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Existence and stability of alternative dust ion acoustic solitary wave solution of the combined MKP-KP equation in nonthermal plasma

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The aim of this paper is to extend the recent work of Sardar *et al.* [Phys. Plasmas **23**, 073703 (2016)] on the stability of the small amplitude dust ion acoustic solitary wave in a collisionless unmagnetized nonthermal plasma in the presence of isothermal positrons. Sardar *et al.* [Phys. Plasmas **23**, 073703 (2016)] have derived a KP (Kadomtsev Petviashvili) equation to study the stability of the dust ion acoustic solitary wave when the weak dependence of the spatial coordinates perpendicular to the direction of propagation of the wave is taken into account. They have also derived a modified KP (MKP) equation to investigate the stability of the dust ion acoustic solitary wave when the coefficient of the nonlinear term of the KP equation vanishes. When the coefficient of the nonlinear term of the KP equation is close to zero, a combined MKP-KP equation more efficiently describes the nonlinear behaviour of the dust ion acoustic wave. This equation is derived in the present paper. The alternative solitary wave solution of the combined MKP-KP equation having profile different from sech^2 or sech is obtained. This alternative solitary wave solution of the combined MKP-KP equation is stable at the lowest order of the wave number. It is found that this alternative solitary wave solution of the combined MKP-KP equation and its lowest order stability analysis are exactly same as those of the solitary wave solution of the MKP equation when the coefficient of the nonlinear term of the KP equation tends to zero. *Published by AIP Publishing.* [http://dx.doi.org/10.1063/1.4972881]

I. INTRODUCTION

The study of different nonlinear wave structures in dusty plasma is an important area of research for last few decades as those plasmas are frequently observed in space plasma environments as well as in laboratory experiments. Several authors[1–14] reported that the propagation properties of nonlinear waves are significantly affected by the presence of highly charged massive dust particles. Depending on different time scales, there can exist different acoustic waves in a typical dusty plasma. For the first time, Shukla and Silin[2] reported that a dusty plasma can support low frequency Dust Ion Acoustic (DIA) waves with phase velocity much smaller (larger) than electron (ion) thermal velocity. DIA waves are basically ion acoustic (IA) waves modified by the presence of heavy dust particulates. Recently, considerable interests are observed in the investigations of IA/DIA solitary structures in four component electron-positron-ion-dust (e-p-i-d) plasma as those plasmas may be found in various astrophysical environments, viz., in the galactic centre,[15] in the insersteller medium,[13,15,16] in the interior regions of accretion disks near neutron stars and magnetars,[17] in dusty cosmological environments such as milky way,[13] in the ionosphere and in the magnetosphere of the Earth,[18–20] in the magnetosphere of the other magnetized planets of our solar system,[20] such as Jupiter[21] and Saturn.[22] Ghosh and Bharuthram[23] considered nonlinear IA waves in a collisionless unmagnetized e-p-i-d plasma with isothermally distributed electrons and positrons. They derived a Korteweg-de Vries (KdV) equation and a modified KdV equation to discuss the nonlinear behaviour of IA waves in such plasmas. Dubinov *et al.*[17] investigated the nonlinear theory of IA waves in a collisionless unmagnetized e-p-i-d plasma with the help of Bernoulli's pseudo-potential method. Saini *et al.*[24] studied the arbitrary amplitude DIA solitary structures in the same e-p-i-d plasma system of Ghosh and Bharuthram[23] by considering the effect of ion temperature. They found the existence of solitary waves of both polarities and rarefactive double layers. Beside these, several authors[25–28] investigated small or arbitrary amplitude IA/DIA solitary structures in different e-p-i-d plasma systems.

Recently, Sardar *et al.*[29] have derived a three-dimensional KP (Kadomtsev Petviashvili) equation to investigate the stability of small amplitude DIA solitary waves in a collisionless unmagnetized dusty plasma consisting of warm adiabatic ions, static negatively charged dust grains, nonthermal electrons, and isothermal positrons when the weak dependence of the spatial coordinates perpendicular to the direction of propagation of the wave is taken into account. It is found that a factor B_1 of the coefficient of the nonlinear term of the KP equation vanishes along different family of curves (see Figure 1 of Sardar *et al.*[29]) in $\mu - p$ parametric plane for different values of β_e and fixed values of other parameters of the plasma system, where β_e is the nonthermal parameter associated with the Cairns distributed[30] nonthermal electrons, p is the ratio of the unperturbed positron number density to the total

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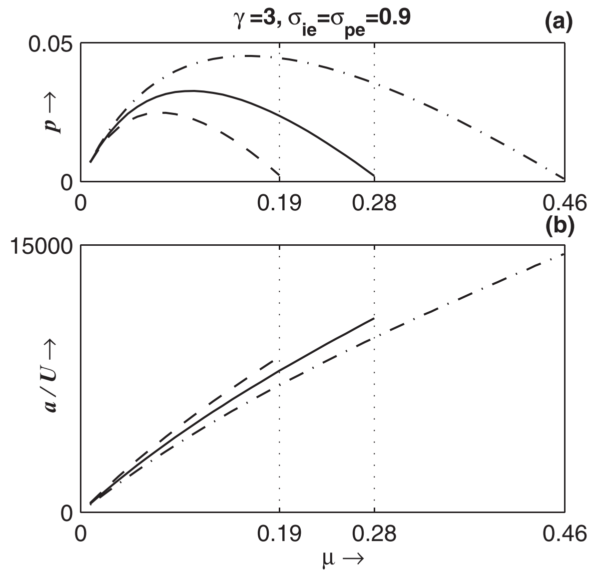

FIG. 1. (a) p is plotted against μ and (b) a/U is plotted against μ for different values of β_e with $\gamma=3$, $\sigma_{ie}=\sigma_{pe}=0.9$ when $B_1=0.0001$. In both lower and the upper panels of this figure, dashed, solid and dashed-dotted curves, respectively, correspond to $\beta_e=0$, $\beta_e=0.2$, and $\beta_e=0.4$.

equilibrium number density of positive charges, and μ is the ratio of the unperturbed electron number density to the total equilibrium number density of positive charges.

When $B_1=0$, in the same paper, Sardar *et al.*[29] have derived a three-dimensional modified KP (MKP) equation and they have discussed the existence and the lowest order stability of the solitary wave solution of the MKP equation. But this MKP equation describes the nonlinear behaviour of DIA waves only when $B_1=0$. When $B_1 \neq 0$ but B_1 is close to zero, then the KP equation cannot describe the nonlinear behaviour of DIA waves because the amplitude of the solitary wave solution defined by the KP equation assumes a very large numerical value when B_1 is close to zero. So, neither KP nor MKP equation can describe the nonlinear behaviour of DIA waves when $B_1 \approx O(\epsilon)$, where ϵ is a small parameter measuring the weakness of dispersion and weakness of nonlinearity. Therefore, in connection with the description of the nonlinear behaviour of small amplitude DIA waves in the present plasma system, we see that there is a gap between the KP and MKP equation when $B_1 \approx O(\epsilon)$.

The present paper is an extension of the recently published paper of Sardar *et al.*[29] in the following directions:

(i) The case has been considered when the coefficient of the nonlinear term of the KP equation derived in the paper of Sardar *et al.*[29] is not equal to zero but it is close to zero. In such a situation, i.e., when $B_1 \approx O(\epsilon)$, a combined MKP-KP equation has been derived which efficiently describes the nonlinear behaviour of DIA waves.

(ii) The method of Malfliet and Hereman[31] has been used to find the alternative solitary wave solution of the combined MKP-KP equation having profile different from $\text{sech}^{2/r}$ for any strictly positive real value of r.

(iii) The condition for the existence of the alternative solitary wave solution of the combined MKP-KP equation has been investigated.

(iv) The small-k perturbation expansion method of Rowlands and Infeld[32–36] has been used to analyse the lowest order stability of the alternative solitary wave solution of the combined MKP-KP equation.

This paper is organized as follows: the basic equations are given in Section II. The KP and the MKP equations are given in Section III. In Section IV, we have derived a combined MKP–KP equation. The alternative solitary wave solution of the combined MKP–KP equation has been investigated in Section V. The stability of the alternative solitary wave solution of the combined MKP–KP equation has been considered in Section VI. Finally, conclusions are given in Section VII.

II. BASIC EQUATIONS

We consider a collisionless unmagnetized unbounded dusty plasma consisting of warm adiabatic ions, static negatively charged dust grains, nonthermal electrons, and isothermal positrons. The nonlinear behaviour of DIA waves in this plasma may be described by the following set of fluid equations, which consist of the equation of continuity of ions, the equation of motion of ion fluid, the pressure equation for ion fluid and the Poisson equation:

$$\frac{\partial n_i}{\partial t} + \nabla.(n_i \mathbf{u}) = 0, \tag{1}$$

$$M_s^2\left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u}\right] + \frac{(1-p)\sigma_{ie}}{n_i}\nabla P + \nabla\phi = 0, \tag{2}$$

$$\frac{\partial P}{\partial t} + (\mathbf{u}.\nabla)P + \gamma P(\nabla.\mathbf{u}) = 0, \tag{3}$$

$$C\nabla^2\phi = n_e - n_i - n_p + \frac{Z_d n_{d0}}{N_0}, \tag{4}$$

where

$$C = \frac{1-p}{M_s^2 - \gamma\sigma_{ie}}, \tag{5}$$

and we have used the following notations:

$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

Here, n_i, n_e, n_p, $\mathbf{u} = (u, v, w)$, P, ϕ, (x, y, z), and t are, respectively, the ion number density, the electron number density, the positron number density, the ion fluid velocity, the ion fluid pressure, the electrostatic potential, the spatial variables, and time, and these quantities have been normalized, respectively, by N_0 $(=n_{i0}+n_{p0}=n_{e0}+Z_d n_{d0})$, N_0, N_0, C_D (linearized velocity of the DIA wave in the present plasma system for long-wave length plane wave perturbation), $n_{i0}K_BT_i$, $\frac{K_BT_e}{e}$, λ_D (Debye length of the present plasma system) and λ_D/C_D, where n_{i0}, n_{p0}, n_{e0}, n_{d0}, T_i, and T_e are, respectively, the unperturbed ion number density, the unperturbed positron number density, the unperturbed electron number density, the constant dust number density, the average unperturbed temperature of ions, the average temperature of nonthermal electrons, and Z_d is the number of electrons residing on the dust grain surface with K_B is the Boltzmann constant, $-e$ is the charge of an electron, and $\gamma(=3)$ is the adiabatic index.

The expression of M_s and the four basic parameters p, μ, σ_{ie}, and σ_{pe} are given by

$$M_s = \sqrt{\gamma\sigma_{ie} + \frac{(1-p)\sigma_{pe}}{p+\mu(1-\beta_e)\sigma_{pe}}}, \tag{6}$$

$$p = \frac{n_{p0}}{N_0}, \quad \mu = \frac{n_{e0}}{N_0}, \quad \sigma_{ie} = \frac{T_i}{T_e}, \quad \sigma_{pe} = \frac{T_p}{T_e}, \tag{7}$$

where T_p is the average temperature of isothermal positrons.

Under the above-mentioned normalization of the dependent variables, the expressions of n_e and n_p can be written as

$$n_e = \mu(1-\beta_e\phi+\beta_e\phi^2)e^{\phi}, \quad n_p = pe^{-\frac{\phi}{\sigma_{pe}}}. \tag{8}$$

The above system of equations are supplemented by the unperturbed charge neutrality condition

$$\frac{n_{i0}}{N_0} = 1-p \quad \text{and} \quad \frac{Z_d n_{d0}}{N_0} = 1-\mu. \tag{9}$$

Expanding n_e and n_p as given by (8) upto ϕ^4, the Poisson equation (4) can be written as

$$C\nabla^2\phi = 1-p+\sum_{i=1}^{4} Q_i\phi^i - n_i, \tag{10}$$

where Q_1, Q_2, Q_3, and Q_4 are given in Appendix A. We have used Equations (1), (2), (3), and (10) to derive the different evolution equations.

III. KP AND MKP EQUATIONS

To derive the different evolution equations, Sardar *et al.*[29] have used the following stretchings of space coordinates and time:

$$\xi = \epsilon(x-Vt), \quad \eta = \epsilon^2 y, \quad \zeta = \epsilon^2 z, \quad \tau = \epsilon^3 t, \tag{11}$$

where V is a constant being independent of space coordinates and time. In fact, V is the dimensionless phase velocity of the DIA wave (normalized by C_D) for long-wave length plane wave perturbation, i.e.

$$V = \lim_{k\to 0}\frac{(\omega/k)}{C_D},$$

where ω and k are, respectively, the wave frequency and wave number of the plane wave perturbation.

A. KP equation

To derive the KP equation describing the nonlinear behaviour of DIA waves in the present plasma system, Sardar *et al.*[29] have used the following perturbation expansions of the dependent variables along with the stretchings (11):

$$f = f^{(0)} + \sum_{j=1}^{\infty}\epsilon^{2j} f^{(j)}, \quad g = g^{(0)} + \sum_{j=1}^{\infty}\epsilon^{2j+1} g^{(j)}, \tag{12}$$

where $f = n_i, P, \phi, u$ with $n_i^{(0)} = 1-p$, $P^{(0)} = 1$, $\phi^{(0)} = 0$, $u^{(0)} = 0$, and $g = v, w$ with $v^{(0)} = w^{(0)} = 0$. Substituting (11) and (12) in the Equations (1), (2), (3), and (10), and equating the coefficient of different powers of ϵ on each side of every equation, one can get a sequence of equations. From these sequence of equations, Sardar *et al.*[29] have derived the following KP equation:

$$\frac{\partial}{\partial\xi}\left[\phi_\tau^{(1)} + AB_1\phi^{(1)}\phi_\xi^{(1)} + \frac{1}{2}AC\phi_{\xi\xi\xi}^{(1)}\right] + \frac{1}{2}AD\left(\phi_{\eta\eta}^{(1)} + \phi_{\zeta\zeta}^{(1)}\right) = 0, \tag{13}$$

where the coefficients A, B_1, and D are given by

$$A = \frac{1}{1-p}\frac{\left(M_s^2V^2-\gamma\sigma_{ie}\right)^2}{VM_s^2}, \tag{14}$$

$$B_1 = \frac{1}{2}\left[(1-p)\frac{3M_s^2V^2+\gamma(\gamma-2)\sigma_{ie}}{\left(M_s^2V^2-\gamma\sigma_{ie}\right)^3} - \left(\mu - \frac{p}{\sigma_{pe}^2}\right)\right], \tag{15}$$

$$D = (1-p)\frac{M_s^2V^2}{\left(M_s^2V^2-\gamma\sigma_{ie}\right)^2}, \tag{16}$$

and the constant V is determined by

$$V^2 = 1. \tag{17}$$

It is shown in Figure 1 of Sardar *et al.*[29] that B_1 vanishes along a family of curves in $\mu - p$ parameter plane for different values of the nonthermal parameter β_e. In this situation, they have derived the following MKP equation.

B. MKP equation

When $B_1 = 0$, Sardar *et al.*[29] have used the following perturbation expansions of the dependent variables:

$$f = f^{(0)} + \sum_{j=1}^{\infty}\epsilon^{j} f^{(j)}, \quad g = g^{(0)} + \sum_{j=1}^{\infty}\epsilon^{j+1} g^{(j)}, \tag{18}$$

where $f = n_i, P, \phi, u$ with $n_i^{(0)} = 1-p$, $P^{(0)} = 1$, $\phi^{(0)} = 0$, $u^{(0)} = 0$, and $g = v, w$ with $v^{(0)} = w^{(0)} = 0$. Substituting (11) and (18) in the Equations (1), (2), (3), and (10), and then, equating the coefficient of different powers of ϵ on each side of every equation, one can get a sequence of equations. From these sequence of equations, Sardar *et al.*[29] have derived the following MKP equation:

$$\frac{\partial}{\partial\xi}\left[\phi_\tau^{(1)} + AB_2\left(\phi^{(1)}\right)^2\phi_\xi^{(1)} + \frac{1}{2}AC\phi_{\xi\xi\xi}^{(1)}\right] + \frac{1}{2}AD\left(\phi_{\eta\eta}^{(1)} + \phi_{\zeta\zeta}^{(1)}\right) = 0, \tag{19}$$

where

$$B_2 = \frac{1-p}{4\left(M_s^2V^2-\gamma\sigma_{ie}\right)^5}\left[15M_s^4V^4+\gamma\left(\gamma^2+13\gamma-18\right)M_s^2V^2\sigma_{ie} + \gamma^2(\gamma-2)(2\gamma-3)\sigma_{ie}^2\right]-\frac{3}{2}Q_3. \tag{20}$$

Here, A and D are given by (14) and (16), respectively, and V is determined by (17).

The MKP equation (19) describes the nonlinear behaviour of DIA waves only when $B_1=0$. When $B_1\neq 0$ but B_1 is close to zero, then the KP equation (13) cannot describe the nonlinear behaviour of DIA waves because the amplitude of the solitary wave solution defined by the KP equation (13) assumes a very large numerical value when B_1 is close to zero. To explain this fact, we choose $B_1=0.0001$, i.e., we take a small numerical value of B_1. Now, it is simple to check that B_1 is a function of p, μ, and β_e, i.e., $B_1=B_1(p,\mu,\beta_e)$ for fixed values of γ, σ_{ie}, and σ_{pe}. Therefore, $B_1=B_1(p,\mu,\beta_e)=0.0001$ gives a functional relation between μ and p for any fixed value of β_e within the physically admissible interval of β_e, i.e., $0\leq\beta_e\leq 0.6$. In Figure 1(a), this functional relation ($B_1(p,\mu,\beta_e)=0.0001$) between μ and p is plotted for different values of β_e with $\gamma=3$, $\sigma_{ie}=0.9$, and $\sigma_{pe}=0.9$. Figure 1(a) shows the existence of a region in the parameter space where $B_1=0.0001$. In Figure 1(b), a/U is plotted against μ for different values of β_e with $\gamma=3$, $\sigma_{ie}=\sigma_{pe}=0.9$ when $B_1=0.0001$, where a is the amplitude of the solitary wave solution defined by the KP equation (13) and U is the dimensionless velocity (normalized by C_D) of the solitary wave solution defined by the KP equation (13). So, in Figure 1(b), a/U is plotted against μ along each curve of Figure 1(a). Again, it can be easily checked that the minimum numerical value of a/U is greater than 350 for any value of β_e. Therefore, each curve of Figure 1(b) clearly shows that the amplitude of the solitary wave solution defined by the KP equation (13) assumes a very large numerical value when $B_1=0.0001$. Again, one can use MKP equation (19) only when $B_1=0$. Consequently, neither KP nor MKP equation can describe the nonlinear behaviour of DIA waves when $B_1\approx O(\epsilon)$. So, a further modification of the MKP equation (19) is necessary. In Section IV, we have derived a combined MKP-KP equation to describe the nonlinear behaviour of DIA waves on the basis of the assumption that $B_1\approx O(\epsilon)$.

IV. COMBINED MKP-KP EQUATION

Here, we take the same stretchings (11) and the same perturbation expansions of the dependent variables as given by (18). But instead of considering $B_1=0$, we assume that $B_1\approx O(\epsilon)$ (Nejoh[37]). Substituting the stretchings (11) and perturbation expansions (18) in the Equations (1), (2), (3), and (10) and finally equating the coefficient of different powers of ϵ on each side of every equation, we get a sequence of equations. At the lowest order ($O(\epsilon)=2$), from the equation of continuity of ions, the x-component of the equation of motion of ion fluid and the pressure equation of ion fluid, we get the following equations:

$$n_i^{(1)} = \frac{1-p}{M_s^2V^2-\gamma\sigma_{ie}}\phi^{(1)}, \tag{21}$$

$$u^{(1)} = \frac{V}{M_s^2V^2-\gamma\sigma_{ie}}\phi^{(1)}, \tag{22}$$

$$P^{(1)} = \frac{\gamma}{M_s^2V^2-\gamma\sigma_{ie}}\phi^{(1)}. \tag{23}$$

From Equation (21) and the Poisson equation (10) at the order ϵ, we get the dispersion relation (17) and this dispersion relation determines the constant V.

At the next order ($O(\epsilon)=3$), Equations (1)–(3) give the following expressions for $n_i^{(2)}$, $u^{(2)}$, $P^{(2)}$, and $\frac{\partial}{\partial\xi}\left(\frac{\partial v^{(1)}}{\partial\eta}+\frac{\partial w^{(1)}}{\partial\zeta}\right)$:

$$n_i^{(2)} = \frac{1-p}{M_s^2V^2-\gamma\sigma_{ie}}\phi^{(2)}+\frac{1-p}{2}\frac{3M_s^2V^2+\gamma(\gamma-2)\sigma_{ie}}{\left(M_s^2V^2-\gamma\sigma_{ie}\right)^3}\left(\phi^{(1)}\right)^2, \tag{24}$$

$$u^{(2)} = \frac{V}{M_s^2V^2-\gamma\sigma_{ie}}\phi^{(2)}+\frac{V}{2}\times\frac{M_s^2V^2+\gamma^2\sigma_{ie}}{\left(M_s^2V^2-\gamma\sigma_{ie}\right)^3}\left(\phi^{(1)}\right)^2, \tag{25}$$

$$P^{(2)} = \frac{\gamma}{M_s^2V^2-\gamma\sigma_{ie}}\phi^{(2)}+\frac{\gamma}{2}\times\frac{(\gamma+2)M_s^2V^2-\gamma\sigma_{ie}}{\left(M_s^2V^2-\gamma\sigma_{ie}\right)^3}\left(\phi^{(1)}\right)^2, \tag{26}$$

$$\frac{\partial}{\partial\xi}\left(\frac{\partial v^{(1)}}{\partial\eta}+\frac{\partial w^{(1)}}{\partial\zeta}\right) = \frac{V}{M_s^2V^2-\gamma\sigma_{ie}}\times\left[\frac{\partial^2\phi^{(1)}}{\partial\eta^2}+\frac{\partial^2\phi^{(1)}}{\partial\zeta^2}\right]. \tag{27}$$

With the help of the expression for $n_i^{(2)}$ given by (24), it is simple to check that the Poisson equation (10) at the order ϵ^2 is identically satisfied since the only nonvanishing term, $-B_1(\phi^{(1)})^2$, is of the order ϵ^3 as $B_1\approx O(\epsilon)$, and therefore, this term has to be included in the next higher order Poisson equation.

At the order ϵ^4, differentiating the continuity equation of ions, the x-component of equation of motion for ions and the pressure equation for ion fluid with respect to ξ, we get three equations for the unknowns $n_{i\xi\xi}^{(3)}(=\frac{\partial^2 n_i^{(3)}}{\partial\xi^2})$, $u_{\xi\xi}^{(3)}(=\frac{\partial^2 u^{(3)}}{\partial\xi^2})$, and $P_{\xi\xi}^{(3)}(=\frac{\partial^2 P^{(3)}}{\partial\xi^2})$. Solving the resulting equations for the unknowns $n_{i\xi\xi}^{(3)}$, $u_{\xi\xi}^{(3)}$, and $P_{\xi\xi}^{(3)}$, we can express $n_{i\xi\xi}^{(3)}$ (as well as $u_{\xi\xi}^{(3)}$ and $P_{\xi\xi}^{(3)}$) as a function of $\phi^{(3)}$, $\phi^{(2)}$, $\phi^{(1)}$ and their different derivatives. The final expression of $n_{i\xi\xi}^{(3)}$ can be written as

$$\begin{aligned} n_{i\xi\xi}^{(3)} &= \frac{1-p}{M_s^2V^2-\gamma\sigma_{ie}}\phi_{\xi\xi}^{(3)}+\frac{2V(1-p)M_s^2}{\left(M_s^2V^2-\gamma\sigma_{ie}\right)^2}\phi_{\xi\tau}^{(1)} \\ &+(1-p)\frac{3M_s^2V^2+\gamma(\gamma-2)\sigma_{ie}}{\left(M_s^2V^2-\gamma\sigma_{ie}\right)^3}\left(\phi^{(1)}\phi^{(2)}\right)_{\xi\xi} \\ &+\frac{(1-p)M_s^2V^2}{\left(M_s^2V^2-\gamma\sigma_{ie}\right)^2}\left[\frac{\partial^2\phi^{(1)}}{\partial\eta^2}+\frac{\partial^2\phi^{(1)}}{\partial\zeta^2}\right] \\ &+M^{(3)}\left[\left(\phi^{(1)}\right)^2\phi_\xi^{(1)}\right]_\xi, \end{aligned} \tag{28}$$

where $M^{(3)}$ is given by

$$M^{(3)} = \frac{1-p}{2(M_s^2V^2-\gamma\sigma_{ie})^5}\Big\{15M_s^4V^4+\gamma(\gamma^2+13\gamma-18) \times M_s^2V^2\sigma_{ie}+\gamma^2(\gamma-2)(2\gamma-3)\sigma_{ie}^2\Big\}, \quad (29)$$

and we have used Equations (21)–(27) to simplify Equation (28). At the order ϵ^3, the Poisson equation, including the term $-B_1(\phi^{(1)})^2$, can be written as

$$C\phi^{(1)}_{\xi\xi} = Q_1\phi^{(3)} - n_i^{(3)} + 2Q_2\phi^{(1)}\phi^{(2)} + Q_3(\phi^{(1)})^3 - B_1(\phi^{(1)})^2. \quad (30)$$

It is important to note that the term, $-B_1(\phi^{(1)})^2$, which was omitted from its previous order equation, is a term of order ϵ^3, and therefore, this term has been included in the Poisson equation at the order ϵ^3. Now, differentiating equation (30) with respect to ξ twice, we get

$$C\phi^{(1)}_{\xi\xi\xi\xi} = Q_1\phi^{(3)}_{\xi\xi} - n^{(3)}_{i\xi\xi} + \frac{\partial^2}{\partial\xi^2}\Big[2Q_2\phi^{(1)}\phi^{(2)} + Q_3\left(\phi^{(1)}\right)^3 - B_1\left(\phi^{(1)}\right)^2\Big]. \quad (31)$$

Eliminating $n^{(3)}_{i\xi\xi}$ from Equations (31) and (28), we get the following combined MKP-KP equation:

$$\frac{\partial}{\partial\xi}\left[\phi^{(1)}_\tau + AB_1\phi^{(1)}\phi^{(1)}_\xi + AB_2\left(\phi^{(1)}\right)^2\phi^{(1)}_\xi + \frac{1}{2}AC\phi^{(1)}_{\xi\xi\xi}\right] + \frac{1}{2}AD\left(\phi^{(1)}_{\eta\eta} + \phi^{(1)}_{\zeta\zeta}\right) = 0. \quad (32)$$

Here, A, B_1, D, V, and B_2 are the same as those given, respectively, by equations (14)–(17) and (20). It is important to note that we have used the condition $B_1 \approx O(\epsilon)$ to eliminate the term $AB_1\frac{\partial^2}{\partial\xi^2}(\phi^{(1)}\phi^{(2)})$ from the final form of (32) since this is a higher order term and this term has to be included in the next higher order equation.

V. SOLITARY WAVE SOLUTION OF THE COMBINED MKP-KP EQUATION

For a solitary wave solution of the combined MKP-KP equation (32), we take following transformation of the independent variables

$$X = \xi - U\tau, \quad \eta' = \eta, \quad \zeta' = \zeta, \quad \tau' = \tau. \quad (33)$$

Here, U is the dimensionless velocity (normalized by C_D) of the travelling wave moving along ξ-axis, i.e., U is the dimensionless velocity of the wave frame.

Under the above changes of the independent variables, the combined MKP-KP equation (32) assumes the following form (in which we drop the primes on the independent variables η, ζ, and τ to simplify the notations):

$$\frac{\partial}{\partial X}\left[-U\phi^{(1)}_X + \phi^{(1)}_\tau + AB_1\phi^{(1)}\phi^{(1)}_X + AB_2\left(\phi^{(1)}\right)^2\phi^{(1)}_X + \frac{1}{2}AC\phi^{(1)}_{XXX}\right] + \frac{1}{2}AD\left(\phi^{(1)}_{\eta\eta} + \phi^{(1)}_{\zeta\zeta}\right) = 0. \quad (34)$$

Now, for the travelling wave solitons of (34), we set

$$\phi^{(1)} = \phi_0(X). \quad (35)$$

Substituting (35) in (34), we get

$$\frac{d^2}{dX^2}\left[-U\phi_0 + \frac{1}{2}AB_1(\phi_0)^2 + \frac{1}{3}AB_2(\phi_0)^3 + \frac{1}{2}AC\frac{d^2\phi_0}{dX^2}\right] = 0. \quad (36)$$

To get the solitary wave solution of Equation (36), we use the following boundary conditions:[38]

$$\frac{d^n\phi_0}{dX^n} \to 0 \text{ as } |X| \to \infty \text{ for all } n = 1, 2, 3, \ldots, \quad (37)$$

together with the condition that the electrostatic potential ϕ_0 vanishes at infinity, i.e.,

$$\lim_{|X|\to\infty} \phi_0(X) = 0. \quad (38)$$

Using the boundary conditions (37) and (38), we can write the Equation (36) as

$$-U\phi_0 + \frac{1}{2}AB_1(\phi_0)^2 + \frac{1}{3}AB_2(\phi_0)^3 + \frac{1}{2}AC\frac{d^2\phi_0}{dX^2} = 0. \quad (39)$$

According to Malfliet and Hereman,[31] we take

$$\phi_0 = a_0\frac{\text{sech}^2\frac{X}{W_1}}{b_0 + c_0\,\text{sech}^2\frac{X}{W_1}} = a_0\frac{\text{sech}^2 p_1X}{b_0 + c_0\,\text{sech}^2 p_1X}, \quad p_1 = \frac{1}{W_1} \quad (40)$$

as a solution of Equation (39). Substituting (40) into (39) and following the same method as given in Malfliet and Hereman,[31] the alternative solitary wave solution (40) of (39) can be put in the form

$$\phi_0 = a_1\frac{S}{\Psi_1}, \quad (41)$$

where

$$S = \text{sech}[2p_1X], \quad \Psi_1 = B_1S + \lambda\sqrt{L}, \quad (42)$$

$$a_1 = 12Cp_1^2, \quad L = B_1^2 + 12B_2Cp_1^2, \quad (43)$$

and $\lambda = \pm 1$. The solution (41) exists if and only if

$$L = B_1^2 + 12B_2Cp_1^2 > 0, \quad (44)$$

and if this condition holds good then U is given by

$$U = 2ACp_1^2. \quad (45)$$

Figure 2(a) can be drawn in a similar way as Figure 1(a), i.e., in Figure 2(a), p is plotted against μ when $B_1 = 0.0001$. In Figure 2(b), L is plotted against μ for different values of β_e with $\gamma = 3$, $\sigma_{ie} = \sigma_{pe} = 0.9$ when $B_1 = 0.0001$, i.e., in Figure 2(b), L is plotted against μ along the each curve of Figure 2(a). Figure 2(b) clearly shows that there exists a value μ_{cr} of μ such that $L > 0$ or $L < 0$ according to whether $\mu > \mu_{cr}$ or

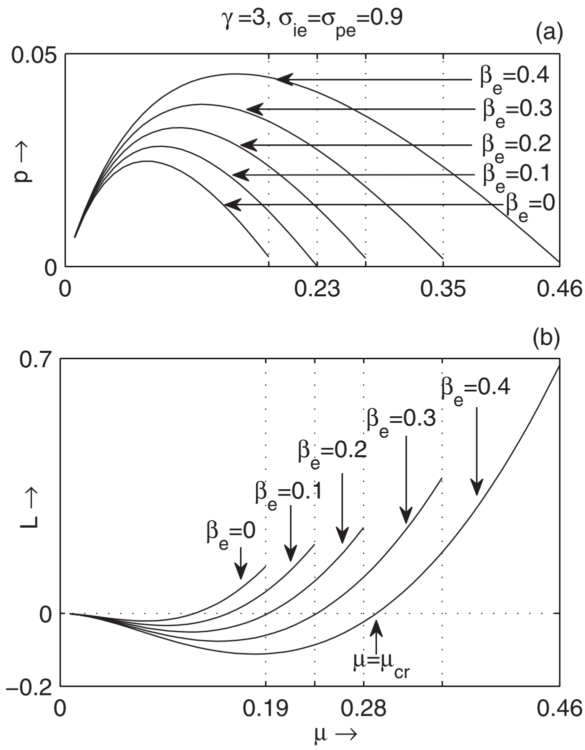

FIG. 2. (a) p is plotted against μ and (b) L is plotted against μ for different values of β_e with $\gamma = 3$, $\sigma_{ie} = \sigma_{pe} = 0.9$ when $B_1 = 0.0001$.

$\mu < \mu_{cr}$ and $L = 0$ at $\mu = \mu_{cr}$. Obviously, in the neighbourhood of $\mu = \mu_{cr}$, L is close to zero, and in this case, the denominator $\Psi_1 = B_1 S + \lambda\sqrt{L}$ of the alternative solitary wave solution (41) is also close to zero, and consequently, this solution cannot describe the nonlinear behaviour of DIA waves because the amplitude of the alternative solitary wave solution defined by the combined MKP-KP equation assumes a very large numerical value. So, if $L = 0$ or $L \approx O(\epsilon)$, then this alternative solitary wave solution (41) fails to describe the nonlinear behaviour of DIA waves. Therefore, if $L > 0$ and L is of moderate numerical value, i.e., $L > 0$ and L is not very close to zero, then only one can use alternative solitary wave solution (41) to describe the nonlinear behaviour of DIA waves when $B_1 \approx O(\epsilon)$. Further investigation is necessary when $L = 0$ or $L \approx O(\epsilon)$. On the other hand, from the expression of L as given in the second equation of (43), we see that the conditions $B_1 \approx O(\epsilon)$ and $B_2 > 0$ always give $L > 0$. Therefore, there exists a region in the parameter space where $L > 0$ and L is of moderate magnitude.

As $B_1 \approx O(\epsilon)$, the numerical value of B_1 is close to zero, and consequently, for the limiting case where $B_1 \to 0$, from the solution (41) of the combined MKP-KP equation, we get the following equation:

$$\lim_{B_1 \to 0} \phi_0 = \lambda\sqrt{\frac{6U}{AB_2}}\,\mathrm{sech}\,\frac{2X}{W_1} \quad \text{with } W_1^2 = \frac{2AC}{U}. \tag{46}$$

Taking $W_1 = 2W$, the above equation can be written as

$$\lim_{B_1 \to 0} \phi_0 = \lambda\sqrt{\frac{6U}{AB_2}}\,\mathrm{sech}\,\frac{X}{W} \quad \text{with } W^2 = \frac{AC}{2U}. \tag{47}$$

Again, it is simple to check that Equations (31) and (32) of Sardar *et al.*[29] can be put in the form of the above equation for $r = 2$, and consequently, the alternative solitary wave solution (41) simply reduces to the solitary wave solution of the MKP equation (19) when $B_1 \to 0$. This is expected because we use the same perturbation expansions of the dependent variables and the same stretching of coordinates and time to derive both MKP and combined MKP-KP equations. The only exception is that we use the critical condition $B_1 = 0$ to derive the MKP equation whereas we use the condition $B_1 \approx O(\epsilon)$ to derive the combined MKP-KP equation. On the other hand, if we take the limit $B_1 \to 0$ on both sides of the combined MKP-KP equation (32), this equation simply reduces to the MKP equation (19) and consequently, it is expected that the solitary wave solution of the combined MKP-KP equation (32) will converge to the solitary wave solution of the MKP equation (19) when $B_1 \to 0$. Therefore, we can conclude that under certain condition [inequality (44)], the solitary wave solution (41) of the combined MKP-KP equation (32) fills the gap between the solitary wave solution (sech^2—profile) of the KP equation and the solitary wave solution (sech—profile) of the MKP equation. In Section VI, we shall consider the stability of the alternative solitary wave solution (41) of the combined MKP-KP equation (32). To simplify the calculations, we have used the following notations:

$$a = \frac{a_1}{B_1}, \quad M = \frac{L}{B_1^2}, \quad \Psi = \frac{\Psi_1}{B_1}. \tag{48}$$

Under the above-mentioned notations, the expression of ϕ_0 as given by Equation (41) can be written as

$$\phi_0 = a\frac{S}{\Psi}. \tag{49}$$

VI. STABILITY OF ALTERNATIVE SOLITARY WAVE

To analyze the stability of the alternative solitary wave solution (41) or equivalently (49) of the Equation (34) by the small-k perturbation expansion method of Rowlands and Infeld,[32–36] we decompose $\phi^{(1)}$ as

$$\phi^{(1)} = \phi_0(X) + q(X, \eta, \zeta, \tau), \tag{50}$$

where $\phi_0(X)$ is the steady state alternative solitary wave solution (41) of the Equation (34) and $q(X, \eta, \zeta, \tau)$ is the perturbed part of $\phi^{(1)}$. Now, for long-wave length plane wave perturbation along a direction having direction cosines l, m, n, we set

$$q(X, \eta, \zeta, \tau) = \bar{q}(X)e^{i\{k(lX+m\eta+n\zeta)-\omega\tau\}}, \tag{51}$$

where k is small and $l^2 + m^2 + n^2 = 1$.

According to small-k perturbation expansion method of Rowlands and Infeld,[32–36] $\bar{q}(X)$ and ω can be expanded as follows:

$$\bar{q}(X) = \sum_{j=0}^{\infty} k^j q^{(j)}(X), \quad \omega = \sum_{j=0}^{\infty} k^j \omega^{(j)} \tag{52}$$

with $\omega^{(0)} = 0$. Substituting (50) into (34) and then linearizing it with respect to q, we get a linear equation for q. Substituting (51) into this linear equation of q, we get an equation of $\bar{q}$. Finally, substituting (52) into the equation of $\bar{q}$ and then equating the coefficient of different powers of k on the both sides of the resulting equation, we get the following equations

$$U\frac{d}{dX}\left(L_X q^{(j)}\right) = M^{(j)}, \quad j = 0, 1, 2, ..., \tag{53}$$

where

$$L_X = -1 + 6\frac{S}{\Psi} + 6(M-1)\frac{S^2}{\Psi^2} + \frac{1}{4p_1^2}\frac{d^2}{dX^2}, \tag{54}$$

and

$$M^{(j)} = \int_{-\infty}^{X} Q^{(j)} dX, \tag{55}$$

and $Q^{(j)}$ for $j = 0, 1, 2$ are given in Appendix B.

Assuming that $q^{(j)}$ and its derivative up to third order vanish as $|X| \to \infty$, the general solution of (53) can be written as

$$q^{(j)} = A_1^{(j)} f + A_2^{(j)} g + A_3^{(j)} h + \frac{2}{AC}\chi^{(j)}, \tag{56}$$

where $A_1^{(j)}$, $A_2^{(j)}$, and $A_3^{(j)}$ are the integration constants and f, g, h, $\chi^{(j)}$ are given by

$$f = \frac{d\phi_0}{dX}, \quad g = f\int\frac{1}{f^2}dX, \quad h = f\int\frac{\phi_0}{f^2}dX,$$
$$\chi^{(j)} = f\int\frac{\int\left(f\int M^{(j)}dX\right)dX}{f^2}dX, \tag{57}$$

where ϕ_0 and $M^{(j)}$ are, respectively, given by Equations (49) and (55). From the expressions of f, g, and h as given by (57), using MATHEMATICA,[39] we get

$$\lim_{|X|\to\infty} f = 0, \quad \lim_{|X|\to\infty} g = \frac{-\lambda}{sign[a]} \times \infty, \quad \lim_{|X|\to\infty} h = -\frac{1}{4p_1^2}, \tag{58}$$

where

$$sign[a] = \begin{cases} 1 & \text{for } a > 0, \\ -1 & \text{for } a < 0. \end{cases} \tag{59}$$

Therefore, to make $q^{(j)}$ bounded, we must have

$$A_2^{(j)} = 0 \quad \text{for } j = 0, 1, 2, ... \tag{60}$$

Consequently, Equation (56) assumes the following form:

$$q^{(j)} = A_1^{(j)} f + A_3^{(j)} h + \frac{2}{AC}\chi^{(j)}. \tag{61}$$

As $Q^{(0)} = 0$, the solution (61) for $j = 0$ can be written as

$$q^{(0)} = A_1^{(0)} f + A_3^{(0)} h. \tag{62}$$

To make $q^{(0)}$ consistent with the boundary condition, i.e., $q^{(0)} \to 0$ as $|X| \to \infty$, we must have $A_3^{(0)} = 0$. Therefore, Equation (62) assumes the following form:

$$q^{(0)} = A_1^{(0)} f. \tag{63}$$

Using (63) and MATHEMATICA,[39] the bounded and consistent solution (61) for $j = 1$ can be written as

$$q^{(1)} = A_1^{(1)} f + iA_1^{(0)}\left[s_1\frac{S^2}{\Psi^2} + s_2\frac{S}{\Psi^2} + s_3 X f\right], \tag{64}$$

where

$$s_1 = \frac{a}{6U}\{6u_1 - au_2\}, \tag{65}$$

$$s_2 = \frac{\lambda a}{12U\sqrt{M}}\{6(1+M)u_1 - a(2u_2 + au_3)\}, \tag{66}$$

$$s_3 = \frac{1}{2U}\{u_1 - 4p_1^2 u_4\}, \tag{67}$$

with

$$u_1 = \omega^{(1)} + 2lU, \quad u_2 = 2lAB_1, \quad u_3 = 2lAB_2, \quad u_4 = 2lAC.$$

Now, for the solution of the Equation (53) to exist, the right hand side of the Equation (53) must be perpendicular to the kernel of the operator adjoint to the operator $\frac{d}{dX}L_X$; this kernel, which must tend to zero, is ϕ_0. Thus, we get the following consistency condition for the existence of the solution of Equation (53):

$$\int_{-\infty}^{\infty} \phi_0 M^{(j)} dX = 0. \tag{68}$$

It is simple to check that the consistency condition (68) is trivially satisfied for $j = 0$ and $j = 1$. Using (63) and (64), the consistency condition (68) for $j = 2$ can be written as

$$\left(\omega^{(1)}\right)^2 = \frac{2UV(m^2+n^2)}{1-N}\chi, \tag{69}$$

where

$$N = \frac{1}{M} \quad \text{and} \quad \chi = 1 - 2\pi_\lambda\sqrt{\frac{N}{1-N}}. \tag{70}$$

Here, π_λ is given by

$$\pi_\lambda = \arctan\left(\frac{\lambda - \sqrt{N}}{\sqrt{1-N}}\right). \tag{71}$$

A. Stability analysis for $B_1 \to 0$

Now as B_1 is small enough, we can consider the limiting case where $B_1 \to 0 \Longleftrightarrow M \to \infty \Longleftrightarrow N \to 0$. For this limiting case, the consistency condition (69) assumes the following form:

$$(\omega^{(1)})^2 = 2UV(m^2 + n^2). \tag{72}$$

This equation is exactly the same as Equation (47) of Sardar *et al.*[29] for $r = 2$, and consequently, if $B_1 \to 0 \Longleftrightarrow M \to \infty \Longleftrightarrow N \to 0$, the first order stability analysis of the solitary wave solution of the combined MKP-KP equation is exactly same as that of the MKP equation as presented in the paper of Sardar *et al.*[29] Therefore, the steady state solitary wave solution (41) of the combined MKP-KP equation (32) and its first order stability analysis are exactly the same as those of the solitary wave solution of the MKP equation (19) if $B_1 \to 0$.

B. Stability analysis for physically admissible values of the parameters of the system satisfying the conditions $B_1 \approx O(\epsilon)$ and $L > 0$

From the definition of M as given by the second equation of (48) and from the definition of N as given by first equation of (70), it is simple to check that $0 \leq N < 1$. Again, for $0 \leq N < 1$, the right hand side of (69) is positive if and only if χ is positive. In Figure 3(a), χ is plotted against N for $\lambda = 1$ whereas for $\lambda = -1$, χ is plotted against N in Figure 3(b). These figures show that χ is positive for $\lambda = 1$ and also for $\lambda = -1$ for any N lying within the interval $0 \leq N < 1$. Therefore, Equation (69) gives a real solution for $\omega^{(1)}$ and consequently, the solitary wave solution (41) of the combined MKP-KP equation (32) is always stable at the lowest order of k.

VII. CONCLUSIONS

1. A three-dimensional KP equation describes the nonlinear behaviour of DIA waves in an unmagnetized collisionless nonthermal dusty plasma when the weak dependence of the spatial coordinates perpendicular to the direction of propagation of the wave is taken into account. When the nonlinear term of the KP equation vanishes, the nonlinear behaviour of the same DIA wave is described by a MKP equation. But neither KP equation nor MKP equation can describe the nonlinear behaviour of DIA waves when the coefficient of the nonlinear term of the KP equation is not exactly equal to zero but it is close to zero. In this case, a combined MKP–KP equation more efficiently describes the nonlinear behaviour of DIA waves. Solitary wave solution of this equation is different from $\text{sech}^{2/r}$— profile. This alternative solitary wave solution fills the gap between the solitary wave solutions of KP and MKP equations having profile sech^2 and sech, respectively.
2. The steady state alternative solitary wave solution of the combined MKP-KP equation is exactly same as the solitary wave solution (sech-profile) of the MKP equation if the coefficient of the nonlinear term of the KP equation tends to zero.
3. The stability analysis of the alternative solitary wave solution of the combined MKP-KP equation is exactly the same as that of the solitary wave solution of the MKP equation when the coefficient of the nonlinear term of the KP equation approaches to zero.
4. The alternative solitary wave solution of the combined MKP-KP equation is always stable.

In this connection, it is important to note that the solitary wave solutions of both KP and MKP equations are stable.

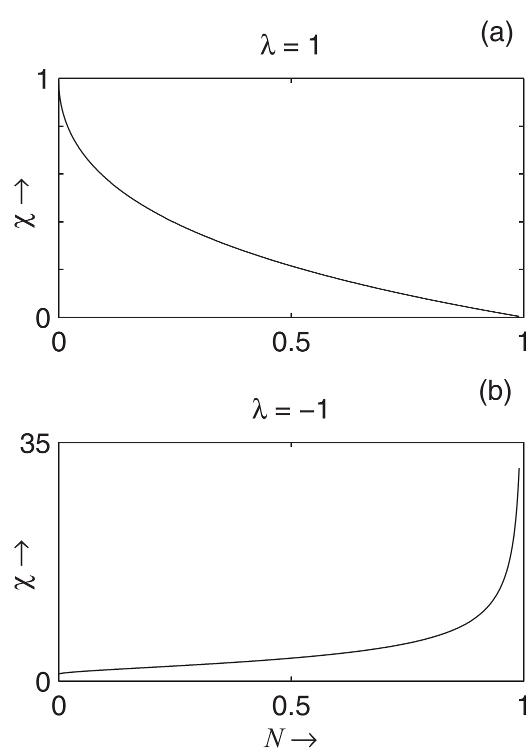

FIG. 3. χ is plotted against N in (a) $\lambda = 1$ and in (b) $\lambda = -1$.

APPENDIX A: Q_i—THE COEFFICIENT OF ϕ^i IN EQUATION (10) FOR $i = 1, 2, 3,$ AND 4

$$Q_1 = \mu(1 - \beta_e) + \frac{p}{\sigma_{pe}}, \tag{A1}$$

$$Q_2 = \frac{1}{2}\left[\mu - \frac{p}{\sigma_{pe}^2}\right], \tag{A2}$$

$$Q_3 = \frac{1}{6}\left[\mu(1 + 3\beta_e) + \frac{p}{\sigma_{pe}^3}\right], \tag{A3}$$

$$Q_4 = \frac{1}{24}\left[\mu(1 + 8\beta_e) - \frac{p}{\sigma_{pe}^4}\right]. \tag{A4}$$

APPENDIX B: $Q^{(j)}$—INTEGRAND OF THE INTEGRATION IN EQUATION (55) FOR $j = 0, 1,$ AND 2

$$Q^{(0)} = 0, \tag{B1}$$

$$Q^{(1)} = \frac{d}{dX}\left[i\left(\omega^{(1)} + 2lU\right)q^{(0)} - 2il\left\{AB_1\phi_0 q^{(0)} + AB_2(\phi_0)^2 q^{(0)} + AC\frac{d^2 q^{(0)}}{dX^2}\right\}\right], \tag{B2}$$

$$Q^{(2)} = -2il\frac{d}{dX}\left[-Uq^{(1)} + AB_1\phi_0 q^{(1)} + AB_2(\phi_0)^2 q^{(1)} + AC\frac{d^2q^{(1)}}{dX^2}\right] + l^2\left[-Uq^{(0)} + AB_1\phi_0 q^{(0)} + AB_2(\phi_0)^2 q^{(0)} + 3AC\frac{d^2q^{(0)}}{dX^2}\right] + \frac{1}{2}AD(m^2+n^2)q^{(0)} + i\frac{d}{dX}\left[\omega^{(2)}q^{(0)} + \omega^{(1)}q^{(1)}\right] - l\omega^{(1)}q^{(0)}. \tag{B3}$$

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