AIP Journal of Applied Physics

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Citation: Journal of Applied Physics **36**, 3388 (1965); doi: 10.1063/1.1703001
View online: http://dx.doi.org/10.1063/1.1703001
View Table of Contents: http://scitation.aip.org/content/aip/journal/jap/36/11?ver=pdfcov
Published by the AIP Publishing

our arguments as,

$$\nabla\times\mathbf{H}=\mathbf{J};\quad \nabla\times\mathbf{E}=-\mu\partial\mathbf{H}/\partial t;$$
$$\mathbf{J}=\mathbf{J}_c+\mathbf{J}_d;\quad \mathbf{J}_c=\sigma(\mathbf{E}-\mathbf{E}_{cH});$$
$$\mathbf{J}_d=\epsilon_0\partial\mathbf{E}/\partial t+(\epsilon-\epsilon_0)(\partial/\partial t)(\mathbf{E}-\mathbf{E}_{dH});\qquad(4)$$
$$\mathbf{E}_{cH}=R_c\mathbf{B}\times\mathbf{J}_c;\quad \mathbf{E}_{dH}=R_d(\epsilon-\epsilon_0)\mathbf{B}\times(\partial/\partial t)(\mathbf{E}-\mathbf{E}_{dH}),$$

where $\mathbf{H}$ is the rf magnetic field vector and μ is the permeability of the medium.

Assuming a time dependence of fields given by $e^{j\omega t}$, it is a simple matter to calculate the propagation constants $\Gamma_\pm$ of the right- and left-handed circularly polarized waves as,

$$\Gamma_\pm^2=j\omega\mu\sigma\left[\frac{1\pm j\mu_{cH}B}{1+(\mu_{cH}B)^2}\right]-\omega^2\mu\epsilon\left[\frac{1\mp\mu_{dH}B}{1-(\mu_{dH}B)^2}\right]-\omega^2\mu\epsilon_0\left[\frac{\pm\mu_{dH}B-(\mu_{dH}B)^2}{1-(\mu_{dH}B)^2}\right].\qquad(5)$$

Barlow, on the other hand, gives the values of $\Gamma_\pm$ as,

$$\Gamma_\pm^2=\frac{j\omega\mu(\sigma+j\omega\epsilon)}{1+B^2(\mu_{cH}+j\mu_{dH})^2}[1\pm j(\mu_{cH}+j\mu_{dH})B],\qquad(6)$$

where $\mu_{cH}=R_c\sigma$ and $\mu_{dH}=R_d\omega(\epsilon-\epsilon_0)$.

We can compare Eqs. (5) and (6) in the limit of small magnetic fields, i.e., when $(\mu_{cH}B)^2\ll1$ and $(\mu_{dH}B)^2\ll1$. Under these conditions Eqs. (5) and (6) reduce, respectively, to

$$\Gamma_\pm^2=j\omega\mu(\sigma+j\omega\epsilon)\mp\omega\mu B[\sigma\mu_{cH}-\omega(\epsilon-\epsilon_0)\mu_{dH}],\qquad(7)$$
$$\Gamma_\pm^2=j\omega\mu(\sigma+j\omega\epsilon)\mp\omega\mu B(\sigma+j\omega\epsilon)(\mu_{cH}+j\mu_{dH}).\qquad(8)$$

On comparing the above two equations it may be noted that the contribution due to μ_{dH} would be reduced by a factor of $[(\epsilon-\epsilon_0)/(\epsilon+j\sigma/\omega)]$ according to our analysis.

The authors are indebted to Professor J. N. Bhar for his kind supervision of the present study.

JOURNAL OF APPLIED PHYSICS VOLUME 36, NUMBER 11 NOVEMBER 1965

Experimental Observation of Faraday Rotation in Artificial Dielectrics

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(Received 20 May 1965)

An experimental investigation of the Faraday rotation at 3 cm in artificial dielectrics composed of paraffin wax and powdered indium antimonide is described. It is found that a rotation which can be ascribed to the dielectric properties of the medium does indeed occur. The experimental results are found to be in reasonable agreement with theoretical predictions made earlier.

1. INTRODUCTION

THE possibility of Faraday rotation in artificial dielectrics has been theoretically examined by Wicher,[1] Rau and Caspari[2] and also by the present authors.[3] Wicher and also Nag and Engineer concluded that Faraday rotation would be exhibited, but Rau and Caspari concluded otherwise. It is needless to mention that if artificial dielectrics exhibit Faraday rotation, a number of practical applications of the phenomenon can be found. Some of the common applications that may be mentioned are phase modulation, amplitude modulation, construction of nonreciprocal transmission elements, etc. However, despite the theoretical study which was made about a decade ago no experimental study has yet been reported. The lack of experimental data is, perhaps, due to the fact that the magnitude of rotation in artificial dielectrics constructed with metallic elements is very small, as the metals have a very low value of Hall coefficient. It was suggested in Ref. 3 that significant rotation may be obtained by using indium antimonide which has a value of Hall coefficient about 10^8 times that for metals. Some calculations were presented, assuming the dielectric to be made of indium antimonide disks and foam rubber and it was found that rotation of a few degrees per meter may be produced. For experiments in bounded media, like waveguides, however, a more convenient form of artificial dielectric may be fabricated by using indium antimonide powder instead of disks. The present authors have made some experiments with artificial dielectrics made with paraffin wax and indium antimonide powder. The experiments show that Faraday rotation indeed occurs in the dielectrics. These experiments are here described.

[1] E. Wicher, J. Appl. Phys. **22**, 827 (1951).
[2] R. R. Rau and M. E. Caspari, Phys. Rev. **100**, 632 (1955).
[3] B. R. Nag and M. H. Engineer, J. Appl. Phys. **36**, 192 (1965).

2. PREPARATION OF THE ARTIFICIAL DIELECTRIC

The artificial dielectric has been prepared by mixing indium antimonide powder with paraffin wax. Polycrystals of indium antimonide having a conductivity of 10^5 mho/m and a Hall coefficient of the order of 10^{-3} m^3/C at room temperature were first powdered. The powdered particles were found to be approximately rectangular in shape and of size lying within 200–400 μ.

The powder was mixed with wax in the volumetric ratio of 1:1. The mixture was heated in a stainless steel container at a temperature slightly higher than the melting point of wax and stirred continuously for about two hours. It was then allowed to solidify, but stirring was continued. The solidified material was then desiccated for about 24 h. The samples were prepared from the mixture, pressing in a die by a ball press.

3. DIELECTRIC CONSTANT AND THE EFFECTIVE CONDUCTIVITY OF THE SAMPLES

In order to determine the dielectric constant and the effective conductivity of the dielectric, a section of a waveguide was filled with the dielectric. One end of the dielectric-filled guide was shorted and a slotted line was terminated by this shorted section. The SWR in the slotted line was then determined at different frequencies. Since the dielectric has a very low loss the guide wavelength λ_g, in the dielectric-filled waveguide corresponding to free space wavelength of λ_0, is given by

$$\lambda_g = \lambda_0/[\epsilon - (\lambda_0/\lambda_c)^2]^{\frac{1}{2}}, \quad (1)$$

where ϵ is the dielectric constant and λ_c is the cutoff wavelength. The input impedance of the shorted dielectric-filled guide has also a maximum and minimum value at frequencies for which it is an odd and even number of quarter guide wavelengths long. The SWR in the slotted line is also a minimum or a maximum at these two frequencies. Hence if λ_1 and λ_2 be the free-space wavelengths at which the SWR is a minimum for two successive lengths l_1 and l_2, one obtains

$$l_1 = (2n+1)\lambda_1/4[\epsilon - (\lambda_1/\lambda_c)^2]^{\frac{1}{2}}, \quad (2)$$

$$l_2 = (2n+3)\lambda_2/4[\epsilon - (\lambda_2/\lambda_c)^2]^{\frac{1}{2}}, \quad (3)$$

where n is an unknown integer. The unknown integer n may be eliminated from Eqs. (2) and (3) and ϵ determined. The value of ϵ measured this way has been found to be 20.4.

The dielectric constant of an artificial dielectric made with a dielectric of dielectric constant ϵ_p may be written as

$$\epsilon = \epsilon_p + (\alpha N)/\epsilon_0, \quad (4)$$

where αN is the polarization due to the metallic elements, N is the concentration of metallic elements, and ϵ_0 is the free-space permittivity. The dielectric constant of paraffin wax being 2.2, one obtains

$$\alpha N = (20\cdot 4 - 2\cdot 2)\epsilon_0.$$

The effective conductivity of the artificial dielectric may also be obtained by noting that if γ_1 is the maximum value of the SWR, then

$$\gamma_1 \text{ or } 1/\gamma_1 = \{[1-(\lambda_1/\lambda_c)^2]^{\frac{1}{2}}/[\epsilon-(\lambda_1/\lambda_c)^2]^{\frac{1}{2}}\} \times \coth\{(\pi\sigma_{\text{eff}} l)/\lambda_1\omega\epsilon_0[\epsilon-(\lambda_1/\lambda_c)^2]^{\frac{1}{2}}\}, \quad (5)$$

where ω is the frequency at which SWR is γ_1 and σ_{eff} is the effective conductivity of the dielectric. If the shift in the position of minimum is zero one uses $(1/\gamma_1)$ in (5), while if the shift is a quarter guide wavelength one uses γ_1. The values of γ_1, ω, and l were found to be 3.75, 9.0 kMc/sec, and 0.56 cm, respectively. The effective conductivity at $(2\pi\times 9\times 10^9)$ rad/sec is thus 2.6 mho/m.

It may be shown[3] that if σ be the conductivity of the metallic elements then

$$\sigma_{\text{eff}} = (\epsilon\epsilon_0/\mu)^{\frac{1}{2}}\cdot[\omega\epsilon\epsilon_0/\mu]^{\frac{1}{2}}\cdot 1/\sqrt{2}d, \quad (6)$$

where 'd' is the diameter of the elements assumed to be circular disks. In obtaining Eq. (6) it is assumed that the disks have thickness much larger than the skin depth in the material forming the disks.

Since σ_{eff} and ϵ are known, one may obtain 'd' from Eq. (6). The value of 'd' is found to be 30 μ which is of the same order as the measured dimensions of the powdered indium antimonide particles. This agreement verifies the formula of Ref. 3 for the attenuation in an artificial dielectric made of conducting elements of finite conductivity.

4. FARADAY ROTATION EXPERIMENT

In the commonly used method for studying Faraday rotation a circular waveguide is filled with the dielectric and a magnetic field is applied along the axis of the guide. The angle of rotation is then found by using a rotary joint at the output end of the circular guide and finding the direction for which the output is maximum. As the equipments necessary for this experiment were not available this method could not be used in our experiments. A method modified from that described by Hambleton and Gartner[4] for the measurement of Hall mobility in semiconductors was used instead.

The narrow side of a waveguide was coupled by a square waveguide at the common junction of two crossed waveguides as shown in Fig. 1. The square waveguide was of dimensions slightly larger than the inner narrow dimensions of the main waveguide, and was completely filled with the artificial dielectric. Terminals 2 and 5 of the system were terminated by matched loads, while matched detectors were connected to the terminals 4 and 6. A matched load with a matching unit was connected at terminal 3. Signal at the experimental frequency (9×10^9 Mc/sec) was fed to terminal 1. If guide 5-6 is exactly perpendicular to 1-2 and if the junction between the square guide and the crossed guide is symmetric all the power appearing at the output of the square waveguide would flow into guide 3-4 only and the detector at 4 would indicate a voltage corresponding to half of this power. The detector at 6 would show no voltage under this condition. But in practice the detector at 6 gives some reading due to misalignment or assymmetry in the junction. This initial voltage at 6 may, however, be annulled by adjusting the matching unit.

Now, if there be Faraday rotation when a magnetic field is applied to the system along the axis of the

[4] B. R. Nag and M. H. Engineer, J. Electron (to be published).

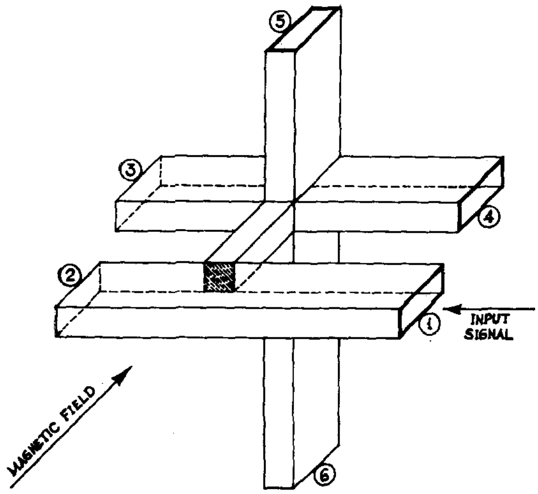

FIG. 1. Common junction of two crossed waveguides.

square waveguide, in addition to the normal TE_{10} mode, TE_{01} mode signals are also excited. Hence, at the output end of the square waveguide the electric vector is rotated and the voltage at 6 increases. Change in output at 6 with magnetic field may also occur due to magnetoresistance effect in the sample. Whether the increase in voltage is due to Faraday rotation or magnetoresistance may, however, be decided by allowing an initial voltage at 6. In case of Faraday rotation the voltage increases or decreases depending on the relative sense of rotation with respect to the initial field. If there be an increase for one direction, there would be a decrease for the opposite direction of the magnetic field. In case of magnetoresistance, on the other hand, the change in voltage would be independent on the direction of the magnetic field. In our experiments the voltage was found to increase for one direction and to decrease equally for the opposite direction of the magnetic field. This definitely indicates that the field is rotated by the magnetic field as in the case of Faraday rotation.

Now, let E_1 be the voltage at 4 and E_2 the voltage at 6 in the presence of the magnetic field.

Hence the angle of rotation is

$$\phi_m=\tan^{-1}(E_2/E_1)\approx(E_2/E_1) \quad \text{since} \quad E_2\ll E_1.$$

Thus, one may obtain ϕ_m by measuring the ratio (E_2/E_1).

In our experiments the magnetic field had a strength of 0.3 Wb/m^2 and the length of the sample was 2×10^{-2} *m*. The ratio E_2/E_1 was found to be 0.074 for both directions of the magnetic field. This data gives a value of ϕ_m equal to 4.3°.

The value of R may also be calculated[4] using the experimental values of (αN) and ϕ_m from the following equation:

$$\phi_m=(4l/\pi^2)(\mu/\epsilon\epsilon_0)^{\frac{1}{2}}B_0(\omega\alpha N)^2R.$$

The calculated value of R is 4×10^{-3} m^3/C which is found to be of the same order as the Hall coefficient of indium antimonide at room temperature.

It may be noted that the Faraday rotation in the unbounded dielectric is $[(\pi^2/4)\cdot(\phi_m/l)]$ and is 530°/m for the experimental sample.

5. DISCUSSION AND CONCLUSION

In the experimental work described in the present paper, it has been found that an artificial dielectric prepared with paraffin wax and indium antimonide powder exhibits Faraday rotation. For composition having a volumetric ratio of the two materials as 1:1, the rotation is 530°/m. The attenuation in the dielectric is also found to be of the same order as obtained by assuming that any power flowing into the indium antimonide particles is absorbed by them. The experiment, thus verifies the theory which was developed by the authors in Ref. (3). However, one point may be raised which needs clarification. Indium antimonide if used alone would have produced a Faraday rotation, due to the tensor nature of the conductivity of the material in the presence of a magnetic field. Hence, one should consider whether the Faraday rotation observed in the present experiment is due to the modification of the signal as it propagates through the indium antimonide particles or due to the modification caused by the fields produced in the dielectric as an effect of the induced polarization on the particles. In other words one may ask whether we should consider the medium studied here to be an artificial dielectric or just a mixture of indium antimonide and the dielectric. In the present case all the indium antimonide particles had thicknesses much larger than the skin depth and, therefore the possibility of propagation of the signals through the particles is ruled out. This is also confirmed by the agreement between the experimental and the theoretical value of σ_{eff}. Thus the effect observed is due to the polarization induced in the particles and is an evidence of Faraday rotation in an artificial dielectric.

The results of the present experiment in addition to verifying the theory of Faraday rotation[3] in artificial dielectrics, make available a new type of material which has the promise of practical applications mentioned earlier. At the present stage the angle of rotation though significant is small. In fact the figure of merit for Faraday rotation is 0.56 deg/dB as compared to 300–700 for ferrites. It is hoped, however, that much improvement may be made by more suitable choice of the composition of the dielectric and by increasing the magnetic field. Availability of a compound having a value of conductivity and Hall coefficient higher than that of indium antimonide evidently would also lead to an increase in the angle of rotation and make the artificial dielectric more suitable for practical exploitation.

ACKNOWLEDGMENT

The authors are indebted to Professor J. N. Bhar for his kind interest in the work.