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# Influence of light on the Debye screening length in III–V, ternary, and quaternary materials

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We study the electron energy spectrum and the Debye screening length (DSL) for III–V, ternary, and quaternary materials in the presence of light waves, whose unperturbed energy band structures are defined by the three-band model of Kane. The solution of the Boltzmann transport equation on the basis of this newly formulated electron dispersion law will introduce new physical ideas and experimental findings in the presence of external photoexcitation. It has been found taking  $n$ -InAs,  $n$ -InSb,  $n$ -Hg<sub>1-x</sub>Cd<sub>x</sub>Te, and  $n$ -In<sub>1-x</sub>Ga<sub>x</sub>As<sub>y</sub>P<sub>1-y</sub> lattice matched to InP, as examples that the DSL decreases with the increase in electron concentration, intensity, and wavelength, respectively in various manners. The strong dependence of the DSL on both light intensity and wavelength reflects the direct signature of light waves which is in contrast as compared with the corresponding bulk specimens of the said materials in the absence of external photoexcitation. The rate of change is totally band structure dependent and is significantly influenced by the presence of the different energy band constants. The classical DSL equation in the absence of light waves has been obtained as a special case of the present analysis under certain limiting conditions and this compatibility is the indirect test of our generalized formalism. We have also suggested an experimental method of determining the DSL in degenerate materials having arbitrary dispersion laws. © 2007 American Institute of Physics. [DOI: 10.1063/1.2785002]

## I. INTRODUCTION

It is well known that the Debye screening length (DSL) of the carriers in semiconductors is a very important quantity characterizing the screening of the Coulomb field of the ionized impurity centers by the free carriers.<sup>1</sup> It affects many of the special features of modern nanodevices, the carrier mobilities under different mechanisms of scattering, and the carrier plasmas in semiconductors.<sup>2</sup> The DSL is a very good approximation to the accurate self-consistent screening in presence of band tails and is also used to illustrate the interaction between the colliding carriers in Auger effect in solids.<sup>1</sup> The classical value of the DSL is equal to  $[\epsilon_{sc}k_B T / (e^2 n_0)]^{1/2}$  ( $\epsilon_{sc}$ ,  $k_B$ ,  $T$ ,  $e$ , and  $n_0$  are the semiconductor permittivity, the Boltzmann's constant, the temperature, the magnitude of the carrier charge, and the electron concentration, respectively), which is valid for both the carriers. In this conventional form, the DSL decreases with increasing electron concentration at a constant temperature and this relation holds only under the condition of carrier nondegeneracy. Since, the performance of the electron devices at the device terminals and the speed of operation of modern switching transistors are significantly influence by the degree of carrier degeneracy present in these devices,<sup>3</sup> the simplest way of analyzing such devices taking into account of the degeneracy of the band is to use the appropriate DSL to express the performance at the device terminal and switching speed in terms of the carrier concentration.<sup>3</sup>

The DSL depends on the density-of-states function which, in turn, significantly affects the different physical properties of III–V, ternary, quaternary, and related compounds having various band structures.<sup>2</sup> It is well known from the fundamental study of Landsberg,<sup>1</sup> that the DSL for electronic materials having degenerate electron concentration is essentially determined by their respective energy band structures. It has, therefore, different values in different materials and varies with the electron concentration, with the magnitude of the reciprocal quantizing magnetic field under magnetic quantization, with the quantizing electric field as in inversion layers, with the nanothickness as in quantum wells, with superlattice period as in the quantum confined superlattices of small gap compounds with graded interfaces having various carrier energy spectra. The nature of these variations has been investigated by in the literature<sup>1–12</sup> and some of the significant features, which have emerged from these studies, are:

- (a) the DSL decreases with increasing electron concentration and such variations are significantly influenced by constants of the energy band spectra;
- (b) the DSL decreases with the magnitude of the quantizing electric field as in inversion layers;
- (c) the DSL oscillates with the inverse quantizing magnetic field under magnetic quantization due to the Shubnikov–de Haas (SdH) effect; and
- (d) the DSL exhibits composite oscillations with the various controlled parameters as in superlattices of non-parabolic compounds with graded interfaces.

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It appears from the literature that the DSL for III–V, ternary, and quaternary materials has yet to be investigated in the presence of light waves. For the purpose of numerical computation, we shall take InAs and InSb as examples of III–V compounds. We shall use  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  and  $\text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$  lattice matched to InP as examples of ternary and quaternary materials, respectively. The III–V compounds find extensive application in Bragg reflectors, distributed feedback lasers,<sup>13</sup> passive filter devices,<sup>14</sup> photorefective materials,<sup>15</sup> and integrated optoelectronics.<sup>16</sup> The ternary alloy  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  is a classic narrow gap compound and is an important optoelectronic material. The band gap of this ternary alloy can be varied to cover the spectral range from 0.8 to over 30  $\mu\text{m}$  by adjusting the alloy composition.<sup>17</sup>  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  finds wide use in infrared detector materials and photovoltaic detector arrays in the 8–12  $\mu\text{m}$  wave bands.<sup>18</sup> The earlier uses have spurred  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  technology for production of high mobility single crystal with specially prepared surfaces and the same compound is ideally suitable for the narrow subband physics because the relevant material constants are within easy experimental reach.<sup>19</sup> The quaternary alloy  $\text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$  lattice matched to InP, also finds extensive application in the fabrication of avalanche photodetectors,<sup>20</sup> heterojunction lasers,<sup>21</sup> light emitting diodes,<sup>22</sup> avalanche photodiodes,<sup>23</sup> field effect transistors, detectors, and other devices. New types of integrated optical devices such as switches, modulators, solar cells, and filters are made from the quaternary systems.<sup>24</sup>

In Sec. II A, the dispersion relation of the conduction electrons of III–V, ternary, and quaternary materials in the presence of light waves has been formulated whose unperturbed conduction electrons obey the three band model of Kane. In the same section, we have studied the dispersion relations for the said materials in the presence of external photoexcitation when the unperturbed energy spectra are defined by the two band model of Kane and that of parabolic energy bands, respectively, for the purpose of relative comparison. In Sec. II B, the expressions of the electron statistics and the DSL have been derived for all the aforementioned cases. In Sec. II C, we have investigated the DSL in the absence of light waves. Section II D contains the suggestion of an experimental method for determining the DSL for semiconductors having arbitrary dispersion laws. The DSL has been numerically investigated by taking  $n$ -InAs and  $n$ -InSb as examples of III–V compounds,  $n$ - $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  as an example of ternary compounds, and  $n$ - $\text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$  lattice matched to InP as an example of quaternary compounds in accordance with the three and the two band models of Kane together with model of parabolic energy bands, respectively, for the purpose of relative assessment.

## II. THEORETICAL BACKGROUND

### A. Simplified formulation of the electron dispersion law in the presence of light waves in III–V, ternary, and quaternary materials

The Hamiltonian ( $\bar{H}$ ) of an electron in the presence of light wave characterized by the vector potential  $\mathbf{A}$  can be written as following:<sup>25</sup>

$$H = [(\bar{p} + e\mathbf{A})^2/2m] + V(\mathbf{r}), \quad (1)$$

in which,  $\bar{p}$  is the momentum operator,  $V(\mathbf{r})$  is the crystal potential, and  $m$  is the free electron mass.

Equation (1) can be expressed as

$$\bar{H} = \bar{H}_0 + \bar{H}', \quad (2)$$

where

$$\bar{H}_0 = \frac{\bar{p}^2}{2m} + V(\mathbf{r}),$$

$$\bar{H}' = \frac{e}{2m} \mathbf{A} \cdot \mathbf{p}. \quad (3)$$

The perturbed Hamiltonian  $\bar{H}'$  can be written as

$$\bar{H}' = \left( \frac{-i\hbar e}{2m} \right) (\mathbf{A} \cdot \nabla), \quad (4)$$

where  $i = \sqrt{-1}$  and  $\mathbf{p} = -i\hbar \nabla$ .

The vector potential ( $\mathbf{A}$ ) of the monochromatic light of plane wave can be expressed as

$$\mathbf{A} = A_0 \boldsymbol{\epsilon}_s \cos(\mathbf{s}_0 \cdot \mathbf{r} - \omega t), \quad (5)$$

where  $A_0$  is the amplitude of the light wave,  $\boldsymbol{\epsilon}_s$  is the polarization vector,  $\mathbf{s}_0$  is the momentum vector of the incident photon,  $\mathbf{r}$  is the position vector,  $\omega$  is the angular frequency of light wave, and  $t$  is the time scale. The matrix element of  $\bar{H}'_{nl}$  between initial state,  $\psi_l(\mathbf{q}, \mathbf{r})$  and final state  $\psi_n(\mathbf{k}, \mathbf{r})$  in different bands can be written as

$$\bar{H}'_{nl} = \frac{e}{2m} \langle n\mathbf{k} | \mathbf{A} \cdot \mathbf{p} | l\mathbf{q} \rangle. \quad (6)$$

Using Eqs. (4) and (5), we can rewrite Eq. (6) as

$$\bar{H}'_{nl} = \left( \frac{-i\hbar e A_0}{4m} \right) \boldsymbol{\epsilon}_s \cdot \{ \langle n\mathbf{k} | \exp(i\mathbf{s}_0 \cdot \mathbf{r}) \nabla | l\mathbf{q} \rangle e^{-i\omega t} \} + \{ \langle n\mathbf{k} | \exp(-i\mathbf{s}_0 \cdot \mathbf{r}) \nabla | l\mathbf{q} \rangle e^{i\omega t} \}. \quad (7)$$

The first matrix element of Eq. (7) can be written as

$$\langle n\mathbf{k} | \exp(i\mathbf{s}_0 \cdot \mathbf{r}) \nabla | l\mathbf{q} \rangle = \int \exp(i[\mathbf{q} + \mathbf{s}_0 - \mathbf{k}] \cdot \mathbf{r}) i\mathbf{q} u_n^*(\mathbf{k}, \mathbf{r}) u_l(\mathbf{q}, \mathbf{r}) d^3r + \int \exp(i[\mathbf{q} + \mathbf{s}_0 - \mathbf{k}] \cdot \mathbf{r}) u_n^*(\mathbf{k}, \mathbf{r}) \nabla u_l(\mathbf{q}, \mathbf{r}) d^3r. \quad (8)$$

The functions  $u_n^* u_l$  and  $u_n^* \nabla u_l$  are periodic. The integral over all space can be separated into a sum over unit cells times an integral over a single unit cell. It is assumed that the wavelength of the electromagnetic wave is sufficiently large so that if  $\mathbf{k}$  and  $\mathbf{q}$  are within the Brillouin zone,  $(\mathbf{q} + \mathbf{s}_0 - \mathbf{k})$  is not a reciprocal lattice vector.

Therefore, we can write Eq. (8) as

$$\begin{aligned}
& \langle n\mathbf{k} | \exp(i\mathbf{s}_0 \cdot \mathbf{r}) \nabla | l\mathbf{k} \rangle \\
&= \left[ \frac{(2\pi)^3}{\Omega} \right] \left\{ i\bar{q} \delta(\mathbf{q} + \mathbf{s}_0 - \mathbf{k}) \delta_{nl} \right. \\
&\quad \left. + \delta(\mathbf{q} + \mathbf{s}_0 - \mathbf{k}) \int_{\text{cell}} u_n^*(\mathbf{k}, \mathbf{r}) \nabla u_l(\mathbf{q}, \mathbf{r}) d^3r \right\} \\
&= \left[ \frac{(2\pi)^3}{\Omega} \right] \left\{ \delta(\mathbf{q} + \mathbf{s}_0 - \mathbf{k}) \right. \\
&\quad \left. \times \int_{\text{cell}} u_n^*(\mathbf{k}, \mathbf{r}) \nabla u_l(\mathbf{q}, \mathbf{r}) d^3r \right\}, \quad (9)
\end{aligned}$$

where  $\Omega$  is the volume of the unit cell and  $\int u_n^*(\mathbf{k}, \mathbf{r}) u_l(\mathbf{q}, \mathbf{r}) d^3r = \delta(\mathbf{q} - \mathbf{k}) \delta_{nl} = 0$ , since  $n \neq l$ .

The delta function expresses the conservation of wave vector in the absorption of light wave and  $\mathbf{s}_0$  is small compared to the dimension of a typical Brillouin zone and we set  $\mathbf{q} = \mathbf{k}$ .

From Eqs. (8) and (9), we can write

$$\bar{H}'_{nl} = \frac{eA_0}{2m} \boldsymbol{\varepsilon}_s \cdot \mathbf{p}_{nl}(\mathbf{k}) \delta(\mathbf{q} - \mathbf{k}) \cos(\omega t), \quad (10)$$

where  $\mathbf{p}_{nl}(\mathbf{k}) = -i\hbar \int u_n^* \nabla u_l d^3r = \int u_n^*(\mathbf{k}, \mathbf{r}) \mathbf{p} u_l(\mathbf{k}, \mathbf{r}) d^3r$ .

Therefore, we can write

$$\bar{H}'_{nl} = \frac{eA_0}{2m} \boldsymbol{\varepsilon}'_s \cdot \mathbf{p}_{nl}(\mathbf{k}), \quad (11)$$

where  $\boldsymbol{\varepsilon}'_s = \boldsymbol{\varepsilon}_s \cos \omega t$ .

When a photon interacts with a semiconductor, the carriers (i.e., electrons) are generated in the bands which are followed by the interband transitions. For example, when the carriers are generated in the valence band, the carriers then make interband transition to the conduction band. The transition of the electrons within the same band, i.e.,  $H'_{mm} = \langle n\mathbf{k} | H' | n\mathbf{k} \rangle$  is neglected. Because, in such a case, i.e., when the carriers are generated within the same bands by photons are lost by recombination within the aforementioned band resulting in zero carriers.

Therefore,

$$\langle n\mathbf{k} | H' | n\mathbf{k} \rangle = 0, \quad (12)$$

where  $n=c$  stands for conduction band and  $l=v$  stands for valence band. The energy equation for the conduction electron can approximately be expressed as

$$\gamma(E) = \left( \frac{\hbar^2 k^2}{2m_c} \right) + \frac{\left( \frac{eA_0}{2m} \right)^2 \langle |\boldsymbol{\varepsilon}_s \cdot \mathbf{p}_{cv}(\mathbf{k})|^2 \rangle_{av}}{E_c(\mathbf{k}) - E_v(\mathbf{k})}, \quad (13)$$

where  $\gamma(E) = E(aE+1)(bE+1)/(cE+1)$ ,  $a=1/E_{g_0}$ ,  $E_{g_0}$  is the unperturbed band gap,  $b=1/(E_{g_0}+\Delta)$ ,  $\Delta$  is the spin-orbit splitting constant in the absence of any field,  $c=1/(E_{g_0}+2\Delta/3)$ ,  $m_c$  is the effective electron mass at the edge of the conduction band in the absence of any field and  $\langle |\boldsymbol{\varepsilon}_s \cdot \mathbf{p}_{cv}(\mathbf{k})|^2 \rangle_{av}$  represents the average of the square of the optical matrix element (OME).

For the three-band model of Kane, we can write

$$\xi_{1k} = E_c(\mathbf{k}) - E_v(\mathbf{k}) = (E_{g_0}^2 + E_{g_0} \hbar^2 k^2 / m_r)^{1/2}, \quad (14)$$

where  $m_r$  is the reduced mass and is given by  $m_r^{-1} = m_c^{-1} + m_v^{-1}$  and  $m_v$  is the effective mass of the heavy hole at the top of the valance band in the absence of any field.

Using Eqs. (A19), (A20), (A23), and (A24) from the Appendix, we can write

$$\begin{aligned}
& \left( \frac{eA_0}{2m} \right)^2 \frac{\langle |\boldsymbol{\varepsilon}_s \cdot \mathbf{p}_{cv}(\mathbf{k})|^2 \rangle_{av}}{E_c(\mathbf{k}) - E_v(\mathbf{k})} \\
&= \left( \frac{eA_0}{2m} \right)^2 \frac{2\pi}{3} |\boldsymbol{\varepsilon}_s \cdot \mathbf{p}_{cv}(0)|^2 \frac{\beta^2}{4} \left( t + \frac{\rho}{\sqrt{2}} \right)^2 \frac{1}{\xi_{1k}} \\
&\quad \times \left\{ \left( 1 + \frac{E_{g_0} - \delta'}{\xi_{1k} + \delta'} \right) + (E_{g_0} - \delta') \right. \\
&\quad \times \left[ \frac{1}{\xi_{1k} + \delta'} - \frac{1}{E_{g_0} + \delta'} \right]^{1/2} \left[ \frac{1}{\xi_{1k} + \delta'} \right. \\
&\quad \left. \left. - \frac{E_{g_0} + \delta'}{(E_{g_0} - \delta')^2} \right]^{1/2} \right\}^2. \quad (15)
\end{aligned}$$

Following Nag,<sup>26</sup> it can be shown that

$$A_0^2 = \frac{I\lambda^2}{2\pi^2 c^3 \sqrt{\varepsilon_{sc} \varepsilon_0}}, \quad (16)$$

where  $I$  is the light intensity of wavelength  $\lambda$ ,  $c$  is the velocity of light, and  $\varepsilon_0$  is the permittivity of vacuum. Thus, the simplified electron energy spectrum in III-V, ternary, and quaternary materials up to the second order in the presence of light waves can approximately be written as

$$\frac{\hbar^2 k^2}{2m_c} = \beta_0(E, \lambda), \quad (17)$$

where

$$\beta_0(E, \lambda) = [\gamma(E) - \theta_0(E, \lambda)],$$

$$\begin{aligned}
\theta_0(E, \lambda) &= \frac{e^2}{96m_r \pi c^3 \sqrt{\varepsilon_{sc} \varepsilon_0}} \frac{E_{g_0} (E_{g_0} + \Delta) \beta^2}{\left( E_{g_0} + \frac{2}{3} \Delta \right)^4} \left( t \right. \\
&\quad \left. + \frac{\rho}{\sqrt{2}} \right)^2 \frac{1}{\phi_0(E)} \left\{ \left( 1 + \frac{E_{g_0} - \delta'}{\phi_0(E) + \delta'} \right) + (E_{g_0} - \delta') \right. \\
&\quad \times \left[ \frac{1}{\phi_0(E) + \delta'} - \frac{1}{E_{g_0} + \delta'} \right]^{1/2} \left[ \frac{1}{\phi_0(E) + \delta'} \right. \\
&\quad \left. \left. - \frac{E_{g_0} + \delta'}{(E_{g_0} - \delta')^2} \right]^{1/2} \right\}^2,
\end{aligned}$$

$$\phi_0(E) = E_{g_0} \left[ 1 + 2 \left( 1 + \frac{m_c}{m_v} \right) \frac{\gamma(E)}{E_{g_0}} \right]^{1/2}.$$

Thus, under the limiting condition  $\mathbf{k} \rightarrow 0$ , from Eq. (17), we observe that  $E \neq 0$  and is positive. Therefore, in the presence of external light waves, the energy of the electron does not tend to zero when  $\mathbf{k} \rightarrow 0$ , whereas for the unperturbed three



band model of Kane,  $\gamma(E) = \hbar^2 k^2 / 2m_c$  in which  $E \rightarrow 0$  for  $\mathbf{k} \rightarrow 0$ . As the conduction band is taken as the reference level of energy, therefore the lowest positive value of  $E$  for  $\mathbf{k} \rightarrow 0$  provides the increased band gap ( $\Delta E_g$ ) of the semiconductor due to photon excitation. The values of the increased band gap can be obtained by computer iteration processes for various values of  $I$  and  $\lambda$ , respectively.

### 1. Special cases

(1) For the two-band model of Kane, we have  $\Delta \rightarrow 0$ . Under this condition,  $\gamma(E) \rightarrow E(1 + \alpha E) = \hbar^2 k^2 / 2m_c$  with  $\alpha = 1/E_{g_0}$ . Since  $\beta \rightarrow 1$ ,  $t \rightarrow 1$ ,  $\rho \rightarrow 0$ , and  $\delta' \rightarrow 0$  for  $\Delta \rightarrow 0$ , from Eq. (17), we can write the energy spectrum of III–V, ternary, and quaternary materials in the presence of external photoexcitation whose unperturbed conduction electrons obey the two band model of Kane as

$$\frac{\hbar^2 k^2}{2m_c} = \omega_0(E, \lambda), \quad (18)$$

where

$$\begin{aligned} \omega_0(E, \lambda) &= E(1 + \alpha E) - B_0(E, \lambda), \\ B_0(E, \lambda) &= \frac{e^2 \Gamma \lambda^2 E_{g_0}}{384 \pi c^3 m_r \sqrt{\epsilon_{sc} \epsilon_0}} \frac{1}{\phi_1(E)} \left\{ \left[ 1 + \frac{E_{g_0}}{\phi_1(E)} \right] \right. \\ &\quad \left. + E_{g_0} \left[ \frac{1}{\phi_1(E)} - \frac{1}{E_{g_0}} \right] \right\}^2, \\ \phi_1(E) &= E_{g_0} \left\{ 1 + \frac{2m_c E(1 + \alpha E)}{m_r E_{g_0}} \right\}^{1/2}. \end{aligned}$$

(2) In the case of relatively wide band gap semiconductor, we can write  $a \rightarrow 0$ ,  $b \rightarrow 0$ ,  $c \rightarrow 0$ , and  $\gamma(E) \rightarrow E$ .

Thus, from Eq. (18), we can find

$$\frac{\hbar^2 k^2}{2m_c} = \rho_0(E, \lambda), \quad (19)$$

$$\rho_0(E) = E - \frac{e^2 \Gamma \lambda^2}{96 \pi c^3 m_r \sqrt{\epsilon_{sc} \epsilon_0}} \left[ 1 + \left( \frac{2m_c}{m_r} \right) \frac{E}{E_g} \right]^{-3/2}. \quad (20)$$

### B. Formulation of the DSL in the presence of light waves in III–V, ternary, and quaternary materials

The DSL can, in general,<sup>4</sup> be written as

$$L_D = \left[ \frac{e^2}{\epsilon_{sc}} \left( \frac{\partial n_0}{\partial E_F} \right) \right]^{-1/2}, \quad (21)$$

where  $E_F$  is the Fermi energy in the presence of light waves as measured from the edge of conduction band in the absence of any field in the vertically upward direction. Therefore, the formulation of the DSL requires an expression of electron statistics, which, in turn, is determined by the density-of-states function:

(i) Using Eq. (17), the density-of-states function for III–V, ternary, and quaternary materials in the presence of

light waves whose unperturbed conduction electrons obey the three band model of Kane can be written as

$$D_0(E) = 4\pi \left( \frac{2m_c}{\hbar^2} \right)^{3/2} \sqrt{\beta_0(E, \lambda)} \beta_0'(E, \lambda), \quad (22)$$

where  $\beta_0'(E, \lambda) = \partial / \partial E [\beta_0(E, \lambda)]$ .

Combining Eq. (22) with the Fermi–Dirac occupation probability factor  $f_0$  and using the generalized Sommerfeld's lemma,<sup>27</sup> the electron concentration can be written as

$$n_0 = (3\pi^2)^{-1} \left( \frac{2m_c}{\hbar^2} \right)^{3/2} [M_1(E_F, \lambda) + N_1(E_F, \lambda)], \quad (23)$$

where

$$M_1(E_F, \lambda) = [\beta_0(E_F, \lambda)]^{3/2},$$

$$N_1(E_F, \lambda) = \sum_{r=1}^{s'} L(r, 0) M_1(E_F, \lambda),$$

$$L(r, J) = [2(k_B T)^{2r} (1 - 2^{1-2r}) \zeta(2r)] (\partial^{2r+J} / \partial E_F^{2r+J}),$$

$r$  and  $J$  are the sets of real positive integers and  $\zeta(2r)$  is the Zeta function of order  $2r$ .<sup>28</sup>

Using Eqs. (21) and (23) we get

$$L_D = \left\{ (3\pi^2)^{-1} \left( \frac{2m_c}{\hbar^2} \right)^{3/2} \frac{e^2 [p_1(E_F, \lambda) + q_1(E_F, \lambda)]}{\epsilon_{sc}} \right\}^{-1/2}, \quad (24)$$

where  $p_1(E_F, \lambda) = \partial / \partial E_F [M_1(E_F, \lambda)]$  and  $q_1(E_F, \lambda) = \partial / \partial E_F [N_1(E_F, \lambda)]$ .

(ii) The expressions of  $n_0$  and DSL for III–V, ternary, and quaternary materials in the presence of light waves whose unperturbed conduction electrons obey the two band model of Kane can be expressed as

$$n_0 = (3\pi^2)^{-1} \left( \frac{2m_c}{\hbar^2} \right)^{3/2} [M_2(E_F, \lambda) + N_2(E_F, \lambda)] \quad (25)$$

and

$$L_D = \left[ (3\pi^2)^{-1} \left( \frac{2m_c}{\hbar^2} \right)^{3/2} \frac{e^2 [p_2(E_F, \lambda) + q_2(E_F, \lambda)]}{\epsilon_{sc}} \right]^{-1/2}, \quad (26)$$

where

$$M_2(E_F, \lambda) = [\omega_0(E_F, \lambda)]^{3/2},$$

$$N_2(E_F, \lambda) = \sum_{r=1}^{s'} L(r, 0) M_2(E_F, \lambda),$$

$$p_2(E_F, \lambda) = \frac{\partial}{\partial E_F} [M_2(E_F, \lambda)],$$

$$q_2(E_F, \lambda) = \frac{\partial}{\partial E_F} [N_2(E_F, \lambda)].$$

(iii) The expressions of  $n_0$  and DSL for III–V, ternary, and quaternary materials in the presence of light waves whose unperturbed conduction electrons obey the parabolic energy bands can be written as

$$n_0 = (3\pi^2)^{-1} \left( \frac{2m_c}{\hbar^2} \right)^{3/2} [M_3(E_F, \lambda) + N_3(E_F, \lambda)] \quad (27)$$

and

$$L_D = \left[ (3\pi^2)^{-1} \left( \frac{2m_c}{\hbar^2} \right)^{3/2} \frac{e^2 [p_3(E_F, \lambda) + q_3(E_F, \lambda)]}{\epsilon_{sc}} \right]^{-1/2}, \quad (28)$$

where

$$M_3(E_F, \lambda) = [\rho_0(E_F, \lambda)]^{3/2},$$

$$N_3(E_F, \lambda) = \sum_{r=1}^{s'} L(r, 0) M_3(E_F, \lambda),$$

$$p_3(E_F, \lambda) = \frac{\partial}{\partial E_F} [M_3(E_F, \lambda)],$$

$$q_3(E_F, \lambda) = \partial / \partial E_F [N_3(E_F, \lambda)].$$

### C. Formulation of the DSL in the absence of light waves in III–V, ternary, and quaternary materials

(i) In the absence of external photoexcitations, the expressions of the  $n_0$  and the DSL in accordance with the three band model of Kane assume the forms

$$n_0 = (3\pi^2)^{-1} \left( \frac{2m_c}{\hbar^2} \right)^{3/2} [M_4(E_{F_0}) + N_4(E_{F_0})], \quad (29)$$

$$L_D = \left[ (3\pi^2)^{-1} \left( \frac{2m_c}{\hbar^2} \right)^{3/2} \frac{e^2 [p_4(E_{F_0}) + q_4(E_{F_0})]}{\epsilon_{sc}} \right]^{-1/2}, \quad (30)$$

where  $E_{F_0}$  is the Fermi energy in the absence of light waves and is measured from the edge of the conduction band in the vertically upward direction

$$M_4(E_{F_0}) = [\gamma(E_{F_0})]^{3/2},$$

$$N_4(E_{F_0}) = \sum_{r=1}^{s'} L(r, 0) M_4(E_{F_0}),$$

$$p_4(E_{F_0}) = \frac{\partial}{\partial E_{F_0}} [M_4(E_{F_0})],$$

$$q_4(E_{F_0}) = \frac{\partial}{\partial E_{F_0}} [N_4(E_{F_0})].$$

(ii) In accordance with the two band model of Kane, the corresponding expressions of the  $n_0$  and DSL are given by

$$n_0 = (3\pi^2)^{-1} \left( \frac{2m_c}{\hbar^2} \right)^{3/2} [M_5(E_{F_0}) + N_5(E_{F_0})] \quad (31)$$

and

$$L_D = \left[ (3\pi^2)^{-1} \left( \frac{2m_c}{\hbar^2} \right)^{3/2} \frac{e^2 [p_5(E_{F_0}) + q_5(E_{F_0})]}{\epsilon_{sc}} \right]^{-1/2}, \quad (32)$$

where

$$M_5(E_{F_0}) = [E_{F_0}(1 + \alpha E_{F_0})]^{3/2},$$

$$\alpha = \frac{1}{E_{g_0}},$$

$$N_5(E_{F_0}) = \sum_{r=1}^{s'} L(r, 0) M_5(E_{F_0}),$$

$$p_5(E_{F_0}) = \frac{\partial}{\partial E_F} [M_5(E_{F_0})],$$

$$q_5(E_{F_0}) = \frac{\partial}{\partial E_{F_0}} [N_5(E_{F_0})].$$

(iii) Under the constraints  $\Delta \gg E_{g_0}$  or  $\Delta \ll E_{g_0}$  together with the condition  $\alpha E_{F_0} \ll 1$ , Eqs. (31) and (32) assume the forms

$$n_0 = N_C \left[ F_{1/2}(\eta) + \left( \frac{15\alpha k_B T}{4} \right) F_{3/2}(\eta) \right] \quad (33)$$

and

$$L_D = \left\{ \frac{e^2 N_C}{\epsilon_{sc} k_B T} \left[ F_{-1/2}(\eta) + \left( \frac{15\alpha k_B T}{4} \right) F_{1/2}(\eta) \right] \right\}^{-1/2}, \quad (34)$$

where  $N_C = 2(2\pi m_c k_B T / h^2)^{3/2}$ ,  $\eta = E_{F_0} / k_B T$ , and  $F_{t_0}(\eta)$  is the one parameter Fermi–Dirac integral of order  $t_0$  which can be written as<sup>29</sup>

$$F_{t_0}(\eta) = \left[ \frac{1}{\Gamma(t_0 + 1)} \right] \int_0^\infty y^{t_0} [1 + \exp(y - \eta)]^{-1} dy, \quad (35)$$

$y > -1,$

where  $\Gamma(t_0 + 1)$  is the complete Gamma function or for all  $t$ , analytically continued as a complex contour integral around the negative axis

$$F_{t_0}(\eta) = A_{t_0} \int_{-\infty}^{(0+)} y^{t_0} [1 + \exp(-y - \eta)]^{-1} dy, \quad (36)$$

in which  $A_{t_0} = \Gamma(-t_0) / 2\pi\sqrt{-1}$ .

(iv) For relatively wide gap materials  $E_g \rightarrow \infty$  and Eqs. (33) and (34) assume the forms<sup>30</sup>

$$n_0 = N_C F_{1/2}(\eta) \quad (37)$$

and

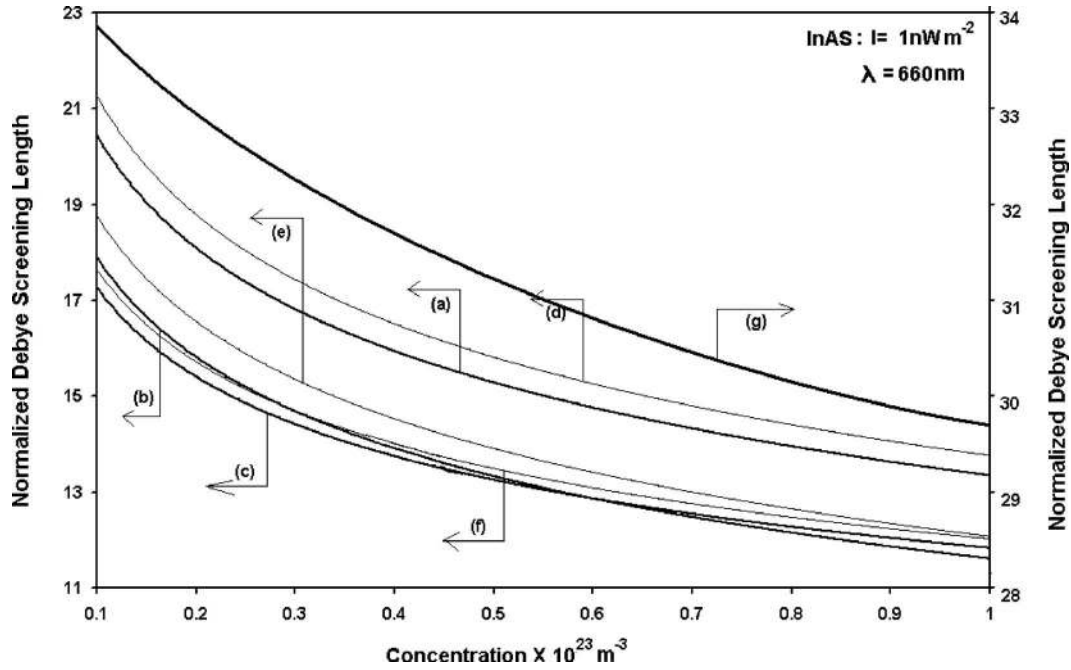


FIG. 1. Plot of the DSL as a function of electron concentration for  $n$ -InAs in the presence of light waves in which curves (a), (b), and (c) represent the three and two band models of Kane and that of the parabolic energy bands, respectively. Curves (d), (e), and (f) represent the same in the absence of external photoexcitation. Plot (g) indicates the classical DSL equation.

$$L_D = \left[ \frac{e^2 N_c}{\epsilon_{sc} k_B T} [F_{-1/2}(\eta)] \right]^{-1/2}. \quad (38)$$

Under the condition of nondegeneracy, Eq. (38) gets simplified to the classical DSL equation.

#### D. Suggestion for the experimental determination of the Debye screening length in materials having arbitrary dispersion laws

It is well known that the thermoelectric power of the electrons in materials in the presence of a classically large magnetic field is independent of the scattering mechanism and is determined only by the dispersion law.<sup>31</sup> The magnitude of the thermoelectric power  $G$  can be expressed as<sup>31</sup>

$$G = \frac{1}{eTn_0} \int_{-\infty}^{\infty} (E - E_F) R(E) \left[ -\frac{\partial f_0}{\partial E} \right] dE, \quad (39)$$

where  $R(E)$  is the total number of states. Following Tsidilkovski,<sup>32</sup> Eq. (43) can be written under the condition of carrier degeneracy as

$$G = \left( \frac{\pi^2 k_B^2 T}{3n_0} \right) \left( \frac{\partial n_0}{\partial E_F} \right). \quad (40)$$

The use of Eqs. (21) and (40) leads to the result

$$L_D = \left( \frac{\pi^2 k_B^2 T \epsilon_{sc}}{3e^3 n_0 G} \right)^{1/2}. \quad (41)$$

Thus, the DSL for degenerate materials can be determined by knowing the experimental values of  $G$ .

From the suggestion for the experimental determination of the DSL for degenerate materials having arbitrary dispersion laws as given by Eq. (41), we observe that for a constant  $T$ , the DSL varies inversely with the square root of  $Gn_0$ .

Only the experimental values of  $G$  for any material as a function of electron concentration will generate the experimental values of the DSL for that range of  $n_0$  for that material. Since  $(Gn_0)^{-1/2}$  decreases with increasing  $n_0$  for constant  $T$ , from Eq. (41) we can conclude that the DSL will decrease with increasing  $n_0$ . This statement provides a compatibility test of our theoretical analysis. Equation (41) provides an experimental check of the DSL and also a technique for probing the band structures of the materials having arbitrary band structures.

### III. RESULT AND DISCUSSIONS

Using the appropriate equations, we have plotted the DSL as a function of electron concentration at  $T=4.2$  K in Figs. 1–4 by using the values of the energy band constants as given in Table I for  $n$ -InAs,  $n$ -InSb,  $n$ -Hg<sub>1-x</sub>Cd<sub>x</sub>Te, and  $n$ -In<sub>1-x</sub>Ga<sub>x</sub>As<sub>y</sub>P<sub>1-y</sub> lattice matched to InP as examples of III–V, ternary, and quaternary materials, respectively, which are used for the purpose of numerical computations in accordance with three and two band models of Kane together with the parabolic energy bands. In Figs. 1–4, we have included the plot the classical DSL equation for the purpose of fixing the reference. From Figs. 1–4, we observe that the DSL decreases from the light off case to the light on case, since the value of the Fermi energy in the presence of light waves becomes larger due to the increase in the carrier concentration as compared with the same in the absence of photoexcitation. Therefore, the numerical magnitude of the DSL in the presence of light is smaller as compared with the same in the light off case in the whole range of the concentration considered, although the DSL decreases with increase in carrier degeneracy.

The combined influence of the energy band constants on

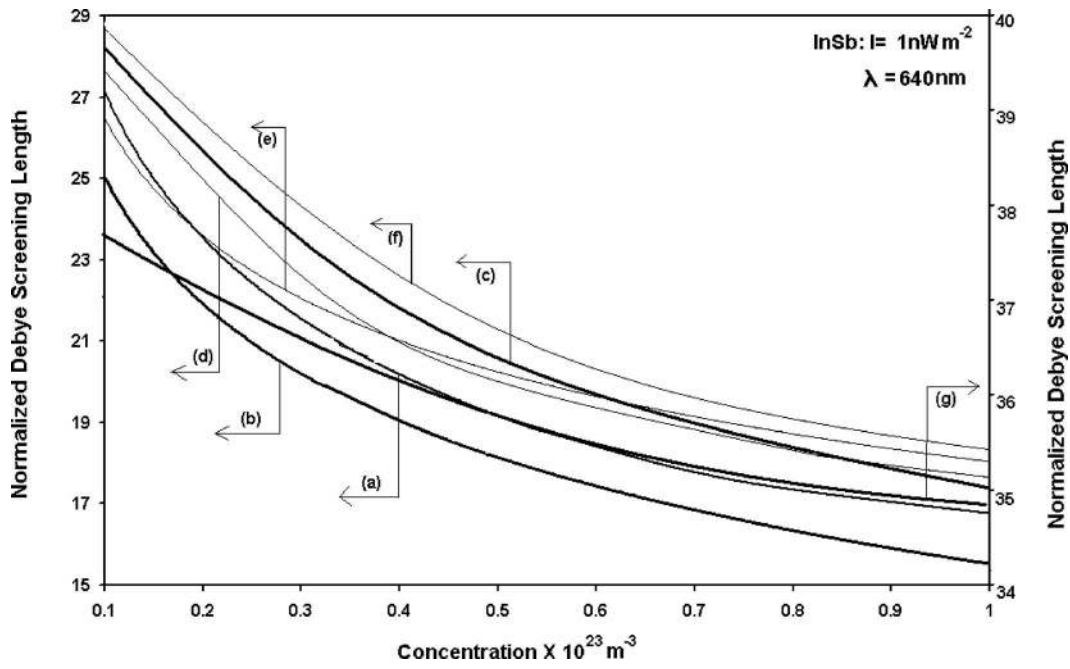


FIG. 2. Plot of the DSL as a function of electron concentration for *n*-InSb for all cases of Fig. 1.

the DSL for *n*-InAs and *n*-InSb can easily be assessed from Figs. 1 and 2. For the purpose of relative assessment, all the plots in the absence of light waves have further been drawn. In Figs. 5–8, we have plotted the DSL as a function of light intensity and we observe that the DSL decreases with increasing light intensity for the materials, whereas in the absence of external photoexcitation the DSL is independent of intensity. In Figs. 9–12, we have plotted the DSL as a function of wavelengths in the visible region for *n*-InAs, *n*-Hg<sub>1-x</sub>Cd<sub>x</sub>Te, and *n*-In<sub>1-x</sub>Ga<sub>x</sub>As<sub>y</sub>P<sub>1-y</sub> lattice matched to InP, respectively, for all types of energy band models. The DSL decreases as the wavelength shifts from red color to violet. In

Figs. 13 and 14, the DSL has been plotted as a function of alloy composition for *n*-Hg<sub>1-x</sub>Cd<sub>x</sub>Te and *n*-In<sub>1-x</sub>Ga<sub>x</sub>As<sub>y</sub>P<sub>1-y</sub> lattice matched to InP, respectively, in which all the cases of Fig. 1 have further been plotted for the purpose of relative comparison. The plots of the Fig. 13 are valid for *x* > 0.17, since for *x* < 0.17, the band gap becomes negative in *n*-Hg<sub>1-x</sub>Cd<sub>x</sub>Te leading to a semimetallic state. The plots of Fig. 14 exhibit the variation of the DSL with the alloy composition for *n*-In<sub>1-x</sub>Ga<sub>x</sub>As<sub>y</sub>P<sub>1-y</sub> lattice matched to InP. In Figs. 5–14, we have further included the classical DSL equation for the purpose of fixation of the reference.

The influence of light is immediately apparent from the

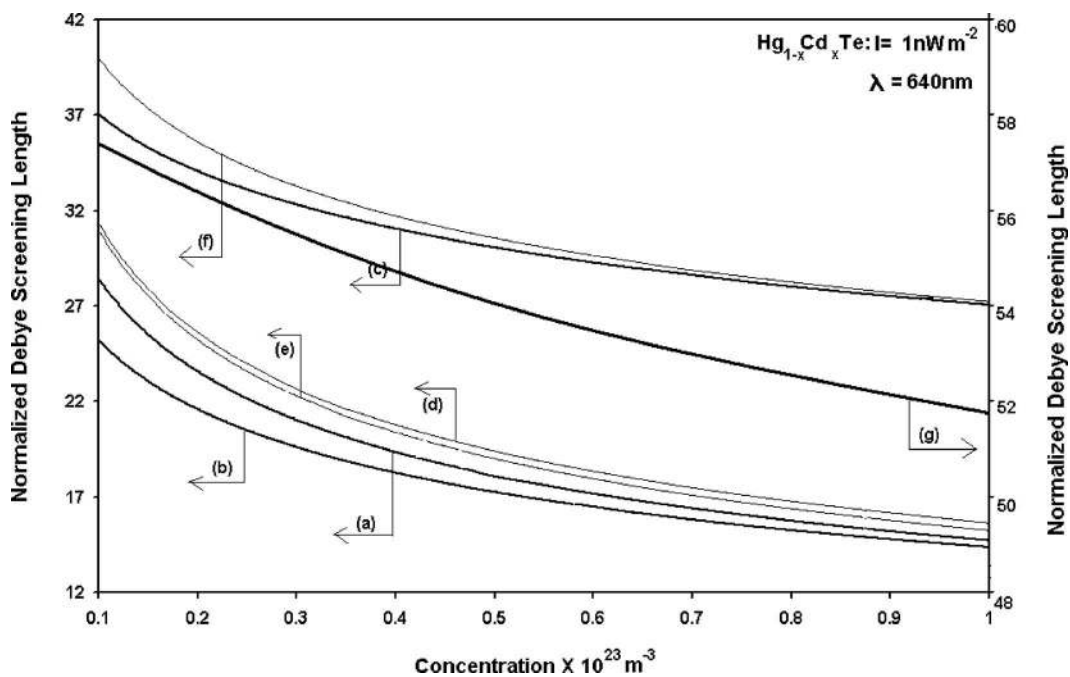


FIG. 3. Plot of the DSL as a function of electron concentration for *n*-Hg<sub>1-x</sub>Cd<sub>x</sub>Te for all cases of Fig. 1.



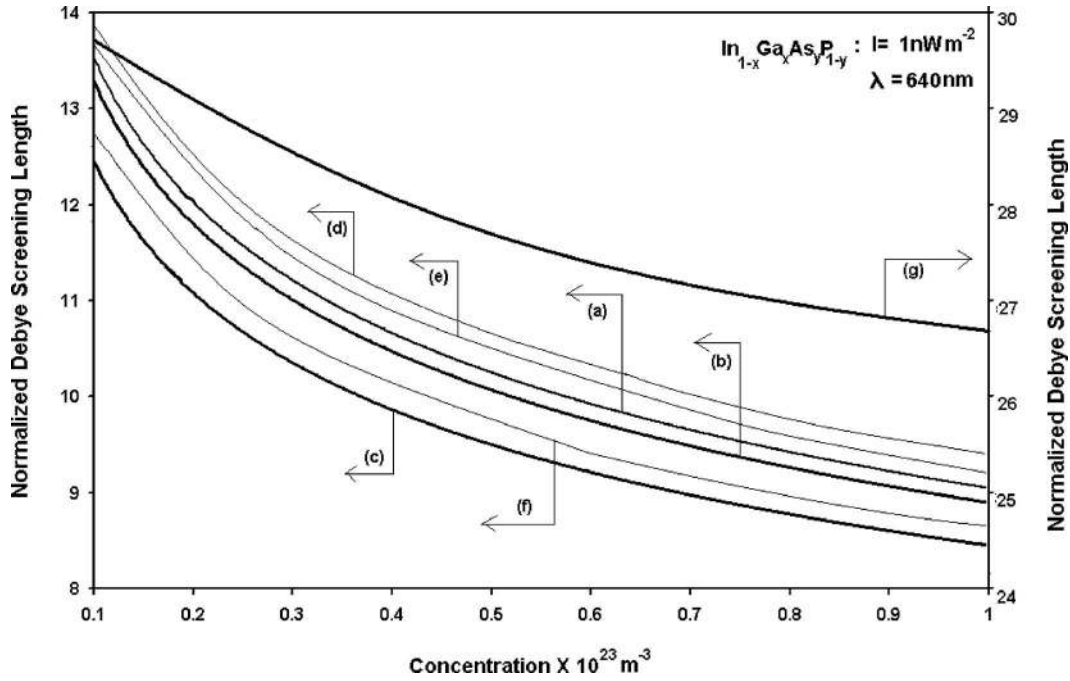


FIG. 4. Plot of the DSL as a function of electron concentration for  $n\text{-In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$  lattice matched to InP for all cases of Fig. 1.

plots in Figs. 5–12 since the DSL depends strongly on  $I$  and  $\lambda$  which is in direct contrast as compared with the corresponding cases for the bulk specimens of the said compounds in the absence of external photoexcitation. The variations of the DSLs in Figs. 5–12 reflect the direct signature of the light wave on the electronic, optic and the other band structure dependent properties of semiconducting materials in the presence of light waves and the photon assisted transport for the corresponding semiconductor devices. Although the DSL tends to decrease with the intensity and the wavelength but the rate of decrease is totally band structure de-

pendent. The numerical values of the DSL are greatest for ternary materials and least for quaternary compounds.

We note that our basic Eq. (24) covers various materials having different energy band structures. In this article, the concentration, the alloy composition, the light intensity, and the wavelength dependences of the DSL for  $n\text{-InAs}$ ,  $n\text{-InSb}$ ,  $n\text{-Hg}_{1-x}\text{Cd}_x\text{Te}$ , and  $n\text{-In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$  lattice matched to InP have been studied. Thus, we have covered a wide class of optoelectronic and allied compounds whose energy band structures are defined by the three and two band models of Kane in the absence of photon field. Under certain limiting conditions, all the results of the DSLs for different materials having various band structures lead to the well-known classical DSL equation. This indirect test not only exhibits the mathematical compatibility of our formulation but also shows the fact that our simple analysis is a more generalized one, since one can obtain the corresponding results for the relatively wide gap nondegenerate materials having parabolic energy bands under certain limiting conditions from our present derivation. Our experimental suggestion for the determination of the screening length is valid for materials having arbitrary dispersion relations. Since the experimental curves of  $n_0$  versus  $G$  are not available in the literature to the best of our knowledge for the present generalized systems, we cannot compare our theoretical formulation with the proposed experiment although, the generalized analysis as presented in this context can be checked when the experimental investigation of  $G$  for the present system would appear in the literature.

We can conclude that the influence of the presence of an external photoexcitation is to change radically the original band structure of the material. Because of this change, the photon field causes to increase the band gap of semiconductor. Our method is not at all related to the density of states (DOS) technique as used in the literature.<sup>34</sup> From the **E-k**

TABLE I. Values of the energy band constants (Refs. 30, 33, and 35–37).

(a) InAs, (b) InSb, (c)  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$ :

$$E_g = [-0.302 + 1.93x + 5.35(10)^{-4}(1-2x)T - 0.810x^2 + 0.832x^3] \text{ eV, valid from } x=0 \text{ to } x=0.6 \text{ and } T=4 \text{ K to } T=300 \text{ K,}$$

$$m_c = \{[3\hbar^2 E_{g0}(x, T)]/[4P^2(x)]\}, P^2(x) = [(\hbar^2/2m_0)(18+3x)]^a \text{ and } \Delta = (0.63 + 0.24x - 0.27x^2) \text{ eV}$$

$$\epsilon_s(x) = [20.262 - 14.812x + 5.2795x^2] \epsilon_0^b \text{ and (d) } \text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y} \text{ lattice matched to InP: } E_g = [1.337 - 0.73y + 0.13y^2] \text{ eV, } \Delta = (0.114 + 0.26y - 0.22y^2) \text{ eV, and } m_c = (0.080 - 0.039y)m_0:$$

(a) InAs:  $E_g = 0.418 \text{ eV}$ ,  $\Delta = 0.38 \text{ eV}$ ,  $m_c = 0.026m_0$ ,  $m_v = 0.45m_0$  and  $T = 4.2 \text{ K}$ ,  $\epsilon_{sc} = 12.25\epsilon_0^c$

(b) InSb:  $E_g = 0.235 \text{ eV}$ ,  $\Delta = 0.803 \text{ eV}$ ,  $m_c = 0.0145m_0$ ,  $m_v = 0.43m_0$ ,  $T = 4.2 \text{ K}$ ,  $\epsilon_{sc} = 15.68\epsilon_0^c$

(c)  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$ :  $E_g = 0.0596042 \text{ eV}$ ,  $\Delta = 0.6672 \text{ eV}$ ,  $m_c = 0.00596042m_0$ ,  $x = 0.2$ ,  $m_v = 0.4m_0$ ,  $T = 4.2 \text{ K}$ ,  $\epsilon_{sc} = 12.642\epsilon_0^c$

(d)  $\text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$  lattice matched to InP:  $E_g = 1.0045 \text{ eV}$ ,  $\Delta = 0.189 \text{ eV}$ ,  $m_c = 0.0605m_0$ ,  $y = 0.5$ ,  $m_v = 0.724m_0$ ,  $T = 4.2 \text{ K}$ ,  $\epsilon_{sc} = 11.82\epsilon_0^d$

<sup>a</sup>Reference 35.

<sup>b</sup>Reference 36.

<sup>c</sup>Reference 30.

<sup>d</sup>Reference 37.

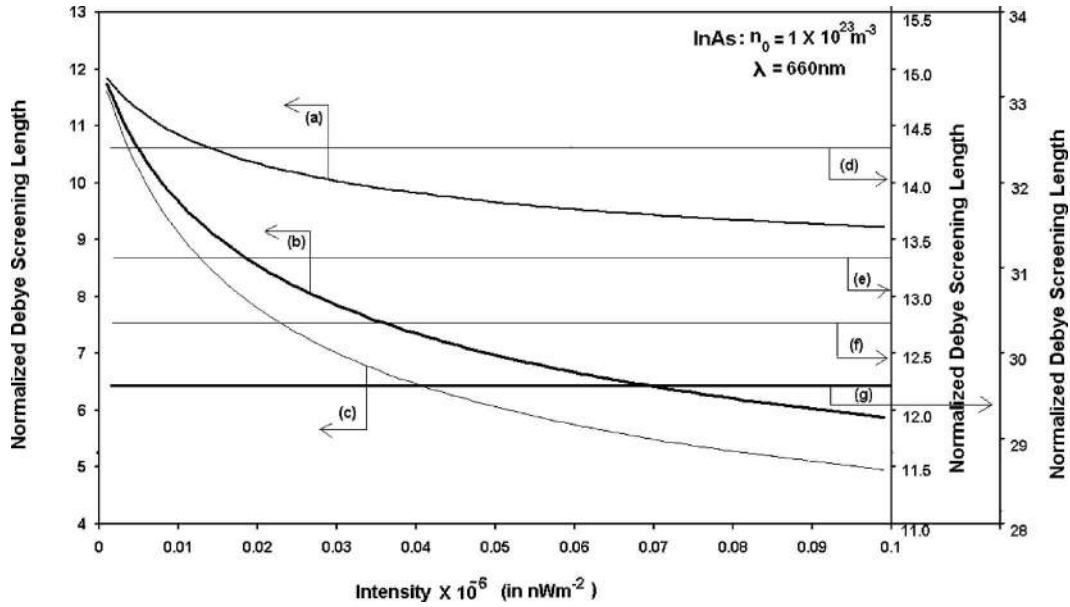


FIG. 5. Plot of the DSL as a function of light intensity for *n*-InAs in which curves (a), (b), and (c) represent the three and two band models of Kane and that of parabolic energy bands, respectively. Curves (d), (e), and (f) represent the same in the absence of external photoexcitation. Plot (g) indicates the classical DSL equation.

dispersion relation, we can obtain the DOS, but the DOS technique as used in the literature<sup>34</sup> cannot provide the **E-k** dispersion relation. Therefore, our study is more fundamental than those of the existing literature because the Boltzmann transport equation, which controls the study of the charge transport properties of semiconductor devices, can be solved if and only if the **E-k** dispersion relation is known. We wish to note that we have not considered the many body effects in this simplified theoretical formalism due to the lack of availability in the literature of proper analytical techniques for including them for the generalized systems as considered in this article. Our simplified approach will be useful for the purpose of comparison when methods of tackling the formidable problem after inclusion of the many body effects for

the present generalized systems appear. It is worth remarking in this context that from our simple theory under certain limiting conditions we get the well-known result of the DSL for wide gap materials having parabolic energy bands. The inclusion of the said effects would certainly increase the accuracy of the results, although the qualitative features of the DSL discussed in this article would not change in the presence of the aforementioned effect. Besides, our suggestion for the experimental determination of the DSL is independent of the inclusion of the said effect. We have not considered other types of optoelectronic materials and other external variables in order to keep the presentation brief. The influence of energy band models and the various band con-

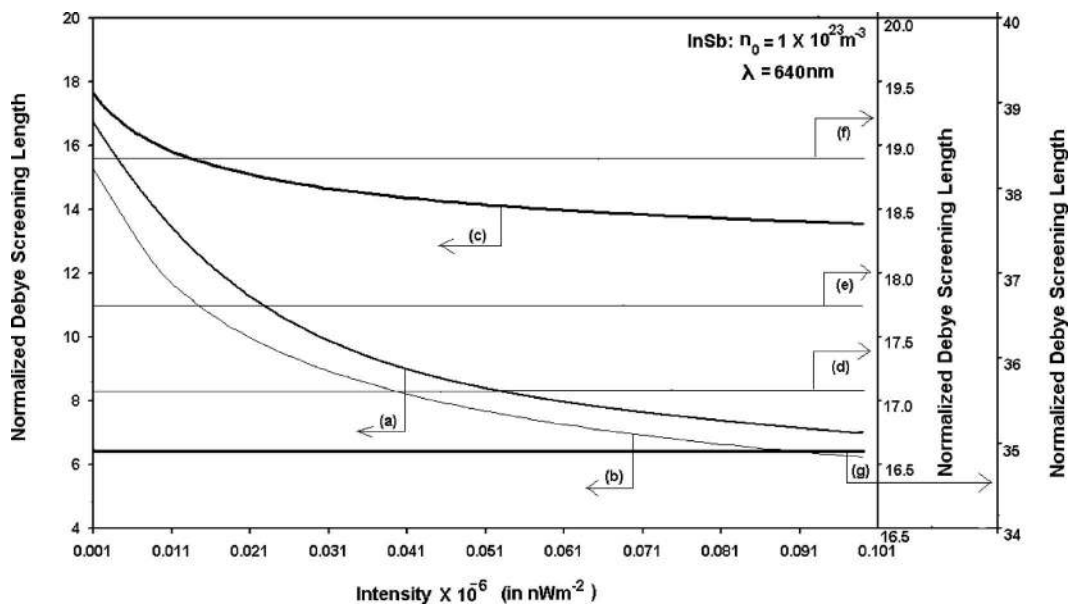


FIG. 6. Plot of the DSL as a function of light intensity for *n*-InSb for all cases of Fig. 5.

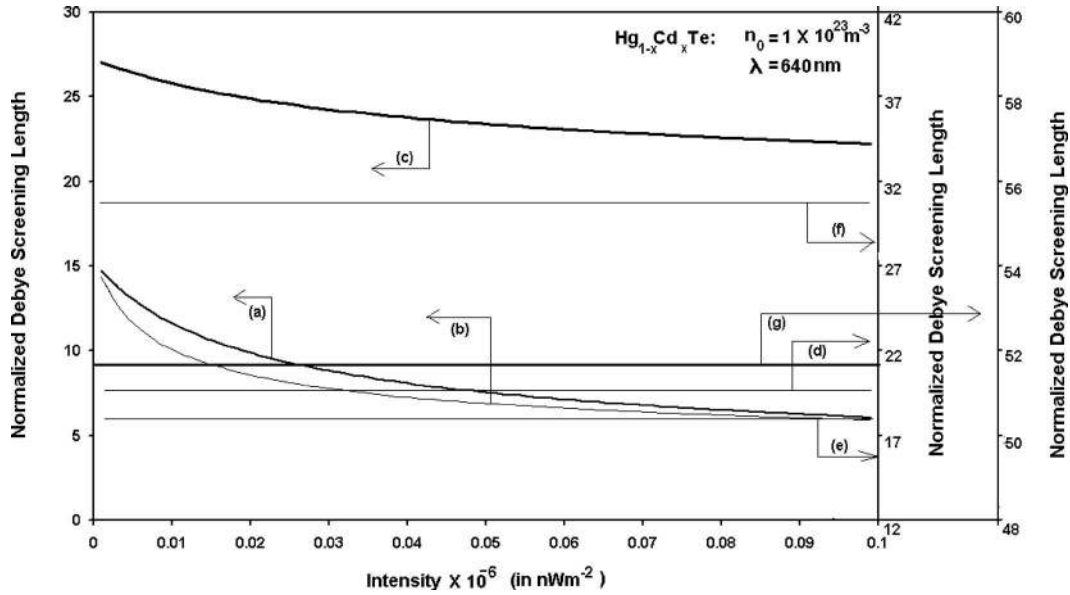


FIG. 7. Plot of the DSL as a function of light intensity for  $n\text{-Hg}_{1-x}\text{Cd}_x\text{Te}$  for all cases of Fig. 5.

stants on the DSL for different materials can also be studied from all the figures of this article.

The numerical results presented in this article would be different for other materials but the nature of variation would be unaltered. The theoretical results as given here would be useful in analyzing various other experimental data related to this phenomenon. Finally, we can write that this theory can be used to investigate the Einstein relation, the Burstein Moss shift, the effective electron mass, the carrier contribution to the elastic constants, the specific heat, and other different transport coefficients of modern semiconductor devices operated under the influence of external photon field in addition to the suggestion for the experimental determination of the DSL for materials having arbitrary carrier energy spectra.

**APPENDIX**

The doubly degenerate wave functions  $u_1(\mathbf{k}, \mathbf{r})$  and  $u_2(\mathbf{k}, \mathbf{r})$  can be expressed as<sup>30</sup>

$$u_1(\mathbf{k}, \mathbf{r}) = a_{k+}[(is)\downarrow'] + b_{k+} \left[ \frac{X' - iY'}{\sqrt{2}} \uparrow' \right] + c_{k+}[Z'\downarrow, ] \tag{A1}$$

and

$$u_2(\mathbf{k}, \mathbf{r}) = a_{k-}[(is)\uparrow'] - b_{k-} \left[ \frac{X' + iY'}{\sqrt{2}} \downarrow' \right] + c_{k-}[Z'\uparrow, ], \tag{A2}$$

where  $s$  is the  $s$ -type atomic orbital in both unprimed and primed coordinates,  $\downarrow'$  indicates the spin down function in the primed coordinates

$$a_{k\pm} \equiv \beta \{ [E_{g_0} - (\gamma_{0k\pm})^2(E_{g_0} - \delta')]^{1/2} (E_{g_0} + \delta')^{-1/2} \},$$

$$\beta \equiv \{ [6(E_{g_0} + 2\Delta/3)(E_{g_0} + \Delta)] / \chi \}^{1/2},$$

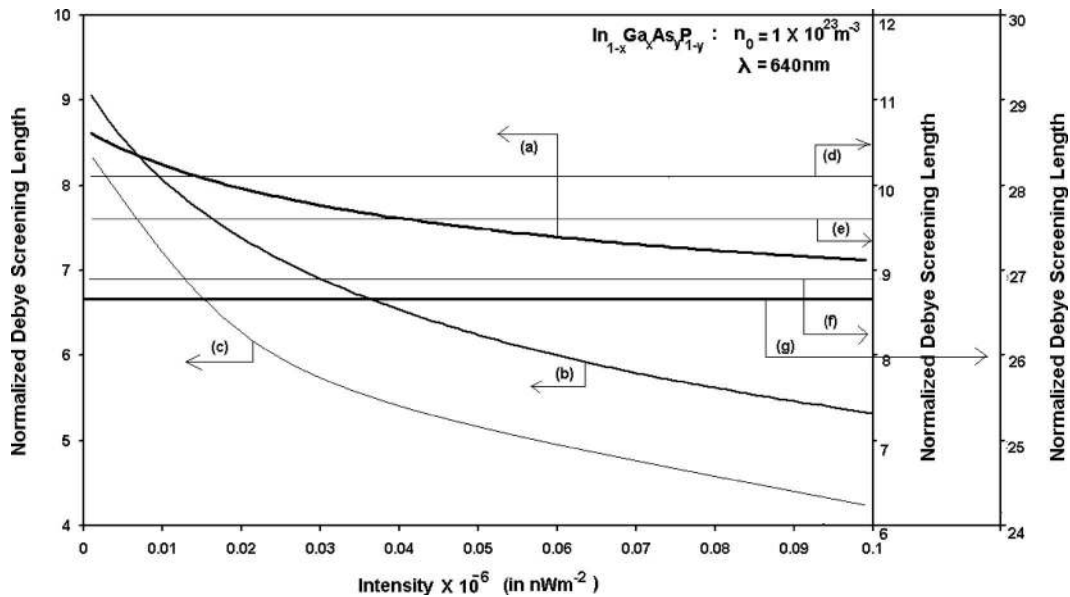


FIG. 8. Plot of the DSL as a function of light intensity for  $n\text{-In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$  lattice matched to InP for all cases of Fig. 5.

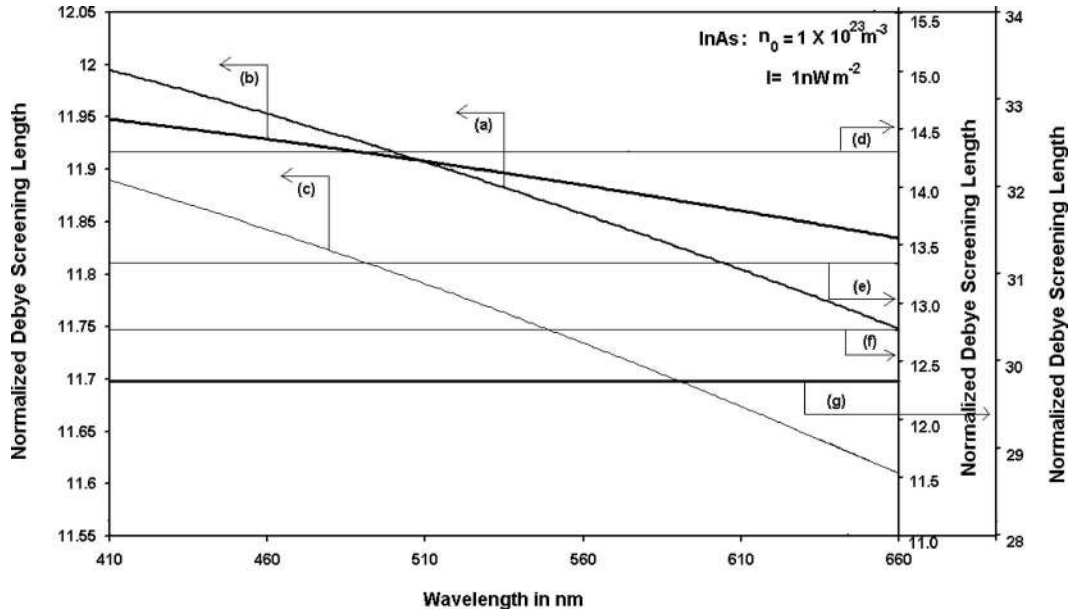


FIG. 9. Plot of the DSL as a function of wavelength for  $n$ -InAs in which curves (a), (b), and (c) represent the three and two band models of Kane and that of parabolic energy bands, respectively. Curves (d), (e), and (f) represent the same in the absence of external photoexcitation. Plot (g) indicates the classical DSL equation.

$$\chi \equiv (6E_{g_0}^2 + 9E_{g_0} \Delta + 4\Delta^2),$$

$$\gamma_{0k\pm} \equiv \left[ \frac{(\xi_{1k} \mp E_{g_0})}{2(\xi_{1k} + \delta')} \right]^{1/2},$$

$$\xi_{1k} = E_c(\mathbf{k}) - E_v(\mathbf{k}) = E_{g_0} \left[ 1 + 2 \left( 1 + \frac{m_c}{m_v} \right) \frac{\gamma(E)}{E_{g_0}} \right]^{1/2},$$

$\delta' = (E_{g_0}^2 \Delta)(\chi)^{-1}$ ,  $X'$ ,  $Y'$ , and  $Z'$  are the  $p$ -type atomic orbitals in the primed coordinates,  $\uparrow'$  indicates the spin-up function in the primed coordinates,  $b_{k\pm} \equiv \rho \gamma_{0k\pm}$ ,  $\rho \equiv (4\Delta^2/3\chi)^{1/2}$ ,  $c_{k\pm} \equiv t \gamma_{0k\pm}$ , and  $t \equiv [6(E_{g_0} + 2\Delta/3)^2/\chi]^{1/2}$ .

We can, therefore, write the expression for the OME as

$$\text{OME} = \bar{p}_{cv}(\mathbf{k}) = \langle u_1(\mathbf{k}, \mathbf{r}) | \bar{p} | u_2(\mathbf{k}, \mathbf{r}) \rangle. \tag{A3}$$

Since the photon vector has no interaction in the same band for the study of interband optical transition, we can therefore write

$$\langle S | \bar{p} | S \rangle = \langle X | \bar{p} | X \rangle = \langle Y | \bar{p} | Y \rangle = \langle Z | \bar{p} | Z \rangle = 0$$

and

$$\langle X | \bar{p} | Y \rangle = \langle Y | \bar{p} | Z \rangle = \langle Z | \bar{p} | X \rangle = 0.$$

There are finite interactions between the conduction band and the valance band and we can obtain

$$\langle S | P | X \rangle = \hat{i} \cdot \mathbf{P} = \hat{i} \cdot \mathbf{P}_x,$$

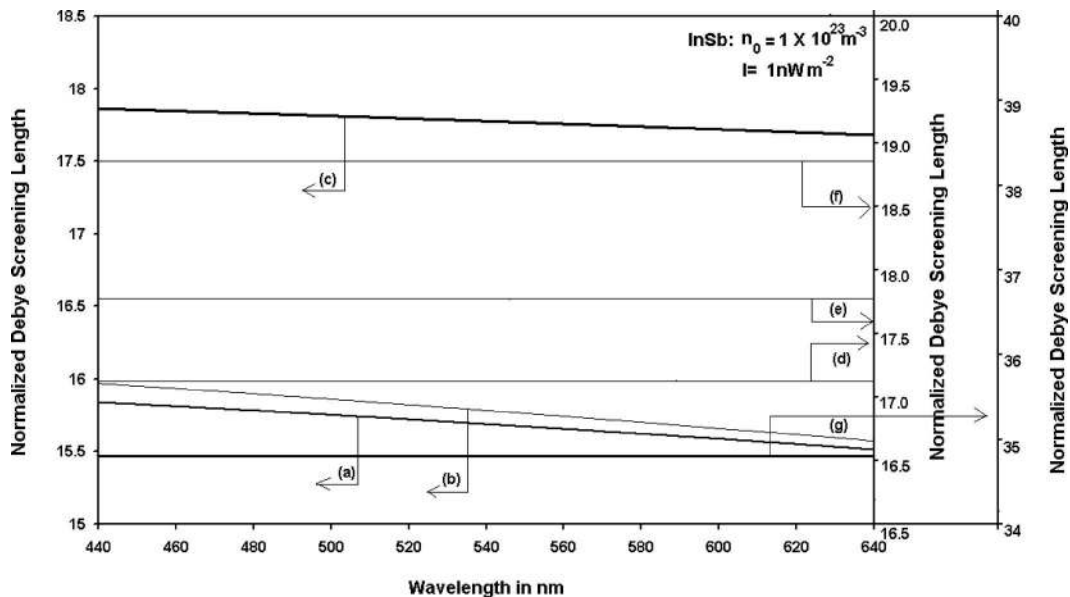


FIG. 10. Plot of the DSL as a function of wavelength for  $n$ -InSb for all cases of Fig. 9.



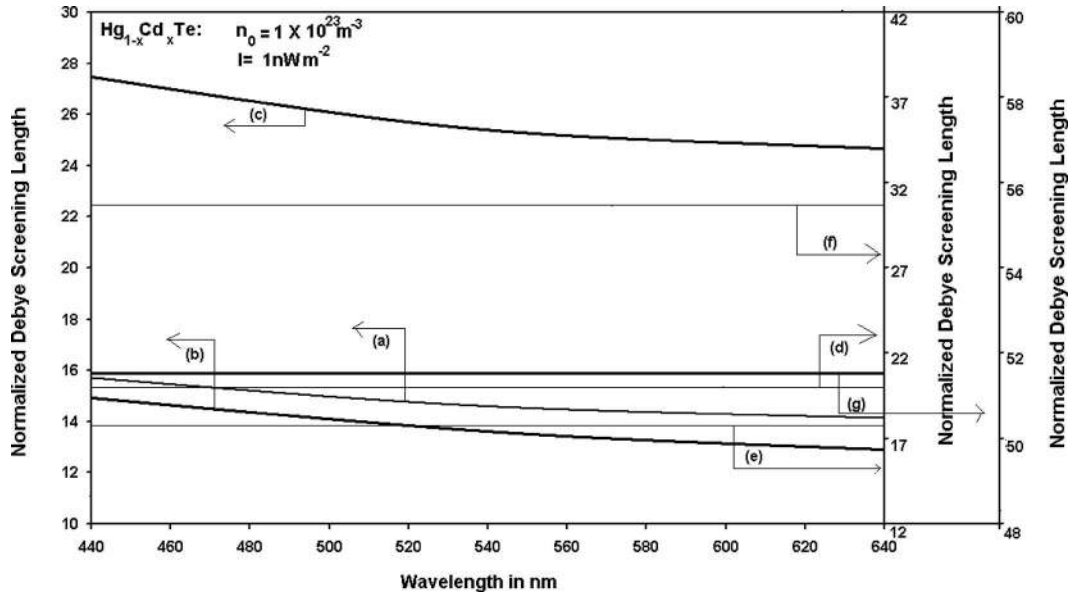


FIG. 11. Plot of the DSL as a function of wavelength for  $n\text{-Hg}_{1-x}\text{Cd}_x\text{Te}$  for all cases of Fig. 9.

$$\langle S|P|Y \rangle = \hat{j} \cdot \mathbf{P} = \hat{j} \cdot \mathbf{P}_y,$$

$$\langle S|P|Z \rangle = \hat{k} \cdot \mathbf{P} = \hat{k} \cdot \mathbf{P}_z,$$

where  $\hat{i}$ ,  $\hat{j}$ , and  $\hat{k}$  are the unit vectors along  $x$ ,  $y$ , and  $z$  axes, respectively.

It is well known that

$$\begin{bmatrix} \uparrow' \\ \downarrow' \end{bmatrix} = \begin{bmatrix} e^{-i\phi/2} \cos \theta/2 & e^{i\phi/2} \sin \theta/2 \\ -e^{-i\phi/2} \sin \theta/2 & e^{i\phi/2} \cos \theta/2 \end{bmatrix} \begin{bmatrix} \uparrow \\ \downarrow \end{bmatrix},$$

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \phi & \cos \phi & 0 \\ \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}.$$

Besides, the spin vector can be written as

$$\mathbf{S} = \frac{\hbar}{2} \boldsymbol{\sigma},$$

where

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

$$\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix},$$

$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$

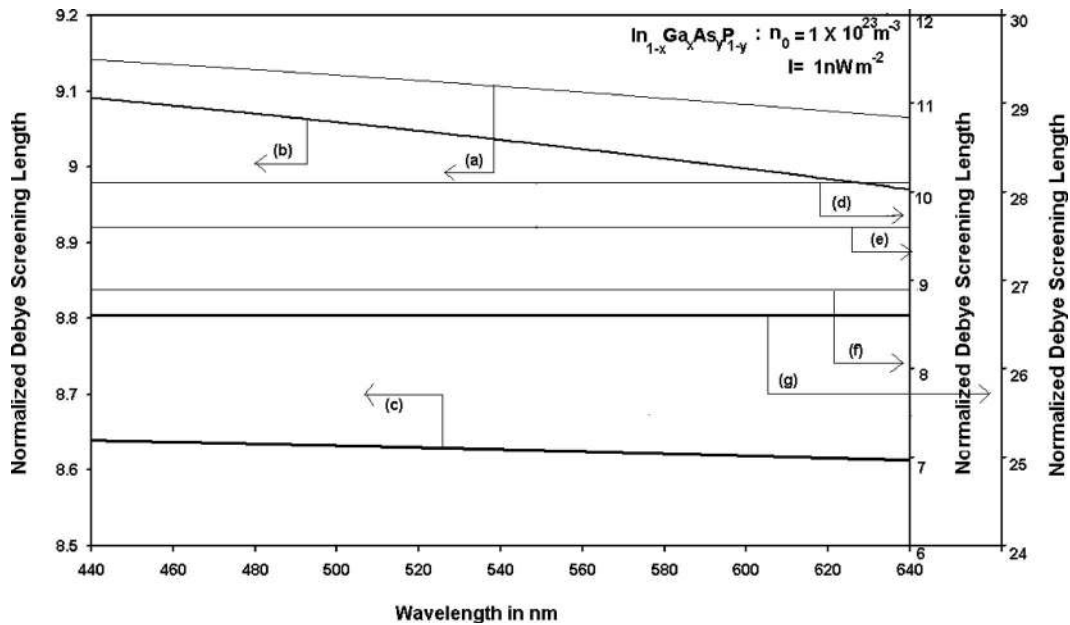


FIG. 12. Plot of the DSL as a function of wavelength for  $n\text{-In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$  lattice matched to InP for all cases of Fig. 9.

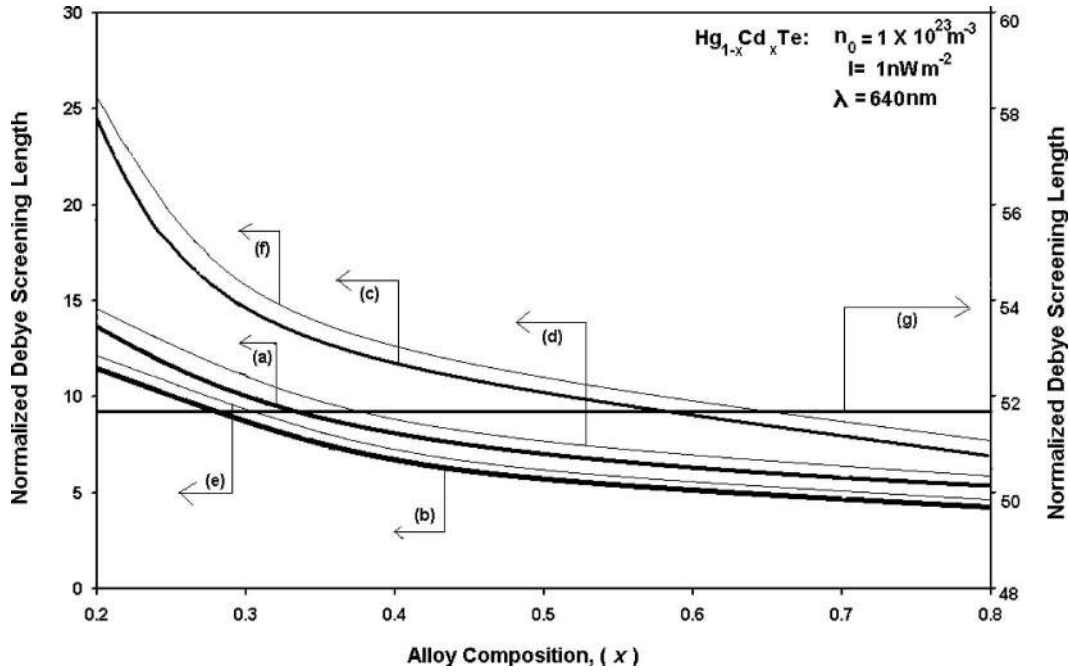


FIG. 13. Plot of the DSL as a function of alloy composition for  $n$ - $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  in the presence of light waves in which curves (a), (b), and (c) represent the three and two band models of Kane and that of parabolic energy band, respectively. Curves (d), (e), and (f) represent the same in the absence of external photoexcitation. Plot (g) indicates the classical DSL equation.

$$i = \sqrt{-1}.$$

From earlier, we can write

$$P_{CV}(\mathbf{k}) = \langle u_1(\mathbf{k}, \mathbf{r}) | P | u_2(\mathbf{k}, \mathbf{r}) \rangle = \left\langle \left\{ a_{k_+} [(iS) \downarrow'] \right. \right. \\ \left. \left. + b_{k_+} \left[ \left( \frac{X' - iY'}{\sqrt{2}} \right) \uparrow' \right] + c_{k_+} [Z' \downarrow'] \right\} | P \right. \\ \left. \times \left\{ a_{k_-} [(iS) \uparrow'] - b_{k_-} \left[ \left( \frac{X' + iY'}{\sqrt{2}} \right) \downarrow' \right] \right. \right. \\ \left. \left. + c_{k_-} [Z' \uparrow'] \right\} \right\rangle.$$

Using the earlier relations, we get

$$P_{CV}(\mathbf{k}) = \langle u_1(\mathbf{k}, \mathbf{r}) | P | u_2(\mathbf{k}, \mathbf{r}) \rangle = \frac{b_{k_+} a_{k_-}}{\sqrt{2}} \{ \langle (X' - iY') | P | iS \rangle \\ \times \langle \uparrow' | \uparrow' \rangle \} + c_{k_+} a_{k_-} \{ \langle Z' | P | iS \rangle \langle \downarrow' | \uparrow' \rangle \} \\ - \frac{a_{k_+} b_{k_-}}{\sqrt{2}} \{ \langle iS | P | (X' + iY') \rangle \langle \downarrow' | \downarrow' \rangle \} \\ + a_{k_+} c_{k_-} \{ \langle iS | P | Z' \rangle \langle \downarrow' | \uparrow' \rangle \}. \quad (\text{A4})$$

From Eq. (A4), we can write

$$\langle (X' - iY') | P | iS \rangle = \langle (X') | P | iS \rangle - \langle (iY') | P | iS \rangle \\ = \int u_X^* P S - \int -i u_Y^* P i u_X \\ = i \langle X' | P | S \rangle - \langle Y' | P | S \rangle.$$

From the earlier relations, for  $X'$ ,  $Y'$ , and  $Z'$ , we get

$$|X'\rangle = \cos \theta \cos \phi |X\rangle + \cos \theta \sin \phi |Y\rangle - \sin \theta |Z\rangle.$$

Thus,

$$\langle X' | P | S \rangle = \cos \theta \cos \phi \langle X | P | S \rangle + \cos \theta \sin \phi \langle Y | P | S \rangle \\ - \sin \theta \langle Z | P | S \rangle = P \hat{r}_1,$$

where  $\hat{r}_1 = \hat{i} \cos \theta \cos \phi + \hat{j} \cos \theta \sin \phi - \hat{k} \sin \theta$ .

Similarly, we obtain

$$|Y'\rangle = -\sin \phi |X\rangle + \cos \phi |Y\rangle + 0 |Z\rangle.$$

Thus,

$$\langle Y' | P | S \rangle = -\sin \phi \langle X | P | S \rangle + \cos \phi \langle Y | P | S \rangle + 0 \langle Z | P | S \rangle \\ = P \hat{r}_2,$$

where  $\hat{r}_2 = -\hat{i} \sin \phi + \hat{j} \cos \phi$  so that  $\langle (X' - iY') | P | S \rangle = P(i\hat{r}_1 - \hat{r}_2)$

Thus,

$$\frac{a_{k_+} b_{k_-}}{\sqrt{2}} \langle (X' - iY') | P | S \rangle \langle \downarrow' | \uparrow' \rangle = \frac{a_{k_+} b_{k_-}}{\sqrt{2}} P(i\hat{r}_1 - \hat{r}_2) \\ \times \langle \downarrow' | \uparrow' \rangle. \quad (\text{A5})$$

Now since

$$\langle iS | P | (X' + iY') \rangle = i \langle S | P | X' \rangle - \langle S | P | Y' \rangle = P(i\hat{r}_1 - \hat{r}_2),$$

we can write

$$-\frac{a_{k_+} b_{k_-}}{\sqrt{2}} \{ \langle iS | P | (X' + iY') \rangle \langle \downarrow' | \downarrow' \rangle \} = -\frac{a_{k_+} b_{k_-}}{\sqrt{2}} P(i\hat{r}_1 - \hat{r}_2) \\ \times \langle \downarrow' | \downarrow' \rangle. \quad (\text{A6})$$

Similarly, we get

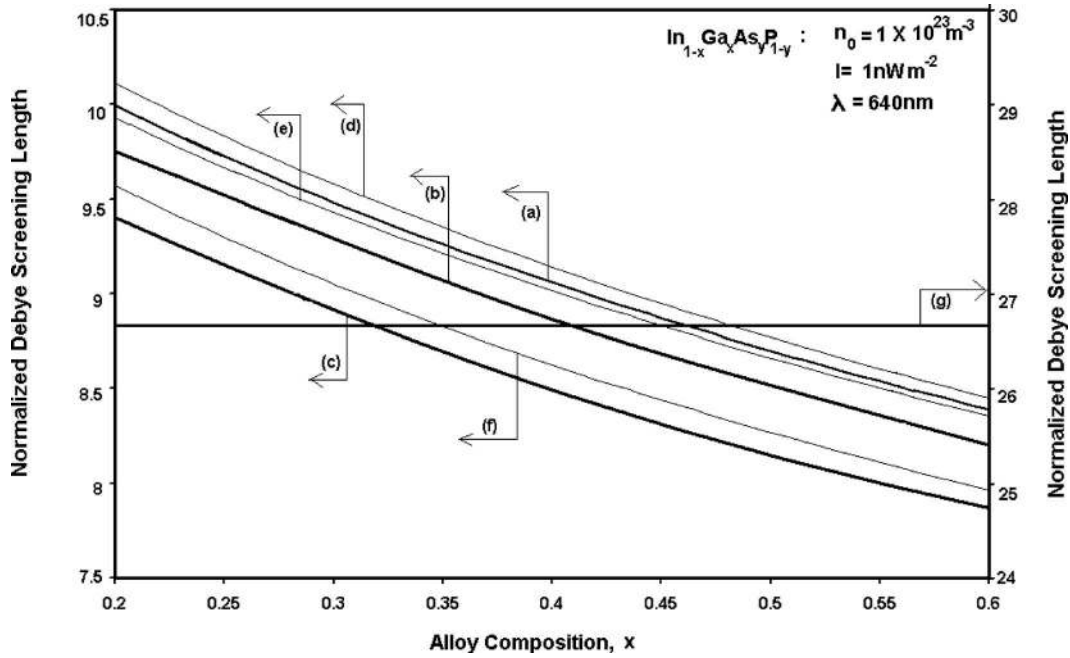


FIG. 14. Plot of the DSL as a function of alloy composition for  $n$ - $\text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$  lattice matched to InP for all cases of Fig. 13.

$$\langle Z' \rangle = \sin \theta \cos \phi \langle X \rangle + \sin \theta \sin \phi \langle Y \rangle + \cos \theta \langle Z \rangle,$$

so that

$$\begin{aligned} \langle Z' | P | iS \rangle &= i \langle Z' | P | S \rangle = iP \{ \sin \theta \cos \phi \hat{i} + \sin \theta \sin \phi \hat{j} \\ &+ \cos \theta \hat{k} \} = iP \hat{r}_3, \end{aligned}$$

where  $\hat{r}_3 = \hat{i} \sin \theta \cos \phi + \hat{j} \sin \theta \sin \phi + \hat{k} \cos \theta$ .

Thus,

$$c_{k_+} a_{k_-} \langle Z' | P | iS \rangle \langle \downarrow' | \uparrow' \rangle = c_{k_+} a_{k_-} iP \hat{r}_3 \langle \downarrow' | \uparrow' \rangle. \quad (\text{A7})$$

Similarly, we can write

$$c_{k_-} a_{k_+} \langle iS | P | Z' \rangle \langle \downarrow' | \uparrow' \rangle = c_{k_-} a_{k_+} iP \hat{r}_3 \langle \downarrow' | \uparrow' \rangle. \quad (\text{A8})$$

Therefore, we obtain

$$\begin{aligned} \frac{a_{k_-} b_{k_+}}{\sqrt{2}} \{ \langle (X' - iY') | P | S \rangle \langle \uparrow' | \uparrow' \rangle \} - \frac{a_{k_+} b_{k_-}}{\sqrt{2}} \{ \langle iS | P | (X' \\ + iY') \rangle \langle \downarrow' | \downarrow' \rangle \} &= \frac{P}{\sqrt{2}} (-a_{k_+} b_{k_-} \langle \downarrow' | \downarrow' \rangle + a_{k_-} b_{k_+} \langle \uparrow' | \uparrow' \rangle) \\ &\times (i\hat{r}_1 - \hat{r}_2). \end{aligned} \quad (\text{A9})$$

Also, we can write

$$\begin{aligned} c_{k_+} a_{k_-} \langle Z' | P | iS \rangle \langle \downarrow' | \uparrow' \rangle + c_{k_-} a_{k_+} \langle iS | P | Z' \rangle \langle \downarrow' | \uparrow' \rangle \\ = iP (c_{k_+} a_{k_-} + c_{k_-} a_{k_+}) \hat{r}_3 [ \langle \downarrow' | \downarrow' \rangle ]. \end{aligned} \quad (\text{A10})$$

Combining Eqs. (A9) and (A10), we find

$$\begin{aligned} P_{CV}(\mathbf{k}) &= \frac{P}{\sqrt{2}} (i\hat{r}_1 - \hat{r}_2) \{ (b_{k_+} a_{k_-}) \langle \uparrow' | \uparrow' \rangle - (b_{k_-} a_{k_+}) \langle \downarrow' | \downarrow' \rangle \} \\ &+ iP \hat{r}_3 (c_{k_+} a_{k_-} - c_{k_-} a_{k_+}) \langle \downarrow' | \uparrow' \rangle. \end{aligned} \quad (\text{A11})$$

From the earlier relations, we obtain

$$\uparrow' = e^{-i\phi/2} \cos(\theta/2) \uparrow + e^{i\phi/2} \sin(\theta/2) \downarrow,$$

$$\downarrow' = -e^{-i\phi/2} \sin(\theta/2) \uparrow + e^{i\phi/2} \cos(\theta/2) \downarrow. \quad (\text{A12})$$

Therefore,

$$\begin{aligned} \langle \downarrow' | \uparrow' \rangle &= [-e^{-i\phi/2} \sin(\theta/2) \uparrow + e^{i\phi/2} \cos(\theta/2) \downarrow]^* [ \\ &-e^{-i\phi/2} \cos(\theta/2) \uparrow + e^{i\phi/2} \sin(\theta/2) \downarrow] \\ &= [-e^{i\phi/2} \sin(\theta/2) \uparrow^* + e^{-i\phi/2} \cos(\theta/2) \downarrow^*] \\ &\times [-e^{-i\phi/2} \cos(\theta/2) \uparrow + e^{i\phi/2} \sin(\theta/2) \downarrow] \\ &= -\sin(\theta/2) \cos(\theta/2) \langle \uparrow | \uparrow \rangle + e^{-i\phi} \cos^2(\theta/2) \langle \downarrow | \downarrow \rangle \\ &- e^{i\phi} \sin^2(\theta/2) \langle \uparrow | \downarrow \rangle + \sin(\theta/2) \cos(\theta/2) \langle \downarrow | \downarrow \rangle. \end{aligned}$$

Therefore,

$$\begin{aligned} \langle \downarrow' | \uparrow' \rangle_x &= -\sin(\theta/2) \cos(\theta/2) \langle \uparrow | \uparrow \rangle_x + e^{-i\phi} \cos^2(\theta/2) \\ &\times \langle \downarrow | \downarrow \rangle_x - e^{i\phi} \sin^2(\theta/2) \langle \uparrow | \downarrow \rangle_x \\ &+ \sin(\theta/2) \cos(\theta/2) \langle \downarrow | \downarrow \rangle_x. \end{aligned} \quad (\text{A13})$$

But we know from before that

$$\langle \uparrow | \uparrow \rangle_x = 0,$$

$$\langle \downarrow | \uparrow \rangle = \frac{1}{2},$$

$$\langle \downarrow | \uparrow \rangle_x = \frac{1}{2},$$

$$\langle \downarrow | \downarrow \rangle_x = 0.$$

Thus, from Eq. (A13), we get

$$\begin{aligned} \langle \downarrow' | \uparrow' \rangle_x &= \frac{1}{2} [e^{-i\phi} \cos^2(\theta/2) - e^{i\phi} \sin^2(\theta/2)] = \frac{1}{2} [(\cos \phi \\ &- i \sin \phi) \cos^2(\theta/2) - (\cos \phi \\ &+ i \sin \phi) \sin^2(\theta/2)] = \frac{1}{2} [\cos \phi \cos \theta - i \sin \phi]. \end{aligned} \quad (\text{A14})$$

Similarly, we obtain

$$\langle \downarrow' | \uparrow' \rangle_y = \frac{1}{2} [i \cos \phi + \sin \phi \cos \theta]$$

and

$$\langle \downarrow' | \uparrow' \rangle_z = \frac{1}{2} [-\sin \theta].$$

Therefore,

$$\begin{aligned} \langle \downarrow' | \uparrow' \rangle &= \hat{i} \langle \downarrow' | \uparrow' \rangle_x + \hat{j} \langle \downarrow' | \uparrow' \rangle_y + \hat{k} \langle \downarrow' | \uparrow' \rangle_z \\ &= \frac{1}{2} \{ (\cos \theta \cos \phi - i \sin \phi) \hat{i} + (i \cos \phi \\ &+ \sin \phi \cos \theta) \hat{j} - \sin \theta \hat{k} \} = \frac{1}{2} \{ (\cos \theta \cos \phi) \hat{i} \\ &+ (\sin \phi \cos \theta) \hat{j} - \sin \theta \hat{k} \} + i \{ -\hat{i} \sin \phi \\ &+ \hat{j} \cos \phi \} = \frac{1}{2} [\hat{r}_1 + i \hat{r}_2] = -\frac{1}{2} i [\hat{r}_1 - \hat{r}_2]. \end{aligned}$$

Similarly, we can write

$$\langle \uparrow' | \uparrow' \rangle = \frac{1}{2} [i \sin \theta \cos \phi + \hat{j} \sin \theta \sin \phi + \hat{k} \cos \theta] = \frac{1}{2} \hat{r}_3,$$

$$\langle \downarrow' | \downarrow' \rangle = -\frac{1}{2} \hat{r}_3.$$

Using these, we can write [from Eq. (A11)],

$$\begin{aligned} P_{CV}(\mathbf{k}) &= \frac{P}{\sqrt{2}} (i \hat{r}_1 - \hat{r}_2) \{ (a_{k-} b_{k+}) \langle \uparrow' | \uparrow' \rangle - (b_{k-} a_{k+}) \langle \downarrow' | \downarrow' \rangle \} \\ &+ iP \hat{r}_3 \{ (c_{k+} a_{k-} - c_{k-} a_{k+}) \langle \downarrow' | \uparrow' \rangle \} = \frac{P}{2} \hat{r}_3 (i \hat{r}_1 - \hat{r}_2) \\ &\times \left\{ \left( \frac{a_{k-} b_{k+}}{\sqrt{2}} + \frac{b_{k-} a_{k+}}{\sqrt{2}} \right) \right\} + \frac{P}{2} \hat{r}_3 (i \hat{r}_1 - \hat{r}_2) \\ &\times \{ (c_{k+} a_{k-} + c_{k-} a_{k+}) \}. \end{aligned}$$

Thus,

$$\begin{aligned} P_{CV}(\mathbf{k}) &= \frac{P}{2} \hat{r}_3 (i \hat{r}_1 - \hat{r}_2) \left\{ a_{k+} \left( \frac{b_{k-}}{\sqrt{2}} + c_{k-} \right) + a_{k-} \left( \frac{b_{k+}}{\sqrt{2}} \right. \right. \\ &\left. \left. + c_{k+} \right) \right\}. \end{aligned} \quad (\text{A15})$$

We can write that

$$|\hat{r}_1| = |\hat{r}_2| = |\hat{r}_3| = 1,$$

$$P \hat{r}_3 = P_x \sin \theta \cos \phi \hat{i} + P_y \sin \theta \sin \phi \hat{j} + P_z \cos \theta \hat{k},$$

where

$$P = \langle S | P | X \rangle = \langle S | P | Y \rangle = \langle S | P | Z \rangle,$$

$$\langle S | P | X \rangle = \int u_C^*(0, \mathbf{r}) P u_{VX}(0, \mathbf{r}) d^3 r = P_{CVX}(0),$$

$$\langle S | P | Y \rangle = P_{CVY}(0),$$

$$\langle S | P | Z \rangle = P_{CVZ}(0).$$

Thus,

$$P = P_{CVX}(0) = P_{CVY}(0) = P_{CVZ}(0) = P_{CV}(0),$$

where  $P_{CV}(0) = \int u_C^*(0, \mathbf{r}) P u_V(0, \mathbf{r}) d^3 r = P$ .

For a plane polarized light wave, we have the polarization vector  $\boldsymbol{\varepsilon}_s = \hat{k}$ , when the light wave vector is traveling along the  $z$  axis. Therefore, for a plane polarized light wave, we have considered  $\boldsymbol{\varepsilon}_s = \hat{k}$ .

Then, from Eq. (A15),

$$[\boldsymbol{\varepsilon}'_s \cdot \mathbf{P}_{CV}(\mathbf{k})] = \mathbf{k} \cdot \frac{P}{2} \hat{r}_3 (i \hat{r}_1 - \hat{r}_2) [A(\mathbf{k}) + B(\mathbf{k})] \cos \omega t, \quad (\text{A16})$$

$$A(\mathbf{k}) = a_{k-} \left( \frac{b_{k+}}{\sqrt{2}} + c_{k+} \right),$$

$$B(\mathbf{k}) = a_{k+} \left( \frac{b_{k-}}{\sqrt{2}} + c_{k-} \right). \quad (\text{A17})$$

Thus,

$$\begin{aligned} |\boldsymbol{\varepsilon}'_s \cdot \mathbf{p}_{cv}(\mathbf{k})|^2 &= \left| \hat{k} \cdot \frac{P}{2} \hat{r}_3 \right|^2 |i \hat{r}_1 - \hat{r}_2|^2 [A(\mathbf{k}) \\ &+ B(\mathbf{k})]^2 \cos^2 \omega t = \frac{1}{4} P_z^2 \cos^2 \theta [A(\mathbf{k}) \\ &+ B(\mathbf{k})]^2 \cos^2 \omega t. \end{aligned} \quad (\text{A18})$$

So, the average value of  $|\bar{\boldsymbol{\varepsilon}}_s \cdot \bar{\mathbf{p}}_{cv}(\bar{\mathbf{k}})|^2$  for a plane polarized light wave is given by

$$\begin{aligned} \langle |\boldsymbol{\varepsilon}_s \cdot \mathbf{p}_{cv}(\mathbf{k})|^2 \rangle_{av} &= \frac{2}{4} P_z^2 [A(\mathbf{k}) \\ &+ B(\mathbf{k})]^2 \left( \int_0^{2\pi} d\phi \int_0^\pi \cos^2 \theta \sin \theta d\theta \right) \\ &\times \left( \frac{1}{2} \right) = \frac{2\pi}{3} P_z^2 [A(\mathbf{k}) + B(\mathbf{k})]^2, \end{aligned} \quad (\text{A19})$$

where  $P_z^2 = (1/2) |\mathbf{k} \cdot \mathbf{p}_{cv}(0)|^2$  and

$$|\mathbf{k} \cdot \mathbf{p}_{cv}(0)|^2 = \frac{m^2 E_{g_0} (E_{g_0} + \Delta)}{4m_r \left( E_{g_0} + \frac{2}{3} \Delta \right)}. \quad (\text{A20})$$



We shall express  $A(\mathbf{k})$  and  $B(\mathbf{k})$  in terms of constants of the energy spectra in the following way.

Substituting  $a_{k_{\pm}}$ ,  $b_{k_{\pm}}$ ,  $c_{k_{\pm}}$ , and  $\gamma_{0k_{\pm}}$  in  $A(\mathbf{k})$  and  $B(\mathbf{k})$  in Eq. (A17) we get

$$A(\mathbf{k}) = \beta \left( t + \frac{\rho}{\sqrt{2}} \right) \left\{ \left( \frac{E_{g_0}}{E_{g_0} + \delta'} \right) \gamma_{0k_+}^2 - \gamma_{0k_+}^2 \gamma_{0k_-}^2 \left( \frac{E_{g_0} - \delta'}{E_{g_0} + \delta'} \right) \right\}^{1/2}, \quad (\text{A21})$$

$$B(\mathbf{k}) = \beta \left( t + \frac{\rho}{\sqrt{2}} \right) \left\{ \left( \frac{E_{g_0}}{E_{g_0} + \delta'} \right) \gamma_{0k_-}^2 - \gamma_{0k_+}^2 \gamma_{0k_-}^2 \left( \frac{E_{g_0} - \delta'}{E_{g_0} + \delta'} \right) \right\}^{1/2}, \quad (\text{A22})$$

in which,

$$\gamma_{0k_+}^2 = \frac{\xi_{1k} - E_{g_0}}{2(\xi_{1k} + \delta')} = \frac{1}{2} \left[ 1 - \left( \frac{E_{g_0} + \delta'}{\xi_{1k} + \delta'} \right) \right]$$

and

$$\gamma_{0k_-}^2 = \frac{\xi_{1k} + E_{g_0}}{2(\xi_{1k} + \delta')} = \frac{1}{2} \left[ 1 + \left( \frac{E_{g_0} - \delta'}{\xi_{1k} + \delta'} \right) \right].$$

Substituting  $x = \xi_{1k} + \delta'$  in  $\gamma_{0k_{\pm}}$ , we can write

$$A(\mathbf{k}) = \beta \left( t + \frac{\rho}{\sqrt{2}} \right) \left\{ \left( \frac{E_{g_0}}{E_{g_0} + \delta'} \right) \frac{1}{2} \left( 1 - \frac{E_{g_0} + \delta'}{x} \right) - \frac{1}{4} \left( \frac{E_{g_0} - \delta'}{E_{g_0} + \delta'} \right) \left( 1 - \frac{E_{g_0} + \delta'}{x} \right) \left( 1 + \frac{E_{g_0} - \delta'}{x} \right) \right\}^{1/2}.$$

Thus,

$$A(\mathbf{k}) = \frac{\beta}{2} \left( t + \frac{\rho}{\sqrt{2}} \right) \left\{ 1 - \frac{2a_0}{x} + \frac{a_1}{x^2} \right\}^{1/2},$$

where  $a_0 = (E_{g_0}^2 + \delta'^2)(E_{g_0} + \delta')^{-1}$  and  $a_1 = (E_{g_0} - \delta')^2$ .

After tedious algebra, one can show that

$$A(\mathbf{k}) = \frac{\beta}{2} \left( t + \frac{\rho}{\sqrt{2}} \right) (E_{g_0} - \delta') \left[ \frac{1}{\xi_{1k} + \delta'} - \frac{1}{E_{g_0} + \delta'} \right]^{1/2} \left[ \frac{1}{\xi_{1k} + \delta'} - \frac{(E_{g_0} + \delta')}{(E_{g_0} - \delta')^2} \right]^{1/2}. \quad (\text{A23})$$

Similarly, from Eq. (A22), we can write

$$B(\mathbf{k}) = \beta \left( t + \frac{\rho}{\sqrt{2}} \right) \left\{ \left( \frac{E_{g_0}}{E_{g_0} + \delta'} \right) \frac{1}{2} \left( 1 + \frac{E_{g_0} - \delta'}{x} \right) - \frac{1}{4} \left( \frac{E_{g_0} - \delta'}{E_{g_0} + \delta'} \right) \left( 1 - \frac{E_{g_0} + \delta'}{x} \right) \left( 1 + \frac{E_{g_0} - \delta'}{x} \right) \right\}^{1/2}.$$

So that, finally we get

$$B(\mathbf{k}) = \frac{\beta}{2} \left( t + \frac{\rho}{\sqrt{2}} \right) \left( 1 + \frac{E_{g_0} - \delta'}{\xi_{1k} + \delta'} \right). \quad (\text{A24})$$

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