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Physics Letters B 634 (2006) 557–561
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Gauge dependence of the infrared behaviour of massless QED$_3$

Indrajit Mitra [a,*], Raghunath Ratabole [b], H.S. Sharatchandra [b]

[a] Theory Group, Saha Institute of Nuclear Physics, 1/AF Bidhan-Nagar, Kolkata 700064, India
[b] The Institute of Mathematical Sciences, CIT Campus, Taramani PO, Chennai 600113, India

Received 28 October 2005; received in revised form 30 January 2006; accepted 6 February 2006

Available online 13 February 2006

Editor: M. Cvetič

Abstract

Using the Zumino identities it is shown that in a class of non-local gauges, massless QED$_3$ has an infrared behaviour of a conformal field theory with a continuously varying anomalous dimension of the fermion. In the usual Lorentz gauge, the fermion propagator falls off exponentially for a large separation, but this apparent fermion mass is a gauge artifact.

PACS: 11.15.-q

Massless QED in $2+1$ dimensions (and in general for space-time dimensions $2 < d < 4$) has very interesting features. It is not just super-renormalizable, it is ultraviolet finite. The usual perturbation expansion in the fine structure constant (which is now a dimensionful parameter) has severe infrared (iR) divergences which become worse as the number of loops increases. So QED$_3$ provides an ideal platform for tackling iR divergences. This has led to an extensive study of this model [1]. It is clear that the usual perturbation theory can make sense only by some kind of resummation. In a $1/N$ expansion, N being the number of fermion flavours, there is a resummation of chains of one-loop vacuum polarization diagrams on every photon propagator. This changes the iR behaviour of the photon propagator from being inversely quadratic to inversely linear in momentum. Thus the iR divergence is softened. Even after this there are logarithmic iR divergences. (Throughout this Letter we are concerned with the iR divergences in Green functions for non-exceptional Euclidean momenta and not the additional iR divergences for real processes.) The problem is to sum them up and extract the iR behaviour of the Green functions. We could handle this problem in the following way. We have shown in Ref. [2] that for a particular value (chosen to each order in $1/N$) of the gauge parameter in a specific non-local gauge, the logarithmic iR divergences are absent. As a consequence the iR behaviour of the Green functions to all orders is known. The limiting behaviour is a conformal field theory where the photon has non-canonical scaling dimension one for the entire range of d, in contrast to the engineering dimension $(d-2)/2$. The fermion continues to have the canonical dimension $(d-1)/2$.

This behaviour for the fermions is of course gauge-dependent and special to this gauge. The Green functions involving only the photons is gauge invariant and the scaling dimension one would be valid in any gauge. Our specific choice of gauge has the advantage of extracting this information without being cluttered by the powers of logarithms in the intermediate stages of the calculations.

Thus we know the iR behaviour of the Green functions to all orders for a particular choice of the gauge parameter in a specific non-local gauge. It is instructive to know how the logs add up for other values of gauge parameter and also in other gauges. In this Letter, we predict the behaviour to all orders without detailed calculations.

This is done using the Zumino identities [3] which exactly relate the Green functions in one choice of gauge to those in another. Such a relation is also called the LKF transformation [4]; this name has been mostly used for relation between various conventional covariant gauges (for example, between the

* Corresponding author.
E-mail addresses: indrajit.mitra@saha.ac.in (I. Mitra), raghu@imsc.res.in (R. Ratabole), sharat@imsc.res.in (H.S. Sharatchandra).

0370-2693
doi:10.1016/j.physletb.2006.02.016

Landau gauge and the Feynman gauge). We are interested in a more general class of gauges, including non-local choices. Although the relevant relation is contained in Zumino's paper [3], we rederive it in a way which is suitable for our purpose.

This far, our discussion has been restricted to a non-local gauge. It is also of interest to know how the Green functions behave in a conventional gauge such as the (local) Lorentz gauge. With our choice of the non-local gauge the longitudinal part of the propagator has the iR behaviour $q_\mu q_\nu/q^3$, which is softer than that in the Lorentz gauge, $q_\mu q_\nu/q^4$. This feature led to only log iR divergences in fermionic Green functions which further could be canceled to each order by adjusting the gauge parameter at that order. In contrast, in the Lorentz gauge, the iR divergences become increasingly worse with the number of loops. A resummation seems to be beyond reach. We find out the iR behaviour in this case also using the Zumino identities [5].

Consider the generating functional of the Green functions in a general non-local gauge parametrized by function $g(x,y)$

$$Z = \frac{\int \exp(-\frac{1}{2}\partial A\cdot g\cdot\partial A + j\cdot A + \bar\eta\cdot\psi + \eta\cdot\bar\psi)}{\int \exp(-\frac{1}{2}\partial A\cdot g\cdot\partial A)}. \tag{1}$$

Here $\int$ stands for the measure

$$\int \mathcal{D}\psi\,\mathcal{D}\bar\psi\,\mathcal{D}A\exp(-S), \tag{2}$$

where S is the gauge-invariant Euclidean action. Also, j_μ is the source for the vector potential A_μ, and the Grassmann variables η and $\bar\eta$ are the sources for the fermions. We have used the notation

$$\partial A = \partial^\mu A^\mu, \tag{3}$$

while the $\cdot$ operation signifies the inner product in the Hilbert space, involving integration over spacetime variables and summation of discrete (spin and/or internal) labels. For example,

$$j\cdot A = \int d^3x\, j^\mu(x)A^\mu(x), \tag{4}$$

$$\partial A\cdot g\cdot A = \int d^3x\,d^3y\,\partial^\mu A^\mu(x)g(x,y)\partial^\nu A^\nu(y). \tag{5}$$

Varying the gauge function g by an infinitesimal amount δg, we get

$$\delta Z = -\frac{1}{2}\big(\langle\partial A\cdot\delta g\cdot\partial A\rangle_{j,\eta,\bar\eta} - Z\langle\partial A\cdot\delta g\cdot\partial A\rangle\big), \tag{6}$$

where

$$\langle\mathcal{O}\rangle_{j,\eta,\bar\eta} = \frac{\int \exp(-\frac{1}{2}\partial A\cdot g\cdot\partial A + j\cdot A + \bar\eta\cdot\psi + \eta\cdot\bar\psi)\mathcal{O}}{\int \exp(-\frac{1}{2}\partial A\cdot g\cdot\partial A)} \tag{7}$$

is the expectation value of operator $\mathcal{O}$ in presence of sources j, η and $\bar\eta$. We take $\langle\mathcal{O}\rangle$ without any suffix to denote the corresponding expectation value when the sources are set to zero. As a consequence of Ward identities, the correlations involving longitudinal photons in Eq. (6) can be related to pure fermion correlations. This leads to the Zumino identities which we now derive in a form suitable for our purpose.

Invariance of Z under the following change of integration variables (corresponding to an infinitesimal gauge transformation) in the numerator of Eq. (1)

$$\delta A^\mu(x) = -\partial^\mu\epsilon(x),$$
$$\delta\psi(x) = ie\epsilon(x)\psi(x), \qquad \delta\bar\psi(x) = -ie\epsilon(x)\bar\psi(x) \tag{8}$$

gives the basic Ward identity

$$\big\langle\big(\partial^2 g\cdot\partial A + \partial j + ie(\bar\eta\psi - \eta\bar\psi)\big)_x\big\rangle_{j,\eta,\bar\eta} = 0 \tag{9}$$

where we have used the notation $\mathcal{O}_x$ to mean $\mathcal{O}(x)$. Also, $\partial^2 g$ can be regarded as the product of the symmetric matrices $(\partial^2)_{xy} = \delta^{(3)}(x-y)\partial_x^2$ and g_{xy} (note that Eq. (1) picks out the symmetric part of g). Applying the operation $\partial_y^\mu(\delta/\delta j^\mu(y))$ on Eq. (9), we get

$$\big\langle\partial A_y\big(\partial^2 g\cdot\partial A + \partial j + ie(\bar\eta\psi - \eta\bar\psi)\big)_x\big\rangle_{j,\eta,\bar\eta} = \big(\partial_x^2\delta_{xy}\big)Z, \tag{10}$$

where δ_{xy} stands for the Dirac-delta function $\delta^{(3)}(x-y)$. Setting the sources to zero in the above equation, we get

$$\big\langle\partial A_y\big(\partial^2 g\cdot\partial A\big)_x\big\rangle = \partial_x^2\delta_{xy}. \tag{11}$$

This is the conventional Ward identity for the longitudinal part of the photon propagator. From the difference of Eq. (10) and (Z times) Eq. (11), we get

$$\big\langle\partial A_y\big(\partial A + \big(\partial^2 g\big)^{-1}\cdot\big(\partial j + ie(\bar\eta\psi - \eta\bar\psi)\big)\big)_x\big\rangle_{j,\eta,\bar\eta}$$
$$- Z\langle\partial A_y\partial A_x\rangle = 0. \tag{12}$$

In the last step, we multiplied with the appropriate element of the matrix $(\partial^2 g)^{-1}$, which is to be regarded as the inverse of the matrix $\partial^2 g$. Using Eq. (12) in Eq. (6), we get

$$\delta Z = \frac{1}{2}\big\langle\partial A\cdot\delta g\cdot\big(\partial^2 g\big)^{-1}\cdot\big(\partial j + ie(\bar\eta\psi - \eta\bar\psi)\big)\big\rangle_{j,\eta,\bar\eta}. \tag{13}$$

This is not yet the convenient form for our use. We proceed to eliminate ∂A from Eq. (13) and arrive at an equation involving fermion correlations only. We apply the operation $\bar\eta_\alpha(y)(\delta/\delta\bar\eta_\alpha(y))$ on Eq. (9) to get

$$\big\langle\big(\partial^2 g\cdot\partial A + \partial j + ie(\bar\eta\psi - \eta\bar\psi)\big)_x(\bar\eta\psi)_y\big\rangle_{j,\eta,\bar\eta}$$
$$= -ie\big\langle(\bar\eta\psi)_x\big\rangle_{j,\eta,\bar\eta}\delta_{xy} \tag{14}$$

and the operation $\eta_\alpha(y)(\delta/\delta\eta_\alpha(y))$ to get

$$\big\langle\big(\partial^2 g\cdot\partial A + \partial j + ie(\bar\eta\psi - \eta\bar\psi)\big)_x(\eta\bar\psi)_y\big\rangle_{j,\eta,\bar\eta}$$
$$= ie\big\langle(\eta\bar\psi)_x\big\rangle_{j,\eta,\bar\eta}\delta_{xy}. \tag{15}$$

The difference of Eq. (14) and Eq. (15) yields

$$\big\langle\big(\partial^2 g\cdot\partial A + \partial j + ie(\bar\eta\psi - \eta\bar\psi)\big)_x(\bar\eta\psi - \eta\bar\psi)_y\big\rangle_{j,\eta,\bar\eta}$$
$$= -ie\big\langle(\bar\eta\psi + \eta\bar\psi)_x\big\rangle_{j,\eta,\bar\eta}\delta_{xy}. \tag{16}$$

Multiplying with $(\partial^2 g)_{zx}^{-1}$ and integrating over x, we obtain,

$$-\big\langle\partial A_z(\bar\eta\psi - \eta\bar\psi)_y\big\rangle_{j,\eta,\bar\eta}$$
$$= \big\langle\big(\big(\partial^2 g\big)^{-1}\cdot\big(\partial j + ie(\bar\eta\psi - \eta\bar\psi)\big)\big)_z(\bar\eta\psi - \eta\bar\psi)_y\big\rangle_{j,\eta,\bar\eta}$$
$$+ ie\big(\partial^2 g\big)_{zy}^{-1}\big\langle(\bar\eta\psi + \eta\bar\psi)_y\big\rangle_{j,\eta,\bar\eta}. \tag{17}$$

Also from Eq. (9),

$$-\langle \partial A_x \rangle_{j,\eta,\bar{\eta}} = \langle \left(\left(\partial^2 g \right)^{-1} \cdot \left(\partial j + ie(\bar{\eta}\psi - \eta\bar{\psi}) \right) \right)_x \rangle_{j,\eta,\bar{\eta}}. \tag{18}$$

Using Eqs. (18) and (17) in Eq. (13), and using the fact that $(\partial^2 g)^{-1}$ is a symmetric matrix, we have finally

$$\begin{aligned}\delta Z = \frac{1}{2}\big[& \langle \left(\partial j + ie(\bar{\eta}\psi - \eta\bar{\psi})\right) \cdot \delta\left(\partial^2 g \partial^2\right)^{-1} \\ & \cdot \left(\partial j + ie(\bar{\eta}\psi - \eta\bar{\psi})\right) \rangle_{j,\eta,\bar{\eta}} \\ & - e^2 \delta\left(\partial^2 g \partial^2\right)^{-1}_{00} \langle \bar{\eta}\cdot\psi + \eta\cdot\bar{\psi} \rangle_{j,\eta,\bar{\eta}} \big].\end{aligned} \tag{19}$$

Here we have used $(\partial^2 g)^{-1} \cdot \delta g \cdot (\partial^2 g)^{-1} = -\delta(\partial^2 g \partial^2)^{-1}$, which can be easily checked by going over to the momentum space: $(-k^2 g(k))^{-1} \delta g(k) (-k^2 g(k))^{-1} = -(1/k^2)\delta(1/g(k)) \times (1/k^2)$. (We will come across explicit examples in momentum space later in this Letter.) Also, $(\partial^2 g \partial^2)^{-1}_{00}$ appears in Eq. (19) as follows:

$$\begin{aligned}& \int d^3x \left(\left(\partial^2 g\right)^{-1} \cdot \delta g \cdot \left(\partial^2 g\right)^{-1} \right)_{xx} \langle \left(\bar{\eta}\psi + \eta\bar{\psi}\right)_x \rangle \\ & \quad = -\int d^3x\, \delta\left(\partial^2 g \partial^2\right)^{-1}_{xx} \langle (\bar{\eta}\psi + \eta\bar{\psi})_x \rangle \\ & \quad = -\delta\left(\partial^2 g \partial^2\right)^{-1}_{00} \langle (\bar{\eta}\cdot\psi + \eta\cdot\bar{\psi}) \rangle\end{aligned} \tag{20}$$

since $(\partial^2 g \partial^2)^{-1}_{xy}$ depends only on the difference $x - y$ for a translation-invariant gauge function g.

The dependence of all Green functions on the function g_{xy} is contained in Eq. (19). The simplest case of the photon propagator is obtained by applying the operation $\delta^2/(\delta j^\mu(x)\delta j^\nu(y))$ on Eq. (19) and then setting the sources to zero:

$$\delta \Delta_{\mu\nu}(x, y) = \partial^\mu_x \partial^\nu_y \delta\left(\partial^2 g \partial^2\right)^{-1}_{xy}. \tag{21}$$

This is consistent with the Ward identity Eq. (11).

We now obtain the *dependence of the fermion propagator* on the choice of the gauge function g. Applying $\delta^2/(\delta\bar{\eta}_\gamma(x) \times \delta\eta_\delta(y))$ to Eq. (19) and setting all the sources to zero, we get

$$\delta S_{\gamma\delta}(x, y) = -\delta F_{xy} S_{\gamma\delta}(x, y) \tag{22}$$

where F_{xy} stands for

$$F_{xy} = e^2 \left(\left(\partial^2 g \partial^2\right)^{-1}_{00} - \left(\partial^2 g \partial^2\right)^{-1}_{xy} \right). \tag{23}$$

Integrating this equation, we relate the fermion propagator evaluated with two different gauge functions:

$$S_{\gamma\delta}(x, y) = \exp\left[-\left(F - F^0\right)_{xy} \right] S^0_{\gamma\delta}(x, y). \tag{24}$$

Here S and S^0 stand for the fermion propagators in the gauges g and g_0 respectively; F^0 is related to g^0 by Eq. (23).

We have shown in Ref. [2] that if we choose a particular non-local gauge

$$g = \frac{1}{\alpha}\left(1 + \frac{\mu}{\sqrt{-\partial^2}} \right) \tag{25}$$

(with $\mu = Ne^2/8$) then it is possible to choose the gauge parameter α to each order in $1/N$ such that there are no logarithmic corrections to the fermion propagator and other Green functions. As a consequence, for this particular value of gauge parameter (call it α_0) the iR behaviour of the fermion propagator is that of the free theory with no anomalous dimension:

$$S^{\alpha_0}(x, y) \sim \frac{\not{x} - \not{y}}{|x - y|^3}. \tag{26}$$

The iR behaviour for other values of gauge parameter α within the same non-local gauge then follows from Eqs. (24) and (25). We now have $F_{xy} = \alpha f(x - y)$ where f is formally the matrix $f_{xy} = e^2((\partial^2(1 + \mu/\sqrt{-\partial^2})\partial^2)^{-1}_{00} - (\partial^2(1 + \mu/\sqrt{-\partial^2})\partial^2)^{-1}_{xy})$. It is convenient to represent f by the Fourier transform

$$f(x - y) = e^2 \int \frac{d^3k}{(2\pi)^3} \frac{(1 - e^{ik\cdot(x-y)})}{k^2(k^2 + \mu k)}. \tag{27}$$

(This is obtained by inserting the completeness relation for the momentum eigenstates in $\langle x|(\partial^2(1 + \mu/\sqrt{-\partial^2})\partial^2)^{-1}|y\rangle$. Note that the factor of $(2\pi)^3$ in Eq. (27) is consistent with that in the Fourier transform of $1_{xy} = \delta(x - y)$.) We now get the propagator for the gauge parameter α:

$$S^\alpha(x, y) = \left(\frac{1}{\lambda(x, y)} \right)^{\alpha - \alpha_0} S^{\alpha_0}(x, y), \tag{28}$$

where

$$\lambda(x, y) = \exp\left[f(x - y) \right]. \tag{29}$$

The integral in Eq. (27) is finite at both the ends $k \to \infty$ and $k \to 0$. For $k \to 0$, finiteness follows from $\mu k \gg k^2$ and $\exp(ik\cdot(x - y)) \approx 1 + ik\cdot(x - y)$ (actually, by symmetry, it is the $O(k^2)$ term in the exponential which contributes). Now, as $|x - y| \to \infty$, $k\cdot(x - y)$ is no longer small, and the integral develops a logarithmic divergence as $k \to 0$. Thus, $1/|x - y|$ serves as an infrared cutoff for k, and for $|x - y| \to \infty$, we expect the integral to behave as $\kappa \ln|x - y|$ where κ is a constant. This can be explicitly verified, and the constant of proportionality extracted, as follows. We have

$$\begin{aligned} f(x) &= \frac{e^2}{2\pi^2} \int_0^\infty \frac{dk}{k^2 + \mu k} \left(1 - \frac{\sin(k|x|)}{k|x|} \right) \\ &= \frac{e^2}{2\pi^2 \mu} \int_0^\infty \frac{du}{\rho u^2 + u} \left(1 - \frac{\sin u}{u} \right),\end{aligned} \tag{30}$$

where $\rho = 1/(\mu|x|)$. Then,

$$\begin{aligned}\kappa &= -\lim_{\rho\to 0} \rho \frac{df}{d\rho} \\ &= \lim_{\rho\to 0} \frac{e^2}{2\pi^2\mu} \int_0^\infty \frac{dv}{(v + 1)^2} \left(1 - \rho \frac{\sin(v/\rho)}{v} \right) \\ &= \frac{e^2}{2\pi^2\mu} \int_0^\infty \frac{dv}{(v + 1)^2} = \frac{4}{\pi^2 N}\end{aligned} \tag{31}$$

which is finite and non-zero. This gives the iR ($|x| \to \infty$) behaviour

$$f(x) \sim \frac{4}{\pi^2 N} \ln\left(\mu|x|\right), \tag{32}$$

$$\lambda(x,y)\sim|x-y|^{\frac{4}{\pi^2N}}. \tag{33}$$

Thus the iR behaviour of the fermion propagator for an arbitrary value of gauge parameter α in our non-local gauge is given by

$$S^\alpha(x,y)\sim\frac{\not x-\not y}{|x-y|^{3+\frac{4(\alpha-\alpha_0)}{\pi^2N}}}. \tag{34}$$

A special case of Eq. (34) is that the power of $|x-y|$ is $3-8/(3\pi^2N)$ to the leading order in $1/N$ in the Landau gauge, which is obtained by using $\alpha=0$ and $\alpha_0=2/3$.[1] This value was obtained earlier by Aitchison et al. [5]. However these authors use a different non-local gauge function (the small momentum limit of our gauge function) in the LKF transformation equation, and so need to regularize an ultraviolet infinity and also put an ultraviolet cutoff scale. Our method is free from these complications.

Eq. (34) suggests that the iR behaviour for other values of α is again a CFT, albeit with a non-zero anomalous dimension for the fermion. We may check this by obtaining the dependence of the *four-fermion Green function*

$$S_{\gamma_1,\gamma_2;\delta_1\delta_2}(x_1,x_2;y_1,y_2)=\left.\frac{\delta^4Z}{\delta\bar\eta_{\gamma_1}(x_1)\delta\bar\eta_{\gamma_2}(x_2)\delta\eta_{\delta_1}(y_1)\delta\eta_{\delta_2}(y_2)}\right|_{j=\eta=\bar\eta=0} \tag{35}$$

on α. From Eq. (19) we get

$$\delta S_{\gamma_1,\gamma_2;\delta_1\delta_2}(x_1,x_2;y_1,y_2)=\big[\delta\big(F_{x_1x_2}+F_{y_1y_2}-F_{x_1y_2}-F_{x_2y_1}-F_{x_1y_1}-F_{x_2y_2}\big)\big]S_{\gamma_1,\gamma_2;\delta_1\delta_2}(x_1,x_2;y_1,y_2). \tag{36}$$

The solution to this equation can be cast in the form

$$S^\alpha_{\gamma_1,\gamma_2;\delta_1\delta_2}(x_1,x_2;y_1,y_2)=\left[\frac{\lambda(x_1,x_2)\lambda(y_1,y_2)}{\lambda(x_1,y_1)\lambda(x_1,y_2)\lambda(x_2,y_1)\lambda(x_2,y_2)}\right]^{\alpha-\alpha_0}\times S^{\alpha_0}_{\gamma_1,\gamma_2;\delta_1\delta_2}(x_1,x_2;y_1,y_2). \tag{37}$$

Using Eq. (33), we may write

$$S^\alpha_{\gamma_1,\gamma_2;\delta_1\delta_2}(x_1,x_2;y_1,y_2)=\frac{1}{|x_1-y_1|^{\frac{4(\alpha-\alpha_0)}{\pi^2N}}|x_2-y_2|^{\frac{4(\alpha-\alpha_0)}{\pi^2N}}}\left(\frac{\rho}{\eta}\right)^{\frac{4(\alpha-\alpha_0)}{\pi^2N}}\times S^{\alpha_0}_{\gamma_1,\gamma_2;\delta_1\delta_2}(x_1,x_2;y_1,y_2) \tag{38}$$

for the iR behaviour. Here ρ and η are the conformal invariant cross-ratios

$$\rho=\frac{|x_1-x_2||y_1-y_2|}{|x_1-y_1||x_2-y_2|},\qquad \eta=\frac{|x_1-y_2||x_2-y_1|}{|x_1-y_1||x_2-y_2|}. \tag{39}$$

As S^{α_0} has a structure required by conformal invariance (in the infrared), Eq. (38) implies that S^α also has such a structure [6]. This is consistent with an anomalous dimension $4(\alpha-\alpha_0)/(\pi^2N)$ for the fermion. It is interesting that the angular dependence of the scattering amplitude is modified by a simple factor given by a power of ρ/η when one changes the gauge parameter.

We now address the gauge dependence of the *three-point fermion-photon Green function*

$$V_{\mu;\gamma,\delta}(z;x,y)=\left.\frac{\delta^3Z}{\delta j^\mu(z)\delta\bar\eta_\gamma(x)\delta\eta_\delta(y)}\right|_{j,\eta,\bar\eta=0}. \tag{40}$$

From Eqs. (19) and (40) it follows that

$$\delta V_{\mu;\gamma,\delta}(z;x,y)=-\delta F(x,y)V_{\mu;\gamma,\delta}(z;x,y)+(i/e)\partial^\mu_z\big(\delta F(z,x)-\delta F(z,y)\big)S_{\gamma\delta}(x,y). \tag{41}$$

For the part of the 3-point function corresponding to a longitudinal photon, we get a simpler equation by applying ∂^μ_z on Eq. (41):

$$\delta\partial^\mu_z V_{\mu;\gamma,\delta}(z;x,y)=-\delta F(x,y)\partial^\mu_z V_{\mu;\gamma,\delta}(z;x,y)-ie\big(\delta\big(\partial^2g\big)^{-1}_{zx}-\delta\big(\partial^2g\big)^{-1}_{zy}\big)S_{\gamma\delta}(x,y). \tag{42}$$

This is satisfied by the Ward identity

$$\partial^\mu_z V_{\mu;\gamma,\delta}(z;x,y)=-ie\big[\big(\partial^2g\big)^{-1}_{zx}-\big(\partial^2g\big)^{-1}_{zy}\big]S_{\gamma\delta}(x,y) \tag{43}$$

following from Eq. (9). On the other hand for the part $\tilde V$ of V that relates to a transverse photon,

$$\tilde V_{\mu;\gamma,\delta}(z;x,y)=\left(\eta_{\mu\nu}-\frac{\partial^z_\mu\partial^z_\nu}{\partial^2_z}\right)V_{\nu;\gamma,\delta}(z;x,y), \tag{44}$$

Eq. (41) leads to

$$\delta\tilde V_{\mu;\gamma,\delta}(z;x,y)=-\delta F(x,y)\tilde V_{\mu;\gamma,\delta}(z;x,y). \tag{45}$$

Its solution is

$$\tilde V^\alpha_{\mu;\gamma,\delta}(z;x,y)=\left(\frac{1}{\lambda(x,y)}\right)^{\alpha-\alpha_0}\tilde V^{\alpha_0}_{\mu;\gamma,\delta}(z;x,y), \tag{46}$$

which is consistent with an anomalous dimension as given in Eqs. (28) and (34) for the fermion and a gauge-invariant anomalous dimension for the photon.

We have shown that the iR behaviour in a class of non-local gauges parametrized by a parameter α is given by a CFT with the fermion anomalous dimension depending on the parameter α. Using this we now obtain *the iR behaviour in the usual class of local gauges*. This turns out to be very instructive regarding attempts to resum iR divergences of the perturbation theory.

We obtain the fermion propagator in the local gauge corresponding to the Lorentz gauge term $-(1/(2\alpha))(\partial A)^2$ by comparing with that for the non-local gauge

$$-\frac{1}{2\alpha}(\partial A)\cdot\left(1+\frac{\mu}{\sqrt{-\partial^2}}\right)\cdot(\partial A) \tag{47}$$

[1] α is related to the gauge parameter ξ used in Ref. [2] by $\alpha=1-\xi$.

with the same gauge parameter α. For the local Lorentz gauge

$$F^L(x,y) = \alpha e^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^4}\left(1 - e^{ik\cdot(x-y)}\right)$$
$$= \frac{\alpha e^2}{2\pi^2} \int_0^\infty \frac{dk}{k^2}\left(1 - \frac{\sin(k|x-y|)}{k|x-y|}\right)$$
$$= \frac{\alpha e^2}{2\pi^2}|x-y| \int_0^\infty du\, \frac{1}{u^2}\left(1 - \frac{\sin u}{u}\right). \tag{48}$$

The integral can be evaluated by rewriting the integrand as $(u - \sin u)(1/u^3)$, integrating by parts twice, and using $\int_0^\infty du\,(\sin u/u) = \pi/2$. We thus find $F^L(x,y) = \alpha e^2|x-y|/(8\pi)$. Using Eq. (24) the iR behaviour in the local gauge then comes out as

$$S^L(x,y) \sim \exp\left[-\frac{\alpha e^2}{8\pi}|x-y|\right] \frac{\not x - \not y}{|x-y|^{3-\frac{4\alpha_0}{\pi^2 N}}}. \tag{49}$$

(It may be noted that starting from our non-local gauge, we reach the local Lorentz gauge through infinitesimal changes of the gauge function by (formally) varying μ from $Ne^2/8$ to zero, at a fixed value of α. As μ is decreased, we pass from the $\mu k \gg k^2$ regime to the $k^2 \gg \mu k$ regime, and finally reach $F^L(x,y)$ smoothly.)

Now $\alpha > 0$ for the contribution of the gauge-fixing term to be of the correct sign to make the Euclidean functional integral converge. Thus Eq. (49) tells us that the fermion propagator falls off exponentially as if the fermion has developed a mass $\alpha e^2/(8\pi)$! However, this apparent mass is spurious, since fermion mass cannot be dynamically generated in perturbation theory (indeed, the propagator of Eq. (49) is proportional to $\not p$ in momentum space).

This strikingly illustrates the pitfalls in resumming iR divergences in perturbation theory. There are severe iR divergences in the local Lorentz gauge, because the longitudinal part of the photon propagator has a $1/k^2$ behaviour in the iR. The cumulative effect is an apparent mass term in gauge non-invariant Green functions. The apparent mass is a gauge artifact; it does not appear in gauge-invariant correlation functions.

In this Letter, we determined how the iR logarithms of massless QED_3 add up for arbitrary values of the gauge parameter in a non-local gauge and also in the usual Lorentz gauge. We demonstrated by studying various correlation functions that the iR behaviour in the non-local gauge is that of a CFT with a continuously varying anomalous dimension for the fermion. We also demonstrated the pitfalls in summing the severe iR divergences of the usual Lorentz gauge (the fermion propagator falls off exponentially as if there is a fermion mass, which is actually a gauge artifact); thus it is the non-local gauge which is suitable for studying the iR behaviour of this theory. The implications of the calculation in the non-local gauge for the important issue of anomalous dimension of the gauge-invariant dressed fermion will be presented elsewhere [7].

Acknowledgements

We thank A. Bashir and A. Davydychev for useful communications. I.M. thanks IMSc, Chennai for hospitality during the course of this work.

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