Physics of Plasmas

Existence and stability of solitary kinetic Alfvén, ion-acoustic and electron-acoustic waves in a two electron temperature plasma

Debalina Chakraborty and K. P. Das

Citation: Phys. Plasmas 10, 2236 (2003); doi: 10.1063/1.1568750
View online: http://dx.doi.org/10.1063/1.1568750
View Table of Contents: http://pop.aip.org/resource/1/PHPAEN/v10/i6
Published by the American Institute of Physics.

PHYSICS OF PLASMAS VOLUME 10, NUMBER 6 JUNE 2003

Existence and stability of solitary kinetic Alfvén, ion-acoustic and electron-acoustic waves in a two electron temperature plasma

Debalina Chakraborty
Department of Mathematics, Charu Chandra College, 22 Lake Road, Calcutta 700 029, India

K. P. Das
Department of Applied Mathematics, University of Calcutta, 92 Acharya Prafulla Chandra Road, Calcutta 700 009, India

(Received 5 August 2002; accepted 24 February 2003)

The Korteweg–de Vries–Zakharov–Kuznetsov equations are derived for kinetic Alfvén, ion-acoustic and electron-acoustic waves in a two electron temperature magnetized plasma. The solitary wave solutions of these equations propagating obliquely to the external uniform magnetic field are obtained. The existence of these solitary wave solutions in the parameter space is investigated and studies are made on the stabilities of these solitary waves by the Rowlands–Infeld method. The kinetic Alfvén solitons are found to be stable. The instability criterion and maximum growth rate of instability become the same for both electron-acoustic and ion-acoustic solitons. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1568750]

I. INTRODUCTION

Large amplitude magnetic field perturbations have been observed in nature, for example in the solar wind.[1] The existence of these perturbations cannot be explained by exact magnetohydrodynamic (MHD) theory, since exact large amplitude Alfvén waves are known to decay into ion-acoustic waves over distances that are of the order of an astronomical unit.[2] Growing interest in this topic has led many authors,[3–7] to investigate the existence of an exact solitary Alfvén wave. Due to absence of dispersion in MHD theory, kinetic effects, like the finite ion Larmor radius effect and the finite electron inertial length effect, have been included. These latter two parameter ranges are called, respectively, the "kinetic regime" and "inertial regime." The dispersion effects introduced by these two regimes combined with nonlinearity lead to the formation of solitary Alfvén waves known as solitary kinetic Alfvén waves. Solitary kinetic Alfvén waves in the kinetic regime were obtained by Hasegawa and Mima[3] and Yu and Shukla,[4] and in the inertial regime by Shukla *et al.*,[5] Kalita and Kalita,[6] and Das *et al.*[7] Very recently Bandyopadhyay and Das[8] have obtained solitary kinetic Alfvén waves in the kinetic regime for nonthermal plasmas. Such a type of nonthermal plasmas stems from recent observations of solitary structures with density depletion made by the Freja Satellite (Dovner *et al.*[9]) and solitary structures in such plasmas without magnetic field perturbations have been obtained by Cairns *et al.*[10,11] and Mamun and Cairns.[12]

In some plasmas the presence of two electron populations are often encountered. This type of two electron population magnetized plasma can support three types of waves—kinetic Alfvén, ion-acoustic, and electron-acoustic waves, when both kinetic and inertial regimes are considered. A preliminary study of waves in a two electron temperature magnetized plasma was made by Treumann *et al.*[13] in view of its applications to conditions in the solar corona. Such two electron temperature plasmas as also present in the geophysical environment, at auroral latitudes for example, where a mixture of cold ionosphere and warm magnetospheric plasma is commonly observed.[14] A systematic study of the three types of solitary waves, viz., kinetic Alfvén, ion-acoustic and electron-acoustic waves, which can exist in a two electron temperature magnetized plasmas, has recently been made by Berthomier and Pottelette.[15] The most interesting case of two electron population plasma, in which one of the components is cold, such that its inertia cannot be neglected, while the other is warm enough to neglect its inertial effect, was taken by Berthomier and Pottelette.[15] Assuming that a solitary wave propagates obliquely to the external magnetic field, the governing equations for two electron population magnetized plasma have been reduced to the motion of an imaginary particle moving in the field of a Kirchhoff–Sagdeev pseudopotential. Then they have investigated the existence of different types of solitary waves by observing the nature of the pseudopotential. It is known that in a single electron plasma both kinetic Alfvén and ion-acoustic solitary waves are compressive. But in a two electron temperature they have shown that both compressive and rarefactive solitary waves can exist.

It is of natural interest to see how weakly nonlinear and weakly dispersive waves in a two electron temperature magnetized plasma evolve in space–time and to investigate the existence and stability of solitary waves propagating obliquely to the external magnetic field. These are the matters of investigation in the present paper. Appropriate Korteweg–de Vries–Zakharov–Kuznetsov (KdV–ZK) equations are derived, which describe the evolution of weakly nonlinear and weakly dispersive wave in a two electron temperature magnetized plasma in both the kinetic and internal regimes. Solitary wave solutions of these equations for kinetic Alfvén, electron-acoustic and ion-acoustic waves propagating obliquely to the external magnetic field are obtained and their existence in parameter space is investigated.

Regions in the $\alpha_c\mu$-parameter plane for some different values of θ are shown in the figures, where compressive and rarefactive solitons can exist. Here α_c is the square of the ratio of cold electron thermal speed and Alfvén speed, μ is the ratio of unperturbed number densities of hot and cold electrons, and θ is the ratio of temperatures of hot and cold electrons. Finally, the stability of the solitons are studied by the small-k perturbation expansion method of Rowlands and Infeld.[16–19] It is found that kinetic Alfvén solitons are stable. The instability conditions and maximum growth rate of the instability are obtained for both electron-acoustic and ion-acoustic solitons and they are found to be the same for both electron-acoustic and ion-acoustic solitons. Consequently the new matters of this paper, which are not present in previous investigations, consist of the derivation of KdV–ZK equations for weakly nonlinear and weakly dispersive ion-acoustic, electron-acoustic, and kinetic Alfvén waves in a two electron temperature magnetized plasma and the study of the existence and stability of solitary wave solutions of these equations.

II. BASIC EQUATIONS

We consider a collisionless magnetized plasma consisting of cold ions and two populations of electrons having two different temperatures. The electrons with higher temperature are called hot electrons and those having lower temperature are called cold electrons. The external uniform magnetic field B_0 is supposed to be directed along the z-axis. The plasma-β is supposed to be very small and the characteristic frequency is much smaller than the ion-cyclotron frequency. Due to this low-β assumption, both the electron populations are considered as magnetic and we assume a Boltzmann distribution for the hot electrons, but we keep inertia terms in the momentum conservation equation for the cold electrons. The ion transverse motion is supposed to be solely due to polarization drift. This type of plasma can support three types of waves,[15] which are the kinetic Alfvén wave, ion-acoustic wave, and electron-acoustic wave. The nonlinear behavior of these waves is described by the following equations, which consist of Ampère's law in the parallel direction, quasineutrality condition, Boltzmann's distribution for hot electrons, continuity equations for both hot and cold electrons, parallel component of momentum conservation equation for cold electron fluid, equation of continuity and parallel component of momentum conservation equation for ion fluid,[15]

$$\nabla_\perp^2 \frac{\partial}{\partial z}(\varphi-\psi)=\frac{\partial j_z}{\partial t}, \tag{1}$$

$$n_i=\frac{n_c+\mu n_h}{1+\mu}, \tag{2}$$

$$n_h=\exp\left(\frac{\psi}{\theta}\right), \tag{3}$$

$$\frac{\partial n_h}{\partial t}+\frac{\partial}{\partial z}(n_h v_h)=0, \tag{4}$$

$$\frac{\partial n_c}{\partial t}+\frac{\partial}{\partial z}(n_c v_c)=0, \tag{5}$$

$$\frac{\partial v_c}{\partial t}+v_c\frac{\partial v_c}{\partial z}=\alpha_c\left(\frac{\partial \psi}{\partial z}-\frac{1}{n_c}\frac{\partial n_c}{\partial z}\right), \tag{6}$$

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i v_{ix})+\frac{\partial}{\partial y}(n_i v_{iy})+\frac{\partial}{\partial z}(n_i v_{iz})=0, \tag{7}$$

$$\frac{\partial v_{iz}}{\partial t}+v_{ix}\frac{\partial v_{iz}}{\partial x}+v_{iy}\frac{\partial v_{iz}}{\partial y}+v_{iz}\frac{\partial v_{iz}}{\partial z}=-\alpha_c q\frac{\partial \psi}{\partial z}, \tag{8}$$

where ion transverse velocity $\mathbf{v}_{i\perp}$ and parallel current density j_z are given by

$$v_{ix}=-\frac{\partial^2\varphi}{\partial x\,\partial t},\quad v_{iy}=-\frac{\partial^2\varphi}{\partial y\,\partial t}, \tag{9}$$

$$j_z=n_i v_{iz}-\frac{n_c v_c+\mu n_h v_h}{1+\mu}. \tag{10}$$

The electric field components are determined from the two potentials ϕ and ψ according to

$$\mathbf{E}_\perp=-\mathbf{\nabla}_\perp\varphi,\quad E_z=-\frac{\partial\psi}{\partial z} \tag{11}$$

(Kadomtsev[20]), where $\perp$ and z indicate components perpendicular and parallel to the external magnetic field. In the above equations n_i, n_c, and n_h are, respectively, the ion number density, cold electron, and hot electron number densities; $\mathbf{v}_i$ is the ion fluid velocity: v_c and v_h are, respectively, the parallel component of velocity of cold and hot electron fluids; $\mu=n_{ho}/n_{co}$, $\theta=T_h/T_c$, $\alpha_c=(V_{ec}/V_A)^2$, $q=m_e/m_i$, n_{ho}, and n_{co} being, respectively, the unperturbed hot and cold electron number densities. T_h and T_c being, respectively, the temperatures of hot and cold electrons, $V_{ec}=(k_BT_c/m_e)^{1/2}$ being the cold electron thermal speed, $V_A=(B_o^2/4\pi m_i n_{io})^{1/2}$ being the Alfvén velocity and n_{io} being the unperturbed number density of ions. The above equations have been written in dimensionless form by normalizing perpendicular space coordinates (x,y) by ρ_i, parallel space coordinate (z) by c/ω_i, time (t) by $1/\Omega_i$, magnetic field $(\mathbf{B})$ by B_o, electric potentials (φ, ψ) by k_BT_c/e, electron and ion number densities (n_h,n_c,n_i) by (n_{ho},n_{co},n_{io}) parallel ion and electron fluid velocities (v_{iz},v_h,v_c) by V_A, perpendicular ion velocities (v_{ix},v_{iy}) by c, where $\rho_i=c_s/\Omega_i$, the ion Larmor radius, $\omega_i=(4\pi e^2 n_{io}/m_i)^{1/2}$, the ion plasma frequency, $\Omega_i=eB_o/m_ic$ the ion gyrofrequency, $c_s=(k_BT_e/m_i)^{1/2}$ the ion-acoustic speed.

Replacing n_i, n_c, n_h by $1+n$, $1+n_c$, $1+n_h$, respectively, the governing equations (1)–(8) become as follows, where we keep nonlinear terms on the right-hand side:

$$\nabla_\perp^2\frac{\partial}{\partial z}(\varphi-\psi)-\frac{\partial v_{iz}}{\partial t}+\frac{1}{1+\mu}\frac{\partial}{\partial t}(v_c+\mu v_h)=A, \tag{12}$$

$$n-\frac{1}{1+\mu}(n_c+\mu n_h)=0, \tag{13}$$

$$n_h-\frac{\psi}{\theta}=B, \tag{14}$$

$$\frac{\partial n_h}{\partial t}+\frac{\partial v_h}{\partial z}=C, \tag{15}$$

$$\frac{\partial n_c}{\partial t}+\frac{\partial v_c}{\partial z}=D, \tag{16}$$

$$\frac{\partial v_c}{\partial t}-\alpha_c\frac{\partial \psi}{\partial z}+\alpha_c\frac{\partial n_c}{\partial z}=E, \tag{17}$$

$$\frac{\partial n}{\partial t}-\frac{\partial}{\partial t}\nabla_\perp^2\varphi+\frac{\partial v_{iz}}{\partial z}=F, \tag{18}$$

$$\frac{\partial v_{iz}}{\partial t}+\alpha_c q\frac{\partial \psi}{\partial z}=G. \tag{19}$$

The expressions for the nonlinear terms A,B,C, etc., appearing on the right-hand side of the above equations are given by

$$A=\frac{\partial}{\partial t}(nv_{iz})-\frac{1}{1+\mu}\frac{\partial}{\partial t}(n_c v_c+\mu n_h v_h), \tag{20}$$

$$B=\frac{\psi^2}{2\theta^2}+\cdots, \tag{21}$$

$$C=-\frac{\partial}{\partial z}(n_h v_h), \tag{22}$$

$$D=-\frac{\partial}{\partial z}(n_c v_c), \tag{23}$$

$$E=-v_c\frac{\partial v_c}{\partial z}+\alpha_c n_c\frac{\partial n_c}{\partial z}, \tag{24}$$

$$F=\frac{\partial}{\partial x}\left(n\frac{\partial^2\phi}{\partial x\partial t}\right)+\frac{\partial}{\partial y}\left(n\frac{\partial^2\varphi}{\partial y\partial t}\right)-\frac{\partial}{\partial z}(nv_{iz}), \tag{25}$$

$$G=-v_{iz}\frac{\partial v_{iz}}{\partial z}+\frac{\partial^2\varphi}{\partial x\partial t}\frac{\partial v_{iz}}{\partial x}+\frac{\partial^2\varphi}{\partial y\partial t}\frac{\partial v_{iz}}{\partial y}. \tag{26}$$

III. LINEAR DISPERSION RELATION

Neglecting nonlinear terms and assuming space–time dependence of the dependent variables to be of the form $\exp i(k_x x+k_y y+k_z z-\omega t)$, Eqs. (12)–(19) produce the following linear dispersion relation:

$$(\omega^2-k_z^2)\left[\frac{\mu}{\theta}\omega^4-\omega^2k_z^2\alpha_c\left\{1+\frac{\mu}{\theta}+q(1+\mu)\right\}+k_z^4\alpha_c^2q(1+\mu)\right]=\omega^2k_\perp^2k_z^2(1+\mu)(\omega^2-\alpha_c k_z^2). \tag{27}$$

For long wavelength, i.e., for small k_z, $k_\perp$, ω, the term on the right-hand side of (27) corresponds to the dispersive effect. Neglecting this dispersive effect, the dispersion relation becomes

$$(\omega^2-k_z^2)\left[\frac{\mu}{\theta}\omega^4-\omega^2k_z^2\alpha_c\left\{1+\frac{\mu}{\theta}+q(1+\mu)\right\}+k_z^4\alpha_c^2q(1+\mu)\right]=0. \tag{28}$$

The dispersion relation gives the following three values of ω^2, which corresponds to three modes of propagation,

$$\omega^2=k_z^2,\quad \omega^2=V_1^2k_z^2,\quad \omega^2=V_2^2k_z^2, \tag{29}$$

where

$$V_1^2=\alpha_c\left(1+\frac{\theta}{\mu}\right)+\frac{\theta^2}{\mu^2}.\frac{\alpha_c q(1+\mu)}{1+\frac{\theta}{\mu}},$$

$$V_2^2=\frac{\theta}{\mu}.\frac{\alpha_c q(1+\mu)}{1+\frac{\theta}{\mu}}. \tag{30}$$

In deriving these expressions for V_1^2 and V_2^2 we have neglected terms containing squares and higher degree of q. The three dispersion relations (29) become, respectively, the following in dimensional form, where in the second expression order q terms have been neglected:

$$\omega^2=V_A^2k_z^2,\quad \omega^2=\frac{k_z^2V_{eh}^2}{\mu}\left(1+\frac{\mu}{\theta}\right),\quad \omega^2=\frac{k_z^2k_BT_{ef}}{m_i}, \tag{31}$$

and consequently they correspond, respectively, to Alfvén wave, electron-acoustic wave and ion-acoustic wave. Hence the dispersion relations $\omega^2=k_z^2$, $\omega^2=V_1^2k_z^2$, $\omega^2=V_2^2k_z^2$ in dimensionless form corresponds, respectively, to kinetic Alfvén, electron-acoustic, and ion-acoustic waves. The expressions for V_{eh}^2 and T_{ef} appearing in (31) are given by

$$V_{eh}^2=\frac{k_BT_h}{m_e},\quad T_{ef}=\frac{k_B(n_{ho}+n_{co})T_hT_c}{T_cn_{ho}+T_hn_{co}}. \tag{32}$$

IV. DERIVATION OF KdV–ZK EQUATIONS

We make the following stretchings of space coordinates and time in order to derive the KdV–ZK equations for the kinetic Alfvén wave, electron-acoustic wave and ion-acoustic wave:[7,12,21,22]

$$\zeta=\in^{1/2}(z-V_ot),\quad \xi=\in^{1/2}x,\quad \eta=\in^{1/2}y,$$
$$\tau=\in^{3/2}t. \tag{33}$$

The small parameter $\in$ appearing here measures the weakness of dispersion and V_o is a constant. Further the perturbation expansion of the dependent variables are made as follows:

$$f=\in f^{(1)}+\in^2f^{(2)}+\in^3.f^{(3)}+\cdots,$$
$$\varphi=\varphi^{(1)}+\in\varphi^{(2)}+\in^2\varphi^{(3)}+\cdots, \tag{34}$$

where f stands for n, n_c, n_h, ψ, v_{iz}, v_c, v_h.

Substituting the stretchings (33) and the expansions (34) in the governing equations (12)–(19) and then equating coefficients of like powers of $\in$ on both sides, we get a sequence of equations.

From the lowest order equations of these sequences we get the following:

$$n_c^{(1)}=(1+\mu)\left[1-\frac{\frac{\mu}{\theta}(V_0^2-\alpha_c)}{V_0^2\frac{\mu}{\theta}-\alpha_c\left(1+\frac{\mu}{\theta}\right)}\right]n^{(1)},$$

$$n_h^{(1)}=\frac{n^{(1)}}{\theta}\cdot\frac{(1+\mu)(V_0^2-\alpha_c)}{V_0^2\frac{\mu}{\theta}-\alpha_c\left(1+\frac{\mu}{\theta}\right)},$$

$$v_c^{(1)}=V_0(1+\mu)\left[1-\frac{\frac{\mu}{\theta}(V_0^2-\alpha_c)}{V_0^2\frac{\mu}{\theta}-\alpha_c\left(1+\frac{\mu}{\theta}\right)}\right]n^{(1)},$$

$$v_h^{(1)}=\frac{V_0}{\theta}\cdot\frac{(1+\mu)(V_0^2-\alpha_c)}{V_0^2\frac{\mu}{\theta}-\alpha_c\left(1+\frac{\mu}{\theta}\right)}n^{(1)},$$

$$\psi^{(1)}=\frac{(1+\mu)(V_0^2-\alpha_c)}{V_0^2\frac{\mu}{\theta}-\alpha_c\left(1+\frac{\mu}{\theta}\right)}n^{(1)},$$

$$v_{iz}=\frac{\alpha_c q}{V_0}\,\frac{(1+\mu)(V_0^2-\alpha_c)}{V_0^2\frac{\mu}{\theta}-\alpha_c\left(1+\frac{\mu}{\theta}\right)}n^{(1)},$$

$$\left(\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}\right)\varphi^{(1)}=\left[1-\frac{\alpha_c q(1+\mu)(V_0^2-\alpha_c)}{V_0^2\left\{V_0^2\frac{\mu}{\theta}-\alpha_c\left(1+\frac{\mu}{\theta}\right)\right\}}\right]n^{(1)}. \tag{35}$$

At this order we also get the following equation determining the constant V_0:

$$(V_0^2-1)\left[\frac{\mu}{\theta}V_0^4-V_0^2\alpha_c\left\{1+\frac{\mu}{\theta}+q(1+\mu)\right\}+\alpha_c^2 q(1+\mu)\right]=0. \tag{36}$$

This equation becomes the same as Eq. (28), if we set $V_0^2=\omega^2/k_z^2$, and consequently the three roots for V_0^2 are

$$V_0^2=1,\quad V_1^2,\quad V_2^2, \tag{37}$$

which correspond, respectively, to kinetic Alfvén wave, electron-acoustic wave, and ion-acoustic wave.

At the next order we get the following equations from (12)–(19):

$$\left(\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}\right)\frac{\partial\varphi^{(2)}}{\partial\zeta}+V_0\frac{\partial v_{iz}^{(2)}}{\partial\zeta}-\frac{V_0}{1+\mu}\frac{\partial}{\partial\zeta}(v_c^{(2)}+\mu v_h^{(2)})=a^{(2)}, \tag{38}$$

$$n^{(2)}-\frac{1}{1+\mu}n_c^{(2)}-\frac{\mu}{1+\mu}n_h^{(2)}=0, \tag{39}$$

$$n_h^{(2)}-\frac{1}{\theta}\psi^{(2)}=b^{(2)}, \tag{40}$$

$$-V_0\frac{\partial n_h^{(2)}}{\partial\zeta}+\frac{\partial v_h^{(2)}}{\partial\zeta}=c^{(2)}, \tag{41}$$

$$-V_0\frac{\partial n_c^{(2)}}{\partial\zeta}+\frac{\partial v_c^{(2)}}{\partial\zeta}=d^{(2)}, \tag{42}$$

$$-V_0\frac{\partial n_c^{(2)}}{\partial\zeta}-\alpha_c\frac{\partial\psi^{(2)}}{\partial\zeta}+\alpha_c\frac{\partial n_c^{(2)}}{\partial\zeta}=e^{(2)}, \tag{43}$$

$$-V_0\frac{\partial n^{(2)}}{\partial\zeta}+V_0\left(\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}\right)\frac{\partial\varphi^{(2)}}{\partial\zeta}+\frac{\partial v_{iz}^{(2)}}{\partial\zeta}=f^{(2)}, \tag{44}$$

$$-V_0\frac{\partial v_{iz}^{(2)}}{\partial\zeta}+\alpha_c q\frac{\partial\psi^{(2)}}{\partial\zeta}=g^{(2)}, \tag{45}$$

where $a^{(2)}$, $b^{(2)}$, etc., are contributions from the respective nonlinear terms in Eqs. (20)–(26) and they have been expressed in terms of two independent variables $n^{(1)}$ and $\varphi^{(1)}$. These expressions are given in the Appendix.

We differentiate the two equations (39) and (40) with respect to ζ and solve these two equations together with Eqs. (41)–(45) for

$$\frac{\partial n_h^{(2)}}{\partial\zeta},\ \frac{\partial n_c^{(2)}}{\partial\zeta},\ \frac{\partial v_h^{(2)}}{\partial\zeta},\ \frac{\partial v_c^{(2)}}{\partial\zeta},\ \frac{\partial v_{iz}^{(2)}}{\partial\zeta},\ \frac{\partial\psi^{(2)}}{\partial\zeta},\ \left(\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}\right)\frac{\partial\varphi^{(2)}}{\partial\zeta},$$

to express them in terms of $n^{(2)}$, $n^{(1)}$, and $\varphi^{(1)}$. Substituting these solutions in Eq. (38), it is found that the resulting equation does not involve the terms containing $n^{(2)}$ as its coefficient vanishes due to Eq. (36) satisfied by V_0. The resulting equation then assumes the following forms in the two cases that may arise:

Case I: $V_0^2=1$ (kinetic Aflvén wave)

$$\frac{\partial n^{(1)}}{\partial\zeta}-\Lambda_4\frac{\partial}{\partial\zeta}\left(\frac{\partial^2 n^{(1)}}{\partial\xi^2}+\frac{\partial^2 n^{(1)}}{\partial\eta^2}\right)-\frac{1}{2}n^{(1)}\frac{\partial n^{(1)}}{\partial\zeta}-\Lambda_3\left(\frac{\partial^2\varphi^{(1)}}{\partial\xi\partial\zeta}\frac{\partial n^{(1)}}{\partial\xi}+\frac{\partial^2\varphi^{(1)}}{\partial\eta\partial\zeta}\frac{\partial n^{(1)}}{\partial\eta}\right)=0. \tag{46}$$

Case II: $V_0^2=V_1^2$ or V_2^2 (electron-acoustic or ion-acoustic-wave),

$$\frac{\partial n^{(1)}}{\partial\tau}+\Lambda_1\left(\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}\right)\frac{\partial n^{(1)}}{\partial\zeta}+\Lambda_2 n^{(1)}\frac{\partial n^{(1)}}{\partial\zeta}=0. \tag{47}$$

In writing this equation we have used the fact that in this case $\varphi^{(1)}=0$, since in this case Eq. (35) becomes

$$\left(\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}\right)\varphi^{(1)}=0. \tag{48}$$

The coefficients of Eqs. (46) and (47) are given by

$$\Lambda_1=-\frac{(1+\mu)pV_0}{2(1-V_0^2)}\left[\frac{V_0^2}{V_0^2-\alpha_c}\left(1-\frac{\mu}{\theta}p\right)-1\right]^{-1},$$

$$\Lambda_2=\frac{V_0}{2}\left[(1+\mu)\left\{p^2\frac{\mu}{\theta^2}+\left(1-\frac{\mu}{\theta}p\right)^2\frac{(3V_0^2-\alpha_c)}{(V_0^2-\alpha_c)}\right\}-3\right]\times\left[\frac{V_0^2}{V_0^2-\alpha_c}\left(1-\frac{\mu}{\theta}p\right)-1\right]^{-1},$$

$$\Lambda_3=\frac{1}{2-2\alpha_c qp(1+\mu)},\qquad \Lambda_4=\frac{(1+\mu)p}{2-2\alpha_c qp(1+\mu)}, \tag{49}$$

where

$$p=(V_0^2-\alpha_c)\Bigg/\left[V_0^2\frac{\mu}{\theta}-\alpha_c\left(1+\frac{\mu}{\theta}\right)\right]. \tag{50}$$

A. KdV–ZK equation for the kinetic Alfvén wave

For $V_0^2=1$, i.e., for the kinetic Alfvén wave, Eq. (35) becomes

$$\left(\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}\right)\varphi^{(1)}=\frac{1}{2\Lambda_3}n^{(1)}. \tag{51}$$

The two coupled equations (46) and (51) constitute the KdV–ZK equation for the kinetic Alfvén wave.

Setting

$$\Lambda_3\varphi^{(1)}=\Phi^{(1)}, \tag{52}$$

the two coupled equations constituting the KdV–ZK equation for the kinetic Alfvén wave can be written as

$$\frac{\partial n^{(1)}}{\partial\tau}-\Lambda_4\frac{\partial}{\partial\zeta}\left(\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}\right)n^{(1)}-\frac{1}{2}n^{(1)}\frac{\partial n^{(1)}}{\partial\zeta}-\left(\frac{\partial^2\Phi^{(1)}}{\partial\xi\partial\zeta}\frac{\partial n^{(1)}}{\partial\xi}+\frac{\partial^2\Phi^{(1)}}{\partial\eta\partial\zeta}\frac{\partial n^{(1)}}{\partial\eta}\right)=0, \tag{53}$$

$$\frac{\partial^2\Phi^{(1)}}{\partial\xi^2}+\frac{\partial^2\Phi^{(1)}}{\partial\eta^2}=\frac{1}{2}n^{(1)}. \tag{54}$$

B. KdV–ZK equation for ion-acoustic and electron-acoustic wave

Equation (47) constitutes the KdV–ZK equation for the electron-acoustic or ion-acoustic wave according to whether we set $V_0^2=V_1^2$ or $V_0^2=V_2^2$.

Setting

$$n^{(1)}=\frac{1}{\Lambda_2}N, \tag{55}$$

the KdV–ZK equation for the electron-acoustic or ion-acoustic wave can be written as

$$\frac{\partial N}{\partial\tau}+\Lambda_1\left(\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}\right)\frac{\partial N}{\partial\zeta}+N\frac{\partial N}{\partial\zeta}=0 \tag{56}$$

depending on whether V_0 appearing in Λ_1 becomes equal to V_1^2 or V_2^2.

V. EXISTENCE OF SOLITARY WAVES

In order to investigate the existence and stability of solitary waves of the coupled Eqs. (53) and (54) for the kinetic Alfvén wave, and Eq. (56) for electron-acoustic and ion-acoustic waves, propagating obliquely at an angle α with the external magnetic field and lying in the (x,z) plane, we rotate the coordinate axes (ξ,ζ) through an angle α keeping the η-axis fixed, so that the ζ-axis coincides with the direction of propagation of the solitary waves. The relations between these new and old coordinates then become

$$\zeta'=\zeta\cos\alpha+\xi\sin\alpha,\quad \xi'=-\zeta\sin\alpha+\xi\cos\alpha, \tag{57}$$

where (ξ',ζ') are the new coordinates. With this change of variables the two coupled Eqs. (53) and (54) for the kinetic Alfvén wave becomes as follows in which we drop primes on ζ' and ξ':

$$\begin{aligned}&\frac{\partial n^{(1)}}{\partial\tau}-\beta n^{(1)}\frac{\partial n^{(1)}}{\partial\zeta}-\gamma\frac{\partial^3 n^{(1)}}{\partial\zeta^3}-\delta\frac{\partial^2\Phi^{(1)}}{\partial\zeta^2}\frac{\partial n^{(1)}}{\partial\zeta}+a_1\frac{\partial^3 n^{(1)}}{\partial\zeta^2\partial\xi}\\&+a_2\frac{\partial^3 n^{(1)}}{\partial\xi^2\partial\zeta}+a_3\frac{\partial^3 n^{(1)}}{\partial\xi^3}+a_4\frac{\partial^3 n^{(1)}}{\partial\zeta\partial\eta^2}+a_5\frac{\partial^3 n^{(1)}}{\partial\xi\partial\eta^2}\\&+b_1 n^{(1)}\frac{\partial n^{(1)}}{\partial\xi}+b_2\frac{\partial^2\Phi^{(1)}}{\partial\xi\partial\zeta}\frac{\partial n^{(1)}}{\partial\zeta}+b_3\frac{\partial^2\Phi^{(1)}}{\partial\xi^2}\frac{\partial n^{(1)}}{\partial\zeta}\\&+b_4\frac{\partial^2\Phi^{(1)(1)}}{\partial\zeta^2}\frac{\partial n^{(1)}}{\partial\xi}+b_5\frac{\partial^2\Phi^{(1)}}{\partial\zeta\partial\xi}\frac{\partial n^{(1)}}{\partial\xi}+b_6\frac{\partial^2\Phi^{(1)}}{\partial\xi^2}\frac{\partial n^{(1)}}{\partial\xi}\\&+b_7\frac{\partial^2\Phi^{(1)}}{\partial\xi\partial\eta}\frac{\partial n^{(1)}}{\partial\eta}+b_8\frac{\partial^2\Phi^{(1)}}{\partial\zeta\partial\eta}\frac{\partial n^{(1)}}{\partial\eta}=0,\end{aligned} \tag{58}$$

$$\frac{1}{2}n^{(1)}=\lambda\frac{\partial^2\Phi^{(1)}}{\partial\zeta^2}+c_1\frac{\partial^2\Phi^{(1)}}{\partial\xi\partial\zeta}+c_2\frac{\partial^2\Phi^{(1)}}{\partial\xi^2}+\frac{\partial^2\Phi^{(1)}}{\partial\eta^2}, \tag{59}$$

where

$$\begin{aligned}&\beta=\tfrac{1}{2}\cos\alpha,\quad \gamma=\Lambda_4\cos\alpha\sin^2\alpha,\quad \delta=\sin^2\alpha\cos\alpha,\\&a_1=-\Lambda_4(2\sin\alpha\cos^2\alpha-\sin^3\alpha),\\&a_2=-\Lambda_4(\cos^3\alpha-2\sin^2\alpha\cos\alpha),\\&a_3=\Lambda_4\sin\alpha\cos^2\alpha,\quad a_4=-\Lambda_4\cos\alpha,\\&a_5=\Lambda_4\sin\alpha,\quad b_1=\tfrac{1}{2}\sin\alpha,\\&b_2=-\sin\alpha(\cos^2\alpha-\sin^2\alpha),\\&b_3=\sin^2\alpha\cos\alpha,\quad b_4=-\sin\alpha\cos^2\alpha,\\&b_5=-\sin\alpha(\cos^2\alpha-\sin^2\alpha),\quad b_7=\sin\alpha,\\&b_8=-\cos\alpha,\quad \lambda=\sin^2\alpha,\quad c_1=2\sin\alpha\cos\alpha,\\&c_2=\cos^2\alpha.\end{aligned} \tag{60}$$

Also the single Eq. (56) for the ion-acoustic wave and for the electron-acoustic wave becomes the following in the new coordinate system defined by (57). Here also we drop primes on ζ and ξ,

$$\begin{aligned}&\frac{\partial N}{\partial\tau}+\beta' N\frac{\partial N}{\partial\zeta}+\gamma'\frac{\partial^3 N}{\partial\zeta^3}+a_1'\frac{\partial^3 N}{\partial\zeta^2\partial\xi}+a_2'\frac{\partial^3 N}{\partial\zeta\partial\xi^2}\\&+a_3'\frac{\partial^3 N}{\partial\xi^3}+a_4'\frac{\partial^3 N}{\partial\xi\partial\eta^2}+a_5'\frac{\partial^3 N}{\partial\zeta\partial\eta^2}+b_1' N\frac{\partial N}{\partial\xi}=0,\end{aligned} \tag{61}$$

where

$$\begin{aligned}&\beta'=\cos\alpha,\quad \gamma'=\Lambda_1\cos\alpha\sin^2\alpha,\\&a_1'=\Lambda_1(2\sin\alpha\cos^2\alpha-\sin^3\alpha),\\&a_2'=\Lambda_1(\cos^3\alpha-2\sin^2\alpha\cos\alpha),\\&a_3'=-\Lambda_1\sin\alpha\cos^2\alpha,\quad a_4'=-\Lambda_1\sin\alpha,\\&a_5'=\Lambda_1\cos\alpha,\quad b_1'=-\sin\alpha.\end{aligned} \tag{62}$$

A. Solitary kinetic Alfvén wave

We now find solitary wave solutions of the coupled Eqs. (58) and (59) propagating along the ζ-axis, i.e., propagating in the direction making an angle α with the direction of the external magnetic field and lying in the (ξ,ζ)-plane. For solitary wave solutions, setting

$$n^{(1)}=N_0(z),\quad \Phi^{(1)}=\Phi_0(z), \tag{63}$$

where

$$z=\zeta-u_0\tau,\quad (u_0>0) \tag{64}$$

in Eqs. (58) and (59), we get the following two coupled equations for N_0 and Φ_0:

$$u_0\frac{dN_0}{dz}+\kappa N_o\frac{dN_o}{dz}+r\frac{d^3N_o}{dz^3}=0, \tag{65}$$

$$\frac{1}{2}N_o=\lambda\frac{d^2\Phi_o}{dz^2}, \tag{66}$$

where

$$\kappa=\beta+\frac{\delta}{2\lambda}=\cos\alpha. \tag{67}$$

The solitary wave solution of (65) is

$$N_o=a\,\mathrm{sech}^2\,pz, \tag{68}$$

where

$$a=-\frac{3u_o}{\cos\alpha},\quad p=\sqrt{-u_o/(4\Lambda_4\cos\alpha\sin^2\alpha)}. \tag{69}$$

For this soliton to exist, the expression under the square-root sign in the expression for p must be positive. This imposes the following restrictions on the propagation of solitary waves. In the regions of parameter space where $\Lambda_4>0$, the propagation of kinetic Alfvén solitary waves is possible, if the angle α made by the direction of propagation of the solitary wave with the external magnetic field lies in the interval $\pi/2<\alpha<3\pi/2$, and in this case the soliton is compressive. In the regions of parameter space where $\Lambda_4<0$, the propagation of kinetic Alfvén solitons is possible if $(-\pi/2)<\alpha<\pi/2$ and in this case the soliton is rarefactive. The regions in the $\alpha_c\mu$-parameter space where Λ_4 is positive and negative for some different values of θ are shown in Fig. 1, where α_c has been replaced by $\log_{10}(100\alpha_c)$.

B. Solitary electron-acoustic and ion-acoustic waves

Setting

$$N=\bar{N}_0(z),$$

where z is given by (64), in Eq. (61) we get

$$-u_0\frac{d\bar{N}_0}{dz}+\beta'\bar{N}_0\frac{d\bar{N}_0}{dz}+\gamma'\frac{d^3\bar{N}_0}{dz^3}=0. \tag{70}$$

The solitary wave solution of this equation is

$$\bar{N}_0=\frac{3u_0}{\beta'}\mathrm{sech}^2\sqrt{\frac{u_0}{4\gamma'}}z \tag{71}$$

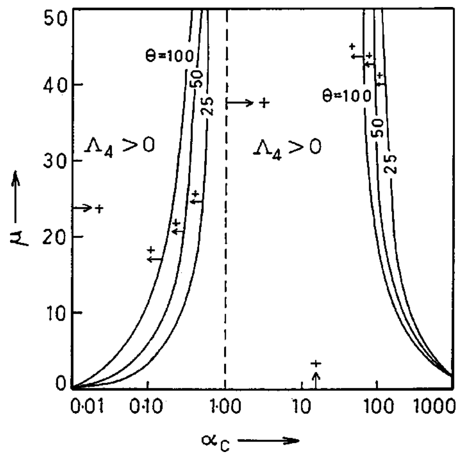

FIG. 1. The regions in the $\alpha_c\mu$-plane where $\Lambda_4>0$ are shown. The sign $\rightarrow+$ against a curve indicates the side of the curve where $\Lambda_4>0$. In the rest of the regions $\Lambda_4<0$.

and consequently from (55) we get the following solitary wave solution for $n^{(1)}$:

$$n^{(1)}=\bar{a}\ \mathrm{sech}^2\ \bar{p}z, \tag{72}$$

where

$$\bar{a}=\frac{3u_0}{\Lambda_2\cos\alpha},\quad \bar{p}=\sqrt{u_0/(4\Lambda_1\cos\alpha\sin^2\alpha)}. \tag{73}$$

Equation (72) gives solitary wave solution for the electron-acoustic wave or the ion-acoustic wave according to whether we take $V_0^2=V_1^2$ or V_2^2 appearing in the expressions for Λ_1 and Λ_2.

The solitary wave solutions (68) and (72) exhibit no symmetry in propagation, though there are symmetries in the basic set of equations. That symmetries cannot be expected in the equations like KdV or KdV–ZK equations has been explained by Jeffrey and Kakutani.[23]

For the existence of solitary wave solutions the expression under the square-root sign in (73) must be positive. This imposes the following restrictions on the propagation of solitary electron-acoustic and ion-acoustic waves.

1. Electron-acoustic wave

Taking $V_0^1=V_1^2$, we find that the expressions for Λ_1 and Λ_2 given in (49) become

$$\Lambda_1\approx\frac{\alpha_c^{1/2}(1+\mu)}{2\left[1-\alpha_c\left(1+\frac{\theta}{\mu}\right)\right]\left(1+\frac{\theta}{\mu}\right)^{1/2}},$$

$$\Lambda_2\approx-\frac{\alpha_c\left(\frac{\theta}{\mu}\right)^3B(1+\mu)^2}{q\left(1+\frac{\theta}{\mu}\right)}<0, \tag{74}$$

which have been approximated by the use of the fact that $q\ll1$. In the expression for Λ_2, B is given by

$$B=\left\{\frac{\mu}{\theta^2}+\frac{\mu^2}{\theta^2}\left(\frac{2\mu}{\theta}+1\right)\right\}\frac{\left(1+\frac{\mu}{\theta}\right)^2}{(1+\mu)^2}>0. \tag{75}$$

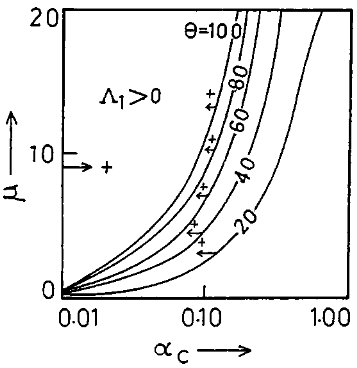

FIG. 2. The regions in the $\alpha_c\mu$-plane where $\Lambda_1>0$ are shown with the same notations as in Fig. 1.

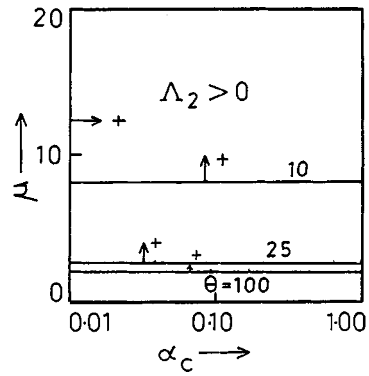

FIG. 3. The regions in the $\alpha_c\mu$-plane where $\Lambda_2>0$ are shown with the same notations as in Fig. 1.

Due to these expressions for Λ_1 and Λ_2, we find that in the regions of parameter space, where $\Lambda_1>0$, propagation of the electron-acoustic solitary wave is possible if α lies within $-(\pi/2)<\alpha<\pi/2$ and in this case the soliton becomes rarefactive. In the regions of parameters space, where $\Lambda_1<0$, propagation of the electron-acoustic wave is possible, if α lies within $\pi/2<\alpha<3\pi/2$ and in this case the electron-acoustic solitary wave is compressive. The regions in $\alpha_c\mu$-parameter space where Λ_1 is positive and negative are shown in Fig. 2.

2. Ion-acoustic wave

The expressions for Λ_1 and Λ_2 given in (49), with $V_0^2=V_2^2$ become

$$\Lambda_1\approx\frac{\alpha_c^{1/2}q^{1/2}(1+\mu)^{3/2}}{2\left(1+\frac{\theta}{\mu}\right)^{3/2}}\cdot\left(\frac{\theta}{\mu}\right)^{3/2}>0,$$

$$\Lambda_2\approx\frac{\alpha_c^{1/2}q^{1/2}(1+\mu)^{1/2}}{2\left(1+\frac{\theta}{\mu}\right)^{1/2}}\cdot\left(\frac{\theta}{\mu}\right)^{1/2}\times\left[3-\frac{(1+\mu)\left(1+\frac{\theta^2}{\mu}\right)}{\mu\left(1+\frac{\theta}{\mu}\right)^2}\right], \tag{76}$$

which have been approximated as in the case of the electron-acoustic wave by the use of the fact that $q\ll 1$. By the use of the expressions for Λ_1 and Λ_2, it is found that the propagation of ion-acoustic solitons is possible if α lies within $-(\pi/2)<\alpha<\pi/2$, and the solution in the case is compressive in the region of parameter space where $\Lambda_2>0$ and is rarefactive in the region of parameter space where $\Lambda_2<0$. The regions in $\alpha_c\mu$ parameter space where Λ_2 is positive and negative are shown in Fig. 3, where α_c has been replaced by $\log_{10}(100\alpha_c)$.

VI. STABILITY ANALYSIS

A. The kinetic Alfvén wave

In order to make a stability analysis of the kinetic Alfvén soliton given by (68) we set

$$n^{(1)}=N_0(z)+u(z,\xi,\eta,\tau),$$

$$\Phi^{(1)}=\Phi_0(z)+\psi(z,\xi,\eta,\zeta), \tag{77}$$

in the two Eqs. (58) and (59) and then linearize with respect to u and ψ. Thus we get the following two equations:

$$-u_0\frac{\partial u}{\partial z}+\frac{\partial u}{\partial\tau}+\beta N_0\frac{\partial u}{\partial z}-\beta\frac{dN_0}{dz}u-\gamma\frac{\partial^3 u}{\partial z^3}-\delta\frac{d^2\Phi_0}{dz^2}\frac{\partial u}{\partial z}-\delta\frac{dN_0}{dz}\frac{\partial^2\psi}{dz^2}+a_1\frac{\partial^3 u}{\partial z^2\partial\xi}+a_2\frac{\partial^3 u}{\partial\xi^2\partial z}+a_3\frac{\partial^3 u}{\partial\xi^3}+a_4\frac{\partial^3 u}{\partial z\partial\eta^2}+a_5\frac{\partial^3 u}{\partial\xi\partial\eta^2}+b_1N_0\frac{\partial u}{\partial\xi}+b_2\frac{dN_0}{dz}\frac{\partial^2\psi}{\partial\xi\partial z}+b_3\frac{dN_0}{dz}\frac{\partial^2\psi}{\partial\xi^2}+b_4\frac{d^2\Phi_0}{dz^2}\frac{\partial u}{\partial\xi}=0, \tag{78}$$

$$\frac{1}{2}u=\lambda\frac{\partial^2\psi}{\partial z^2}+c_1\frac{\partial^2\psi}{\partial\xi\partial z}+c_2\frac{\partial^2\psi}{\partial\xi^2}+\frac{\partial^2\psi}{\partial\eta^2}. \tag{79}$$

Following Rowlands and Infeld[16–18] we set

$$u=\bar{u}(z)\exp i\{k(l\xi+m\eta+nz)-\omega\tau\},$$

$$\psi=\bar{\psi}(z)\exp i\{k(l\xi+m\eta+nz)-\omega\tau\}, \tag{80}$$

for long wavelength plane wave perturbation given in a direction with direction cosines l, m, n, where k is small and $l^2+m^2+n^2=1$, and make the following perturbation expansions for $\bar{u}(z)$, $\bar{\psi}(z)$, and ω:

$$\bar{u}(z)=q_0(z)+kq_1(z)+k^2q_2(z)+\cdots,$$

$$\bar{\psi}(z)=\psi_0(z)+k\psi_1(z)+k^2\psi_2(z)+\cdots,$$

$$\omega=k\omega_1+k^2\omega_2+\cdots. \tag{81}$$

Substituting (80) together with expansion (81) in Eqs. (78) and (79) and then equating coefficients of k^0, k^1, k^2 on both sides of each equation, we get, respectively, zeroth, first, and second order equations.

1. Zeroth order equations

The two equations, that we get at this order, are

$$u_0 q_0 + \kappa N_0 q_0 + \gamma \frac{d^2 q_0}{dz^2} = 0, \tag{82}$$

$$\frac{1}{2} q_0 = \lambda \frac{d^2 \psi_0}{dz^2}, \tag{83}$$

where the first equation has been written after eliminating $d^2\psi_0/dz^2$ and $d^2\Phi_0/dz^2$ by the help of Eqs. (83) and (66), respectively, and then integrating the resulting equation once with respect to z, under the assumption that both q_0 and its derivatives vanish at $|z| \rightarrow \infty$.

Now, following the same procedure as in Bandyopadhyay and Das[8] we get the following solutions of the above two equations for q_0 and $d\psi_0/dz$, which remain bounded and tend to zero as $|z| \rightarrow \infty$:

$$q_0 = A_1 R S^2, \quad \frac{d\psi_0}{dz} = -\frac{A_1}{4\lambda p} S^2. \tag{84}$$

Here $S = \operatorname{sech} pz$, $R = \tanh pz$, and A_1 is an arbitrary constant.

2. First order equations

The two equations at this order can be written as

$$u_0 q_1 + \kappa N_0 q_1 + \gamma \frac{d^2 q_1}{dz^2} = Q_1, \tag{85}$$

$$\frac{1}{2} q_1 = \lambda \frac{d^2 \psi_1}{dz^2} + 2in\lambda \frac{d\psi_0}{dz} + ilc_1 \frac{d\psi_0}{dz}, \tag{86}$$

where

$$Q_1 = \lambda_1 S^2 + \lambda_2 S^2 R^2, \tag{87}$$

λ_1 and λ_2 being given by

$$\lambda_1 = \frac{iA_1 \bar{a}_2}{2p}, \quad \lambda_2 = \frac{iA_1 \bar{b}_2 u_0}{2p}$$

with

$$\bar{a}_2 = (\omega_1 + nu_0) + \frac{u_0 l}{\sin\alpha \cos\alpha}$$

and

$$\bar{b}_2 = -\frac{3}{\sin\alpha\cos\alpha}(n \sin\alpha \cos\alpha + l\cos^2\alpha). \tag{88}$$

Equation (85) has been written after eliminating $d^2\psi_1/dz^2$ and $d^2\Phi_0/dz^2$ by the use of Eqs. (86) and (66), respectively, and then integrating once with respect to z under the assumption that q_1 and its derivative vanish at $|z| \rightarrow \infty$.

Due to Eq. (65), dN_0/dz is a solution of the homogeneous part of Eq. (85). Therefore the solution of the latter equation can be written as

$$q_1 = c_1(z) \frac{dN_0}{dz}. \tag{89}$$

Substituting this in (85), we get a linear equation for $c_1'(z)$, which can be solved very easily for $c_1'(z)$. Here a prime implies differentiation with respect to z. Integrating this latter equation we get $c_1(z)$. Thus we get the following solution for $q_1(z)$, which remains bounded and tends to zero as $|z| \rightarrow \infty$:

$$q_1 = B_1 R S^2 - \frac{iA_1}{6p}\left(\frac{3\bar{a}_2}{u_0} + \bar{b}_2\right) S^2 - \frac{iA_1}{2}\left(\frac{\bar{a}_2}{u_0} + \bar{b}_2\right) zRS^2. \tag{90}$$

3. Second order equation

From the two equations at this order we get the following equations for q_2 after eliminating $d^2\psi_2/dz^2$ between the two equations and using Eq. (66) to eliminate $d^2\Phi_0/dz^2$:

$$u_0 \frac{dq_2}{dz} + \kappa \frac{d}{dz}(N_0 q_2) + \gamma \frac{d^3 q_2}{dz^3} = Q_2, \tag{91}$$

where

$$Q_2 = -i\omega_2 q_0 - \Lambda_4 \nu_1 \frac{dq_0}{dz} - \nu_2 \frac{dN_0}{dz} \psi_0 - i(nu_0 + \omega_1) q_1 + i\Lambda_4 \nu_3 \frac{d^2 q_1}{dz^2} + \frac{i\nu_4}{2\sin\alpha} N_0 q_1 + inu_5 \frac{dN_0}{dz}\frac{d\psi_1}{dz}, \tag{92}$$

ν_j's being given by

$$\begin{aligned}
\nu_1 &= l^2(\cos^3\alpha - 2\sin^2\alpha\cos\alpha) + m^2\cos\alpha \\
&\quad + 3n^2\cos\alpha\sin^2\alpha + 2nl(2\sin\alpha\cos^2\alpha - \sin^3\alpha), \\
\nu_2 &= l^2\cos\alpha + m^2\cos\alpha + nl\sin\alpha, \\
\nu_3 &= l(2\sin\alpha\cos^2\alpha - \sin^3\alpha) + 3n\cos\alpha\sin^2\alpha, \\
\nu_4 &= l(\sin^2\alpha - \cos^2\alpha) - 2n\sin\alpha\cos\alpha, \\
\nu_5 &= l\sin\alpha.
\end{aligned} \tag{93}$$

Now for the solution of Eq. (91) for q_2 to exist, its right-hand side must be perpendicular to the kernel of the operator,

$$L = u_0 \frac{d}{dz} + \kappa \frac{d}{dz} N_0 + \gamma \frac{d^3}{dz^3}.$$

This kernel, which tends to zero as $|z| \rightarrow \infty$ is S^2. Therefore the condition for the existence of the solution of (91) to exist gives

$$\int_{-\infty}^{\infty} Q_2 S^2 dz = 0. \tag{94}$$

Performing all the integrals appearing in this equation after substituting for q_0, q_1, ψ_0, ψ_1 given by (84), (90), (86), we find the following quadratic equation determining ω_1:

$$15\Omega^2 + 10 B_l \sin\alpha \Omega + C_l \sin\alpha = 0, \tag{95}$$

where

$$\Omega=(\omega_l+nu_0)\sin\alpha\cos\alpha/u_0,$$

$$B_l=l(2\sin^2\alpha-\cos^2\alpha)-3n\sin\alpha\cos\alpha,$$

$$C_l=l^2(5\sin^4\alpha-10\sin^2\alpha\cos^2\alpha)$$
$$-8m^2\cos^2\alpha+15n^2\sin^2\alpha\cos^2\alpha$$
$$+10ln\sin\alpha\cos\alpha(\cos^2\alpha-2\sin^2\alpha). \tag{96}$$

The discriminant Δ of Eq. (95) simplifies to

$$\Delta=20(5l^2+24m^2\cos^2\alpha).$$

This being positive the kinetic Alfvén wave is stable with respect to long wavelength plane wave perturbation.

B. Electron-acoustic and ion-acoustic wave

Setting

$$N=\bar{N}_0(z)+v(z,\xi,\eta,\tau)$$

in Eq. (61), where $\bar{N}_0(z)$ is given by (71) and $z=\zeta-u_0\tau$, and then linearizing we get the following equation for v:

$$-u_0\frac{\partial v}{\partial z}+\frac{\partial v}{\partial \tau}+\beta'\bar{N}_0\frac{\partial v}{\partial z}+\beta'\frac{d\bar{N}_0}{dz}v+\gamma'\frac{\partial^3 v}{\partial z^3}$$
$$+a_1'\frac{\partial^3 v}{\partial z^2\partial\xi}+a_2'\frac{\partial^3 v}{\partial z\partial\xi^2}+a_3'\frac{\partial^3 v}{\partial\xi^3}+a_4'\frac{\partial^3 v}{\partial\xi\partial\eta^2}$$
$$+a_5'\frac{\partial^3 v}{\partial z\partial\eta^2}+b_1\bar{N}_0\frac{\partial v}{\partial\xi}=0. \tag{97}$$

For long-wavelength plan-wave perturbation given in a direction with direction cosines, l,m,n we set

$$v=\bar{v}(z)\exp i\{k(l\xi+m\eta+nz)-\omega\tau\} \tag{98}$$

when k is small and $l^2+m^2+n^2=1$ and also make the following perturbation expansions for $\bar{v}(z)$:

$$\bar{v}(z)=v_0(z)+kv_1(z)+k^2v_2(z)+\cdots \tag{99}$$

and the same expansion (81) for ω.

Substituting into (97) the expression (98) for v and then the expansions (99) and (81) for $\bar{v}(z)$ and ω and then equating coefficients of like power of k on both sides, we get a sequence of equations for v_j's. Then proceeding in the same way as in the case of kinetic Alfvén wave, we find the following solutions for v_0 and v_1:

$$v_0=A_1\bar{R}\bar{S}^2, \tag{100}$$

$$v_1=B_1^1\bar{R}\bar{S}^2-\frac{iA_1}{6\bar{p}}\left(\frac{3\bar{a}_2'}{u_0}-\bar{b}_2'\right)\bar{S}^2$$
$$+\frac{iA_1}{2\bar{p}}\left(\frac{\bar{a}_2'}{u_0}-\bar{b}_2'\right)\bar{p}z\bar{R}\bar{S}^2, \tag{101}$$

where

$$\bar{S}=\operatorname{sech}\bar{p}z,\quad \bar{R}=\tanh\bar{p}z,$$
$$\bar{a}_2'=(\omega_1+nu_0)+\frac{u_0l}{\sin\alpha\cos\alpha},\quad \bar{b}_2'=3n+3l\cot\alpha. \tag{102}$$

The equation for v_2 becomes

$$-u_0\frac{dv_2}{dz}+\beta'\frac{d}{dz}(\bar{N}_0v_2)+\gamma'\frac{d^3v_2}{dz^3}=Q_2', \tag{103}$$

where

$$Q_2'=i\omega_2v_0+\Lambda_1\nu_1\frac{dv_0}{dz}+i(\omega_1+nu_0)v_1$$
$$-i\nu_2\bar{N}_0v_1-i\Lambda_1\nu_3\frac{d^3v_1}{dz^3}, \tag{104}$$

ν_j's being given by

$$\nu_1=(-2\sin^2\alpha\cos\alpha+\cos^3\alpha)l^2+m^2\cos\alpha$$
$$+3n^2\sin^2\alpha\cos\alpha+2nl(2\sin\alpha\cos^2\alpha-\sin^3\alpha),$$
$$\nu_2=n\cos\alpha-l\sin\alpha,$$
$$\nu_3=3n\sin^2\alpha\cos\alpha+l(2\sin\alpha\cos^2\alpha-\sin^3\alpha). \tag{105}$$

Now for the solution of Eq. (104) to exist, its right-hand side must be perpendicular to the kernel of the adjoint of the operator,

$$-u_0\frac{d}{dz}+\beta'\frac{d}{dz}(\bar{N}_0)+\gamma'\frac{d^3}{dz^3}.$$

This kernel, which must tend to zero as $|z|\to\infty$, is $\bar{S}^2$. Therefore, the condition for the existence of the solution of (104) gives

$$\int_{-\infty}^{\infty}Q_2'\bar{S}^2dz=0. \tag{106}$$

Substituting here the expressions for Q_2' given by (105) and performing integration, we get the following equation determining ω_1:

$$\Omega^2+B_1'\Omega+C_1'=0, \tag{107}$$

where $B_1'=(2l/3)(5\sin^2\alpha+2\cos^2\alpha)-2n\sin\alpha\cos\alpha$,

$$C_1'=\frac{l^2}{3}(7\sin^4\alpha+4\sin^2\alpha\cos^2\alpha)$$
$$+\tfrac{4}{15}m^2\cos^2\alpha+n^2\sin^2\alpha\cos^2\alpha$$
$$-\tfrac{2}{3}nl\sin\alpha\cos\alpha(2\cos^2\alpha+5\sin^2\alpha). \tag{108}$$

The discriminant Δ' Eq. (107) is

$$\Delta'=\tfrac{16}{5}(5l^2-3m^2\cos^2\alpha). \tag{109}$$

Hence there is instability, if the direction cosines *l, m, n* of the long wavelength plane-wave perturbation satisfy the following condition:

$$\frac{m^2}{l^2}>\frac{5}{3\cos^2\alpha}. \tag{110}$$

Obviously, this instability condition remains the same for both electron-acoustic and ion-acoustic solitons. If this insta-

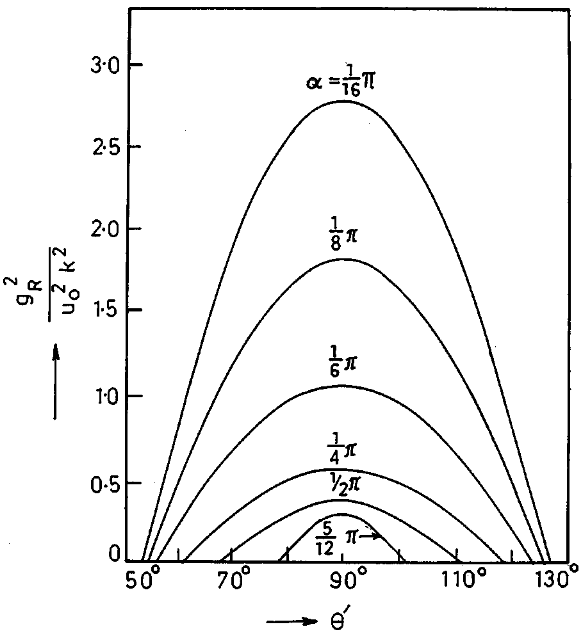

FIG. 4. A plot of the square of dimensionless growth rate of instability against θ' for some different values of α.

bility condition is satisfied, then the growth rate g_R of instability is given by the following and is the same for both electron-acoustic and ion-acoustic solitons,

$$g_R^2 = \frac{4k^2u_0^2}{45\sin^2\alpha\cos^2\alpha}(3m^2\cos^2\alpha - 5l^2). \tag{111}$$

For perturbation in a plane through the ζ-axis that makes an angle θ' with the (ξ,ζ) plane this square of the growth rate of instability becomes

$$g_R^2 = \frac{4k^2u_0^2(1-n^2)}{45\sin^2\alpha\cos^2\alpha}(3\sin^2\theta'\cos^2\alpha - 5\cos^2\theta'). \tag{112}$$

Hence the growth rate of instability attains maximum when $n=0$, i.e., when the perturbation is given in a plane perpendicular to the direction of propagation of the solitary wave. If the direction of the perturbation given in this plane makes an angle θ' with the ξ-axis, then the growth rate of the instability g_R is given by

$$\frac{g_R^2}{k^2u_0^2} = \frac{4}{45\sin^2\alpha\cos^2\alpha}(3\sin^2\theta'\cos^2\alpha - 5\cos^2\theta'). \tag{113}$$

The square of the dimensionless growth rate of the instability $g_R/(ku_0)$ given by (113) has been plotted in Fig. 4, against θ' for same different values of α. It is interesting to note here that both electron-acoustic and ion-acoustic solitons have the same dimensionless growth rate of instability.

VII. CONCLUSION

The matters covered in the paper are new and they have not been considered by previous authors. In a two electron temperature magnetized plasma there exists three modes when both kinetic and inertial regimes are considered. These three modes are the kinetic Alfvén, electron-acoustic, and ion-acoustic waves. The KdV–ZK equations for these three waves are derived in this paper. These equations describe the evolution of the corresponding waves when the effects of both weak nonlinearity and weak dispersion are included. Solitary wave solutions of these equations propagating obliquely to the external magnetic field are obtained and their existence in the $\alpha_c\mu$-parameter plane for some different values of θ is studied. Finally, the stabilities of these solitons are investigated by the Rowlands–Infeld method. The kinetic Alfvén solitons are found to be stable, and both electron-acoustic and ion-acoustic solitons have the same instability criterion and maximum growth rate of instability. As in space plasmas, where the presence of two electron populations are often encountered, the present investigation may be of some application in such cases.

ACKNOWLEDGMENT

The authors are grateful to the referee for his helpful comments.

APPENDIX

$$a^{(2)} = \left(\frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2}\right)\frac{\partial\psi^{(1)}}{\partial\zeta} - \frac{1}{1+\mu}\frac{\partial}{\partial\tau} \times (v_c^{(1)} + \mu v_h^{(1)}) - V_0\frac{\partial}{\partial\zeta}(n^{(1)}v_{iz}^{(1)}) + \frac{V_0}{1+\mu}\frac{\partial}{\partial\zeta}(n_c^{(1)}v_c^{(1)} + \mu n_h^{(1)}v_h^{(1)}),$$

$$b^{(2)} = \frac{(1+\mu)^2}{2\theta^2}p^2 n^{(1)^2}, \quad c^{(2)} = -\frac{\partial n_h^{(1)}}{\partial\tau} - \frac{\partial}{\partial\tau}(n_h^{(1)}v_h^{(1)}),$$

$$d^{(2)} = -\frac{\partial n_c^{(1)}}{\partial\tau}\frac{\partial}{\partial\tau}(n_c^{(1)}v_c^{(1)}),$$

$$e^{(2)} = -\frac{\partial v_c^{(1)}}{\partial\tau} - v_c^{(1)}\frac{\partial v_c^{(1)}}{\partial\tau} + \alpha_c n_c^{(1)}\frac{\partial n_c^{(1)}}{\partial\zeta},$$

$$f^{(2)} = -\frac{\partial n^{(1)}}{\partial\tau} + \left(\frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2}\right)\frac{\partial\varphi^{(1)}}{\partial\tau} - V_0\frac{\partial}{\partial\xi}\left(n^{(1)}\frac{\partial^2\varphi^{(1)}}{\partial\xi\partial\zeta}\right) - V_0\frac{\partial}{\partial\eta}\left(n^{(1)}\frac{\partial^2\varphi^{(1)}}{\partial\eta\partial\zeta}\right) - \frac{\partial}{\partial\tau}(n^{(1)}v_{iz}^{(1)}),$$

$$g^{(2)} = -\frac{\partial v_{iz}^{(1)}}{\partial\tau} - v_{iz}^{(1)}\frac{\partial v_{iz}^{(1)}}{\partial\zeta} - V_0\left(\frac{\partial^2\varphi^{(1)}}{\partial\xi\partial\zeta}\frac{\partial v_{iz}^{(1)}}{\partial\zeta} + \frac{\partial^2\varphi^{(1)}}{\partial\eta\partial\zeta}\frac{\partial v_{iz}^{(1)}}{\partial\eta}\right).$$

[1]J. W. Belcher and L. Davis, J. Geophys. Res. **76**, 3534 (1971).
[2]R. Z. Sagdeev and A. A. Galeev, *Nonlinear Plasma Theory* (Benjamin, Reading, 1969), p. 8.
[3]A. Hasegawa and K. Mima, Phys. Rev. Lett. **37**, 690 (1976).
[4]M. Y. Yu and P. K. Skukla, Phys. Fluids **21**, 1457 (1978).

[5]P. K. Shukla, H. U. Rahaman, and R. P. Sharma, J. Plasma Phys. **28**, 125 (1982).
[6]M. K. Kalita and B. C. Kalita. J. Plasma Phys. **35**, 267 (1986).
[7]K. P. Das, L. P. J. Kamp, and F. w. Sluijter, J. Plasma Phys. **41**, 171 (1989).
[8]A. Bandyopadhyay and K. P. Das, Phys. Plasmas **7**, 3227 (2000).
[9]P. O. Dovner, A. I. Eriksson, R. Böstrom, and B. Holback, Geophys. Res. Lett. **21**, 1827 (1994).
[10]R. A. Cairns, A. A. Mamun, R. Bingham, R. Dendy, R. Böstrum, P. K. Shukla, and C. Nairn, Geophys. Res. Lett. **22**, 2709 (1995).
[11]R. A. Cairns, A. A. Mamun, R. Bingham, and P. K. Shukla, Phys. Scr., T **T63**, 80 (1996).
[12]A. A. Mamun and R. A. Cairns, J. Plasma Phys. **56**, 175 (1996).
[13]R. A. Treumann, M. Güdel, and A. O. Benz, Astron. Astrophys. **236**, 242 (1990).
[14]R. A. Treumann and W. Baumjohann, *Advance Space Plasma Physics* (Imperial College Press, London, 1997), p. 277.
[15]M. Berthomier and R. Pottelette, Phys. Plasmas **6**, 467 (1999).
[16]G. Rowlands, J. Plasma Phys. **3**, 567 (1969).
[17]E. Infeld, J. Plasma Phys. **8**, 105 (1972).
[18]E. Infeld and G. Rowlands, J. Plasma Phys. **10**, 293 (1973).
[19]V. E. Zakharov and A. M. Rubenchik, Sov. Phys. JETP **38**, 494 (1974).
[20]B. B. Kadomtsev, *Plasma Turbulence* (Academic, New York, 1965), p. 82.
[21]E. Infeld, J. Plasma Phys. **33**, 171 (1985).
[22]K. P. Das and F. Verheest, J. Plasma Phys. **412**, 139 (1989).
[23]A. Jeffrey and K. Kakutani, SIAM Rev. **14**, 582 (1972).