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GLOBULAR CLUSTERS IN THE MILKY WAY AND DWARF GALAXIES: A DISTRIBUTION-FREE STATISTICAL COMPARISON

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ABSTRACT

It has been found that globular clusters (GCs) in dwarf galaxies and those in the Milky Way (MW) outer halo mostly have the same parent distributions, while GCs in the MW disk and inner halo have a different origin from those in dwarf galaxies. Thus, these dwarf galaxies did not play a crucial role in the formation of the Galactic disk or inner halo. In order to investigate this phenomenon in a more objective manner, a statistical comparison of the GCs of our Galaxy and those of neighboring dwarf galaxies has been carried out by a multivariate nonparametric method. For the various parameters of GCs in the MW and in dwarf galaxies, the multivariate Gaussian assumption fails, so a non-parametric method of comparison (instead of multivariate analysis of variance [MANOVA]) has been chosen. The test is performed on GCs of the MW disk, inner halo, and outer halo separately, with GCs from neighboring dwarf galaxies Canis Major, Fornax, and Sculptor, and the LMC dwarf irregular galaxy. The test is also performed for GCs from dwarf spheroidal galaxies in the neighborhood of M31: M33, NGC 147, NGC 185, and NGC 205.

Subject headings: methods: statistical — galaxies: dwarf — globular clusters: general

1. INTRODUCTION

Recent developments in the study of Milky Way globular clusters (GCs) have generated considerable excitement with the suggestion that the halo system of globular clusters is composed of two subsystems (Zinn 1993; van den Bergh 1993, 1994; Mackey & van den Bergh 2005; Chattopadhyay & Chattopadhyay 2007, hereafter CC07): (1) the inner halo, consisting of comparatively older GCs with a metallicity gradient, in a flattened spatial distribution around the galactic disk, and (2) the outer halo, consisting of comparatively younger GCs having a negligible metallicity gradient, and high retrograde orbits.

The possibility that the halo globular cluster system comprises two distinct populations is a significant development. Zinn (1993) suggested that the younger halo GCs may have an accretion origin, as envisaged by Searle & Zinn (1978). Several others have attempted to identify candidates for accreted GCs. Approaches have included studies of the retrograde motion of some metal-poor GCs (Rodgers & Paltoglou 1984), the variation of horizontal branch morphology with metallicity in the younger (or outer) halo (Zinn 1993), the Oosterhoff class (van den Bergh 1993), the size–perigalactic distance relation (van den Bergh 1995), associations in phase space (Lynden-Bell & Lynden-Bell 1995), spectroscopy of GCs in the Fornax dwarf galaxy (Strader et al. 2003), the age-metallicity relationship of GCs in the Canis Major dwarf galaxy (Forbes et al. 2004), comparison of the GCs of the Sculptor group with Milky Way GCs (Olsen et al. 2004), kinematical and chemical abundances of stars of Galactic halo and Local Group dwarf galaxies (Geisler et al. 2007), and of the luminosity distribution (van den Bergh 2007; Sharina et al. 2005). Many authors have seen evidence for such accretion to the Milky Way from, e.g., the Sagittarius dwarf galaxy GCs, Terzan 7, Terzan 8, Arp 2, and M54 (Ibata et al. 1995; Layden & Sarajedini 2000). In this light the phase-space distribution of GCs are compared by Bellazzini et al. (2003a, 2003b). Recently, Martin et al. (2004) presented evidence for a disrupted dwarf galaxy, the "Monoceros Ring," discovered by Newberg et al. (2002). Crane et al. (2003) and Frinchaboy et al. (2004) compared the phase-space distribution of Galactic GCs with M giant stars in the Monoceros Ring and have identified NGC 2298, NGC 2808, NGC 5286, Pal 1, and BH 176 of the MW as accreted GCs of that dwarf galaxy.

All these previous studies have taken one parameter at a time for comparison, thus neglecting the joint effects of the remaining ones. In the present work we have compared the GCs of Milky Way belonging to disk and inner and outer halo separately with all the GCs of neighboring dwarf galaxies, but under a multivariate setup.

In § 2 we discuss the various samples under consideration. In § 3 the methods are given, and the results and discussions are presented in § 4.

2. DATA SET

Our analysis is based on nine samples of GCs in the Milky Way, Sculptor dwarf, Fornax, Canis Major, LMC, M33, NGC 147, NGC 185, and NGC 205 dwarf galaxies.

Sample 1.—This consists of 142 GCs of the Milky Way taken from the catalog of Harris (1996, and 2003 update) having nonzero parameter values for absolute visual magnitude in the *R* and *V* bands (M_{T_1}), (M_V), color ($B-V$), horizontal branch ratio (HBR), metallicity ([Fe/H]), concentration parameter (c), core radius (R_c), central surface brightness (μ_V), age (Chaboyer et al. 1992), Lick indices Mg *b*, Mg 1, and Mg 2, and abundance ratios [Mg/Fe], [Ca/Fe], [Ti/Fe], and [α/Fe] (Pritzi et al. 2005).

Sample 2.—This consists of 36 GCs in Sculptor Group of dwarf galaxies (Olsen et al. 2004). The parameters used are M_{T_1}, $[\mathrm{Fe/H}]_{(C-T_1)}$, and [Mg/Fe].

Sample 3.—This consists of five GCs in the Fornax dwarf galaxy (Mackey & Gilmore 2004). The parameters used are [Fe/H], HBR, R_c, μ_V, age, and abundance ratios [Mg/Fe] and [Ca/Fe] (Letarte et al. 2006).

Sample 4.—This consists of four GCs in the Canis Major dwarf galaxy (Forbes et al. 2004). The parameters are [Fe/H], HBR, R_{GC}, M_V, and age.

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TABLE 1
Results for the Multivariate Normal Test

Galaxy	Galaxy Compared	Parameter Set	p Value	Outcome
MW Cluster 1	Sculptor Dwarf	$(M_{T_1},\ \text{[Fe/H]}_{(C-T_1)})$	5.59×10^{-5}	Rejected
MW Cluster 2	Canis Major Dwarf	([Fe/H], HBR)	5.32×10^{-8}	Rejected
	Fornax Dwarf	([Fe/H], R_c)	3.584×10^{-10}	Rejected
	Sculptor Dwarf	$(M_{T_1},\ \text{[Fe/H]}_{(C-T_1)})$	2.418×10^{-5}	Rejected
MW Cluster 3	Canis Major Dwarf	([Fe/H], HBR)	0.002693	Rejected
	Fornax Dwarf	([Fe/H], HBR)	0.0009282	Rejected

Sample 5.—This consists of 23 GCs of LMC used in the paper by Mackey & Gilmore (2003). The parameters used are M_V, $(B-V)$, c, R_c, [Fe/H], μ_V, and age. We have converted the visual magnitudes from Mackey & Gilmore (2003) to absolute visual magnitudes using a distance of 49 kpc (Sparke & Gallagher 2000) for the LMC.

Sample 6.—This consists of four GCs in M33 (Larsen et al. 2002). The parameters used are R_c, μ_V, and color $(V-I)$.

Sample 7.—This consists of three GCs in dwarf spheroidal galaxy NGC 147 (Sharina et al. 2006). The parameters used are M_V, $(B-V)$, [Fe/H], Mg 1, Mg 2, Mg b, and $[\alpha/\text{Fe}]$.

Sample 8.—This consists of six GCs in dwarf spheroidal galaxy NGC 185 (Sharina et al. 2006). The parameters used are M_V, $(B-V)$, [Fe/H], Mg 1, Mg 2, Mg b, and $[\alpha/\text{Fe}]$.

Sample 9.—This consists of three GCs in dwarf spheroidal galaxy NGC 205 (Sharina et al. 2006). The parameters used are M_V, $(B-V)$, [Fe/H], Mg 1, Mg 2, Mg b, and $[\alpha/\text{Fe}]$.

3. METHOD

In studying the compatibility among data with a multivariate setup, the equality of location measures (mean, median, mode, etc.) and dispersion measures (standard deviation, range, etc.) are of interest. If the joint distribution of the parameters under consideration is multivariate normal, then the MANOVA test is appropriate; otherwise, use of a multivariate two-sample nonparametric method is a better option. Thus, we first carry out a normality test for some of the parameter sets; the results are shown in Table 1. In all situations the test shows that the distributions of different parameter sets are not multivariate normal. Thus, we use the nonparametric test, appropriate for a multivariate set. Since the size of the sample in some cases (e.g., samples 3 and 4) is small, combinations of two parameters at a time are preferred. These are listed in Tables 2–9. A short description of the multivariate nonparametric test used is given below.

Let

$$X^{(k)}_{\alpha} = (X^{(k)}_{1\alpha}, \ldots, X^{(k)}_{p\alpha})',$$
$$\alpha = 1, \ldots, n_k, \quad k = 1, \ldots, c,$$

be a set of independent vector-valued random values, where c is the total number of populations, n_k is the sample size of the kth population, and p is the total number of parameters. The cumulative distribution function (c.d.f.) of $X^{(k)}_{\alpha}$ is denoted by $F_k(x)$. The set of admissible hypotheses designates that each $F_k(x)$ belongs to same class of distribution functions Ω. The hypothesis to be tested, say H_0, specifies that

$$H_0 : F_1(x) = \ldots = F_c(x) = F(x), \quad \forall\, x,$$
$$\text{where } F \in \Omega.$$

The alternative to H_0 is the hypothesis that each $F_k(x)$ belongs to Ω but that H_0 does not hold. To avoid the problem of ties, it is assumed that the class Ω is the class of all continuous distribution functions. Here we pay particular attention to translation-type alternatives. For translation-type alternatives, we let

$$F_k(x) = F(x + \delta_k), \quad \forall\, k = 1, \ldots, c, \quad F \in \Omega,$$

and we are interested in testing (the reversed null hypothesis)

$$H^1_0 : \delta_1 = \ldots = \delta_c = 0$$

against the alternative that $\delta_1, \ldots, \delta_c$ are not all equal. We use the "basic rank permutation principle" given by Puri & Sen (1970). Let us rank the N i-variate observations $X^{(k)}_{i\alpha}$, $\alpha = 1, \ldots, n_k$, $k = 1, \ldots, c$ in ascending order of magnitude, and let $R^{(k)}_{i\alpha}$ denote the rank of $X^{(k)}_{i\alpha}$ in this set. The observation vector $X^{(k)}_{\alpha} = (X^{(k)}_{1\alpha}, \ldots, X^{(k)}_{p\alpha})'$ then gives rise to the rank vector $R^{(k)}_{\alpha} = (R^{(k)}_{1\alpha}, \ldots, R^{(k)}_{p\alpha})'$, $\alpha = 1, \ldots, n_k$, $k = 1, \ldots, c$. The N rank vectors corresponding to the N observation vectors, $N = n_1 + n_2 + \ldots. + n_c$, can be represented by the rank matrix

$$R^{p\times N}_N = \begin{pmatrix} R^{(1)}_{11} & . & . & . & R^{(1)}_{1n_1} & . & . & . & R^{(c)}_{1n_c} \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ R^{(1)}_{p1} & . & . & . & R^{(1)}_{pn_1} & . & . & . & R^{(c)}_{pn_c} \end{pmatrix}.$$

Each row of this matrix is a random permutation of the numbers $1, 2, \ldots, N$. Thus, $R^{p\times N}_N$ is a random matrix which can have $(N!)^p$ possible realizations. Two rank matrices of the above form are said to be permutationally equivalent if one can be obtained from the other by a rearrangement of its columns. Thus, a matrix R_N is permutationally equivalent to another matrix R^*_N which has the same column vectors as in R_N, but arranged so that the first row of R^*_N consists of the numbers $1, 2, \ldots, N$ in natural order, i.e.,

$$R^{*p\times N}_N = \begin{pmatrix} 1 & 2 & . & . & . & N \\ R^*_{21} & . & . & . & . & R^*_{2N} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ R^*_{p1} & . & . & . & . & R^*_{pN} \end{pmatrix}.$$

In order to perform a permutation rank order test, we start with a general class of rank scores defined by explicitly known functions of the ranks 1, . . . , N, viz.,

$$E^{(i)}_{N,\alpha} = \left(\frac{\alpha}{N+1}\right),$$
$$1 \le \alpha \le N, \quad i = 1, \ldots, p.$$

Now, replacing the ranks $R^{(k)}_{i\alpha}$ in R_N by $E^{(i)}_{N,R^{(k)}_{i\alpha}}$, for all $i = 1, \ldots, p$, $\alpha = 1, \ldots, n_k$, $k = 1, \ldots, c$, we get a corresponding $p \times N$ matrix of general scores, which we denote E_N. Thus,

$$E_N = \begin{pmatrix} E^{(1)}_{N,R^{(1)}_{11}} & \cdot & \cdot & E^{(1)}_{N,R^{(1)}_{1n_1}} & \cdot & \cdot & E^{(1)}_{N,R^{(c)}_{1n_c}} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ E^{(p)}_{N,R^{(1)}_{p1}} & \cdot & \cdot & E^{(p)}_{N,R^{(1)}_{pn_1}} & \cdot & \cdot & E^{(p)}_{N,R^{(c)}_{pn_c}} \end{pmatrix}$$

We then consider the average rank scores for each i (=1, . . . , p) of the c samples, defined by

$$T^{(k)}_{N_i} = \frac{1}{n_k}\sum_{\alpha=1}^{n_k} E^{(i)}_{N,R^{(k)}_{i\alpha}},$$
$$k = 1, \ldots, c, \quad i = 1, \ldots, p.$$

Then, by straightforward computation,

$$v_{ij}(R^*_N) = \frac{1}{N}\sum_{k=1}^{c}\sum_{\alpha=1}^{n_k} E^{(k)}_{N\alpha,i}E^{(q)}_{N\alpha,j} - \overline{E}^{(i)}_N\overline{E}^{(j)}_N$$

can be obtained, where $E^{(k)}_{N\alpha,i}$ is the value of $E^{(i)}_{N,S}$ associated with the rank $S = R^{(k)}_{i\alpha}$, and

$$\overline{E}^{(i)}_N = \sum_{\alpha=1}^{N} E^{(i)}_{N,\alpha}/N, \quad i = 1, \ldots, p.$$

Now denoting

$$V(R^*_N) = (v_{ij}(R^*_N))_{i,j=1,\ldots,p},$$

and following the structure of the test asymptotically equivalent to the likelihood ratio test based on Lawley-Hotelling's generalized T^2 statistic, we take as our test statistic $\mathcal{L}_N$,

$$\mathcal{L}_N = \sum_{k=1}^{c} n_k\left[\left(T^{(k)}_N - \overline{E}_N\right)V^{-1}(R^*_N)\left(T^{(k)}_N - \overline{E}_N\right)'\right],$$

where $V^{-1}(R^*_N) = (v_{ij}(R^*_N))^{-1}$, $T^{(k)}_N = (T^{(k)}_{N1}, \ldots, T^{(k)}_{Np})$, and $\overline{E}_N = (\overline{E}^{(1)}_N, \ldots, \overline{E}^{(p)}_N)$.

Here p is the number of parameters, c is the number of populations, $N = \sum_{i=1}^{c} n_i, n_i$ is the sample size of the ith sample, $i = 1, \ldots, c$, $m_H = c - 1$, and $m_E = n - c$. The statistic $\mathcal{L}_N$ can be approximated by $m_E cF$, where F follows the F distribution with a, b degrees of freedom. Here

$$a = pm_H, \quad b = 4 + \frac{a+2}{B-1}, \quad c = \frac{a(b-2)}{b(m_E - p - 1)},$$

where

$$B = \frac{(m_E + m_H - p - 1)(m_E - 1)}{(m_E - p - 3)(m_E - p)}.$$

This approximation was done by McKeon (1974). In order to compute the value of the statistic, we have used the R-code program.

TABLE 2

Nonparametric Tests: MW Cluster 1, 2, and 3 vs. Canis Major Dwarf GCs (Level of Significance = 0.05)

Parameter Set	MW Cluster 1 vs. CMa Dwarf	MW Cluster 2 vs. CMa Dwarf	MW Cluster 3 vs. CMa Dwarf
([Fe/H], HBR).........	Rejected	Accepted	Accepted
([Fe/H], $R_{\rm GC}$)...........	Rejected	Rejected	Accepted
([Fe/H], M_V)............	Rejected	Accepted	Accepted
([Fe/H], age)............	Rejected	Rejected	Accepted
(HBR, $R_{\rm GC}$)..............	Rejected	Accepted	Accepted
(HBR, M_V)...............	Rejected	Accepted	Accepted
(HBR, age)...............	Accepted	Rejected	Accepted
($R_{\rm GC}$, M_V)	Rejected	Accepted	Accepted
($R_{\rm GC}$, age).................	Accepted	Rejected	Accepted
(M_V, age)	Accepted	Rejected	Accepted
Overall......................	70% rejection	50% rejection	50% acceptance

4. RESULTS AND DISCUSSIONS

In previous studies (Forbes et al. 2004), the origin of Canis Major GCs was concluded to be different from that of MW GCs on the basis of two-point correlation studies of different parameters such as age, metallicity, galactocentric distance, magnitude, horizontal branch morphology, and half-mass radius. The present approach differs from these in that a nonparametric technique is used for the comparison, which is more appropriate in a multivariate analysis, allowing more than one parameter to be used simultaneously for the required comparison. Furthermore, our comparison is carried out between different groups of GCs of the MW (classified according to CC07) and the Canis Major dwarf galaxy, rather than taking all GCs of the MW together as a single sample. This makes the analysis more precise, as the nature of the GCs of MW disk and inner and outer halo are completely different. Disk GCs have high metallicity, low core radii, are close to the Galactic center, and have substantial rotation. Inner halo GCs have the lowest metallicity, low core radii, and very small rotation. Outer halo GCs are farthest from the Galactic center, with still lower metallicity, high core radii, and substantial velocity dispersion (CC07; Zinn 1993). It is clear from Table 2 that disk GCs of the MW (cluster 1) are completely different in almost all respects (in 7 out of 10 cases the equality of location test fails). The inner halo is similar in all other respect except age and metallicity, while outer halo (cluster 3) are similar in all respect. Forbes et al. (2004) also concluded that the age-metallicity relation (AMR) of Canis Major dwarf galaxy GCs is completely distinct from that of Milky Way GCs. Here it can be seen from Table 2 that the test for age-metallicity combination has been rejected for both clusters 1 and 2, but is accepted for cluster 3 of Milky Way GCs with Canis Major GCs. So the present analysis indicates that MW outer halo GCs have the same origin as Canis Major dwarf galaxy GCs. The same conclusion has been drawn by many authors for other dwarf galaxies (Mackey & Gilmore 2004); it is proved in a more objective way in the present analysis. As a verification, when the analysis is carried out with disk GCs (cluster 1) of the MW with those in Sample 4, the test fails in the majority of cases (Col. [2] of Table 2, 70% rejection).

TABLE 3

Nonparametric Tests: MW Cluster 1, 2, and 3 vs. Fornax Dwarf GCs (Level of Significance = 0.05)

Parameter Set	MW Cluster 1 vs. Fornax Dwarf	MW Cluster 2 vs. Fornax Dwarf	MW Cluster 3 vs. Fornax Dwarf
(HBR, [Fe/H])	Rejected	Rejected	Rejected
(HBR, R_c)	Rejected	Rejected	Accepted
(HBR, μ_V)	Rejected	Rejected	Rejected
(HBR, age)	Accepted	Accepted	Accepted
([Fe/H], R_c)	Rejected	Rejected	Rejected
([Fe/H], μ_V)	Rejected	Rejected	Rejected
([Fe/H], age)	Accepted	Accepted	Accepted
(R_c, μ_V)	Accepted	Rejected	Rejected
(R_c, age)	Accepted	Accepted	Accepted
(μ_V, age)	Accepted	Rejected	Accepted
([Mg/Fe], R_c)	Accepted	Accepted	Accepted
([Mg/Fe], μ_v)	Accepted	Accepted	Accepted
([Ca/Fe], R_c)	Accepted	Accepted	Accepted
([Ca/Fe], μ_v)	Accepted	Accepted	Accepted
Overall	36% rejection	50% rejection	65% acceptance

A comparison among GCs of LMC, Fornax, and Sagittarius dwarf spheroidal galaxies has been carried out by Mackey & Gilmore (2004). They studied only the core radii of GCs in these three galaxies. The three distributions match within the limits of measurements as a result of a Kolmogorov-Smirnov (K-S) test. That is why a multivariate nonparametric test is performed including more than one parameter. It is found that in more than half of the cases (9 out of 14; see Table 3) the test is accepted for MW cluster 3 GCs and Fornax dwarf galaxy GCs. For MW cluster 2 GCs there is dissimilarity in horizontal branch morphology, metallicity, core radii, and central surface brightness parameters (rejection is 50%), while the ages and abundances are comparable. Thus, it is more likely that the GCs of the outer halo (cluster 3) of the MW may have been accreted from the Fornax dwarf galaxy. However, since there is also some acceptance for disk GCs, a firmer conclusion could be drawn if the sample sizes under consideration are comparable as well as moderate.

Mackey & Gilmore (2004) have compared the core radii of GCs in LMC, Fornax, and Sagittarius with those in the younger halo (outer halo in our case) of the Milky Way and found that they have similar distribution. Hence, it becomes interesting to compare GCs in the LMC and MW in different regions separately in a multivariate set up. CC07 have made a classification of LMC GCs, and there the optimum number of homogeneous groups is two (clusters 1 and 2). GCs in cluster 1 are metal-poor and younger (mean age $\sim 10^{8.09}$ yr), compared to GCs in cluster 2, which are metal-rich and older (mean age $\sim 10^{10.20}$ yr). The comparison is made for LMC clusters 1 and 2 with Milky Way clusters 1, 2, and 3. It is found that (Table 4) LMC cluster 1 GCs are different in almost all respects from MW GCs, but cluster 2 (i.e., the older GCs of the LMC) has some resemblance with the GCs in the disk part of the MW (MW cluster 1; 67% acceptance). This result differs from the univariate approach, and it is very likely that the old GCs of the LMC comprise a disk system. The study will be more conclusive if the observed sample size of LMC GCs (23 in the present case) is comparable to the number of MW GCs ($\sim$142), which it is not in the present case.

Similar comparisons have been carried out with M33 GCs (Larsen et al. 2002) and GCs of dwarf spheroidal galaxies NGC 147, NGC 185, and NGC 205 (Sharina et al. 2006), which are considered to be satellite galaxies of the central massive galaxy M31. The results have been listed in Tables 5, 6, 7, and 8, respectively. The GCs of M33 closely resemble GCs of the MW outer halo, while the GCs of M31's other satellite galaxies are similar to the three subsystems of the MW; the similarity is greatest (100%) in the case of the outer halo. Here also the sample sizes are small. The values of the heavy element abundance ratio ([Mg/Fe]) of the GCs are not available, but can be calculated using Lick indices (Thomas et al. 2003). Since the values of [α/Fe] are already listed there, we have used this parameter for comparison.

Olsen et al. (2004) carried out a comparison of GCs in the Sculptor group and MW. They found that the luminosity functions are similar, but GCs in the Sculptor group are comparatively metal poor. In the present study the nonparametric test shows (Table 9) that cluster 3 GCs are similar to Sculptor group GCs, i.e., the joint distribution (luminosity, metallicity) of GCs in both galaxies have the same parent distribution, strengthening the evidence for this general trend. Some observations (5 GCs) of heavy element abundance ratio (e.g., [Mg/Fe]) are available for Sculptor dwarf galaxy GCs (Olsen et al. 2004). The same abundance ratio and many more are available for some GCs (40)

TABLE 4

Nonparametric Tests: MW vs. LMC GCs (Level of Significance = 0.05)

Parameter Set	MW Cluster 1 vs. LMC Cluster 1	MW Cluster 2 vs. LMC Cluster 1	MW Cluster 3 vs. LMC Cluster 1	MW Cluster 1 vs. LMC Cluster 2	MW Cluster 2 vs. LMC Cluster 2	MW Cluster 3 vs. LMC Cluster 2
(R_c, age)	Rejected	Rejected	Rejected	Accepted	Rejected	Rejected
(R_c, c)	Accepted	Rejected	Rejected	Accepted	Accepted	Rejected
(R_c, μ_v)	Accepted	Rejected	Rejected	Rejected	Rejected	Rejected
(R_c, [Fe/H])	Rejected	Rejected	Rejected	Rejected	Accepted	Rejected
(R_c, $M_V(t)$)	Accepted	Rejected	Rejected	Accepted	Rejected	Rejected
(age, c)	Rejected	Rejected	Rejected	Accepted	Rejected	Rejected
(age, μ_v)	Rejected	Rejected	Rejected	Accepted	Rejected	Rejected
(age, [Fe/H])	Rejected	Rejected	Rejected	Accepted	Rejected	Accepted
(age, $M_V(t)$)	Rejected	Rejected	Rejected	Accepted	Rejected	Accepted
(c, μ_v)	Accepted	Rejected	Rejected	Accepted	Accepted	Rejected
(c, [Fe/H])	Rejected	Rejected	Rejected	Rejected	Accepted	Accepted
(c, $M_V(t)$)	Rejected	Rejected	Rejected	Accepted	Rejected	Accepted
(μ_v, [Fe/H])	Rejected	Rejected	Rejected	Rejected	Accepted	Rejected
(μ_v, $M_V(t)$)	Accepted	Rejected	Rejected	Accepted	Accepted	Rejected
([Fe/H], $M_V(t)$)	Rejected	Rejected	Rejected	Rejected	Accepted	Accepted
Overall	67% rejection	100% rejection	100% rejection	67% acceptance	53% rejection	67% rejection

TABLE 5

Nonparametric Tests: MW vs. M33 GCs (Level of Significance = 0.05)

Parameter Set	MW Cluster 1 vs. M33	MW Cluster 2 vs. M33	MW Cluster 3 vs. M33
(r_c, μ_0)	Accepted	Accepted	Rejected
$(r_c, V-I)$	Rejected	Accepted	Accepted
$(\mu_0, V-I)$	Rejected	Accepted	Accepted
Overall	67% rejection	100% acceptance	67% acceptance

in the MW also (Pritzi et al. 2005). A multivariate comparison including the abundance ratio shows that Sculptor dwarf galaxy GCs are similar in all respect to the outer halo GCs of the Milky Way (last column of Table 9), whereas the rejection is pronounced for MW disk and inner halo GCs.

Thus, we conclude that GCs in the outer halo do not play a part in building the stellar mass of the disk and inner halo, but rather are accreted from neighboring dwarf spheroidal galaxies. This is also suggested by Mackey & Gilmore (2004) on the basis of a single parameter (HBR) in case of the Fornax dwarf galaxy, and by Olsen et al. (2004) for the Sculptor dwarf, as discussed above. Geisler et al. (2007) studied the kinematics and detailed chemical abundances of stars in some relevant Galactic GCs as well as Local Group dwarf galaxies (e.g., Fornax and others). They found that outer halo red HB clusters tend to have large eccentricities and inhabit the area of the Lee diagram ([Fe/H] vs. HBR) populated by dwarf Spheroidal stars, favoring an extragalactic origin, and a detailed abundance analysis (α vs, [Fe/H], [O/Fe] vs, [Fe/H]) shows that they are different from the Galactic halo. Sgr appears to be the only possible exception. However, at least some of the metal-poor halo may have come from typical dSphs, and a portion of the intermediate-metallicity and metal-rich halo could have come from very massive systems such as Sgr. The metallicity of the GCs associated with the Sgr dwarf galaxy is faint (van den Bergh 2007). The majority of the GCs in the outer halo of the Galaxy are also subluminous; this may be due to the cannibalism of faint cluster-rich dwarf subsystems such as Sgr. Sharina et al. (2005) studied 57 low surface brightness dwarf galaxies ($-10 > M_v > -16$) and compared them with the outer halo GCs of the Galaxy. They found that the mean integral colors of GCs in dSphs coincides with the corresponding value of Galactic GCs. The color distribution of dIrrs shows bimodality near $(V-I)_0 = 0.5$ and 1.0 mag. The detected GCs have visual magnitudes between $M_v = -10$ and -5 mag. There is an excess population of faint GCs with $M_v > -6.5$ in both dSph and dIrr

TABLE 6

Nonparametric Tests: MW vs. NGC 147 GCs (Level of Significance = 0.05)

Parameter Set	MW Cluster 1 vs. NGC 147	MW Cluster 2 vs. NGC 147	MW Cluster 3 vs. NGC 147
$(M_V, B-V)$	Rejected	Rejected	Accepted
(M_V, [Fe/H])	Accepted	Accepted	Accepted
(M_V, Mg 1)	Accepted	Accepted	. . .
(M_V, Mg 2)	Accepted	Accepted	. . .
(M_V, Mg b)	Accepted	Accepted	. . .
(M_V, [α/Fe])	Accepted	Accepted	Accepted
($B-V$, [Fe/H])	Rejected	Rejected	Accepted
($B-V$, Mg 1)	Accepted	Accepted	. . .
($B-V$, Mg 2)	Accepted	Accepted	. . .
($B-V$, Mg b)	Accepted	Accepted	. . .
($B-V$, [α/Fe])	Accepted	Accepted	Accepted
([Fe/H], Mg 1)	Accepted	. . .	. . .
([Fe/H], Mg 2)	Accepted	. . .	. . .
([Fe/H], Mg b)	Accepted	. . .	. . .
([Fe/H], [α/Fe])	Accepted	Accepted	Accepted
(Mg 1, Mg 2)	Accepted	Accepted	. . .
(Mg 1, Mg b)	Accepted	Accepted	. . .
(Mg 2, Mg b)	. . .	Accepted	. . .
Overall	88% acceptance	93% acceptance	100% acceptance

TABLE 7

Nonparametric Tests: MW vs. NGC 185 GCs (Level of Significance = 0.05)

Parameter Set	MW Cluster 1 vs. NGC 185	MW Cluster 2 vs. NGC 185	MW Cluster 3 vs. NGC 185
$(M_V, B-V)$	Rejected	Rejected	Accepted
(M_V, [Fe/H])	Rejected	Accepted	Accepted
(M_V, Mg 1)	Accepted	Accepted	. . .
(M_V, Mg 2)	Accepted	Accepted	. . .
(M_V, Mg b)	Accepted	Accepted	. . .
(M_V, [α/Fe])	Accepted	Accepted	Accepted
($B-V$, [Fe/H])	Rejected	Accepted	Accepted
($B-V$, Mg 1)	Accepted	Accepted	. . .
($B-V$, Mg 2)	Accepted	Accepted	. . .
($B-V$, Mg b)	Accepted	Accepted	. . .
($B-V$, [α/Fe])	Accepted	Accepted	Accepted
([Fe/H], Mg 1)	Accepted	Accepted	. . .
([Fe/H], Mg 2)	Accepted	Accepted	. . .
([Fe/H], Mg b)	Accepted	Accepted	. . .
([Fe/H], [α/Fe])	Accepted	Accepted	Accepted
(Mg 1, Mg 2)	Accepted	Accepted	. . .
(Mg 1, Mg b)	Accepted	Accepted	. . .
(Mg 1, [α/Fe])	Accepted	. . .	. . .
(Mg 2, Mg b)	Accepted	Accepted	. . .
(Mg 2, [α/Fe])	Accepted	. . .	. . .
(Mg b, [α/Fe])	Accepted	. . .	. . .
Overall	86% acceptance	94% acceptance	100% acceptance

TABLE 8

Nonparametric Tests: MW vs. NGC 205 GCs (Level of Significance = 0.05)

Parameter Set	MW Cluster 1 vs. NGC 205	MW Cluster 2 vs. NGC 205	MW Cluster 3 vs. NGC 205
$(M_V, B-V)$	Rejected	Accepted	Accepted
(M_V, [Fe/H])	Rejected	Accepted	Accepted
(M_V, Mg 1)	Accepted	Accepted	. . .
(M_V, Mg 2)	Accepted	Accepted	. . .
(M_V, Mg b)	Accepted	Accepted	. . .
(M_V, [α/Fe])	Accepted	Accepted	Accepted
($B-V$, [Fe/H])	Rejected	Accepted	Accepted
($B-V$, Mg 1)	Accepted	Accepted	. . .
($B-V$, Mg 2)	Accepted	Accepted	. . .
($B-V$, Mg b)	Accepted	Accepted	. . .
($B-V$, [α/Fe])	Accepted	Accepted	Accepted
([Fe/H], Mg 1)	Accepted	Accepted	. . .
([Fe/H], Mg 2)	Accepted	Accepted	. . .
([Fe/H], Mg b)	Accepted	Accepted	. . .
([Fe/H], [α/Fe])	Accepted	Accepted	Accepted
(Mg 1, Mg 2)	Accepted	Accepted	. . .
(Mg 1, Mg b)	Accepted	Accepted	. . .
(Mg 1, [α/Fe])	Accepted	. . .	. . .
(Mg 2, Mg b)	Accepted	Accepted	. . .
(Mg 2, [α/Fe])	Accepted	. . .	. . .
(Mg b, [α/Fe])	Accepted	. . .	. . .
Overall	86% acceptance	100% acceptance	100% acceptance

TABLE 9

NONPARAMETRIC TESTS: MW CLUSTER 1, 2 AND 3 VS. SCULPTOR DWARF GCs (LEVEL OF SIGNIFICANCE = 0.05)

Parameter Set	MW Cluster 1 vs. Sculptor Dwarf	MW Cluster 2 vs. Sculptor Dwarf	MW Cluster 3 vs. Sculptor Dwarf
(M_{T_1}, [Fe/H]$_{(C-T_1)}$)	Rejected	Rejected	Accepted
(M_{T_1}, [Mg/Fe])	Accepted	Rejected	Accepted
([Fe/H]$_{(C-T_1)}$, [Mg/Fe])	Rejected	Rejected	Accepted
Overall	67% rejection	100% rejection	100% acceptance

galaxies. The structural parameters measured using King profiles are similar to the structural parameters of outer halo GCs of the Milky Way, M31, and recently discovered "faint fuzzy" clusters of lenticular galaxies. All these results strongly support the present multivariate analysis.

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