $O(1/N)$ vertex.

We now adjust ξ at $O(N^{-n})$ so that the contribution from the $O(1/N)$ vertex cancels this log divergence (Fig. 3). Then the vertex is iR finite to $O(N^{-n-1})$. As a consequence of the Ward identity, $Z_1 = Z_2$, the fermion self-energy will also be finite to this order, thus completing the proof.

Here it is to be emphasized that there is a major difference with respect to the case of uV divergences. Once counterterms are in place to remove the uV divergences, all subintegrations have $\delta < 0$, and the loop integrations can be carried out in any order with a unique result (i.e., absolute convergence). But in the present situation the self-energy insertions on a propagator have to be evaluated first, and fed into the other loop integrations (i.e., the integrals are only conditionally convergent). (See Appendix B.) As an example, consider Fig. 1 with higher order vacuum polarization insertions. We do not have the luxury of carrying out the d^3l integration first. When the vacuum polarization insertions are evaluated, each of them is proportional to l , while each photon propagator is proportional to $1/l$. So the problem referred to in Fig. 1 is now absent.

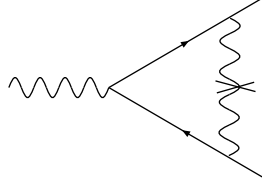


Figure 3: Contribution from adjustment of the gauge parameter ξ which cancels the logarithmic divergence from higher order vertex.

We have shown that with a specific choice of the gauge parameter in Eq. (7) (to each order in $1/N$), all Green functions $G(\{Q\}, \rho \rightarrow 0)$ are finite. These are the Green functions of a theory which uses the photon propagator $(\delta_{\mu\nu} - \xi q_\mu q_\nu / q^2) / (\mu q)$ and the usual QED rules otherwise. The one-loop vacuum polarization is not to be included in corrections to this photon propagator. Indeed the photon propagator is just this one-loop contribution. Thus this corresponds to simply the functional integral

$$\int \prod_i \mathcal{D}\psi^i \mathcal{D}\bar{\psi}^i \mathcal{D}A_\mu \exp[i \int \sum_i \bar{\psi}^i (i\not{\partial} - eA) \psi^i]. \quad (8)$$

Integrating over A_μ , we have

$$\int \prod_i \mathcal{D}\psi^i \mathcal{D}\bar{\psi}^i \prod_x \delta(\sum_i \bar{\psi}^i \gamma^\mu \psi^i) \exp[i \int \sum_i \bar{\psi}^i i\not{\partial} \psi^i]. \quad (9)$$

This is the Amati-Testa model [20]. If we scale the photon field $A_\mu \rightarrow (1/e)A_\mu$ in Eq. (2), the Green functions are formally of the QED theory

$$\mathcal{L} = -\frac{1}{4e^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \sum_i \bar{\psi}^i (i\not{\partial} - A) \psi^i \quad (10)$$

in the limit $e \rightarrow \infty$. Thus $1/e$ plays the role of ρ in our scaling Eq. (6).

The coupling constant e provides the interpolation of the Green functions from the uV to the iR behaviour: $e \rightarrow 0$ gives the free theory of photons and fermions as expected from the asymptotic freedom, while $e \rightarrow \infty$ gives the infrared limit of the Green functions. This latter limit is a non-trivial scale and conformal invariant theory with non-canonical scaling dimension for the photon. In fact we have a line of fixed points labelled by N (regarded as a continuous parameter). Integrating over $\psi^i, \bar{\psi}^i$ in Eq. (8), we get

$$\int \mathcal{D}A_\mu \exp[N \text{Tr} \ln(1 - \frac{1}{i\not{\partial}} A)]. \quad (11)$$

We see that N plays the role of $1/\hbar$, and $N \rightarrow \infty$ can be interpreted as the semi-classical limit of the theory. The Green functions in this limit are obtained as follows: Consider only the tree diagrams built from the theory

$$\mathcal{L} = - \sum_{2,4,\dots}^{\infty} \frac{1}{n} \text{Tr}(\frac{1}{i\not{\partial}} A)^n. \quad (12)$$

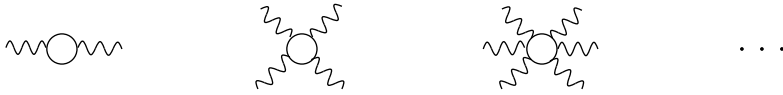


Figure 4: The terms in the effective Lagrangian for a class of conformal invariant field theories in dimensions $2 < d < 4$.

(See Fig. 4.) The $n = 2$ term gives the inverse of the propagator, while the other terms give the effective vertices. Thus we have an explicit non-trivial conformal invariant theory in 2+1 dimensions.

The other conformal field theories corresponding to other $N < \infty$ are obtained by using the propagator and effective vertices of the Lagrangian given in Fig. 4, and including the loop corrections with a factor of $1/N$ for every loop. Thus $1/N$ provides a marginal operator that takes us from the simplest conformal field theory to the entire class labelled by N . It is of great interest to construct these conformal field theories explicitly.

It is very interesting that the analysis of this paper is valid for all (Euclidean) dimensions $2 < d < 4$. The theory is uV finite in this range: $\delta = 4 - (4 - d)L - (3/2)F - B$. From the one-loop vacuum polarization, we now get $\Pi^{(1)}(q^2) \sim q^{d-4}$ for $q \rightarrow 0$. Thus the iR behaviour of our photon propagator will be q^{2-d} . Then the iR superficial degree of divergence is given by

$$\Delta = B + \frac{d-1}{2}F - d. \quad (13)$$

For the fermion self-energy, $\Delta = -1$, which in the absence of self-mass gives iR log as in $d = 3$. Same is the case for the vertex function which has $\Delta = 0$. Again we can choose a gauge such that these are iR finite and there are no log corrections. The iR limit is a conformal field theory where the photon has non-canonical scaling dimension one for the entire range of d , in contrast to the engineering dimension $(d-2)/2$.

In this paper, we demonstrated that the infrared behaviour given by the leading order in $1/N$ is not modified by logarithmic corrections in higher orders. Our technique gives finite Green functions for the Amati-Testa model, and results in a non-trivial conformal field theory for each N in all space-time dimensions $2 < d < 4$.

We have used the technique of choosing the value of the gauge parameter ξ such that the logarithmic infrared divergence in the electron self-energy and the vertex function is removed to all orders. This choice of ξ will therefore simplify the calculation of any gauge-invariant quantity (not necessarily a Green function) in which these insertions occur. Examples of such gauge-invariant quantities are gauge-invariant composite operators, related to the response functions of condensed matter systems [21]. Moreover, since for this value of ξ we know the infrared behaviour of a Green function to all orders, the determination of the infrared behaviour of a gauge-variant Green function to all orders for *any* value of ξ becomes possible. This has implications for the anomalous dimension of the dressed gauge-invariant fermion. These calculations will be presented elsewhere.

Acknowledgment

The LaTeX codes for the figures in this paper were generated primarily using JaxoDraw [22].

Appendices

A Logarithmically divergent part in vertex correction to $O(1/N)$

The $O(1/N)$ correction to the QED vertex, with incoming fermion momentum p and outgoing fermion momentum $p + q$, is given by

$$e\Gamma_\mu^{(1)}(p + q, p) = \int \frac{d^3l}{(2\pi)^3} e\gamma_\sigma \frac{1}{\not{p} + \not{q} + \not{l}} e\gamma_\mu \frac{1}{\not{p} + \not{l}} e\gamma_\rho \frac{\delta_{\sigma\rho} - \xi l_\sigma l_\rho / l^2}{l^2 + \mu l}. \quad (14)$$

(We use the Euclidean space Feynman rules and gamma matrix algebra of Ref. [18].) To evaluate these integrals with the modified photon propagator, we may use the spectral representation

$$\frac{1}{l^2 + \mu l} = \frac{2\mu}{\pi} \int_0^\infty dM \frac{1}{(M^2 + \mu^2)(l^2 + M^2)} \quad (15)$$

(see Ref. [1]), and

$$\frac{1}{l^2(l^2 + \mu l)} = \frac{2\mu}{\pi} \int_0^\infty dM \frac{1}{M^2(M^2 + \mu^2)} \left(\frac{1}{l^2} - \frac{1}{l^2 + M^2} \right) \quad (16)$$

(as obtained from Eq. (15)). However here we consider only the iR behaviour. Choose $p_\mu = \rho P_\mu$ and $q_\mu = \rho Q_\mu$. Also let $l_\mu = \rho L_\mu$. Then,

$$\Gamma_\mu^{(1)}(\rho(P + Q), \rho P) = e^2 \int \frac{d^3L}{(2\pi)^3} \gamma_\sigma \frac{1}{\not{P} + \not{Q} + \not{L}} \gamma_\mu \frac{1}{\not{P} + \not{L}} \gamma_\rho \frac{\delta_{\sigma\rho} - \xi L_\sigma L_\rho / L^2}{\rho L^2 + \mu L}. \quad (17)$$

For $\rho \rightarrow 0$, this is iR finite but logarithmically uV divergent. Let the divergent part be $C \ln \rho$. The coefficient C is obtained by the action of $[\rho(d/d\rho)]_{\rho=0}$ on the R.H.S. of Eq. (17). Thus,

$$C = -e^2 \left[\rho \int \frac{d^3L}{(2\pi)^3} \gamma_\sigma \frac{1}{\not{P} + \not{Q} + \not{L}} \gamma_\mu \frac{1}{\not{P} + \not{L}} \gamma_\rho \frac{\delta_{\sigma\rho} - \xi L_\sigma L_\rho / L^2}{(\rho L + \mu)^2} \right]_{\rho=0}. \quad (18)$$

The problem with this form is that we have ρ times an integral which diverges at $\rho = 0$. To get around this problem, let us replace L by l/ρ in the integral. Then setting $\rho = 0$, we obtain

$$C = -e^2 \int \frac{d^3l}{(2\pi)^3} \gamma_\sigma \frac{\not{l}}{l^2} \gamma_\mu \frac{\not{l}}{l^2} \gamma_\rho \frac{\delta_{\sigma\rho} - \xi l_\sigma l_\rho / l^2}{(l + \mu)^2}. \quad (19)$$

The numerator of the integrand is $l^2(1 - \xi)\gamma_\mu - 2\not{l}_\mu$. By symmetry, \not{l}_μ may be replaced with $(1/3)l^2\gamma_\mu$. Doing the angular integration, we arrive at

$$\begin{aligned} C &= \frac{e^2}{2\pi^2}(\xi - \frac{1}{3})\gamma_\mu \int_0^\infty dl \frac{1}{(l + \mu)^2} \\ &= \frac{4}{\pi^2 N}(\xi - \frac{1}{3})\gamma_\mu. \end{aligned} \quad (20)$$

Thus, for the gauge choice $\xi = 1/3$, there is no log in the $O(1/N)$ vertex. As expected, this is the same gauge in which the $O(1/N)$ self-energy is also free from logarithm [8].

B Absence of infrared divergences for hard and non-exceptional Euclidean external momenta

Consider a Green function for which the external momenta are Euclidean, non-exceptional and of $O(1)$, while the internal photon lines are inversely linear in momentum. We follow the standard power-counting arguments for Euclidean momenta [19, 23] to show that the Green function is free from iR divergences.

The hard momenta of the external lines flow through some of the internal lines also, and all these hard internal lines must be connected due to the non-exceptional nature of the external momenta. Since a hard internal line does not contribute to the iR degree of divergence, it may be contracted to a point for the present purpose. Thus we arrive at a reduced diagram in which all the external lines are joined at a single point. Out of this single point, let b soft internal boson lines and f soft internal fermion lines come out and join to a subdiagram S consisting entirely of soft internal lines.

In addition to usual QED vertices, S can also contain composite vertices arising out of the contraction of hard loops which are not connected to the hard external lines. In general, such a composite vertex can have m boson and n fermion lines. Let the number of such a vertex in S be V_{mn} .

Therefore we have to calculate the iR superficial degree of divergence of a diagram without any external lines, which contains one composite vertex having b boson and f fermion lines, (say) V elementary vertices, and also V_{mn} composite vertices with various values of m and n . Taking i_B and i_F to be the number of internal boson and fermion lines, we have

$$\Delta = i_B + i_F - 3L, \quad (21)$$

$$L = i_B + i_F - (1 + V + \sum_{m,n} V_{mn}) + 1, \quad (22)$$

$$2i_B = b + V + \sum_{m,n} mV_{mn}, \quad (23)$$

$$2i_F = f + 2V + \sum_{m,n} nV_{mn} \quad (24)$$

leading to

$$\Delta = -b - f + \sum_{m,n} (3 - m - n)V_{mn}. \quad (25)$$

It then appears that the cases where (m, n) are $(2, 0)$, $(0, 2)$ and $(1, 2)$, can lead to iR divergence by making positive or logarithmic contribution to Δ . But these are precisely the self-energy and vertex insertions explicitly addressed in this paper. Thus, if the subintegration involved in the composite vertex is performed first, the problem disappears, as will be explained now. The vertex correction is rendered finite by a choice of non-local gauge. Chiral symmetry ensures that the fermion self-energy insertion is proportional to \not{p} , and the proportionality constant is finite in the same non-local gauge. Thus Δ is actually diminished (and not increased) by one due to a fermion self-energy insertion. The same change in Δ happens for a photon self-energy insertion.

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