



International Conference on Computational Heat and Mass Transfer-2015

# Fibre-reinforced Magneto-thermoelastic Rotating Medium With Fractional Heat Conduction

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## Abstract

In this article, a new mathematical model of magneto-thermoelasticity has been constructed to investigate the transient phenomena in the context of a new consideration of heat conduction with fractional order, where the anisotropic fibre-reinforced medium is rotating with a uniform angular velocity. The governing equations for Green-Naghdi theory have been formulated and solved analytically employing the normal mode analysis. Numerical results for the temperature, displacement and stresses are obtained and have been depicted graphically. According to the numerical results and corresponding graphical representations, conclusions about this new theory have been constructed. Finally, some comparisons have been shown to estimate the effect of magnetic field, reinforcement and the presence of non-local fractional parameter on all the studied fields.

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Peer-review under responsibility of the organizing committee of ICCHMT – 2015

**Keywords:** Fibre reinforcement; magneto-thermoelasticity; fractional order heat equation; rotating medium; normal mode analysis.

## 1. Introduction

In recent years, the theory of magneto-thermoelasticity which deals the interactions among the strain, temperature and magnetic field has drawn the attention of several researchers due to its extensive uses in diverse fields, such as geophysics, for understanding the effects of the earth's magnetic field on seismic waves, damping of acoustic waves in a magnetic field, emission of electromagnetic radiations from nuclear devices etc.

Fibre reinforcement is widely used in engineering structures due to its superiority over structural materials in applications in requiring high strength and stiffness in light weight components [1]. Consequently, the characterisation of their mechanical behavior is of particular importance for structural designing using these materials. In recent years, fibre-reinforced composite (FRC) laminated structures have been widely used in the aeronautical mechanical, civil, petrochemical and other engineering industries [2]. Their components are often exposed to high temperature as well as moisture. The analysis of stress and deformation of fibre reinforced composite materials has been an important subject of solid mechanics since last three decades [3].

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In view of the fact that most of the bodies like the earth, the moon and the other planets have an angular velocity, as well as earth behaves like a huge magnet, it is more important to study the propagation of thermoelastic waves in a rotating medium under the influence of a magnetic field. So, the attempts are being made to study the propagation of finite thermoelastic waves in an infinite elastic medium rotating with an angular velocity. Recently, several authors studied the wave propagation in thermoelastic and magneto-thermoelastic media in the context of generalized theories.

During recent years, several interesting problems in engineering have been modeled by using fractional calculus to study physical processes in the area of heat conduction, diffusion, mechanics of solids, electromagnetism etc. In such cases, researchers are using a generalized thermoelasticity theory based on an anomalous heat conduction equation having non-local fractional order operators [4]. Employing the fractional heat conduction, recently, several problems have been solved by Sur and Kanoria [5], Ezzat et. al [6] etc.

In the present analysis the propagation of plane waves in a fibre-reinforced anisotropic rotating thermoelastic medium has been studied under the influence of magnetic field where the heat conduction has been considered employing the Green Naghdi models [7] in the context of a new Taylor's series expansion of time-fractional order. Normal mode analysis technique is employed onto the resulting equations and finally, the physical fields have been presented graphically to study the effect of rotation, magnetic field and the presence of the non-local fractional parameter has been reported also.

## 2. Formulation of the Problem

Let us consider a perfectly conducting thermoelastic half-space  $x \geq 0$  which is rotating with a uniform angular velocity  $\Omega = \Omega \mathbf{n}$ , where  $\mathbf{n}$  is the unit vector in the direction of the axis of rotation. The adjacent free space is assumed to be permeated by a uniform magnetic field  $\mathbf{H} (0, 0, H_0)$  acting parallel to the boundary  $y = 0$ . This produces an induced magnetic field  $\mathbf{h} (0, 0, h)$  and induced electric field  $\mathbf{E} (E_1, E_2, 0)$  which satisfies the linearized equations of electro-magnetism and are valid for slowly moving media. If we take the coordinate axis fixed in the rotating medium, the displacement equation of motion in the rotating frame of reference has two additional terms - the centripetal acceleration  $\Omega \times (\Omega \times \mathbf{u})$  due to the time varying motion only and the coriolis acceleration  $2\Omega \times \dot{\mathbf{u}}$  where  $\mathbf{u}$  is the dynamic displacement vector measured from a steady-state deformed position where the deformation is assumed to be very small.

The equations of motion are given by

$$\sigma_{ij,j} + \mu_0(\mathbf{J} \times \mathbf{H})_i = \rho[\ddot{\mathbf{u}} + \Omega \times (\Omega \times \mathbf{u}) + 2\Omega \times \dot{\mathbf{u}}]_i, \quad (1)$$

The constitutive equation for a fibre-reinforced linearly thermoelastic anisotropic medium whose preferred direction is that of a unit vector  $\mathbf{a}$  is [1]

$$\sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km} \delta_{ij} + a_i a_j e_{kk}) + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) - \beta a_k a_m e_{km} a_i a_j - \beta_{ij}(T - T_0) \delta_{ij}, \quad (2)$$

where  $\sigma_{ij}$  are the stress components,  $e_{ij}$  are the strain components,  $\lambda, \mu_T$  are the elastic constants,  $\alpha, \beta, (\mu_L - \mu_T)$  are reinforcement parameters,  $\delta_{ij}$  is the kronecker delta,  $\alpha_{11}, \alpha_{22}$  are the coefficient of linear thermal expansion,  $T$  is the temperature over the reference temperature  $T_0$ ,  $\bar{a} \equiv (a_1, a_2, a_3)$ ,  $a_1^2 + a_2^2 + a_3^2 = 1$ .

The heat conduction equation corresponding to fractional order theory of Green Naghdi types II and III are given by assuming the fibre direction as  $(1, 0, 0)$

$$\left(1 + \frac{\tau_T^\alpha}{\alpha!}\right) \left[ K_{11}^* \frac{\partial^2 T}{\partial x^2} + K_{22}^* \frac{\partial^2 T}{\partial y^2} + K_{11} \frac{\partial^2 \dot{T}}{\partial x^2} + K_{22} \frac{\partial^2 \dot{T}}{\partial y^2} \right] = \left(1 + \frac{\tau_q^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} + \frac{\tau_q^{2\alpha}}{2\alpha!} \frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\right) \left[ \rho C_E \ddot{T} + T_0 \left( \beta_{11} \frac{\partial \ddot{u}}{\partial x} + \beta_{22} \frac{\partial \ddot{v}}{\partial y} \right) \right]. \quad (3)$$

The stress components are given by

$$\sigma_{xx} = (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta) \frac{\partial u}{\partial x} + (\lambda + \alpha) \frac{\partial v}{\partial y} - \beta_{11}(T - T_0), \quad (4)$$

$$\sigma_{yy} = (\lambda + 2\mu_T) \frac{\partial v}{\partial y} + (\lambda + \alpha) \frac{\partial u}{\partial x} - \beta_{22}(T - T_0), \tag{5}$$

$$\sigma_{xy} = \mu_T \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right). \tag{6}$$

The equations of motion along  $x$  and  $y$  are directions are

$$(A_{11} + \rho R_H^2) \frac{\partial^2 u}{\partial x^2} + (A_{12} + \rho R_H^2) \frac{\partial^2 v}{\partial x \partial y} + A_{13} \frac{\partial^2 v}{\partial y^2} - \beta_{11} \frac{\partial T}{\partial x} = \rho \left( 1 + \frac{R_H^2}{c^2} \right) \frac{\partial^2 u}{\partial t^2} - \rho \Omega^2 u - 2\rho \Omega \frac{\partial v}{\partial t}, \tag{7}$$

$$(A_{22} + \rho R_H^2) \frac{\partial^2 v}{\partial y^2} + (A_{12} + \rho R_H^2) \frac{\partial^2 u}{\partial x \partial y} + A_{13} \frac{\partial^2 v}{\partial x^2} - \beta_{22} \frac{\partial T}{\partial y} = \rho \left( 1 + \frac{R_H^2}{c^2} \right) \frac{\partial^2 v}{\partial t^2} - \rho \Omega^2 v + 2\rho \Omega \frac{\partial u}{\partial t}, \tag{8}$$

with

$$A_{11} = \lambda + 2(\alpha + \mu_T) + 4(\mu_L - \mu_T) + \beta, \quad A_{12} = \alpha + \lambda + \mu_L, \quad A_{13} = \mu_L, \quad A_{22} = \lambda + 2\mu_T, \quad R_H^2 = \frac{\mu_0 H_0^2}{\rho},$$

$$c^2 = \frac{1}{\varepsilon_0 \mu_0}, \quad \beta_{11} = (2\lambda + 3\alpha + 4\mu_L - 2\mu_T + \beta)\alpha_{11} + (\lambda + \alpha)\alpha_{22}, \quad \beta_{22} = (2\lambda + \alpha)\alpha_{11} + (\lambda + 2\mu_T)\alpha_{22},$$

Introducing the following non-dimensional variables

$$x' = c_1 \chi x, \quad y' = c_1 \chi y, \quad u' = c_1 \chi u, \quad v' = c_1 \chi v, \quad t' = c_1^2 \chi t, \quad \chi = \frac{\rho C_E}{K_{11}},$$

$$\sigma'_{ij} = \frac{\sigma_{ij}}{\rho c_1^2}, \quad h' = \frac{h}{H_0}, \quad c_1^2 = \frac{A_{11}}{\rho}, \quad \Omega' = \frac{\Omega}{c_1^2 \chi}, \quad T' = \frac{(T - T_0)\beta_{11}}{\rho c_1^2},$$

thereafter omitting primes, the above equations can be written in non-dimensional form as follows

$$\sigma_{xx} = \frac{\partial u}{\partial x} + B_1 \frac{\partial v}{\partial y} - T, \tag{9}$$

$$\sigma_{yy} = B_2 \frac{\partial v}{\partial y} + B_1 \frac{\partial u}{\partial x} - B_3 T, \tag{10}$$

$$\sigma_{xy} = B_4 \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \tag{11}$$

$$B_5 \frac{\partial^2 u}{\partial x^2} + B_6 \frac{\partial^2 v}{\partial x \partial y} + B_4 \frac{\partial^2 u}{\partial y^2} - \frac{\partial T}{\partial x} = \xi \frac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega \frac{\partial v}{\partial t}, \tag{12}$$

$$B_7 \frac{\partial^2 v}{\partial y^2} + B_6 \frac{\partial^2 u}{\partial x \partial y} + B_4 \frac{\partial^2 v}{\partial x^2} - \frac{\partial T}{\partial y} = \xi \frac{\partial^2 v}{\partial t^2} - \Omega^2 v + 2\Omega \frac{\partial u}{\partial t}, \tag{13}$$

$$\left( 1 + \frac{\tau_q^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) \left[ \varepsilon_{21} \frac{\partial^2 T}{\partial x^2} + \varepsilon_{22} \frac{\partial^2 T}{\partial y^2} + \varepsilon_{31} \frac{\partial^2 T}{\partial x^2} + \varepsilon_{32} \frac{\partial^2 T}{\partial y^2} \right] = \left( 1 + \frac{\tau_q^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} + \frac{\tau_q^{2\alpha}}{2\alpha!} \frac{\partial^{2\alpha}}{\partial t^{2\alpha}} \right) \left[ \ddot{T} + \varepsilon_{11} \frac{\partial \ddot{u}}{\partial x} + \varepsilon_{12} \frac{\partial \ddot{v}}{\partial y} \right], \tag{14}$$

where

$$(B_1, B_2, B_4) = \frac{1}{A_{11}}(\lambda + \alpha, A_{22}, \mu_T), \quad B_3 = \frac{\beta_{22}}{\beta_{11}}, \quad B_5 = 1 + \eta, \quad B_6 = B_1 + B_4 + \eta, \quad B_7 = B_2 + \eta, \quad \xi = 1 + \frac{R_H^2}{c^2},$$

$$\eta = \frac{R_H^2}{c_1^2}, \quad (\varepsilon_{11}, \varepsilon_{12}) = \frac{T_0 \beta_{11}}{\rho A_{11} C_E}(\beta_{11}, \beta_{22}), \quad (\varepsilon_{21}, \varepsilon_{22}) = \frac{1}{\rho C_E}(K_{11}^*, K_{22}^*), \quad \varepsilon_{31} = c_1^2, \quad \varepsilon_{32} = \frac{c_1^2 K_{22}}{K_{11}}.$$

Employing the normal mode analysis, defined as

$$(u, v, T, \sigma_{ij})(x, y, t) = (u^*, v^*, T^*, \sigma_{ij}^*)(x) \exp(\omega t + iby), \quad (15)$$

and after simplification, we have

$$(D^6 - AD^4 + BD^2 - C)v^*(x) = 0. \quad (16)$$

where

$$c_0 = B_5, \quad c_2 = b^2 B_4 + (\xi - 1)\Omega^2, \quad c_3 = ibB_6, \quad c_4 = 2\omega\Omega, \quad c_5 = B_4, \quad c_6 = b^2 B_7 + \xi\omega^2 - \Omega^2, \quad c_7 = ibB_3,$$

$$c_8 = \left(1 + \frac{\tau_T^\alpha}{\alpha!} \omega^\alpha\right) (\varepsilon_{21} + \varepsilon_{31}\omega), \quad c_9 = \omega^2 \left(1 + \frac{\tau_q^\alpha}{\alpha!} \omega^\alpha + \frac{\tau_q^{2\alpha}}{2\alpha!} \omega^{2\alpha}\right) + \varepsilon_{22} b^2 (1 + \omega) \left(1 + \frac{\tau_T^\alpha}{\alpha!} \omega^\alpha\right),$$

$$c_{10} = \varepsilon_{11} \omega^2 \left(1 + \frac{\tau_q^\alpha}{\alpha!} \omega^\alpha + \frac{\tau_q^{2\alpha}}{2\alpha!} \omega^{2\alpha}\right), \quad c_{11} = \frac{ib\varepsilon_{12}}{\varepsilon_{11}}, \quad c_{12} = c_0 c_5 c_7 c_8,$$

$$c_{13} = (c_0 c_7 - c_3)(c_5 c_9 + c_6 c_8) + c_2 c_5 c_7 c_8 + c_3 c_8 (c_5 c_6 + c_3 c_7) + c_5 (c_7 c_{10} - c_3 c_9),$$

$$c_{14} = (c_0 c_7 - c_3)(c_6 c_9 - c_7 c_{11}) + c_2 c_7 (c_5 c_9 + c_6 c_8) + c_4^2 c_7 c_8 + (c_7 c_{10} - c_3 c_9)(c_5 c_6 + c_3 c_7),$$

$$c_{15} = c_2 c_7 (c_6 c_9 - c_7 c_{11}) + c_4^2 c_7 c_9,$$

$$A = \frac{c_{13}}{c_{12}}, \quad B = \frac{c_{14}}{c_{12}}, \quad C = \frac{c_{15}}{c_{12}}.$$

From which, the corresponding solution is obtained in terms of normal modes as

$$v^*(x) = \sum_{n=1}^3 M_n(b, \omega) \exp(-k_n x). \quad (17)$$

Where  $k_n^2$  are roots of the equation

$$k^6 - Ak^4 + bk^2 - C = 0. \quad (18)$$

And similarly, the expressions of displacement and temperatures are obtained as

$$u^*(x) = \sum_{n=1}^3 M'_n(b, \omega) \exp(-k_n x), \quad (19)$$

$$T^*(x) = \sum_{n=1}^3 M''_n(b, \omega) \exp(-k_n x), \quad (20)$$

where  $M_n$ ,  $M'_n$  and  $M''_n$ , are defined as

$$M'_n(b, \omega) = H_{1n} M_n(b, \omega), \quad n = 1, 2, 3,$$

$$M''_n(b, \omega) = H_{2n} M_n(b, \omega), \quad n = 1, 2, 3,$$

where

$$H_{1n} = \frac{-c_5 k_n^3 + (c_5 c_6 + c_3 c_7) k_n - c_4 c_7}{(c_0 c_7 - c_3) k_n^2 - c_4 k_n - c_2 c_7}, \quad n = 1, 2, 3,$$

$$H_{2n} = \frac{(c_0 c_{11} - c_3 c_{10}) k_n^2 + c_{10} c_{11} k_n - c_2 c_{11}}{c_0 c_8 k_n^4 - (c_{10} + c_2 c_8 + c_6 c_9) k_n^2 + c_2 c_9}, \quad n = 1, 2, 3.$$

Using (17), (19) and (20), we arrive at the stress components in terms of normal modes as follows

$$\sigma_{xx}^*(x) = \sum_{n=1}^3 H_{3n} M_n(b, \omega) \exp(-k_n x), \quad (21)$$

$$\sigma_{yy}^*(x) = \sum_{n=1}^3 H_{4n} M_n(b, \omega) \exp(-k_n x), \tag{22}$$

$$\sigma_{xy}^*(x) = \sum_{n=1}^3 H_{5n} M_n(b, \omega) \exp(-k_n x), \tag{23}$$

where

$$\begin{aligned} H_{3n} &= -k_n H_{1n} + B_1 i b - H_{2n}, \quad n = 1, 2, 3, \\ H_{4n} &= B_2 i b H_{1n} - B_3 H_{2n}, \quad n = 1, 2, 3, \\ H_{5n} &= B_4 (i b - k_n H_{1n}), \quad n = 1, 2, 3. \end{aligned}$$

### 3. Applications

In this section, the general solutions for temperature and stresses will be used to yield the response of a uniform time-dependent loading  $f(y, t)$ . The boundary conditions for the problem may be taken as

#### 3.1. Thermal boundary condition

The thermal boundary condition for the present problem is given by

$$T = 0 \quad \text{at} \quad x = 0.$$

#### 3.2. Mechanical boundary condition

$$\sigma_{xx} = f(y, t) \quad \text{at} \quad x = 0; \quad \sigma_{xy} = 0 \quad \text{at} \quad x = 0.$$

Substituting the expressions of the variables considered into the above boundary conditions, and after simplifications, we obtain the solutions of the thermophysical quantities in terms of normal modes.

### 4. Numerical Results and Discussions

In order to study the influence of the magnetic field and reinforcement on the wave propagation in the rotating medium, we now present the obtained results in the form of their graphical representations. The material constants are given by

$$\begin{aligned} \rho &= 2660 \text{Kg} \cdot \text{m}^{-3}, \quad \lambda = 5.65 \times 10^{10} \text{N} \cdot \text{m}^{-2}, \quad \mu_T = 2.46 \times 10^{10} \text{N} \cdot \text{m}^{-2}, \quad \mu_L = 5.66 \times 10^{10} \text{N} \cdot \text{m}^{-2}, \\ \alpha &= -1.28 \times 10^{10} \text{N} \cdot \text{m}^{-2}, \quad \beta = 0.015 \times 10^{-4} \text{N} \cdot \text{m}^{-2}, \quad \alpha_{11} = 0.017 \times 10^{-4} \text{K}^{-1}, \quad H_0 = 10, \\ \epsilon_0 &= 0.3, \quad \alpha_{22} = 0.015 \times 10^{-4} \text{K}^{-1}, \quad C_E = 0.787 \times 10^3 \text{J} \cdot \text{Kg}^{-1} \cdot \text{K}^{-1}, \\ K_{11} &= 0.0921 \times 10^3 \text{J} \cdot \text{m}^{-1} \cdot \text{s}^{-1} \cdot \text{K}^{-1}, \quad K_{22} = 0.0963 \times 10^3 \text{J} \cdot \text{m}^{-1} \cdot \text{s}^{-1} \cdot \text{K}^{-1}, \quad \mu_0 = 0.1, \quad b = 2, \\ K_{11}^* &= 0.13 \times 10^3 \text{J} \cdot \text{m}^{-1} \cdot \text{s}^{-1} \cdot \text{K}^{-1}, \quad K_{22}^* = 0.3 \times 10^3 \text{J} \cdot \text{m}^{-1} \cdot \text{s}^{-1} \cdot \text{K}^{-1}, \quad \omega = \omega_0 + i\xi, \quad \omega_0 = 2, \quad \xi = 1, \end{aligned}$$

Fig. 1 depicts the variation of the temperature  $T$  versus  $x$  when  $y = 0.1, t = 0.1$ , the fractional parameter  $\alpha = 1$  and in absence and presence of magnetic field and rotation respectively. From the figure, it is seen that  $T$  vanishes on  $x = 0$  which satisfies the thermal boundary condition of our problem. The temperature of the body shows an increasing effect in  $0 < x < 0.5$  and then it approaches to zero. The magnitude of the temperature is increasing due to the presence of the magnetic field. Whenever the body is rotating, the temperature of the body increases than that of

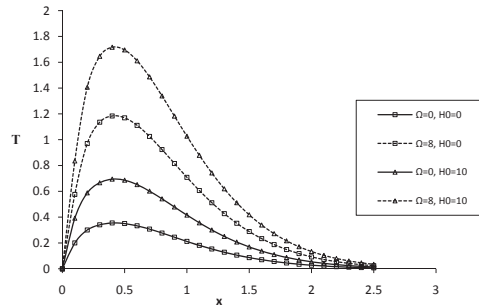


Fig. 1. Variation of  $T$  versus  $x$  for  $\alpha = 1$ ,  $t = 0.1$ .

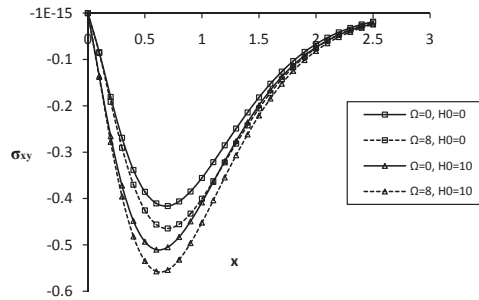


Fig. 2. Variation of  $\sigma_{xy}$  versus  $x$  for  $\alpha = 1$ ,  $t = 0.1$ .

the absence of the rotation of the body. Whenever the medium is rotating with an uniform angular velocity, we found two additional terms in the displacement equation of motion- the centripetal acceleration and coriolis acceleration, which influences the thermoelastic response which increases the magnitude of the temperature  $T$ .

Fig. 2 is plotted to study the variation of the stress component  $\sigma_{xy}$  versus  $x$  for the same set of parameters. As seen from the figure,  $\sigma_{xy}$  vanishes on the plane  $x = 0$  satisfying the mechanical boundary condition of the problem. Further, the stress is compressive in nature near the plane  $x = 0$  and with the increase of the distance  $x$ ,  $\sigma_{xy}$  diminishes to zero value which is quite plausible. Whenever the medium is rotating, the magnitude of  $\sigma_{xy}$  increases. The maximum magnitude is found to be occurred near  $x = 0.6$ . The presence of the magnetic field will also increase the magnitude of  $\sigma_{xy}$ . In presence of the magnetic field ( $H_0 = 10$ ), the induced magneto-mechanical loading increases the magnitude of the thermophysical quantities which is quite plausible.

Fig. 3 is plotted to study the variation of the stress component  $\sigma_{xx}$  versus  $x$  for the same set of parameters. Here,  $\sigma_{xx}$  attains the maximum magnitude on the plane  $x = 0$  which compiles the boundary condition and therefore it validates the correctness of the numerical code prepared in our problem. Further, it is seen that the magnitude of  $\sigma_{xx}$  decays and approaches to zero value with the increase of the distance  $x$ . Due to the presence of the rotation of the body,  $\sigma_{xx}$  decays sharply and almost disappears after  $x = 0.7$  whereas in absence of the rotation of the body, the decay of  $\sigma_{xx}$  becomes slower.

Fig. 4 depicts the variation of the displacement component  $u$  versus the distance  $x$  for  $\Omega = 0$ ,  $H_0 = 0$  and time  $t = 0.1$  for different values of the fractional parameter  $\alpha$ . As seen from the figure,  $u$  increases at first in  $0 < x < 0.5$  and then the magnitude decays with the increase of  $x$ . Also it is seen that as the magnitude of  $\alpha$  increases, the magnitude of  $u$  also increases both for WFR (with reinforcement) and NFR (non-reinforcement) cases. This is because the reinforcement parameters influence the thermoelastic responses for different values of the non-local fractional parameter. Also, the magnitude of  $u$  corresponding to WFR is larger than that of NFR for different values of  $\alpha$ . Further, it is observed that the magnitude of  $u$  decays sharply for  $\alpha = 1.5$  (WFR) than that of  $\alpha = 1.0$  (WFR), which decays faster than that of  $\alpha = 0.5$  (WFR). Since, fractional operator is a non-local operator, so we may conclude that our present

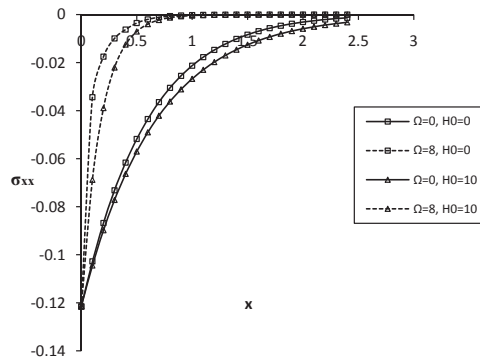


Fig. 3. Variation of  $\sigma_{xx}$  versus  $x$  for  $\alpha = 1, t = 0.1$ .

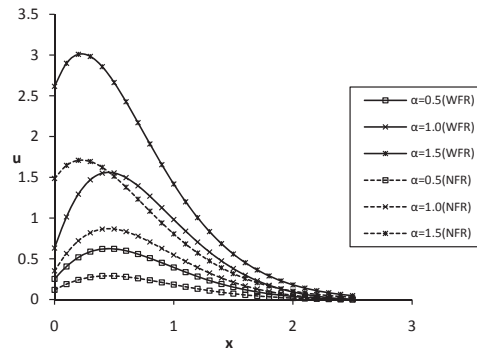


Fig. 4. Variation of  $u$  versus  $x$  for  $t = 0.1, \Omega = 0.0, H_0 = 0$ .

state depends on its historical states also. As seen from fig. 4, a small fractional order smooths the curves of the displacement and with the increase of the fractional parameter as  $\alpha \rightarrow 1$ , it complies our real situation.

### 5. Conclusions

In the present analysis, a new mathematical treatment has been presented to analyze the magneto-thermoelastic wave propagation in a fibre-reinforced rotating medium where the heat conduction equation consists of a new Taylor’s series expansion with fractional operators. Though the figures are self-explanatory in exhibiting the different peculiarities which occur in the propagation of waves, yet the following remarks may be added.

1. The presence of the thermophysical quantities increase with the increase of the non-local fractional parameter.
2. The magnitude of the thermophysical quantities increase whenever the body is rotating with an uniform angular velocity.
3. The presence of reinforcement has the tendency to increase the magnitude of the physical quantities.
4. Here, all the results corresponding to the case when  $\alpha = 1, \Omega = 0$  complies with the results of the existing literature [3].

### Acknowledgements

We are grateful to Professor S. C. Bose of the department of Applied Mathematics, University of Calcutta for his kind help and guidance in the preparation of the paper. We also express our sincere thanks to the reviewers for their suggestions for the improvement of the paper in the present form.

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