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initial application of junction bias during a measurement the emission current will show an initial decrease with a time constant of a few minutes. If the junction bias is removed the emission current will recover to the higher value in a subsequent measurement. The time constant for the decrease of this emission current is dependent upon the external pressure which may indicate that some of the surface charge is due to an increased adsorption of negatively charged ions on the surface when the junction current is on.

SUMMARY

The emission efficiency of hot electrons from a silicon surface is exponentially dependent upon the thickness of the oxide on the silicon. The characteristic thickness associated with this decrease is 4 Å, and the decrease can be accounted for by a constant field in the oxide of 4×10^6 V/cm caused by a surface charge density of $8\times10^{12}/\text{cm}^2$. Changes in emission efficiency due to interface scattering and internal scattering in the oxide layer constitute at most 15% of the observed effect and correspond to an attenuation distance in the oxide of about 35 Å. We find also that the surface charge density and the oxide growth rate can be modified by the presence of electrons injected into the oxide from the reverse biased n–p junction.

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Faraday Rotation in Artificial Dielectrics

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It is shown from an analysis of the propagation characteristics of a plane polarized wave in an artificial dielectric that Faraday rotation would be produced. Numerical results for n-type InSb indicate that the rotation should be experimentally observable. Some practical applications of the phenomenon are suggested.

I. INTRODUCTION

THE characteristics of propagation of a plane-polarized wave through an artificial dielectric in a magnetic field were first analyzed by Wicher.[1] He concluded that the plane of polarization would rotate as the wave propagated through the dielectric. This rotation is similar to the Faraday rotation in dielectrics at optical frequencies and was therefore given the same name. Expressions were also given by the author for the angle of rotation in terms of the physical parameters of the metallic elements constituting the dielectric. In a later communication, Rau and Caspari[2] by extending their analysis of Faraday rotation in semiconductors to the case of artificial dielectrics concluded that no rotation would be produced in artificial dielectrics.

In this paper the problem is re-examined. It is first shown that the contradictory conclusions are due to differences in the role given to Hall field in Maxwell's equations. The proper form of Maxwell's equations including Hall field is then established by considering the expressions for currents in conductors in a magnetic field. It is found from these equations that an artificial dielectric would produce Faraday rotation, but the sense of rotation would be opposite to that given by Wicher.

An experimental arrangement for verification of the above conclusion is suggested and some possible applications of the phenomenon are discussed.

[1] E. Wicher, J. Appl. Phys. **22**, 1327 (1951).
[2] R. R. Rau and M. E. Caspari, Phys. Rev. **100**, 632 (1955).

II. MAXWELL'S EQUATIONS INCLUDING HALL FIELD

The dielectric constant of an artificial dielectric is given by[3,4]

$$\epsilon_r = 1 + \alpha N/\epsilon_0, \tag{1}$$

where N is the number of metallic elements per unit volume, α is the polarizability of the elements, and ϵ_0 is the free-space permittivity.

The artificial dielectric is assumed to be made up of thin circular conducting disks embedded in a dielectric-like foam rubber, which has a dielectric constant nearly equal to unity. For this artificial dielectric α is given by

$$\alpha = (16/3)a^3\epsilon_0, \tag{2}$$

where a is the radius of the disks.

Equation (1) is derived by using the relation between

[3] W. E. Kock, Bell System Tech. J. **27**, 58 (1948).
[4] The conditions required to be satisfied by the dielectric in order that Eq. (1) be applicable are discussed in Ref. (3). It is assumed here that the dielectric satisfies these conditions.

TABLE I. Maxwell's equations including Hall field.

Author	Material	Maxwell's equations[a]
Wicher	Artificial dielectric	$\nabla\times\mathbf{H}=\mathbf{J}$; $\nabla\times\mathbf{E}=-\mu_0(\partial\mathbf{H}/\partial t)$; $\mathbf{J}=\mathbf{J}_m+\epsilon_0(\partial\mathbf{E}/\partial t)$; $\mathbf{J}_m=\alpha N(\partial/\partial t)(\mathbf{E}+\mathbf{E}_H)$; $\mathbf{E}_H=R_c\mathbf{B}\times\alpha N(\partial\mathbf{E}/\partial t)$.
Rau and Caspari	Semiconductor	$\nabla\times\mathbf{H}=\mathbf{J}$; $\nabla\times(\mathbf{E}+\mathbf{E}_H)=-\mu_0(\partial\mathbf{H}/\partial t)$; $\mathbf{J}=\mathbf{J}_c+\epsilon(\partial/\partial t)(\mathbf{E}+\mathbf{E}_H)$; $\mathbf{J}_c=\sigma\mathbf{E}$; $\mathbf{E}_H=R_c\mathbf{B}\times\mathbf{J}_c$.
Rau and Caspari	Artificial dielectric	$\nabla\times\mathbf{H}=\mathbf{J}$; $\nabla\times(\mathbf{E}+\mathbf{E}_H)=-\mu_0(\partial\mathbf{H}/\partial t)$; $\mathbf{J}=\mathbf{J}_m+\epsilon_0(\partial/\partial t)(\mathbf{E}+\mathbf{E}_H)$; $\mathbf{J}_m=\alpha N(\partial/\partial t)(\mathbf{E}+\mathbf{E}_H)$; $\mathbf{E}_H=R_c\mathbf{B}\times\mathbf{J}_m$.
Barlow	Semiconductor	$\nabla\times\mathbf{H}=\mathbf{J}$; $\nabla\times\mathbf{E}=-\mu_0(\partial\mathbf{H}/\partial t)$; $\mathbf{J}=\mathbf{J}_c+\mathbf{J}_d$; $\mathbf{J}_c=\sigma(\mathbf{E}+\mathbf{E}_H)$; $\mathbf{J}_d=\epsilon(\partial/\partial t)(\mathbf{E}+\mathbf{E}_H)$; $\mathbf{E}_H=R_c\mathbf{B}\times\mathbf{J}_c+[(\epsilon-\epsilon_0)/\epsilon]R_d\mathbf{B}\times\mathbf{J}_d$.

[a] Meaning of symbols: $\mathbf{E}$ = electric field vector, $\mathbf{H}$ = magnetic field vector, $\mathbf{J}$ = current density vector, μ_0 = free-space permeability, ϵ_0 = free-space permittivity, $\mathbf{J}_m$ = current due to metallic elements, α = polarizability due to metallic elements, $\mathbf{E}_H$ = Hall field vector, R_c = Hall coefficient of the metal or for conduction current, $\mathbf{B}$ = steady magnetic field; ϵ = semiconductor permittivity, $\mathbf{J}_c$ = carrier component of semiconductor current-density vector, $\mathbf{J}_d$ = dielectric component of current-density vector, R_d = Hall coefficient for dielectric current, σ = conductivity.

polarization produced by the metallic elements and the incident electric field as given below.

$$\mathbf{P}=\alpha N\mathbf{E}, \tag{3}$$

where $\mathbf{P}$ is the polarization produced by the metallic elements.

In the presence of a magnetic field, this relation is modified due to Hall effect. Analysis of the propagation characteristics of electromagnetic waves in an artificial dielectric in the presence of a magnetic field therefore requires consideration of the Hall field. Propagation characteristics in the presence of Hall field has been studied by Wicher,[1] by Rau and Caspari,[2] and by Barlow.[5] In Table I are given Maxwell's equations as written by these authors.

From Table I it is evident that Wicher assumes that Hall field modifies only the current component due to metallic elements. Rau and Caspari assume that only the dielectric current component is modified, while Barlow assumes that both carrier as well as dielectric current components are modified. As a result of these differences in the assumptions, the expressions derived by the above authors for the propagation constants of circularly polarized waves are very different, as shown in Table II.

The information contained in Table II shows that the three analyses lead to contradictory results in regard to Faraday rotation. For artificial dielectrics, Wicher's analysis predicts a Faraday rotation while that of Rau and Caspari predicts none. In the case of semiconductors, Rau and Caspari's analysis predicts a rotation which does not agree with what follows from Barlow's analysis even when the dielectric current Hall coefficient R_d is assumed to be zero.

It is thus evident that in order to analyze the rotation in artificial dielectrics the role of Hall field in Maxwell's equations should be correctly established. In analyzing Faraday rotation in semiconductors, however, this problem is somewhat simplified since one may derive directly the expression for the conductivity tensor by considering the motion of free carriers in the presence of the magnetic field. In the case of artificial dielectrics, on the other hand, analysis in terms of Hall field appears to be more convenient. However, while introducing Hall field in Maxwell's equations, one has to be sure that the resultant equations are consistent with those obtained from considerations of free-carrier motion. This point appears to have been given little attention in the earlier communications mentioned. In the following sections Maxwell's equations are developed including Hall field by considering the motion of free carriers in the presence of a magnetic field.

III. MAXWELL'S EQUATIONS INCLUDING HALL FIELD AS OBTAINED BY CONSIDERING THE MOTION OF CARRIERS

Let us consider a semiconductor sample, to which an electric field is applied[6] in the x and y directions and a magnetic field is applied in the z direction. If one assumes that the effect of momentum relaxation time

TABLE II. Propagation constants for circularly polarized waves.

Author	Material	Propagation constant[a]
Wicher	Artificial dielectric	$\Upsilon_\pm^2=-\omega^2\mu_0\epsilon_0[1+(\alpha N/\epsilon_0)]\mp\omega^3\mu_0R_cB\alpha^2N^2$.
Rau and Caspari	Semiconductor	$\Upsilon_\pm^2=-\omega^2\mu_0\epsilon+\dfrac{(j\omega\mu_0\sigma\mp\omega\mu_0\sigma\mu_{cH}B)}{1+(\mu_{cH}B)^2}$.
Same	Artificial dielectric	$\Upsilon_\pm^2=-\omega^2\mu_0\epsilon_0[1+(\alpha N/\epsilon_0)]$.
Barlow	Semiconductor	$\Upsilon_\pm^2=j\omega\mu_0a\pm\omega\mu_0b$; $a=\dfrac{(\sigma+j\omega\epsilon)}{\{1+B^2(\mu_{cH}+j\mu_{dH})^2\}}$; $b=a(\mu_{cH}+j\mu_{dH})B$.

[a] Meanings of symbols: $\Upsilon_\pm$ = propagation constants for right- and left-handed circularly polarized waves, respectively; $\mu_{cH}=R_c\sigma$ = Hall mobility of free carriers, $\mu_{dH}=R_d\omega(\epsilon-\epsilon_0)$ = effective Hall mobility of bound electrons.

[5] H. E. M. Barlow, Proc. IEE **108**, 349 (1961).

[6] In the case of an incident electromagnetic wave the electric field induced in the medium due to the variation of magnetic induction is referred to as the applied field.

is negligible, the average velocities of the carriers, V_x and V_y, in the x and y directions, respectively, are given by

$$V_x=\mu[E_x+(\mu_{cH}/\mu)V_yB], \quad (4)$$

$$V_y=\mu[E_y-(\mu_{cH}/\mu)V_xB], \quad (5)$$

where μ is the conductivity mobility and μ_{cH} is the Hall mobility.

The above assumption is substantially correct even at microwave frequencies for conductors. If, however, the effect of relaxation time is considerable in a particular sample one has only to substitute for μ and μ_{cH} the complex values appropriate for the signal frequency. This does not alter the nature of results derived here from (4) and (5).

The expressions for the current components J_{cx} and J_{cy} are given by

$$J_{cx}=\sigma[E_x+R_cJ_{cy}B] \quad (6)$$

$$J_{cy}=\sigma[E_y-R_cJ_{cx}B]. \quad (7)$$

When the electric field is applied in one direction only, one evaluates the Hall field from (6) and (7) by equating the current in the other direction to zero. But, when electric fields are applied in two directions, one can not dissociate Hall field by equating the transverse current to zero. It, therefore, appears that the current equations in Maxwell's equation should be written directly from (6) and (7). But from analogy with the expression for Hall field when the applied electric field is in one direction, one may call $\mathbf{E}_H$ as written below, the equivalent Hall field:

$$\mathbf{E}_H=R_c\mathbf{B}\times\mathbf{J}_c. \quad (8)$$

One may thus write, using Eq. (6) through Eq. (8),

$$\mathbf{J}_c=\sigma(\mathbf{E}-\mathbf{E}_H). \quad (9)$$

It is of interest to note that the above line of argument leads us to conclude that Hall field modifies only the current to which it is due, and instead of augmenting as assumed in the earlier analyses, it decreases this current.

Using Eq. (9), Maxwell's equations for conducting media in the presence of a magnetic field may be written as

$$\nabla\times\mathbf{H}=\mathbf{J};\quad \nabla\times\mathbf{E}=-\mu_0(\partial\mathbf{H}/\partial t);$$
$$\mathbf{J}=\mathbf{J}_c+\epsilon(\partial\mathbf{E}/\partial t);\quad \mathbf{J}_c=\sigma(\mathbf{E}-\mathbf{E}_H); \quad (10)$$
$$\mathbf{E}_H=R_c\mathbf{B}\times\mathbf{J}_c.$$

It may easily be verified that the above equations lead to results identical to those obtained from conductivity tensor and may therefore be accepted as the correct form of Maxwell's equations including Hall field.

We may now write the equations for the artificial dielectric in the light of the above discussion. In this case, the impression of the magnetic field would change only the polarization of the metallic elements forming the dielectric. It is assumed here that the change produced is of the same nature as the change in motion of free carriers in conductors. One may hence infer that the polarization equation would be modified to

$$\mathbf{P}=\alpha N(\mathbf{E}-\mathbf{E}_H). \quad (11)$$

Thus, Maxwell's equations for the artificial dielectric are

$$\nabla\times\mathbf{H}=\mathbf{J};\quad \nabla\times\mathbf{E}=-\mu_0(\partial\mathbf{H}/\partial t);$$
$$\mathbf{J}=\mathbf{J}_m+\epsilon_0(\partial\mathbf{E}/\partial t);\quad \mathbf{J}_m=\alpha N(\partial/\partial t)(\mathbf{E}-\mathbf{E}_H); \quad (12)$$
$$\mathbf{E}_H=R_c\mathbf{B}\times\mathbf{J}_m.$$

It is of interest to compare at this stage Rau and Caspari's equations with ours. The field $\mathbf{E}+\mathbf{E}_H$ of their equations may be identified with our $\mathbf{E}$ and called $\mathbf{E}_t$. Then, it is found that for semiconductors, since Rau and Caspari assume that the conduction current is due to only $\mathbf{E}$ or $\mathbf{E}_t-\mathbf{E}_H$, their equations are identical to ours. In the case of artificial dielectrics, however, they assume that the polarization of the metallic elements is proportional to $\mathbf{E}_t$. Since like the conduction current carriers, the charges in the metallic elements causing the polarization move under the action of the field $\mathbf{E}_t-\mathbf{E}_H$, we have assumed that the polarization is proportional to $\mathbf{E}_t-\mathbf{E}_H$, instead of $\mathbf{E}_t$. Due to this difference in the assumption, our results for artificial dielectrics differ from those of Rau and Caspari.

IV. FARADAY ROTATION

One may now consider a plane wave with the electric vector polarized in the x direction to be incident on the dielectric. The characteristics relating to its propagation through the dielectric may be analyzed in the usual manner by breaking the plane wave into two circularly polarized waves of opposite senses and combining the waves as they emerge from the dielectric. It may easily be shown that the propagation constants for the two circularly polarized waves are given by

$$Y_\pm^2=-\omega^2\mu_0\epsilon_0[1+(\alpha'N/\epsilon_0)]\pm\omega^3\mu_0\alpha\alpha'N^2R_cB, \quad (13)$$

where

$$\alpha'=\alpha/\{1-(\omega\alpha NR_cB)^2\}.$$

It also follows from Eq. (13) that the plane wave on passing through the medium has the direction of polarization rotated by an angle given by

$$\phi=[\epsilon_0pp'R_cB/2c(1+p')^2]\omega^2z, \quad (14)$$

where $p=\alpha N/\epsilon_0$ and $p'=\alpha'N/\epsilon_0$.

It should be noted that Eq. (14) is the same as that derived by Wicher except for a change in the sign of the rotation and the incorporation of the factor. $\{1-(\omega\alpha NR_cB)^2\}$.

V. DISCUSSION

Though a Faraday rotation in artificial dielectrics was predicted by Wicher about a decade ago, as far as the authors are aware no experimental results have yet appeared. Rau and Caspari's analysis seemed to indicate that the experiment would give a negative result. In view of our analysis it may be of interest to try an experiment for verifying which of the conclusions are correct.

Success of the experiment will depend on the choice of the material forming the artificial dielectric. The material is required to have a large Hall coefficient as well as high conductivity. The former is required in order to produce a measurable rotation, while the latter is required to ensure that the loss introduced by the dielectric is not high.

The rotation may be calculated using Eq. (14). An estimate of the loss in the dielectric may be obtained by assuming that the field penetrating a conducting element is completely absorbed by it. The attenuation constant for the dielectric on the basis of this assumption may be shown to be given by (see Appendix)

$$\alpha_n = 0.208(\omega\epsilon_r\epsilon_0/\sigma)^{\frac{1}{2}}(\epsilon_r - 1)/a, \tag{15}$$

where σ is the conductivity of the material.

Among the available materials n-type InSb appears to be the best choice in view of the criteria mentioned above. In Table III are given the values of ϕ and α_n which will be obtained by using n-type InSb with carrier concentration of $5\times10^{19}/\mathrm{m}^3$ and temperatures below 140°K. The values of R_c and σ are, respectively, assumed to be 10^{-1} m^3/C and 10^2 V/m.[7] The values of ϕ are given for B equal to 0.3 Wb/m^2.

Thus using a dielectric made of InSb disks the rotation should be detectable using lengths of the order of 10 cm in the 3-cm wavelength range, or of the order of 4 cm in the 1-cm wavelength range.

It may be pointed out that the experiment if it gives a positive result, in addition to confirming the theory, will be of importance from the point of view of applications. Some of the possible applications which have been mentioned by Wicher are frequency or phase modulation, and rotation of plane of polarization. In addition, it is also evident that the phenomenon would be of use in measuring the value of R_c (the Hall coefficient of the material constituting the artificial dielectric) at microwave frequencies since in Eq. (14) all the quantities except α and R_c are known. The value of α can be determined by measuring the phase change undergone by the wave in propagating through the dielectric in the absence of the magnetic field using the equation

$$\Gamma^2 = -\omega^2\mu_0\epsilon_0[1+(\alpha N/\epsilon_0)]. \tag{16}$$

[7] C. Hilsum and A. C. Rose-Innes, *Semiconducting III–V Compounds* (Pergamon Press, Oxford, England, 1961), 1st ed., p. 123.

TABLE III. Angle of rotation and attenuation for different spacings of the conducting elements.

(a)	(b)	(c)	(d)	(e)	(f)	(g)
3	1.25	0.375	1.125	0.833	1.15	5.58
3	1.5	0.375	1.125	0.417	2.53	20.43
1	1.25	0.125	0.375	0.277	6.00	50.12
1	1.5	0.125	0.375	0.139	13.15	183.87

[a] Wavelength in centimeters.
[b] Dielectric constant.
[c] Radius of the disks in centimeters.
[d] Separation in the plane perpendicular to the direction of propagation in centimeters.
[e] Separation in centimeters in the direction of propagation.
[f] Attenuation constant in Np/m.
[g] Angle of rotation in deg/m.

It should be noted in this connection, that microwave study of Hall coefficient of high-conductivity semiconductors like InSb, InAs by the conventional methods is difficult and the method suggested here may prove useful.

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APPENDIX

Let **H** be the magnetic field component of the incident wave. Since the disks are to be thin **H** may be assumed to remain unmodified. The power flowing into the conducting elements per unit area is hence given by

$$P_f = \tfrac{1}{2}|\eta H^2|,$$

where $\eta = (\omega\mu_0/2\sigma)^{\frac{1}{2}}(1+j)$, σ is the conductivity of the elements, and μ_0 is the permeability.

It is assumed that the power flowing into the disks is completely absorbed by them. Hence the disks in an unit volume absorb a power P_L given by

$$P_L = \tfrac{1}{2}(\omega\mu_0/2\sigma)^{\frac{1}{2}}H^2e^{-2\alpha_n z}N\pi a^2, \tag{A1}$$

where α_n is the attenuation in Np/m.

The power propagating in the z direction is

$$P_z = \tfrac{1}{2}(\mu_0/\epsilon_r\epsilon_0)^{\frac{1}{2}}H^2e^{-2\alpha_n z}. \tag{A2}$$

Since the rate of change of P_z per unit distance is equal to the power lost, i.e., P_L, one obtains

$$\alpha_n = P_L/2P_z = (\omega\epsilon_r\epsilon_0/\sigma)^{\frac{1}{2}}\cdot(\pi/2\sqrt{2})Na^2. \tag{A3}$$

Combining Eqs. (1) and (A3),

$$\alpha_n = 0.208(\omega\epsilon_r\epsilon_0/\sigma)^{\frac{1}{2}}(\epsilon_r - 1)/a. \tag{A4}$$