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Generation of axial magnetic fields in laser-produced plasmas

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Axial magnetic fields earlier observed in laser-produced plasma experiments can be generated in the case of nonlinear interactions of lasers with under dense plasmas. It has been shown that for high-frequency laser [viz. Nd-glass laser having the wavelength $(\lambda) = 1.06\ \mu$m and the power flux $(P) = 10^{16}$ W/cm^2] the background dc magnetic field would be important only when electron cyclotron frequency is comparable to the laser frequency.

I. INTRODUCTION

There has been considerable interest in the spontaneous generation of magnetic fields in plasma under the action of intense electromagnetic radiations. A great variety of physical mechanisms have been proposed for the generation of such fields in megagauss range. The important mechanisms proposed until recently for creation of laser-generated toroidal magnetic fields are[1] (i) fields due to thermoelectric effects, (ii) fields due to resonance absorptions, and (iii) fields due to magnetic instabilities. Experimental evidence towards the generation of such toroidal fields have been well established.[2,3] Yabe *et al.*[4] have pointed out the mechanism of large-scale magnetic fields in laser-produced plasmas. Such fields have been evaluated from small-scale fluctuation of target surface or laser beam; and the dynamo effect of such small-scale fluctuation leads to large-scale magnetic fields which can be compared to the thermoelectric effects. But not much attention has been devoted to the mechanisms for generation of axial magnetic fields which are the fields directed along the axis of the target normal that is along the laser beam. Briand *et al.*[5] first pointed out the existence of self-generated axial magnetic fields in laser-produced plasmas through dynamo effects. They experimentally measured such fields in the megagauss range by Faraday rotation on the backscattered emission processes. Dragila[6] has explained the physical meaning of dynamo effect in laser-produced plasmas for the production of such fields. These fields can be generated by the action of turbulent dynamo in the presence of ion-acoustic turbulence.[6] Magnetic fields arising from the nonlinear response of laser produced plasmas to optical frequency fields have been studied[7-9] theoretically in terms of nonlinear precessional rotations and frequency shifts of high intensity elliptically polarized laser fields.

In this paper, we shall consider the mechanism which indicates that axial magnetic fields can be generated in a nonlinear medium (even if its density and temperature are not changed). This turns out to be possible if the loss of angular momentum of the system of charged particles in underdense plasmas due to radiation fields can be estimated because in the presence of electromagnetic radiations, the charged particles will gyrate in circular orbits. These gyrations of charged particles of underdense plasmas give rise to the axial magnetic fields along the direction of radiation propagation. In terms of nonlinear angular momentum of charged particles for underdense plasmas, the generation of axial magnetic fields is to be studied. When the radiation fields are strong enough, these axial fields are independent of the large-scale background dc magnetic field. Such generation fields are proportional to the square of the dimensionless intensity of the laser fields for all powers, and plasma densities. But if the background dc magnetic field would be such that the electron cyclotron frequency is comparable to the wave frequency, only then the said dc field would play an important role for the production of more magnetic fields in laser produced plasmas.

II. FORMULATIONS OF THE PROBLEM

A. Basic assumptions and field equations

Consider a uniform electron plasma in which an elliptically polarized electromagnetic wave propagates along the z-axis and is parallel to the background dc magnetic field (H_0). These radiation fields are strong enough so that the average electron motion becomes relativistic but the ion motion is neglected.[10] But the power of the radiation does not exceed the threshold power limit, so the nonlinear instabilities due to stimulated Raman scattering (SRS) driven by the strong radiation can be minimized. Also stimulated Brillouin scattering (SBS) associated with pressure variation would be insignificant through electrostriction, because the mechanism is much slower (relaxation time $\tau = 10^{-8}$ to 10^{-9} s) compared to the nonlinear electronic polarizability $(\tau = 10^{-15}$ s). The effects of nondispersive longitudinal oscillations propagating at sound speed are neglected because thermal velocity in plasmas is assumed to be insignificant.

Under the above assumptions, the following field equations as well as Maxwell equations are to be used to describe the interaction of electromagnetic wave with a plasma:

$$\frac{\partial N}{\partial t} + \nabla\cdot(\mathrm{N}\mathbf{v}) = 0, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + (\mathbf{v}\cdot\nabla)\right)\mathbf{p} = -e\mathbf{E} - e(\mathbf{v}\times\mathbf{H})/c, \tag{2}$$

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \tag{3}$$

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$$\nabla \times \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} - \frac{4\pi e}{c} N v, \quad (4)$$

$$\nabla \cdot \mathbf{E} = 4\pi e (N - N_0), \quad (5)$$

$$\nabla \cdot \mathbf{H} = 0, \quad (6)$$

where $\mathbf{p} = m\gamma v$ is the relativistic mementum and γ is the Lorentz factor of electrons. The other notations have their usual meanings for motion of electrons of charge e and mass m. Also conventional field variables are considered.

B. First-and second-order field solutions

The given field equations are considered for the closed form harmonic wave solutions by successive approximation. In this scheme, the linear solution is regarded as the first-order approximation, the corresponding field variables bear the subscript 1, and the fields in the unperturbed state bear the subscript 0. Accordingly, the nonlinearly excited fields of approximations of the second and third orders bear the subscripts 2 and 3, respectively, and so on.

Let us consider the first harmonic propagating electromagnetic waves of electric fields as

$$E_{1+} = (\alpha e^{-i\theta_R} + \beta e^{i\theta_L})/(m\omega c/e) \quad (7)$$

and

$$E_{1-} = (\alpha e^{i\theta_R} + \beta e^{-i\theta_L})/(m\omega c/e), \quad (8)$$

where the subscripts R and L represent the right- and left-circularly polarized components of electric fields $E_{\pm}$;

$$E_{1+} = E_{1x} \pm iE_{1y}; \theta_R (= K_R z - \omega t)$$

and $\theta_L (= K_L z - \omega t)$ are, respectively, the phase values of the waves; $i = \sqrt{-1}$; $\alpha, \beta (= e[a,b]/m\omega c)$ the dimensionless amplitudes of the waves having frequency ω.

Using linearized wave solutions of (7) and (8) in the field equations of (1)–(6), we have the linearized dispersion relations as

$$n_R^2 = 1 - X/(1 - Y), \quad (9a)$$

$$n_L^2 = 1 - X/(1 + Y), \quad (9b)$$

where

$$n_{R,L} = K_{R,L} c/\omega, \quad Y = \Omega/\omega, \quad \Omega = eH_0/mc.$$

Also the second-order secular free stationary solutions are

$$v_{2Z} = (CS/X)\alpha\beta e^{i(\theta_R + \theta_L)} + \mathrm{CC}, \quad (10a)$$

$$N_2 = [N_0 S(n_R + n_L)/2X]\alpha\beta e^{i(\theta_R + \theta_L)} + \mathrm{CC}, \quad (10b)$$

where

$$S = [(n_R^2 - 1)n_L + (n_L^2 - 1)n_R]/(4 - X);$$

$$X = \omega_p^2/\omega^2; \quad \omega_p^2 = 4\pi e^2 N_0/m,$$

and CC = complex conjugate.

C. Nonlinear polarization

It is evident that the wave at the second harmonic frequency is longitudinal, and there is no power flow associated with it. Hence the propagation energy is affected by nonlinearities only in the third-order approximation.[11] The differential equations for E_+ and E_- correct up to third order, are

$$\left[\left(\frac{\partial}{\partial t} - i\Omega\right)\left(\frac{\partial^2}{\partial t^2} - \frac{c^2\partial^2}{\partial z^2}\right) + \frac{w_p^2\partial}{\partial t}\right]E_+ = -4\pi e N_0 \frac{\partial}{\partial t}\left[v_{2Z}\frac{\partial v_{1+}}{\partial Z} - \frac{ie}{mc} v_{2Z} H_{1+} + \frac{\partial}{\partial t}\frac{v_{1+}^2 v_{1-}}{2c^2} - \frac{1}{N_0}\left(\frac{\partial}{\partial t} - i\Omega\right)N_2 v_{1+}\right] \quad (11)$$

and

$$\left[\left(\frac{\partial}{\partial t} + i\Omega\right)\left(\frac{\partial^2}{\partial t^2} - \frac{c^2\partial^2}{\partial Z^2}\right) + \frac{w_p^2\partial}{\partial t}\right]E_- = -4\pi e N_0 \frac{\partial}{\partial t}\left[v_{2Z}\frac{\partial v_{1-}}{\partial Z} + \frac{ie}{mc} v_{2Z} H_{1-} + \frac{\partial}{\partial t}\frac{v_{1-}^2 v_{1+}}{2c^2} - \frac{1}{N_0}\left(\frac{\partial}{\partial t} + i\Omega\right)N_2 v_{1-}\right]. \quad (12)$$

Right-hand sides of (11) and (12) contain all nonlinear terms which are coming from Lorentz force, plasma current, relativistic electrons motion due to intense radiations, etc. Solving the nonlinear equations (11) and (12) for secular free solution and retaining only the first harmonic terms correct up to third order, we get the nonlinear flow velocity as

$$v_+ = -ic[(\sigma_{\mathrm{NRE}} + \sigma_{\mathrm{RE}})e^{i\theta_R} + (\delta_{\mathrm{NRE}} + \delta_{\mathrm{RE}})e^{i\theta_L}], \quad (13)$$

$$v_- = ic[(\sigma_{\mathrm{NRE}} + \sigma_{\mathrm{RE}}e^{i\theta_R} + (\delta_{\mathrm{NRE}} + \delta_{\mathrm{RE}})e^{i\theta_L}], \quad (14)$$

where

$$\sigma_{\mathrm{NRE}} = [(P_L + Q_L)(\tau_R - 1) + \tau_R T_L]\alpha\beta^2,$$

$$\sigma_{\mathrm{RE}} = -(\tau_R - 1)U_R\alpha,$$

$$\delta_{\mathrm{NRE}} = -[(P_R + Q_R)(\tau_L - 1) + \tau_L T_R]\alpha^2\beta,$$

and

$$\delta_{\mathrm{RE}} = (\tau_L + 1)U_L\beta.$$

The subscripts NRE and RE represent the contributions of nonrelativistic and relativistic effects, respectively, for the system and other symbols are

$$P_L = n_L(n_L^2 - 1)S/X^2, \quad P_R = n_R(n_R^2 - 1)S/X^2,$$

$$Q_L = Sn_L/X, \quad Q_R = Sn_R/X,$$

$$T = wz(1 + Y)(n_L^2 + 1)/X,$$

$$T_R = wz(1 - Y)(n_R^2 - 1)/X,$$

$$w = S(n_R + n_L)/2x, \quad z = (1 - Y)/(1 + Y),$$

$$U_R = [n_R^2 - 1)/2X^3][(n_R^2 - 1)^2\alpha^2 + 2(n_L^2 - 1)^2\beta^2],$$

$$U_L = [(n_L^2 - 1)/2X^3][(2n_R^2 - 1)^2\alpha^2 + 2(n_L^2 - 1)^2\beta^2],$$

$$\tau_R = X/2n_R\delta n_R(1 - Y), \quad \tau_L = X/2n_L\delta n_L(1 + Y),$$

$$\delta n_R = (X/2n_R)[(P_L + Q_L + T_L)\beta^2 - U_R],$$

$$\delta n_L = (X/2n_L)[(P_R + Q_R + T_R)\alpha^2 - U_L].$$

D. Nonlinear dispersion relation

The study of evolution of nonlinear effects, for waves of different frequencies, is possible from Eqs. (11) and (12). Solving these equations, we have the following nonlinear dispersion relations,

$$(n_R^2 - 1)(1 - Y) + X = X[P_L + Q_L + zw)\beta^2 - U_R], \tag{15}$$

$$(n_L^2 - 1(1 + Y) + X = X[P_R + Q_R + z^{-1}w)\alpha^2 - U_L]. \tag{16}$$

Equations (15) and (16) show that the left- and right-circularly polarized waves are coupled together by the nonlinear sources. Moreover, the general behavior of the nonlinear wave processes in the magnetized plasma can be properly understood from these relations.

E. Angular momentum and axial magnetic fields

The expressions of nonlinear electron current densities are $J_+ = eN_0 v_+$ and $J_- = eN_0 v_-$. Due to flow of current densities $J_\pm$ and flow velocities $v_\pm$, the displacement vector acquired due to nonlinear interactions of high-intensity fields with underdense plasmas can be evaluated as $r_\pm$. Hence, the nonlinear increment to the angular momentum per unit mass, average over the time period $2\pi/\omega$, is

$$\langle L \rangle = \langle J_\perp r_\perp \rangle / eN_0. \tag{17}$$

Therefore, the amount of magnetic field generated in the direction of propagation is

$$\langle M \rangle \equiv (4\pi e N_0/c)\langle L \rangle = -(4\pi \epsilon N_0 c/\omega)[(\sigma_{\rm NRE} + \sigma_{\rm RE})^2 - (\delta_{\rm NRE} + \delta_{\rm RE})^2] \text{ G}. \tag{18}$$

III. MAGNETIC FIELD FOR HIGH FREQUENCY

A. Result for high-frequency wave

When high-frequency laser plasma interactions are to be studied, we may have $X \ll 1$, $n_R^2 \simeq 1 \simeq n_L^2$, $Y \ll 1$, $\alpha^2 \gg \beta^2$ and consequently, Eq. (18) can be written as

$$\langle M \rangle = -16\pi c e N_0 \alpha^2/\omega \text{ G}. \tag{19}$$

It is evident from Eqs. (17) and (19) that the magnetic field, due to nonlinear angular momentum is axial, i.e., such a field directed along the axis of the laser beam, is proportional to the plasma densities and the square of the dimensionless intensity of laser fields. The negative sign indicates that the direction of such axial fields is opposite to the direction of propagation. Further, the above equation shows that more intense fields will be generated at the vicinity of the critical density region. In fact, the largest contribution to the nonlinear magnetization is obtained from the relativistic nonlinear corrections to the electron mass, because the nonlinear terms of nonrelativistic origin are comparatively insignificant. Moreover, the figure shows that our calculation fields at $N = 0.3N_c$ are same as that of the experimental results of Briand *et al.*[5] It has also been pointed out that in the region having a density less than 0.3 N_c the generated magnetic fields are decreasing monotonically, whereas in the region having a density greater than $0.3N_c$ the fields are acting reverse. Moreover, they are free from background dc magnetic field (H_0). But it should be noted that the background dc magnetic field may have some impact on those fields if the cyclotron frequeny of electron is comparable to the wave frequency that is $Y \simeq 1$. It could be studied in the next section.

B. Numerical results

We consider some realistic conditions needed to establish the feasibility of the proposed mechanism for generation of an axial magnetic field in experiments with laser produced plasmas, viz, we consider a region of underdense plasma with electron density $N = 0.01N_c$, where $N_c = 10^{21}$ is the critical density relevant to the radiation of Nd-glass laser with the wavelength $\lambda = 1.06\,\mu$m and power flux $P = 10^{16}$ W/cm^2 in the background magnetic field $H_0 = 10^5$ G. Consequently, we have $\omega = 1.78 \times 10^{15}$ s, $\alpha^2 = 0.005$, $X = 10^{-2}$, $Y = 10^{-2}$ and then

$$|\langle M \rangle| = 0.02 \times 10^6 \text{ G}.$$

IV. MAGNETIC FIELD FOR NEAR RESONANT

A. Frequency matching for enhancement of the effects

The first harmonic fields correct up to third order have been considered in this nonlinear process of which the nonlinearly correct dispersion relations (15) and (16) are complicated; and very difficult to interpret at $\omega = \Omega$ in view of the fact that (15) and (16) are multivalued. However, the dispersion relations (9a) and (9b), for linearized approximation, show that n_R will blow off at $\omega = \Omega$, i.e., R mode cannot propagate, whereas L mode remains unaffected. The nonlinearly correct condition of response for maximum energy absorption appears to be a problem of extremization of the sum of energies per unit volume of waves of a range of frequencies around ω absorbed by the medium from nonlinearly evolving fields. For simplicity, near-resonant frequency-matching conditions (i.e., electron cyclotron frequency Ω is comparable to the wave frequency ω) have been studied for which the nonlinear responses can be amplified. Thus,

$$1 + \mu = Y, \tag{20}$$

where

$$\mu = \epsilon/\omega,$$

ϵ is the small frequency difference but not greater than the frequency of the radiation field.

Using (20) and the relation $X \ll 1$ in (9a) and (9b), we have

$$n_R = 1 + X/2\mu \tag{21a}$$

and

$$n_L = 1. \tag{21b}$$

It is evident from (21a) and (21b) that both L and R modes are being allowed to propagate, and they are qualitatively similar but quantitatively they possess little difference.

Considering all the relations used in the Sec. III A and Eq. (20), Eq. (18) gives the expression of magnetization $\langle M_{\rm NR} \rangle$ as

$$\langle M_{\rm NR} \rangle = -\frac{4\pi e N_c}{\omega} \frac{\alpha^2}{\mu^2} \text{ G}. \tag{22}$$

The above expression gives the same interpretation as that of (19) except the fact that, in the near resonant case, the mag-

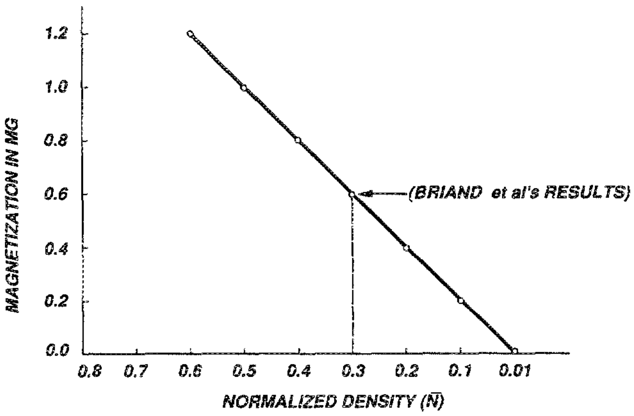

FIG. 1. Variation of magentization with normalized plasma density ($\bar{N} = N_0/N_c$) for $\lambda = 1.06\,\mu$m.

netization varies inversely with the square power of $\mu(=\epsilon/\omega)$, i.e., this magnetization is varying to the square power of λ since ω is inversely proportional to the wavelength λ in the dielectric medium. Hence, at $Y \simeq 1$, the background dc magnetic field has great influence in the generation of more such fields.

B. A numerical estimation

When the numerical data of an earlier section are used for laser plasma interaction in the presence of a dc magnetic field, we have for Nd-glass laser $\lambda = 1.06\ \mu$m, $P = 10^{16}$ W/cm^2 and for plasma in the region $\bar{N} = 0.01\ N_c$ (see Fig. 1). Then for $\epsilon = 1.8 \times 10^{14}$ rad/s and $\Omega = 1.96 \times 10^{15}$ rad/s, where a magnetic field of the order of 10^7 G is necessary. So we have $\mu = 0.101$ using those values of the parameters, Eq. (22) gives

$$\langle M_{\rm NR} \rangle = 1.96 \times 10^6\ {\rm G}.$$

It is evident that in the near resonant case the magnetization is about 10^2 times greater than that of the magnetization for nonresonant case in laser plasma interaction.

V. CONCLUSION

Nonuniform rotation—as observed in astrophysical bodies—is pictured as generating a toroidal field component from an initial poloidal field. To complete the dynamo cycle, there should be other motions which twist the toroidal field in such a way as to yield a new poloidal component.[12,13] This should be studied deeply elsewhere for achieving more axial, i.e., poloidal fields. It seems that the effect of such fields on strong pump waves can be utilized for nuclear spin polarization, and so for the control of the reaction rates and angular distribution of the reaction products. Bhaskar *et al.*[14] (see also Physics Today[15]) discussed the possibility of using polarized plasmas in fusion reactors by polarizing nuclear spin.

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