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Gauge-invariant dressed fermion propagator in massless QED$_3$

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Abstract

The infrared behaviour of the gauge-invariant dressed fermion propagator in massless QED$_3$ is discussed for three choices of dressing. It is found that only the propagator with the isotropic (in three Euclidean dimensions) choice of dressing is acceptable as the physical fermion propagator. It is explained that the negative anomalous dimension of this physical fermion does not contradict any field-theoretical requirement.

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Massless QED_3 in a $1/N$ expansion [1] (N being the number of fermion flavours) has the iR (infrared) behaviour of a conformal field theory in a class of non-local gauges [2, 3]. The photon has a gauge-invariant anomalous dimension 1/2 (scaling dimension one), while the fermion anomalous dimension can have any value depending linearly on the gauge parameter. As the fermion propagator is not gauge-invariant, we may argue that it is not relevant for physical observations. Nevertheless experiments do probe the properties of fermions, and the calculation of the anomalous dimension of gauge-invariant physical fermions in massless QED_3 has become important in relation to the cuprate superconductors [4, 5, 6]. In this paper, we discuss three choices for the gauge-invariant dressed fermion [7] and find that the propagator of only one of them can be identified with the physical fermion propagator. We also explain why the negative anomalous dimension of this physical fermion is allowed from field theory.

It has been proposed [4, 5] that the relevant observable for the physical fermion propagator is the 'stringy ansatz'

$$< \psi_\alpha(x) \exp\left(-ie\int_x^y dz^\mu A^\mu(z)\right)\bar{\psi}_\beta(y) > \tag{1}$$

with a 'Wilson line' connecting the fermion fields. This is gauge invariant, but depends explicitly on the path $\mathcal{C}$ with end-points x and y. Using a straight line connecting the end-points, it has been claimed that for large separation this has a power law behaviour

$$\frac{\not{x} - \not{y}}{|x-y|^{3+\eta}} \tag{2}$$

with an anomalous dimension η. A positive anomalous dimension could lead to a Luttinger type behaviour and possibly explain the ARPES data [8] . Unfortunately, an explicit calculation gives a negative value $\eta = -32/3\pi^2 N$ in the leading order in $1/N$ [9, 4, 10, 11, 12].

However, there are gauge-invariant observables other than the one given in eq. (1). Also, the choice of a straight line path between the end points is unwarranted. For the ultraviolet behaviour, the path is infinitesimally short, and we could expect that its choice does not matter for the leading behaviour. But we are interested in the iR behaviour, $|x-y| \to \infty$, where the path $\mathcal{C}$ becomes infinitely long and it would appear that the leading behaviour depends on the choice.

In addition to these objections, we find a serious problem with the choice of eq. (1). The iR behaviour is not a power law, but has an exponential damping, depending on a cutoff. This has to do with self-energy of the infinitely thin flux line $\mathcal{C}$, and is unavoidable (See Appendix). Thus it appears that we need to consider a more general gauge invariant dressing for the electron than given by eq. (1). We have to now ask which of these dressings is correct and how to compute the correlation function including fluctuations of the flux line.

Long back Dirac [13] presented the gauge-invariant fermion dressed by a photon cloud:

$$\Psi_\alpha(x) = \exp\left(-ie\int d^3z J^\mu(x,z)A^\mu(z)\right)\psi_\alpha(x)\,. \tag{3}$$

Here the current $J^\mu(x,z) \equiv J^\mu(x-z)$ satisfies

$$\partial_z^\mu J^\mu(x,z) = \delta^{(3)}(x-z) \tag{4}$$

as a consequence of which, under a local gauge transformation $A^\mu(x) \to A^\mu(x) - \partial^\mu\theta(x)$,

$$\exp\left(-ie\int d^3z J^\mu(x,z)A^\mu(z)\right) \to e^{-ie\theta(x)}\exp\left(-ie\int d^3z J^\mu(x,z)A^\mu(z)\right). \tag{5}$$

This transformation of the dressing factor compensates for the gauge transformation of $\psi(x)$. Note that many choices of J^μ satisfying eq. (4) are possible. The *isotropic* (in three-dimensional Euclidean space-time) *solution* is an evident choice:

$$J^\mu(x,z) = \partial_x^\mu\phi(|x-z|) = -\partial_z^\mu\phi(|x-z|)\,, \tag{6}$$

$$\phi(|x-z|) = \frac{1}{4\pi|x-z|}\,. \tag{7}$$

As Dirac wanted a dressed operator which is local in time (though non-local in space), he chose (we will refer to this choice as the *Dirac dressing*) [14]

$$J^3(\vec{x},x^3;\vec{z},z^3) = 0, \quad J^i(\vec{x},x^3;\vec{z},z^3) = \delta(x^3-z^3)j^i(\vec{x},\vec{z}) \quad (i=1,2) \tag{8}$$

$$j^i(\vec{x},\vec{z}) = \partial_x^i\phi(|\vec{x}-\vec{z}|) \tag{9}$$

In our case of $(2+1)$-dimensions, we get

$$\phi(|\vec{x}-\vec{z}|) = -\frac{1}{2\pi}\ln|\vec{x}-\vec{z}| \tag{10}$$

In contrast to these, choosing a *straight Wilson line* in eq. (1) means squeezing this Coulomb field around the electron into a line. If we take the line along the 3-direction of the Euclidean space-time, this choice of J^μ is given by

$$J^i(x,z) = 0, \quad J^3(x,z) = \delta^{(2)}(\vec{x}-\vec{z})\theta(z^3-x^3) \tag{11}$$

(this is explained after eq. (15)). It is interesting to note that the current of eq. (11) differs from the isotropic choice of eqs. (6) and (7) by precisely the curl of the vector potential used by Dirac for the magnetic monopole [15]. The infinitely squeezed flux (electric and not magnetic in the present case) is the Dirac string resulting from the singular vector potential.

The two-point correlation function which results from eq. (3) is

$$\mathcal{S}_{\alpha\beta}(x,y) = <\Psi_\alpha(x)\bar{\Psi}_\beta(y)> \tag{12}$$

$$= <\psi_\alpha(x)\exp\left(-ie\int d^3z\mathcal{J}^\mu(z)A^\mu(z)\right)\bar{\psi}_\beta(y)> \tag{13}$$

with

$$\mathcal{J}^\mu(z) \equiv \mathcal{J}^\mu(x,y;z) = J^\mu(x,z) - J^\mu(y,z)\,. \tag{14}$$

From eq. (4), it then follows that

$$\partial_z^\mu \mathcal{J}^\mu(z) = \delta^{(3)}(x-z) - \delta^{(3)}(y-z)\,. \tag{15}$$

[It may be noted that the use of eqs. (11) and (14) in eq. (13) gives

$$< \psi_\alpha(x)\bar{\psi}_\beta(y) \exp\left(-ie\int_{x_3}^{y_3} dz_3 A_3(z)\right) > \,. \tag{16}$$

Thus, as stated before, we have a straight Wilson line which lies along the 3-axis, i.e., x and y are separated only in the 3-direction.]

We may consider all three choices of J^μ mentioned above. It is revealing to express the exponent of eq. (13) using the Fourier components $\tilde{A}^\mu(l)$ of $A^\mu(z)$. The three choices correspond to

$$(1) \quad < \psi_\alpha(x)\bar{\psi}_\beta(y) \exp\left(-ie\int \frac{d^3l}{(2\pi)^3}(e^{il.x} - e^{il.y})\frac{il^\mu}{l^2}\tilde{A}^\mu(l)\right) >, \tag{17}$$

$$(2) \quad < \psi_\alpha(x)\bar{\psi}_\beta(y) \exp\left(-ie\int \frac{d^3l}{(2\pi)^3}(e^{il.x} - e^{il.y})\frac{i\vec{l}}{\vec{l}^2}\cdot\vec{\tilde{A}}(l)\right) >, \tag{18}$$

$$(3) \quad < \psi_\alpha(x)\bar{\psi}_\beta(y) \exp\left(-ie\int \frac{d^3l}{(2\pi)^3}(e^{il.x} - e^{il.y})\frac{il^3}{(l^3)^2}\tilde{A}^3(l)\right) > \,. \tag{19}$$

In the case (3), the electric flux is tightly squeezed in the 3-direction. The case (2) can be obtained from fluctuations of this flux line in the space-like directions, x_3 =constant and y_3 =constant. The case (1) entails fluctuations in all space-time directions. Some details of the derivation of eqs. (17)-(19) are given in the Appendix.

Eq. (1) has been calculated in the leading order by many authors [9, 4, 10, 11, 12] using diverse techniques. Calculation using the temporal gauge, which simplifies the contribution of the Wilson line, yields a $\ln|x-y|$ term. This is supposed to exponentiate to yield a power law. However there is also an iR divergence which is evaded by deforming the contour of integration [12]. When the calculation is performed in the (non-local) covariant gauge, a cut-off dependent term linear in $|x-y|$ appears. This seems to preclude a power-law fall-off [6]. Presuming that this linear term is an artifact of the gauge-noninvariant momentum cut-off and ignoring it, the remaining $\ln|x-y|$ dependence matches with the calculation in the temporal gauge as required by the gauge invariance of the correlation function [12].

In the Appendix, we calculate eq. (1) in the leading order, in the (non-local) covariant gauge, and *with a gauge invariant regulator.* We find that the term linear in $|x-y|$ persists (albeit with a $\ln\Lambda$ coefficient). On exponentiation this gives a correlation function that does not fall off as a power, but falls off exponentially at large distance. In the Appendix, we also show that the Dirac dressing gives a divergent propagator even for finite separation between the fermions. *There are no such problems with the isotropic dressing.*

We now give a physical argument for the isotropic choice of dressing turning out to be the only one which is acceptable. Considering eq. (13) in the path-integral representation, we see that the dressed fermion propagator represents fermion propagation in the presence of a source $\mathcal{J}^\mu$ for the electromagnetic field A^μ. In three dimensions, $\mathcal{J}^\mu$ is in general the sum of a gradient and a curl. Integration by parts shows that the curl part of $\mathcal{J}^\mu$ couples to the field strength $F^{\mu\nu}$; so it is truly a source for the physical degrees of the photon. Thus if $\mathcal{J}^\mu$ (or J^μ) has a curl part, we are computing the propagation of fermions in the presence of an external physical electromagnetic source. *The only dressing that corresponds to physical fermions in the absence of external electromagnetic sources is the one in which* J^μ *is a pure gradient.* This singles out the choice given in eqs. (6) and (7), and so Wilson line fluctuations in space-time have to be permitted.

It has been argued by Kennedy and King [16] (see also Ref. [17]) that *the gauge-invariant correlation function with the isotropic dressing reduces to just the usual fermion propagator in the Landau gauge.* The reason is that the Landau gauge corresponds to simply inserting $\partial A = 0$ constraint in the functional integral [18]:

$$\lim_{\alpha\to 0}\int \mathcal{D}A \exp\left(-\frac{1}{2\alpha}\partial A\cdot\partial A\right)\cdots = \int \mathcal{D}A\delta\left[\partial A\right]\cdots \tag{20}$$

From eqs. (14) and (6),

$$\int d^3z\mathcal{J}^\mu(x,y;z)A^\mu(z) = \int d^3z\left(\phi(|x-z|)-\phi(|y-z|)\right)\partial_z^\mu A^\mu(z) \tag{21}$$

As only the ∂A part is coupled to the sources, the source term vanishes and the dressing factor is unity in the Landau gauge.

This result permits us to read off the iR behaviour of the gauge invariant dressed fermion propagator. For this note that *even with our non-local gauges*, the $\alpha \to 0$ limit corresponds to the constraint $\partial A = 0$ [18]:

$$\lim_{\alpha\to 0}\int \mathcal{D}A \exp\left(-\frac{1}{2\alpha}\partial A\cdot g\cdot\partial A\right)\cdots \propto \int \mathcal{D}A\delta\left[\partial A\right]\cdots \tag{22}$$

As the usual fermion propagator has a power law behaviour with an anomalous exponent $4(\alpha-\alpha_0)/(\pi^2 N)$ [3], we get the iR behaviour of the dressed fermion by simply choosing the Landau gauge value $\alpha = 0$:

$$\mathcal{S}(x-y) \sim \frac{\not{x}-\not{y}}{|x-y|^{3-\frac{4\alpha_0}{\pi^2 N}}} \tag{23}$$

implying an anomalous dimension of (see eq. (2)) $\eta = -4\alpha_0/(\pi^2 N)$. Note that α_0 is a series in $1/N$ with the leading order value 2/3 [19]. Therefore $\eta = -8/(3\pi^2 N)$ to the leading order in $1/N$.

It is to be noted that the anomalous dimension is negative. We now argue that *there is no contradiction with the fact* [20] *that the properties of the spectral function imply a non-negative anomalous dimension.* Note that the physical correlation $\mathcal{S}(x,y)$ of eq. (13) does not involve local operators at only x and y. The physical fermions of eq. (3) are not localized in space-time. The Kallen-Lehmann spectral representation is not valid for correlations between such non-local objects. Of course in the Landau gauge, the physical correlation function does reduce to one between local operators at x and y. But the Hilbert space of the theory in any covariant gauge contains states of negative norm. In that case, even though the Kallen-Lehmann spectral representation is valid, the positivity of the spectral function is not guaranteed.

In this paper, we have discussed three possible choices of the gauge-invariant dressed fermion as candidates for the physical fermions of massless QED_3. It appears from our analysis that the physical fermions possess a negative anomalous dimension.

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Appendix

In this Appendix we present the features of the gauge-invariant fermion propagator of eq. (13) in the leading order in $1/N$. *We treat all the three dressings given in eqs. (17)-(19) in the same way in order to compare and contrast them.* As they are invariant, we should get the same answer in any gauge. However, there are prescription ambiguities when the temporal gauge and the Coulomb gauge are used. Therefore we avoid them and use an explicitly Lorentz covariant gauge. We also avoid gauge non-invariant cut-offs, as they may lead to spurious results. Our regularization is by using sufficient number of higher derivatives in the photon kinetic energy terms.

To find the contributions to the dressed fermion propagator to leading order, consider the numerator of the path-integral representation for the expression (13):

$$\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}A\ \psi_\alpha(x)\bar{\psi}_\beta(y)\exp\left(-ie\int d^3z\mathcal{J}^\mu(z)A^\mu(z)\right)\exp\left(e\int d^3w\,\bar{\psi}(w)\not{A}(w)\psi(w)\right) \tag{24}$$

(here the free part of the QED action is taken to be understood). Expanding the two exponentials upto $O(e^2)$, we find *the following three contributions to the leading order* after performing Wick contractions:

(A) The usual one-loop fermion self-energy is obtained by considering the $O(e^2)$ term from the QED interaction Lagrangian.

(B) The self-energy of the photon cloud is obtained by considering the $O(e^2)$ term from the dressing factor.

(C) The interaction of the photon cloud with the fermion is obtained by considering the $O(e)$ term from each exponential.

It is easy to check that *the net contribution is independent of the gauge parameter* α. The part of the photon propagator $\Delta^{\mu\nu}(x,y)$ depending on α is longitudinal in both x and y. We denote this part as

$$\alpha\partial_x^\mu\partial_y^\nu h(x,y) \tag{25}$$

where $h(x,y)\equiv h(x-y)=h(y-x)$. The contributions (A), (C) and (B) from this part are, respectively,

$$e^2\alpha\int d^3z_1d^3z_2\partial_{z_1}^\mu\partial_{z_2}^\nu h(z_1-z_2)S(x-z_1)\gamma^\mu S(z_1-z_2)\gamma^\nu S(z_2-y)\,, \tag{26}$$

$$-ie^2\alpha\int d^3z_1d^3z_2\mathcal{J}^\nu(z_2)\partial_{z_1}^\mu\partial_{z_2}^\nu h(z_1-z_2)\ S(x-z_1)\gamma^\mu S(z_1-y)\,, \tag{27}$$

$$-\frac{e^2\alpha}{2}\int d^3z_1 d^3z_2 \mathcal{J}^\mu(z_1)\mathcal{J}^\nu(z_2)\partial^\mu_{z_1}\partial^\nu_{z_2}h(z_1-z_2)S(x-y)\,. \tag{28}$$

Using eq. (15), the expressions in eqs. (28) and (27) are, respectively,

$$-e^2\alpha S(x-y)\left(h(0)-h(x-y)\right)\,, \tag{29}$$

$$-ie^2\alpha\int d^3z_1\partial^\mu_{z_1}\left(S(x-z_1)\gamma^\mu S(z_1-y)\right)\left(h(x-z_1)-h(y-z_1)\right)\,. \tag{30}$$

(We use integration by parts wherever it is useful.) Since $i\partial\!\!\!/_x S(x-y) = \delta^{(3)}(x-y) = iS(x-y)\overleftarrow{\partial\!\!\!/}_x$, eq. (30) can be written as

$$2e^2\alpha S(x-y)\left(h(0)-h(x-y)\right), \tag{31}$$

whereas eq. (26) becomes

$$-ie^2\alpha\Bigg(\int d^3z_1(\partial^\mu_{z_1}h(z_1-z_2))\Big|_{z_2=z_1}S(x-z_1)\gamma^\mu S(z_1-y) - \int d^3z_1(\partial^\mu_{z_1}h(y-z_1))S(x-z_1)\gamma^\mu S(z_1-y)\Bigg)\,. \tag{32}$$

Now $\partial^\mu_{z_1}h(z_1-z_2)$ is odd in z_1-z_2 and hence vanishes at $z_1=z_2$. Therefore we are left with the second term which becomes

$$-e^2\alpha S(x-y)\left(h(0)-h(x-y)\right)\,. \tag{33}$$

We see that the sum of eq. (29), eq. (31) and eq. (33) vanishes, showing independence from the gauge parameter.

From the expressions for the leading order contributions, it is also easy to see that in the case of the isotropic dressing, the contribution is simply from the usual fermion self energy in the Landau gauge. Since $\mathcal{J}^\mu$ is a pure gradient in this case, integration by parts in both the contributions eq. (27) and eq. (28) of the photon cloud (with $\Delta^{\mu\nu}(x,y)$ instead of the expression (25)) shows that the dependence on the photon propagator is of the form $\partial^\nu_{z_2}\Delta^{\mu\nu}(z_1,z_2)$. This vanishes in the Landau gauge.

We now demonstrate *the problems with the Wilson line and the Dirac dressing.* For this it is convenient to use the momentum-space representations as given in eqs. (17)-(19). First, we give some details on deriving these formulas. These are obtained by expressing J_μ and A_μ in eqs. (13) and (14) in terms of their Fourier components:

$$\mathcal{S}_{\alpha\beta}(x,y) = <\psi_\alpha(x)\bar{\psi}_\beta(y)\exp\left(-ie\int\frac{d^3l}{(2\pi)^3}(e^{il.x}-e^{il.y})\tilde{J}_\mu(l)\tilde{A}_\mu(l)\right)>\,. \tag{34}$$

For the isotropic case, eqs. (4) and (6) lead to $\tilde{J}_\mu(l) = il_\mu/l^2$. For the Dirac dressing of eq. (8), $\tilde{J}_i(l)$ is independent of l_3 and is the two-dimensional Fourier transform of j_i. Because of isotropy (in two dimensions), $\tilde{J}_i(l) = il_i/\vec{l}^2$. To arrive at the Wilson line case of eq. (19), it is convenient to express $A_3(z)$ in eq. (16) in terms of $\tilde{A}_3(l)$, and perform the integration over z_3.

To obtain the contribution of the photon cloud self-energy, we expand to $O(e^2)$ the exponential of the appropriate expression from eqs. (17)-(19), and use $< \tilde{A}_\mu(l)\tilde{A}_\nu(l') >= (2\pi)^3\delta^{(3)}(l+l')\Delta_{\mu\nu}(l)$, with

$$\Delta_{\mu\nu}(l) = \frac{\delta_{\mu\nu} - (1-\alpha)l_\mu l_\nu/l^2}{(l^2+\mu l)(1+l^2/\Lambda^2)}\,, \tag{35}$$

the photon propagator in the $1/N$ expansion ($\mu = Ne^2/8$). Here we have used a regularization using a higher derivative for the photon propagator [21]. [For this, the following terms are to be added to the Lagrangian:

$$\frac{1}{\Lambda^2}\left[\frac{1}{4}(\partial_\sigma F_{\mu\nu})_x\left(1+\frac{\mu}{\sqrt{-\partial^2}}\right)_{xy}(\partial_\sigma F_{\mu\nu})_y + \frac{1}{2\alpha}(\partial_\sigma\partial_\mu A_\mu)_x\left(1+\frac{\mu}{\sqrt{-\partial^2}}\right)_{xy}(\partial_\sigma\partial_\nu A_\nu)_y\right] \tag{36}$$

(an integration over repeated spacetime index is implied). Note that the first term in eq. (36) is gauge-invariant, while the second term modifies the gauge-fixing term.]

For the Wilson line, which we take to extend from $(0,0,x_3)$ to $(0,0,y_3)$, the contribution of the photon cloud self-energy from eq. (19) to this order is then

$$S(x-y)\exp\left[-\frac{e^2}{2}\int\frac{d^3l}{(2\pi)^3}\frac{\left(e^{il_3x_3}-e^{il_3y_3}\right)\left(e^{-il_3x_3}-e^{-il_3y_3}\right)}{(l_3)^2(l^2+\mu l)(1+l^2/\Lambda^2)}\right]. \tag{37}$$

We have chosen the Feynman gauge to simplify our calculations (also, the part of $\Delta_{\mu\nu}(l)$ proportional to $l_\mu l_\nu$ cancels out between the three contributions). Using spherical coordinates, the exponent in eq. (37) becomes

$$-\frac{e^2}{\pi^2}\int_0^\infty dl\frac{1}{(l^2+\mu l)(1+l^2/\Lambda^2)}\int_0^1 d\eta\frac{4\sin^2(lr\eta/2)}{\eta^2} \tag{38}$$

where $\eta = \cos\theta$ and $r = |x_3 - y_3|$ is the length of the Wilson line. In terms of the new variables $\rho = lr$ and $\xi = lr\eta$, this becomes

$$-\frac{e^2 r}{\pi^2}\int_0^\infty d\rho\frac{1}{(\rho+\mu r)(1+\frac{\rho^2}{\Lambda^2 r^2})}\int_0^\rho d\xi\frac{\sin^2(\xi/2)}{\xi^2} \tag{39}$$

Now the integral over ξ is bounded as $\rho\to\infty$. Therefore with our regularization for the photon propagator, the double integral exists. Call this double integral I (without the $-e^2r/\pi^2$ in front). For $\Lambda r\to\infty$, I diverges logarithmically; so we expect $I\sim\ln(\Lambda r)$ for large r. To confirm this behaviour, we let $(\Lambda r)^{-1}\equiv\epsilon$ and find $-[\epsilon(dI/d\epsilon)]|_{\epsilon=0}$. In this process, after obtaining $-\epsilon(dI/d\epsilon)$, it is convenient to put $\rho=\sigma/\epsilon$ and then take $\epsilon\to 0$. This gives the coefficient of $\ln(\Lambda r)$ in I as

$$2\int_0^\infty d\sigma\frac{\sigma}{(1+\sigma^2)^2}\int_0^\infty d\xi\frac{\sin^2(\xi/2)}{\xi^2}\,, \tag{40}$$

which is finite and non-zero.

Thus there is an overall behaviour e^{-cr} for large r when the stringy ansatz is used for the gauge-invariant dressing. Such a linear term has been seen in earlier calculations [12] in the covariant gauge with a $1/(\mu l)$ photon propagator and a momentum cut-off. But it has been regarded as a spurious effect due to the regularization not being gauge invariant. In our calculation, we have used the full $1/(l^2+\mu l)$ propagator with a gauge invariant regularization, and the linear term survives.

If one chooses the temporal gauge $A^3=0$, it appears that the string can be gauged away. This, of course, does not contradict the fact that for the stringy ansatz J^μ has a curl part coupling to the gauge-invariant degrees of the photon. The gradient part of J^μ couples to $\partial^\mu A^\mu$, and in the temporal gauge $\partial^\mu A^\mu$ gets tuned so that this part of the dressing exactly cancels the gauge-invariant part. The temporal gauge is beset with ambiguities. Otherwise, a careful calculation should give back the term linear in r from the fermion self-energy graph itself.

We now consider the Dirac dressing given in eq. (18). Consider the case $x=(0,0,0)$ and $y=(0,0,r)$, i.e., the propagator at equal spatial coordinates. The dressing is in the spatial planes at $x_3=0$ and $x_3=r$. The contribution of the self-energy of the photon cloud is

$$S(x-y)\exp\left[-2e^2\int\frac{d^3l}{(2\pi)^3}\frac{1}{\vec{l}^2(l^2+\mu l)(1+l^2/\Lambda^2)}\sin^2\frac{l_3 r}{2}\right] \tag{41}$$

where $\vec{l}=(l_1,l_2)$ is the spatial momentum. Holding l_3 fixed at a finite value, we notice immediately the logarithmic iR ($\vec{l}\to 0$) divergence $\int d^2\vec{l}/\vec{l}^2$. This is the infinite self-energy of the two-dimensional Coulomb field in the spatial planes $x_3=0$ and $x_3=r$ of infinite extent. Thus the Dirac dressing does not give a finite propagator even for finite r. (If we consider propagation in spatial direction, taking $x=(0,0,0)$ and $y=(r,0,0)$, we get a finite propagator. The reason is that the two-dimensional Coulomb field from equal and opposite charges falls off rapidly to yield an iR finite self energy.)

It is instructive to see that the contribution from the photon cloud self-energy is free of divergence for the isotropic dressing. This contribution is:

$$S(x-y)\exp\left[-2e^2\alpha\int\frac{d^3l}{(2\pi)^3}\frac{1}{l^2(l^2+\mu l)}\sin^2\frac{l\cdot(x-y)}{2}\right]. \tag{42}$$

The integral in the exponent is finite at both the ends $l\to\infty$ and $l\to 0$. It can be shown [22] that for $|x-y|\to\infty$, the exponent is $\ln(\mu|x-y|)$ upto a constant of proportionality, so that exponentiation leads to a power law behaviour from this contribution.

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[22] We can replace $\sin^2(l \cdot (x-y)/2)$ by $(1/2)(1 - \exp(il \cdot (x-y)))$ in the integral of eq. (42); this reduces it to the integral in $f(x-y)$ of Ref. [3].