Formation Cross Sections of Nucleides Produced from Heavy Elements by High-energy Proton Bombardment

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Theoretical cross sections for the production of elements from ^{209}Bi and natural copper by high-energy proton bombardment have been calculated.

Several attempts[1–13] have been made to calculate the production cross sections of nucleides formed by the bombardment of elements with protons and neutrons. The most widely used method has been the Monte Carlo technique in which the reaction mechanism is assumed to be a two-step one: first the ejection of a few fast nucleons in cascade and then the evaporation of one or more nucleons from the excited nucleus. The application of the Monte Carlo technique, does not always produce satisfactory results: they may differ from the experimental values by a factor of more than three. Rudstam[2, 8] presented an empirical formula and later modified it by taking the charge distribution in the nucleus into account. Even then the results differ considerably from the experimental values. Bertini[10, 13] applied the Monte Carlo method and took the diffuseness of the nuclear surface into account, but this did not much improve the earlier results. Moreover, the Monte Carlo method, being a statistical one, is not strictly applicable in cases where only a few nucleons take part. Our method has no such limitations. The closed semi-empirical formula developed earlier[11] to study spallation cross sections for medium-weight targets is used here to calculate the cross sections for spallation products from heavy targets.

Our theory takes account of the fact that in a high-energy spallation reaction where nucleons or their chuncks are randomly emitted, the ratio of the outgoing neutrons to protons depends on the neutron-proton ratio in the target nucleus. Since the neutrons considerably exceed the protons in heavy target nuclei, it is likely that the emitted particles show a preponderance of neutrons over protons too. Also, neutron emission is favoured compared to proton emission because of the coulomb barrier.

In deriving the formula, the nucleus is assumed to be a Fermi gas consisting of a number of non-interacting nucleons. Following Ref.[11] the cross section is given by

$$\sigma = \sigma_0 \exp\{-K\,\delta^2\} \tag{1}$$

where σ_0 is a constant for a given target element and incident energy, K is a dimensionless parameter independent of target mass number and incident energy, and δ is given by

$$\delta = t - \frac{Z+1}{N}\,u - \frac{Z+1}{N}\,u \exp\left(-\frac{C\,N}{(Z+1)\,u}\right). \tag{2}$$

t protons and u neutrons are emitted in a reaction, C is a non-negative parameter having the same value for medium and heavy elements, and σ_0 can be calculated with the help of the theoretical expressions for the (p, pn) and (p, 2p) reaction cross sections deduced in Refs.[14] and [15]. Thus, the use of an ex-

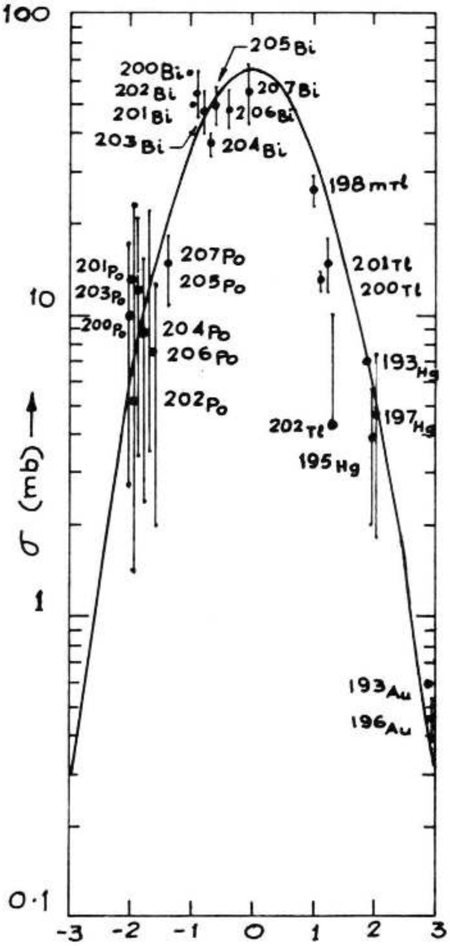

Fig. 1. Formation cross sections, σ(mb), against δ [cf. Eq. (2)] for products from ^{209}Bi bombarded with 380 MeV protons. The points show experimental values (Ref. 17).

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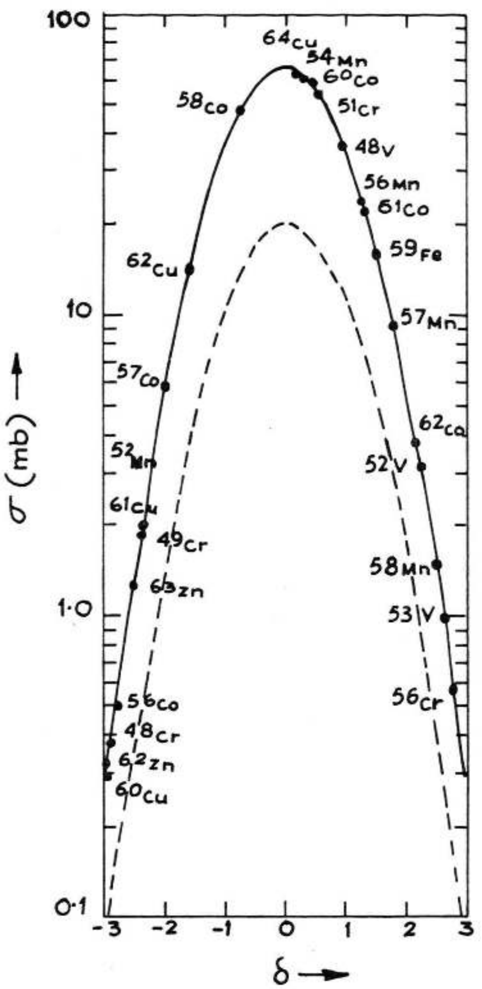

Fig. 2. Theoretical formation cross sections, σ(mb), against δ for the products from ^{65}Cu bombarded with 590 MeV protons. The solid line represents the cross sections of pure ^{65}Cu and the dashed line relates to Cu of natural isotopic composition (30.9% ^{65}Cu).

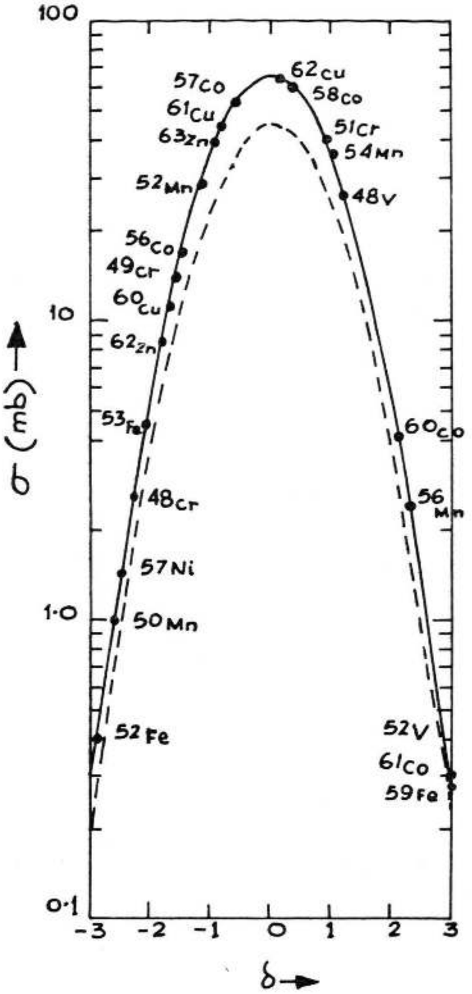

Fig. 3. As in Fig. 2 for the products from ^{63}Cu bombarded with 590 MeV protons. The dashed line relates to Cu of natural isotopic composition (69.1% ^{63}Cu).

perimental cross section for the determination of σ_0, as made in Ref. [11], is avoided here. In the case of a target element consisting of two isotopes, two different values of σ_0 are involved. The actual cross section σ for the product nucleide is then given by

$$\sigma = m\,\sigma_1 + n\,\sigma_2 \tag{3}$$

where σ_1 and σ_2 are the cross sections for the product from each isotope, and m and n are the compositional isotope fractions in the target.

Formation cross sections have been calculated for the target element ^{209}Bi (Table 1) and for natural copper (Table 2) with the help of Eqs. (1), (2) and (3). The results are plotted in Figs. 1, 2, and 3. To obtain the formation cross section of a product from natural copper one has to add the corresponding reduced values given on the dashed lines in Figs. 2 and 3.

As can be seen from the ratios given in the last column of the tables, the calculated cross sections agree fairly well with the experimental data. In some cases the experimental cross sections for the metastable and ground state of the product nucleides are given separately, whereas in others values of the metastable state only are given. Only the sums of the two measured cross sections, when available, are compared with our calculations.

Table 1. Calculated and experimental cross sections for the spallation products from ^{209}Bi bombarded by 380 MeV protons.

Product nucleides Z	A	present calc.	Cross sections (mb) exp. [17]	other calc. [18]	exp. calc.
84	207	20.0	0	32 ± 8	
			(14.6 ± 3.7) [a]		0.73 ± 0.18
	206	15.1	7.7 ± 5.6	9.3 ± 4	0.51 ± 0.33
	205	12.3	12.9 ± 9.4	29 ± 8	1.05 ± 0.76
	204	9.8	8.9 ± 6.5	29 ± 8	0.91 ± 0.60
	203	8.5	12.5 ± 9.1	29 ± 8	1.47 ± 1.10
	202	7.5	5.2 ± 3.8	12 ± 5	0.69 ± 0.48
	201	6.2	13.3 ± 9.7	22 ± 7	2.14 ± 1.60
	200	5.8	10.0 ± 7.3	17 ± 7	1.72 ± 1.29
83	207	65.0	15.7 ± 3.6	37 ± 9	
			(55.0 ± 13) [a]		0.84 ± 0.21
	206	59.5	49.3 ± 5.9	40 ± 10	0.83 ± 0.10
	205	55.0	50.0 ± 7.0	40 ± 10	0.91 ± 0.12
	204	49.7	37.1 ± 3.2	34 ± 9	0.75 ± 0.07
	203	45.0	47.6 ± 7.6	49 ± 11	1.06 ± 0.16
	202	41.0	55.8 ± 9.4	54 ± 12	1.36 ± 0.23
	201	38.4	49.6 ± 4.4	44 ± 10	1.29 ± 0.11
	200	36.3	64.4	30 ± 8	1.77
	199	34.0	68.6 *	30 ± 8	2.02
	198	34.0	60.6 *	47 ± 11	1.78
82	203	60.1	14.0 ± 2.7		0.23 ± 0.09
	201	64.5	24.5 ± 6.6		0.38 ± 0.10
	200	64.5	7.5 ± 5		0.12 ± 0.08
	199	64.1	13.7		0.21
	198	64.3	26.9		0.42
	197	63.2	12.5 (m) *		
81	202	24.0	4.42 ± 5.9		0.18 ± 0.24
	201	27.5	15.1 ± 3.4		0.55 ± 0.12
	200	29.9	13.5 ± 0.8		0.45 ± 0.03
	199	32.2	2.52 ± 1.37		0.08 ± 0.04
	198	34.0	25.8 ± 3.2 (m)		
	196	37.4	62.5 ± 18.4 *		1.67 ± 0.49
	195	39.0	62.3 ± 15.2 *		1.60 ± 0.40
80	197	5.0	4.65 ± 2.83		0.93 ± 0.56
	195	5.4	3.89 ± 1.87		0.72 ± 0.36
	194	6.6	<0.5		—
	193	7.5	~7		0.93
	192	8.0	22.0 *		2.75
	191	8.5	21.9 *		2.57
	190	7.8	39.8 *		5.10
	189	8.5	1.05 *		0.12
79	196	0.32	0.46 ± 0.07		1.44 ± 0.22
	195	0.32	~0		
	194	0.31	1.30 ± 0.17		4.19 ± 0.50
	193	0.40	~0.6		1.50
	192	0.55	14.2 ± 1.3		25.8 ± 2.56
	191	0.55	17.0		30.9

[a] Ref. 18. * Cumulative cross section.
(m) Metastable.

As for the larger divergences between the experimental and calculated values the inherent experimental difficulties of the radiochemical method (Ref. [16]) should be kept in mind. Also, fragmentation and fission processes become important in the reactions where the mass number of the product is far less than that of the target. There are a few experimental values for ^{209}Bi (Table 1) which are markedly higher than the calculated ones. Such values relate to the cumulative yield and not to spallation alone.

There is a large difference between the calculated and experimental [17] data for the products ^{207}Po and ^{207}Bi (Table 1). The value for ^{207}Po, measured at 450 MeV by Pierson et al. [18], agrees better with our calculated value. In the case of ^{207}Bi, the latter authors find a value at 380 MeV which agrees well with our calculated value.

Table 2. Calculated and experimental cross sections for the spallation products from natural copper bombarded by 590 MeV protons.

Product nucleides		Cross sections (mb)			exp. calc.
Z	A	present calc.	exp.[12]	other calc.[12]	
30	63	26.6	3.13 ± 0.4	–	0.12 ± 0.01
	62	5.5	0.81 ± 0.1	8.2	0.15 ± 0.02
29	64	20.1	25.8 ± 3.0	49.1	1.29 ± 0.14
	62	49.5	40.0 ± 4.0	79.8	0.81 ± 0.08
	61	29.6	20.7 ± 2.1	31.4	0.70 ± 0.07
	60	7.63	4.6 ± 0.5	8.62	0.60 ± 0.06
28	57	1.10	1.78 ± 0.24	1.85	1.62 ± 0.21
	56	0.8	0.93 ± 0.25	0.30	1.16 ± 0.29
27	62	1.3	0.48 ± 0.1 (m) 0.61 ± 0.15	0.85	0.84 ± 0.20
	61	7.0	1.3 ± 0.3	4.9	0.19 ± 0.05
	60	21.5	31.7 ± 6.0	15.0	1.47 ± 0.27
	58	57.7	33.0 ± 5.0 (m) 22.0 ± 5.0	5.5	0.95 ± 0.19
	57	36.9	32.3 ± 3.0	27.3	0.87 ± 0.08
	56	11.87	14.8 ± 1.5	8.52	1.24 ± 0.12
	55	1.66	4.0 ± 0.5	1.9	2.41 ± 0.30
26	59	5.44	2.54 ± 0.25	2.40	0.47 ± 0.05
	53	2.94	2.04 ± 0.25	1.92	0.69 ± 0.08
	52	0.27	0.25 ± 0.03	0.33	0.93 ± 0.11
25	58	0.46	0.13 *	0.26	0.28
	57	2.9	0.06 *	1.15	0.02
	56	9.0	5.5 ± 0.7	4.19	0.61 ± 0.08
	54	44.2	31.9 ± 3.2	24.0	0.72 ± 0.07
	52	20.1	5.0 ± 1.0 (m) 12.0 ± 1.2	7.83	0.85 ± 0.09
	50	0.78	0.15 ± 0.03	0.35	0.19 ± 0.04
24	56	0.22	0.06 ± 0.03	0.12	0.27 ± 0.13
	51	31.5	29.0 ± 3.0	18.7	0.92 ± 0.09
	49	9.5	3.21 ± 0.6	1.95	0.34 ± 0.06
	48	2.2	0.51 ± 0.08	0.37	0.23 ± 0.04
23	53	0.33	0.51 ± 0.07	0.26	1.54 ± 0.21
	52	1.20	1.4 ± 0.25	1.13	1.17 ± 0.21
	48	29.7	14.35 ± 2.0	7.09	0.48 ± 0.07

(m) Metastable.

* The uncertainties are unknown in these cases because the value of the branching ratios for the γ-rays used in the analysis is unknown. The values for the yields were obtained assuming a branching ratio of 100%. Thus the values represent the lower limits to the yields (Reference 12).

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