$\Delta B = -\Delta L$, NEUTRON $\to e^- \pi^+, e^- K^+, \mu^- \pi^+$ AND $\mu^- K^+$ DECAY MODES IN $SU(2)_L \times SU(2)_R \times SU(4)^{col}$ OR SO(10)

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It is stressed that within the simplest model SU(4) of color, unifying quarks with leptons, where $B - L$ is a gauge symmetry, a spontaneous breaking of this symmetry leads naturally to baryon–lepton decays like $n \to e^- k^+, e^- K^+, \mu^- \pi^+, \mu^- K^+$ [in contrast to SU(5) where $B - L$ is not gauged]. Our mechanism is crucially tied to the presence of the Higgs multiplet (2, 2, 15) of SU(2) × SU(2) × SU(4), which is needed to account for the observed quark–lepton mass ratios.

1. Ten years after the suggestion was made, it still appears to us that by far the simplest gauge unification of quarks and leptons is realized by uniting three colored quarks of a given flavor with a lepton of the same flavor, under the local gauge symmetry [1], $SU(4)^{col}$. This unification automatically identifies lepton number with the fourth color and introduces the linear combination $B_q - 3L = 3(B - L)$ as a generator, where $B_q \equiv$ quark number = $3B$. The two facts that $B - L$ is introduced as a generator of the local symmetry and that the limits from Eötvos type experiments do not permit a massless gauge particle to remain coupled to $B - L$, imply that the gauge particle coupled to $B - L$ must ultimately acquire a mass spontaneously; the associated "charge" $(B - L)$ must then be violated spontaneously. This in general can bring about a host of $B - L$)-nonconserving processes such as (i) $p \to \ell$ + mesons, (ii) $p \to 3\ell$ + mesons, (iii) $p \to 3\bar{\ell}$ + mesons; (iv) Majorana masses for ν_R and ν_L accompanied by neutrinoless double β-decay, and (v) $n \leftrightarrow \bar{n}$-oscillations. Here ℓ denotes $(e^-, \nu_e)_{L,R}, (\mu^-, \nu_\mu)_{L,R}$ etc., while $\bar{\ell}$ denotes their antiparticles. The relative importance of one or several of these modes and of the $(B - L)$-conserving $p \to e^+\pi^0$-type decays, which are permissible even in the limit of $SU(4)^{col}$ symmetry, will depend, of course, upon the details of the model.

The purpose of this note is to point out that within the $SU(4)^{col}$ gauge-unification; (a) there exists a simple mechanism which naturally emphasizes the $\Delta(B - L) = -2$ decay modes of the proton, i.e. $p \to e^-\pi^+\pi^+, e^-K^+\pi^+, \mu^-K^+\pi^+, \nu_e\pi^+$ etc; (b) even though these processes involve a heavy mass scale of the order of 10^{13} GeV, the rates for these processes, contrary to common expectations, can correspond to proton lifetimes in the range of 10^{31}–10^{33} yr; (c) The mass scale of 10^{13}–10^{14} GeV emerges with $\mathcal{G}$

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$= SU(2)_L \times SU(2)_R \times SU(4)^{col}$, consistent with renormalization group equations for the running coupling constants and the present value of $\sin^2\theta_W \approx 0.22$. The mechanism noted here can be operative within the minimal extension of $\mathcal{G}$, i.e. [2] SO(10), as well.

2. Recall that $SU(4)^{col}$, together with the desire for the quantization of the electric charge, suggests that the low energy symmetry $SU(2)_L \times U(1) \times SU(3)^{col}$ must be extended minimally to the left–right symmetric non-abelian symmetry [1] $\mathcal{G} = SU(2)_L \times SU(2)_R \times SU(4)^{col}$. The fermions of the electron family are represented by: $F_L^e = (2, 1, 4)$, $F_R^e = (1, 2, 4)$, and similarly for the μ and τ families.

Let us list what appears to be the minimal set of Higgs multiplets which are *needed* by the model. This set contains:

$$\phi = (2, 2, 1)\,, \quad \xi = (2, 2, 15)\,,$$
$$\Delta_L = (3, 1, 10)\,, \quad \Delta_R = (1, 3, 10)\,. \tag{1}$$

The associated pattern of the VEV, in obvious notation, is:

$$\langle(\Delta_R)^{I_{3R}=+1}_{\ell\ell}\rangle = v_R\,, \quad \langle\phi\rangle = \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix},$$
$$\langle\xi\rangle = \begin{pmatrix} \delta & 0 \\ 0 & \delta' \end{pmatrix}_{\rm flavor} \times (1, 1, 1, -3)_{\rm color}\,. \tag{2}$$

Here the subscript ℓ denotes leptonic color within the **4** of $SU(4)^c$. For consistency with phenomenology (including $m_Z = m_W/\cos\theta$ etc), one needs the hierarchy:

$$\langle\Delta_R\rangle \gg \langle\phi\rangle \sim \langle\xi\rangle \gg \langle\Delta_L\rangle\,. \tag{3}$$

One can show that such a pattern of VEV can be obtained in accordance with the absolute minimum of the potential [3] [‡1]. Our reasons for believing that the set (3) including (2, 2, 15) is really needed by the model are the following:

(i) First, the multiplet Δ_R is needed to give a heavy Majorana mass [5] to the right-handed neutrino ($M_{\nu R}$) $>$ 100 GeV. Such a mass for ν_R, together with a normal Dirac mass $m_\nu^D \sim 1$ MeV for the (ν_L, ν_R) combination, appears to be the most natural way to account for the lightness of the *physical* neutrinos ($m_{\nu_e} \lesssim 30$ eV) appearing in β decays, which, in this picture, would have a mass $\sim(m_\nu^D)^2/M_{\nu R} \ll m_\nu^D$. Given Δ_R, one, of course, needs to introduce Δ_L to respect left–right symmetry [6]. Note that the VEV of Δ_R, subject to the hierarchy (3), breaks $SU(2)_L \times SU(2)_R \times SU(4)^{col}$ into $SU(2)_L \times U(1) \times SU(3)^{col}$ thereby breaking $SU(4)^{col}$, $B - L$ and left–right symmetry spontaneously.

(ii) The multiplet ϕ (2, 2, 1) with a VEV given by (2) is needed to give Dirac masses to the fermions through standard Yukawa couplings of ϕ and $\tilde{\phi} = \tau_2\phi^*\tau_2$ with the fermions and to break $SU(2)_L \times U(1)$ to $U(1)_{em}$. We expect the larger of (k and k') $\sim$ 300 GeV.

(iii) With only the color singlet ϕ to give Dirac masses to the fermions, however, one obtains: $m_e^{(0)} = m_d^{(0)}$, $m_\mu^{(0)} = m_s^{(0)}$ and $m_\tau^{(0)} = m_b^{(0)}$ in the symmetric limit. Including electroweak and QCD renormalization, these lead to the relation: $(m_e/m_\mu) = (m_d/m_s)$, for the physical masses, which is badly in disagreement with observation. This situation is remedied [‡2] if we introduce an addition scalar multiplet [1] ξ = (2, 2, 15), which is the only other type of multiplet capable of giving Dirac masses to the fermions. We expect (δ, δ') to be of order (k, k').

One may, in general, introduce, in addition to the scalar multiplets listed above, a multiplet [1] (1, 1, 15), which could be used to provide the primary scale $>3 \times 10^5$ GeV for breaking $SU(2) \times SU(2) \times SU(4)^c$ into $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)^c$, which in turn could break into $SU(2)_L \times U(1) \times SU(3)^c$ via $\langle\Delta_R\rangle$ at the second stage of SSB.

It is worth observing that if one did contemplate to embed $SU(2) \times SU(2) \times SU(4)$ within SO(10), then, a desirable minimal set of Higgs multiplets would consist of (54 or 210 or 945) $\oplus$ 126 $\oplus$ 10. Some of these decompose under $SU(2) \times SU(2) \times SU(4)$ as follows:

$$54 = (1,1,1) + (2,2,6) + (3,3,1) + (1,1,20')\,,$$
$$945 = (1,1,15) + (2,2,64) + (2,2,6) + (2,2,6)$$
$$+ \text{ others}\,, \tag{4}$$

[‡1] This has been shown, without the inclusion of (2, 2, 15), by Mohapatra and Senjanovic [4]. A more complete analysis including (2, 2, 15), (2, 2, 64) and (1, 1, 6) suggested here as well as the analogous case in SO(10) involving 945, 126, and 10 of SO(10) will be presented in ref. [3].

[‡2] This is because with contributions from both $\langle\phi\rangle$ and $\langle\xi\rangle$, the masses in the symmetric limit have the form: $m_e^{(0)} = m_e^\phi - 3m_e^\xi$, $m_d^{(0)} = m_e^\phi + m_e^\xi$, etc.

$$126 = (3,1,10) + (1,3,\overline{10}) + (2,2,15) + (1,1,6)\,,$$

$$10 = (2,2,1) + (1,1,6)\,. \qquad \text{(4con'd)}$$

Note that Δ_L, $\bar{\Delta}_R$ and $\xi = (2, 2, 15)$ are contained in the same 126, while $\phi = (2, 2, 1)$ belongs to 10. Thus our minimal set (3) would require the introduction of 126 and 10 of SO(10). This will call for the introduction of (1, 1, 6) in addition to that of Δ_L, $\bar{\Delta}_R$, ξ and ϕ.

In this note, we shall concentrate primarily on the consequences of the minimal set of Higgs multiplets [eq. (1)], which we have argued are needed by the SU(2) × SU(2) × SU(4) model. But we shall remark that the added presence of $(2, 2, 64) \subset 945$ of SO(10) and in a different context $(1, 1, 6) \subset 126$ of SO(10) can play vital roles regarding the complexions of baryon and lepton number violations.

It is straightforward to write down the full Higgs potential including mass and the mutual quartic interaction terms of the set (Δ_L, Δ_R, ϕ and ξ) consistent with the gauge symmetry SU(2) × SU(2) × SU(4), left–right discrete symmetry and renormalizability. We do not exhibit this full potential [3], as it is not essential for our discussions. Instead, we write down a particular quartic interaction term involving $\xi = (2, 2, 15)$, Δ_R and Δ_L [‡3].

$$V_1 = \lambda_1 \epsilon_{\alpha\beta\gamma\delta}\, \epsilon^{i}_{j}\, \epsilon^{i'}_{j'}\, \delta^{\alpha'}_{\gamma'}\, \delta^{\beta'}_{\gamma'} \times [\xi^{ii'}_{\alpha\alpha'*} \cdot \Delta^{\mathrm{P}}_{\mathrm{R},\{\beta,\beta'\}} \cdot \xi^{jj'}_{\gamma\gamma'*} \cdot \Delta^{\mathrm{P}}_{\mathrm{R},\{\delta,\delta'\}} + \mathrm{R} \to \mathrm{L}] + \mathrm{h.c.}\,. \qquad (5)$$

Here (ii') and (jj') denote $SU(2)_{L,R}$ indices corresponding to the (2, 2) representation; $p = +1, 0, -1$ denote $SU(2)_R$ indices for the triplet representation; $(\alpha, \beta, \gamma, \delta)$ denote $SU(4)^{col}$ indices for the **4** representation, while α^*, γ^* correspond to 4^*. The symbol (α, α'^*) stands for **15** of $SU(4)^c$.

‡3 We do not exhibit other invariant interactions, e.g. the one in which the two ξ make the $SU(2)_R$ triplet and the one in which the interaction involves $\xi \cdot \xi \cdot \Delta_R \cdot \Delta_L$. These can also contribute to proton → lepton + meson decay modes. Note that the gauge interactions of SU(2) × SU(2) × SU(4) as well as the invariant Yukawa interactions of ϕ, ξ, and $\Delta_{L,R}$ conserve the global fermion number $F \equiv B_q + L = 3B + L$ (ref. [1]), but the interaction (5) violates F by ±4 units, while conserving $B - L$. Such explicit violation of F is permissible in SU(2) × SU(2) × SU(4), but not in SU(16), since F is a generator of SU(16), though not of SU(2) × SU(2) × SU(4).

To observe the nature of (B, L)-violations it is necessary to write down the Yukawa interactions of $\Delta_{L,R}$ and ξ with the fermions:

$$\mathcal{L}_Y = (h^{\xi}_{ab} \bar{\mathrm{F}}^a_{\mathrm{L}} \cdot \xi \cdot \mathrm{F}^{\mathrm{b}}_{\mathrm{R}} + h'^{\xi}_{ab} \bar{\mathrm{F}}^a_{\mathrm{L}} \cdot \tilde{\xi} \cdot \mathrm{F}^b_{\mathrm{R}} + h^{\Delta}_{ab} \mathrm{F}^{a\mathrm{T}}_{\mathrm{L}} \cdot \bar{C}^1 \mathrm{F}^{\mathrm{b}}_{\mathrm{L}} \cdot \Delta_{\mathrm{L}}) + (\mathrm{L} \to \mathrm{R}) + \mathrm{h.c.} \qquad (6)$$

Here a, b run over family indices: $a = \mathrm{e}, \mu, \tau$; and $\tilde{\xi} = \tau_2 \xi^* \tau_2$. We have written (6) symbolically, suppressing flavor and color indices.

The interactions (5) and (6) together with the VEV (2) induce *spontaneously* the $\Delta(B - L) = -2$, $\Delta(B + L) = 0$, $\Delta F = -2$ decay of the proton into a lepton, i.e. $\mathrm{p} \to \ell$ + mesons ($\ell = \mathrm{e}^-, \mu^-, \nu_e, \nu_\mu$, meson $= \pi$, K, etc.) as shown in fig. 1. Note that such a mechanism can not be implemented by substituting $\xi = (2, 2, 15)$ by $\phi = (2, 2, 1)$, because the upper and lower vertices in fig. 1 must involve the $SU(3)^c$ triplet and octet components (ξ_3 and ξ_8) lying within the **15** of $SU(4)^c$. The magnitude of the amplitude for the six-fermion process of fig. 1 is:

$$A(3\mathrm{q} \to \mathrm{q}\bar{\mathrm{q}}\ell) = [\lambda_1 (h^{\xi})^2 h^{\Delta} \langle \Delta_R \rangle] / (m^2_{\xi_3} m^2_{\xi_8} m^2_{\Delta}) \qquad (7)$$

Here, we have dropped the family index on the Yukawa coupling constants. The quantities m_{ξ_3} and m_{ξ_8} denote the masses of the $SU(3)^c$ triplet and octet components of ξ, which in general may differ from each other subsequent to spontaneous breaking of $SU(4)^c$ via $\langle \Delta_R \rangle \neq 0$.

With a minimal fine tuning of parameters of the Higgs sector, associated with the familiar gauge hierarchy problem, one would ordinarily expect only the color singlet ξ_1 component of ξ to remain light ($m_{\xi_1} \simeq 30$–100 GeV, say), since its neutral member acquires a VEV ~ 100 GeV, but the remaining components within the 15 of $SU(4)^c$ including ξ_3 and ξ_8 to

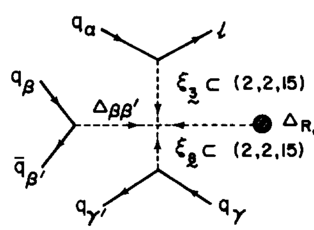

Fig. 1. Mechanism for inducing neutron → $e^-\pi^+$, e^-K^+ type decays.

acquire a relatively heavy mass of order ‡4 $\sim \alpha v_R$. This, however, is not the general situation. For instance, if there exists an additional $SU(3)^c$ triplet χ_3 within a new multiplet χ of SU(4) whose diagonal $(\text{mass})^2$ is much larger than that of ξ_3 [i.e. $m_\chi^2 > (\alpha v_R)^2$] and if there exists a spontaneously induced mixing between ξ_3 and χ_3 with a strength ‡5 $\approx m_\chi m_\xi \sim (\alpha v_R) m_{\chi_3}$, then there could exist a light eigenstate with a large ξ_3 component with positive $(\text{mass})^2$ eigenvalue, which is vanishingly small $\sim (100 \text{ GeV})^2$.

Such a mixing, and an analogous mixing for ξ_8, are indeed brought about simultaneously if we introduce a multiplet $\chi = (2, 2, 64)$, which in the SO(10) context would have its origin within 945. The 64 of $SU(4)^c$ decomposes under $SU(3) \times U(1)_{B-L}$ as follows: $64 = 3(-2/3) + 3^*(2/3) + 6^*(-2/3) + 6(+2/3) + 8(2) + 8(-2) + 15(2/3) + 15(-2/3)$; Note that χ contains a triplet and an octet of $SU(3)^c$ as desired. The presence of χ permits an invariant trilinear coupling between $\xi(2, 2, 15)$, $\chi(2, 2, 64)$ and $\Delta_R(1, 3, 10)$: $[\lambda_t \mathcal{M}(\xi \cdot \chi \cdot \Delta_R) + (R \to L)]$ + h.c. We believe that a natural scale for the mass $\mathcal{M}$ is given by the heaviest scale ‡6 in the theory, which simultaneously is essentially the diagonal mass $m_\chi(2, 2, 64)$. In this case, substituting v_R for $\langle \Delta_R \rangle$ [eq. (2)], in the trilinear coupling, we do obtain a mixing between ξ_3 and χ_3^* as well as between ξ_8 and χ_8 of the type depicted above, whose strengths are of order $\alpha v_R m_\chi$. This can "naturally" ‡6 give light states; which are predominantly ξ_3 and ξ_8.

We, therefore, proceed to discuss the phenomenological implications of the presence of the ξ (2, 2, 15) multiplet together with $\Delta_{L,R}$, ϕ and possibly χ multiplets by permitting the color triplet and octet, (i.e. ξ_3 and ξ_8) components of ξ to be light ($\sim$30–100 GeV, say).

The reader may observe that one can not afford to permit a light color triplet in [8] SU(5), because such a triplet [e.g. the one within the 5 of SU(5)] would lead to fast proton decay. No such problem arises for SU(2) × SU(2) × SU(4), because its basic gauge interactions conserve B and L unlike SU(5). Thus, it is worthwhile to explore the consequences of having a light color triplet as well as an octet like ξ_3 and ξ_8 within SU(2) × SU(2) × SU(4).

Let us proceed to estimate the amplitude (7) by taking typical values of the parameters, viz. $\lambda_1 \sim 10^{-2} - 10^{-3}$, $h^\Delta \sim 10^{-1} - 10^{-3}$, $h^\xi \sim 10^{-3} - 10^{-4}$ and $m_{\Delta R} \sim (v_R/10)$. We obtain:

$$A(3q \to q\bar{q}\ell) \approx (10^{-12} - 10^{-6})/(m_\xi^4 v_R) . \qquad (8)$$

For the case of a light ξ_3 and ξ_8, taking $m_{\xi_3} \sim m_{\xi_8} \sim 100$ GeV, we see that the amplitude will have a canonical strength ‡7 $\sim 10^{-30}$ GeV^{-5} for the proton lifetime to be in the range of $10^{32\pm1}$ yr, if

$$v_R \approx 10^{13\pm3} \text{ GeV} . \qquad (9)$$

v_R from renormalization group equations: Consider the hierarchical descent:

$$SU(2)_L \times SU(2)_R \times SU(4)^c \xrightarrow{\langle\Delta_R\rangle \sim M_X \sim M_R} SU(2)_L \times U(1) \times SU(3)^c \xrightarrow{\langle\phi\rangle \sim \langle\xi\rangle \sim m_W} U(1)^{em} \times SU(3)^c .$$

Here M_X and M_R denote the mass of the leptoquark gauge bosons in $SU(4)^c$ and of W_R respectively. Subject to the boundary conditions that $g_{2L}(M_X) = g_{2R}(M_X)$ and $g_{15}(M_X) = g_3(M_X) = g_4(M_X)$, where $g_{2L,R}$, g_3 and g_4 denote $SU(2)_{L,R}$, $SU(3)^c$ and $SU(4)^c$ coupling constants respectively and g_{15} denotes the normalized coupling constant associated with the 15th generator of $SU(4)^c$, it is straightforward to combine the evolution equations for the coupling constants g_3,

‡4 Here α^2 denotes the typical strength of a quartic interaction involving $\xi = (2, 2, 15)$ and Δ_R fields. For details of this discussion see ref. [3]. For a discussion of an analogous situation in SU(5), and other cases, see ref. [7].

‡5 This would correspond to a fine tuning of a *new* parameter brought about due to the $\xi \cdot \chi \cdot \Delta$ cubic or the related $(1, 1, 15) \cdot \xi \cdot \chi \cdot \Delta$ quartic interaction (see remarks later.) Such a fine tuning is no different in spirit than the familiar fine tuning encountered in grand unification. {see e.g. Buras et al. [7] for SU(5)}.

‡6 This is motivated if we derive the $SU(3)^c \times U(1)_{B-L}$-reduced form of the trilinear coupling $(\lambda_t \mathcal{M}) \xi \cdot \chi \cdot \Delta$ from an SU(2) × SU(2) × $SU(4)^c$-invariant quartic coupling of the type $(\lambda_t)\eta(1, 1, 15) \cdot \xi(2, 2, 15) \cdot \chi(2, 2, 64) \cdot \Delta(1, 3, 10)$, with $\mathcal{M}$ corresponding to the VEV of (1, 1, 15), which in turn could represent the largest scale both for the SU(2) × SU(2) × SU(4) model as well as for the SO(10) model. Note that in the SO(10) context, both $\eta(1, 1, 15)$ and $\chi(2, 2, 64)$ belong to 945, and thus we expect $m_\chi \sim \langle(1, 1, 15)\rangle$.

‡7 To compare the amplitude (7) with that of a four-fermion process (e.g. $3q \to \bar{\ell}$ or $3q \to \ell$), we need to replace $q\bar{q}$ by a dimensional factor m^3 with $m \sim 1$ GeV.

g_2 and g_1 of SU(3)^c, SU(2) and U(1). This yields:

$$(1-2\sin^2\theta_W)-\tfrac{2}{3}\alpha/\alpha_s = 8\pi\alpha(\tfrac{5}{3}b_1-b_{2L}-\tfrac{2}{3}b_3)\ln(M_X/m_W)\,. \qquad (10)$$

Here, $\sin^2\theta_W$, α and α_s are to be evaluated at m_W. The constants $b_{1,2,3}$ denote the standard coefficients determining the β functions for $g_{1,2,3}$. Substituting $\alpha(m_W)=1/128$ and a typical value for $\alpha_s(m_W)=0.10$, and applying one-loop renormalization group analysis [‡8] we obtain:

$$M_X=(2.5, 7.5, 66)\times 10^{13}\ \text{GeV} \qquad (11)$$

for $\sin^2\theta_W=(0.24, 0.23, 0.21)$. We see that $v_R=(M_X/g)$, typically, is predicted to lie in the range $\approx(10^{13}-10^{14})$ GeV. This is precisely the right sort of mass needed to lead to a "canonical" amplitude for proton decay into a single lepton with a lifetime in the range of $10^{31}-10^{33}$ yr. [cf. eq. (9)].

We thus see that the presence of the (2, 2, 15) together with $\Delta_{L,R}$, which in turn permits the invariant interaction (5), emphasizes the $\Delta(B-L)=-2$, $\Delta(B+L)=0$ process, i.e. proton decay into a lepton, with a lifetime, which may well be expected to lie in the range of $10^{31}-10^{33}$ yr, on the basis of renormalization group analysis.

Comparison with dimensional analysis: Note that despite the fact that the process $3q\to q\bar{q}\ell$ is a $(B-L)$-violating process, and we are considering a hierarchy in which SU(2) × SU(2) × SU(4) descends in *one step* to SU(2) × U(1) × SU(3)^c, the amplitude for the process is damped by only one power of the heavy mass scale $\langle\Delta_R\rangle=v_R$, for the case of light ξ_3 and ξ_8 [See eq. (8)]. This circumstance differs from the one encountered in dimensional and operator-analysis [9], which suggests that typically the $(B+L)$-conserving, $(B-L)$-violating proton decay amplitudes (for the case of single stage descent) should be damped by three powers of the heavy mass (M) and should be of order (m_W/M^3). The reasons for this difference may be traced at least in part to the facts that (i) our mechanism involves a spontaneous rather than explicit breaking of $B-L$ through $\langle\Delta_R\rangle\neq 0$, (ii) it permits the color triplet (ξ_3) and the color octet (ξ_8) to remain light [‡9]

Strange meson and muonic $\Delta(B+L)=0$ decay modes: Since fig. 1 involves Yukawa couplings of fermions, one can use diagonal as well as non-diagonal family mixing interactions, together with Cabibbo rotations, to obtain the full complexion of $\Delta(B+L)=0$ modes involving non-strange or strange mesons and the e^- or μ^- lepton, i.e. $n\to e^-\pi^+$, $\mu^-\pi^+$, $\mu^-\pi^+$, μ^-K^+ etc., with competing rates.

$\Delta(B-L)=0$, proton $\to e^+\pi^0$, $\bar{\nu}\pi^+$, $\mu^+\pi^0$, μ^+K^0-decays: Proton $\to e^+\pi^0$ type decay modes will arise within SU(2) × SU(2) × SU(4) at the tree level without involving SSB, if we introduce the scalar multiplet [‡10] $\zeta=(1,1,6)$, which in the SO(16) context arises automatically within 126 together with Δ_L, $\bar{\Delta}_R$ and ξ. We expect (1, 1, 6) to have a mass, characteristic of the heaviest scale in the theory $\sim 10^{13}-10^{14}$ GeV [see eq. (11)]. Thus for a typical Majorana Yukawa coupling constant [‡11] $h_M^\zeta\sim(10^{-1}-10^{-3})$, and a natural mass scale of $10^{12}-10^{14}$ GeV for the (1, 1, 6), the $p\to e^+\pi^0$ type decays arising through (1, 1, 6) exchange can have an inverse partial rate in the canonical range $\sim 10^{31}-10^{33}$ yr. Since (1, 1, 6) exchange involves Yukawa interactions, by using diagonal and non-diagonal family mixing interactions, *one can obtain non-strange and strange modes: i.e. $p\to e^+\pi^0$, $\mu^+\pi^0$, e^+K^0 as well as μ^+K^0*, with the possibility that the rates of strange modes can even surpass those of the non-strange ones depending upon the strengths of diagonal versus non-diagonal Majorana Yukawa couplings.

Proton decays into antileptons would, of course, arise additionally through gauge interactions, if we extend SU(2) × SU(2) × SU(4) into SO(10) (see remarks later). The gauge boson mediated decays will not, however, emphasize the muon and strange particle modes.

‡8 We have included the contributions of light Higgs SU(2)$_L$ doublets in (2, 2, 1) and (2, 2, 15), as well as those of the SU(3)^c triplet, antitriplet and octet in (2, 2, 15) to the β functions for $g_{1,2,3}$ in the momentum range between M_X and m_W. The net contributions of ξ_3, ξ_3^* and ξ_8 to the right side of (10) vanishes. The two-loop analysis is expected to decrease M_X by nearly a factor of 2.

‡9 As we have discussed, ξ_3 and ξ_8 being light is, of course, compatible with the assumption of single stage descent, which is what dimensional analysis assumes.

‡10 This feature is generally known. See e.q. Del Aguila and Ibáñez (ref. [7]).

‡11 Within SO(10), the Yukawa coupling constants of (1, 1, 6), (2, 2, 15) and $\Delta_{L,R}$ are related to each other at the unification scale, since they belong to the same multiplet 126 of SO(10).

n–n̄-oscillations: The introduction of the multiplet $\xi = (2, 2, 15)$ and the interaction (5) forces one to introduce a quartic Δ^4 interaction, for the sake of renormalizability, since such interactions would be induced radiatively anyhow through the interaction (5) [‡12]. The Δ^4 interaction in turn induces $3q \to 3\bar{q}$ or $n-\bar{n}$ oscillations [10] by a diagram analogous to fig. 1. But the corresponding amplitude involves three Δ propagators. With $m_\Delta \sim 10^{13\pm3}$ GeV, as suggested by proton stability [eq. (9)], as well as by RGE for $SU(2) \times SU(2) \times SU(4)$ [eq. (11)], the strength of the $n-\bar{n}$ transition would be too feeble to observe.

SO(10): While we have presented our discussions primarily in the context of $SU(2) \times SU(2) \times SU(4)$, and it remains to be seen whether a preonic theory in the context of supergravity (or a Kaluza–Klein approach) can lead effectively at a composite level, for example, only to $\mathcal{G} = SU(2) \times SU(2) \times SU(4)$, rather than to one of its grand unifying extensions, it is clear that the Higgs multiplets which we have introduced arise together in a compelling manner if we extend $\mathcal{G}$ to SO(10). In this case, a minimal set of 945, 126 and 10 of SO(10) will contain the set (1), the (2, 2, 64) as well as (1, 1, 6), and will suffice to bring forth all the features of *B, L* nonconservation, which we have presented here. These multiplets also suffice to break SO(10) spontaneously as follows:

$$SO(10) \xrightarrow{\langle 945\rangle \sim M_U \sim M_X} SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)^c \xrightarrow{\langle 126\rangle \sim M_R} SU(2)_L \times U(1) \times SU(3)^c .$$

Note, first of all that $(126)^4$ contains singlets of SO(10). One can verify that these singlets do contain the $\xi^2\Delta^2$ interaction of the type (5). We have verified that the renormalization group analysis for the hierarchy mentioned above for SO(10) leads to solutions [3] for $M_R \sim 10^{11}-10^{13}$ GeV with $M_U \sim 10^{15}-10^{16}$ GeV. *Thus our mechanism will induce proton decay into a lepton plus mesons (i.e.* $p \to e^-\pi^+\pi^+$ *etc.) even in SO(10) with a rate in the observable range.* The $\Delta(B-L) = 0$ antilepton mode (i.e. $p \to e^+\pi^0$ etc.), is induced in SO(10) via gauge meson as well as Higgs (1, 1, 6) exchange. The relative importance of the antilepton versus lepton mode will depend on the one hand on the mass M_U and on the other hand upon the Dirac and Majorana type Yukawa coupling constants.
A priori, both types of modes are expected to be relatively important.

To conclude, we see that $SU(4)^{col}$ gauging and the presence of (2, 2, 15), which is needed by the observed fermion spectrum, naturally emphasizes the proton → lepton decay modes. So far, a search for $p \to e^+\pi^0$ type decay mode has been emphasized, especially in the biggest IMB detector [11]. It seems that the same search could be used to detect or set limits on nucleon → lepton decay modes (i.e. $n \to e^-\pi^+, \mu^-\pi^+, e^-K^+, \mu^-K^+$ etc.). We urge such a search, as well as searches for $p \to \bar{\nu}K^+, \mu^+\pi^0$ and μ^+K^0 modes, which we have argued are a priori permissible.

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Addendum. After the submission of this paper it was noted [12] that a mechanism analogous to that of fig. 1 with $q_{\gamma'}\bar{q}_{\beta'}$ being replaced by a $\ell\bar{\ell}$ pair will induce nucleon decays into a lepton + (lepton + antilepton) + mesons, i.e. $p \to \nu + (e^+ + \nu)$, $n \to e^-(e^+\nu)$, $n \to e^-(\nu\bar{\nu})\Pi^+$ etc.; the amplitudes for these processes are a priori comparable to those for processes without the extra lepton pair, i.e. $N \to \ell$ + mesons. There exists an economical choice of the Higgs system consisting of e.g. (2, 2, 6), ϕ, ξ and $\Delta_{R,L}$ for $SU(2) \times SU(2) \times SU(4)^c$ or 54, 126 and 10 for SO(10), which would emphasize nucleon decays into $\ell\ell\bar{\ell}$ + mesons over nucleon decays into ℓ + mesons.

Instead of $\chi(2, 2, 64)$, one may introduce, at least for economy, a second (2, 2, 15) multiplet ξ', which could belong to 120 or a second 126 of SO(10). A mixing between components of ξ and ξ' will be induced via invariant quartic interactions of the type $(\lambda \cdot \xi' \cdot \xi \cdot \Delta_R \cdot \bar{\Delta}_R + R \leftrightarrow L)$, etc., or in SO(10) context via for example $\lambda \cdot (126 \text{ or } 120 \supset \xi') \cdot (126)^2 \cdot (\overline{126})$ + h.c., etc, together with $\langle\Delta_R\rangle \neq 0$. Once again

[‡12] Note, if one introduced only the Δ^4 interaction together with the multiplet $\xi(2, 2, 15)$, but not the $\Delta^2\xi^2$ interaction of type (5), such an interaction will be induced radiatively once the VEV of $\langle\Delta\rangle \neq 0$.

such a mixing can lead to light eigenstates with large ξ_3- and ξ_8-components leaving the main discussion and conclusion of our paper unaltered.

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