Existence and stability of alternative dust ion acoustic solitary waves in a dusty plasma consisting of nonthermal electrons having vortex-like velocity distribution

Sankirtan Sardar, Anup Bandyopadhyay, and K. P. Das

Citation: Physics of Plasmas **24**, 063705 (2017); doi: 10.1063/1.4986090
View online: http://dx.doi.org/10.1063/1.4986090
View Table of Contents: http://aip.scitation.org/toc/php/24/6
Published by the American Institute of Physics

PHYSICS OF PLASMAS **24**, 063705 (2017)

Existence and stability of alternative dust ion acoustic solitary waves in a dusty plasma consisting of nonthermal electrons having vortex-like velocity distribution

Sankirtan Sardar,[1] Anup Bandyopadhyay,[1,a)] and K. P. Das[2]
[1]*Department of Mathematics, Jadavpur University, Kolkata 700032, India*
[2]*Department of Applied Mathematics, University of Calcutta, 92 Acharya Prafulla Chandra Road, Kolkata 700009, India*

(Received 9 April 2017; accepted 29 May 2017; published online 16 June 2017)

The recent work of Sardar *et al.* [Phys. Plasmas **23**, 073703 (2016)] on the existence and stability of the small amplitude dust ion acoustic solitary waves in a collisionless unmagnetized plasma consisting of warm adiabatic ions, static negatively charged dust grains, isothermal positrons, and nonthermal electrons due to Cairns *et al.* [Geophys. Res. Lett. **22**, 2709 (1995)] has been extended by considering nonthermal electrons having a vortex-like velocity distribution due to Schamel [Plasma Phys. **13**, 491 (1971); **14**, 905 (1972)] instead of taking nonthermal electrons. This distribution takes care of both free and trapped electrons. A Schamel's modified Kadomtsev Petviashvili (SKP) equation describes the nonlinear behaviour of dust ion acoustic waves in this plasma system. The nonlinear behaviour of the dust ion acoustic wave is described by the same Kadomtsev Petviashvili (KP) equation of Sardar *et al.* [Phys. Plasmas **23**, 073703 (2016)] when $B=0$, where B is the coefficient of nonlinear term of the SKP equation. A combined SKP-KP equation more efficiently describes the nonlinear behaviour of dust ion acoustic waves when $B \to 0$. The solitary wave solution of the SKP equation and the alternative solitary wave solution of the combined SKP-KP equation having profile different from both sech^4 and sech^2 are stable at the lowest order of the wave number. It is found that this alternative solitary wave solution of the combined SKP-KP equation and its lowest order stability analysis are exactly the same as those of the solitary wave solution of the KP equation when $B \to 0$. *Published by AIP Publishing.*
[http://dx.doi.org/10.1063/1.4986090]

I. INTRODUCTION

The study of different nonlinear wave structures in four component electron-positron-ion-dust (e-p-i-d) plasma is an important area of research as these plasmas are frequently observed in various astrophysical environments, viz., in the galactic centre,[1] in the interstellar medium,[1–3] in the interior regions of accretion disks near neutron stars and magnetars,[4] in dusty cosmological environments such as milky way,[3] in the ionosphere and magnetosphere of the Earth,[5–7] and in the magnetosphere of the other magnetized planets of our solar system,[7] such as Jupiter[8] and Saturn.[9] Depending on different time scales, there can exist two or more acoustic waves in a typical dusty plasma. Dust Acoustic (DA) and Dust Ion Acoustic (DIA) waves are two such acoustic waves in a dusty plasma. DIA waves are basically ion acoustic (IA) waves modified by the presence of heavy dust particulates. For the first time, Shukla and Silin[10] reported that a dusty plasma can support low frequency DIA waves with the phase velocity much smaller (larger) than the electron (ion) thermal velocity. Several authors[4,10–22] investigated small or arbitrary amplitude IA/DIA solitary structures in different e-p-i-d plasma systems.

Sardar *et al.*[19] have investigated the existence and stability of small amplitude DIA solitary waves in a collisionless unmagnetized e-p-i-d plasma consisting of warm adiabatic ions, static negatively charged dust grains, isothermal positrons, and nonthermal electrons due to Cairns *et al.*[23] In this paper, they have derived Kadomtsev Petviashvili (KP) and different modified KP equations to describe the nonlinear behaviour of DIA waves in different regions of the parameter space when the weak dependence of the spatial coordinates perpendicular to the direction of propagation of the wave is taken into account. They have also investigated the stabilities of solitary wave solutions of a more general evolution equation having nonlinear term of the form $\frac{\partial}{\partial \xi}\left[\left(\phi^{(1)}\right)^r \frac{\partial \phi^{(1)}}{\partial \xi}\right]$ for any real positive value of r with the help of the small-k perturbation expansion method of Rowlands and Infeld,[24–28] where $\phi^{(1)}$ is the first order perturbed electrostatic potential. Finally, they have investigated the stabilities of DIA solitary wave solutions of KP and different modified KP equations. In a later paper, Sardar *et al.*[20] have extended their previously published paper[19] in the following directions:

(i) They have considered the case when the coefficient of the nonlinear term of the KP equation derived in the paper of Sardar *et al.*[19] is not equal to zero, but it is close to zero. In such a situation, they have derived a combined MKP-KP equation.

(ii) They have used the method of Malfliet and Hereman[29] to find the alternative solitary wave solution of the combined MKP-KP equation having profile different from sech$^{2/r}$ for any strictly positive real value of r.

a)Electronic mail: abandyopadhyay1965@gmail.com

(iii) The condition for the existence of the alternative solitary wave solution of the combined MKP-KP equation has been investigated.
(iv) They have employed the small - k perturbation expansion method of Rowlands and Infeld[24–28] to analyse the lowest order stability of the alternative solitary wave solution of the combined MKP-KP equation.

In the above mentioned investigations of Sardar *et al.*,[19,20] the velocity distribution function of electrons was taken as[23]

$$f_{e0}(v) = n_{e0}\frac{1+\alpha_e v^4}{\sqrt{2\pi}(1+3\alpha_e)}\exp\left[-\frac{v^2}{2}\right]. \tag{1}$$

Here v is the velocity of the particle in phase space normalized by the average thermal speed of electrons, $\alpha_e(\geq 0)$ is a parameter that determines the proportion of the fast energetic electrons, and n_{e0} is the equilibrium number density of electrons. This model of the velocity distribution function of electrons was considered for the first time by Cairns *et al.*[23] to investigate how the presence of fast energetic electrons changes the properties of ion acoustic waves for both positive and negative density perturbations. In fact, the observations of solitary structures with density depletion made by the Freja Satellite[30] influenced Cairns *et al.*[23] to model this velocity distribution function of electrons which has the property that the number of particles in the neighbourhood of the point $v=0$ is much smaller than the number of particles in the neighbourhood of the point $v=0$ for the case of a Maxwellian distribution. This distribution is often known as Cairns distribution or nonthermal distribution.

In the present paper, we have extended our works[19,20] by taking the following as the velocity distribution function of nonthermal electrons, which is a vortex-like distribution, instead of taking (1) as their distribution function. This distribution takes care of both free and trapped electrons. If we take Cairns distributed nonthermal electrons[23] as given in Eq. (1) in phase space as initial (unperturbed) distribution of electrons, then following Schamel,[31–33] the number density of the non-isothermal electrons can be formulated by the following sequence of equations:

$$n_e(\phi) = \int_{-\infty}^{\infty} f_e(v,\phi)dv, \tag{2}$$

$$f_e(v,\phi) = \begin{cases} f_{e0}(\xi) & \epsilon_e > 0, \\ f_{et}(\xi) & \epsilon_e \leq 0. \end{cases} \tag{3}$$

Here $f_{e0}(\xi)$ and $f_{et}(\xi)$ are analytic functions of the constant of motion $\xi = \pm\sqrt{2|\epsilon_e|}$ (being valid for the ion acoustic regime) and $\epsilon_e = \frac{v^2}{2} - \phi$, where ϕ is the electrostatic potential normalized by $\frac{K_B T_{ef}}{e}$ with K_B being the Boltzmann constant, T_{ef} is the average temperature of free electrons, and $-e$ is the charge of an electron. The free, unperturbed distribution $f_{e0}(v)$ can be taken as any arbitrary, normalizable distribution function, such as a Cairns distribution. In case of Cairns distribution, $f_{e0}(\xi)$ and $f_{et}(\xi)$ are, respectively, given by the following equations:

$$f_{e0}(\xi) = n_{e0}\frac{1+\alpha_e \xi^4}{\sqrt{2\pi}(1+3\alpha_e)}\exp\left[-\frac{\xi^2}{2}\right], \tag{4}$$

$$f_{et}(\xi) = n_{e0}\frac{1+\alpha_e \beta^2 \xi^4}{\sqrt{2\pi}(1+3\alpha_e)}\exp\left[\frac{\beta\xi^2}{2}\right], \tag{5}$$

where $\alpha_e \geq 0$, $\beta^2 = \left(\frac{T_{ef}}{T_{et}}\right)^2$, and T_{et} is the average temperature of trapped electrons.

Integrating over v and making appropriate Taylor series expansions of $f_{et}(\xi)$ and $f_{e0}(\xi)$ about $\xi=0$, the expression of n_e can be written as

$$\begin{aligned}\frac{n_e}{n_{e0}} &= 1 + (1-\beta_e)\phi + \sum_{r=2}^{\infty}\left(1 - 2\beta_e r + \beta_e r^2\right)\frac{\phi^r}{r!} \\ &\quad + \frac{1}{n_{e0}}\sum_{r=2}^{\infty}\left[\frac{2}{r!}\left\{f_{et}^{(r)}(0) - (-1)^{r/2} f_{e0}^{(r)}(0)\right\}\right. \\ &\quad \left.\times\left\{\int_0^{\sqrt{2\phi}}\left(2\phi - v^2\right)^{r/2} dv\right\}\right],\end{aligned} \tag{6}$$

where

$$\beta_e = \frac{4\alpha_e}{1+3\alpha_e} \tag{7}$$

and we have used Eqs. (2)–(4). The following notations have been used to simplify Eq. (6): $\psi^{(r)}(\xi) = \frac{d^r\psi}{d\xi^r}$ with $\psi^{(0)}(\xi) = \psi(\xi)$ and $\psi^{(r)}(0) = \psi^{(r)}(\xi)|_{\xi=0}$.

To derive Eq. (6), we have assumed the following conditions:[33,34]

$$f_{et}(0) = f_{e0}(0) \quad \text{and} \quad f'_{et}(0) = 0. \tag{8}$$

The first condition of Eq. (8) describes the continuity of the electron distribution function at the boundary of trapped electrons, whereas the second condition of Eq. (8) corresponds to the regularity of the distribution function of the trapped electrons at $v = \pm\sqrt{2\phi}$. These two conditions have been extensively discussed by Schamel[33] [Eqs. (17a) and (17b) of Schamel[33]]. Now from the analytic expressions of $f_{e0}(\xi)$ and $f_{et}(\xi)$ as given in Eqs. (4) and (5), respectively, one can easily check that the both conditions of Eq. (8) hold simultaneously. Again, from the analytic expressions of $f_{e0}(\xi)$ and $f_{et}(\xi)$, it is simple to verify that $f_{e0}^{(2r+1)}(0) = 0$ and $f_{et}^{(2r+1)}(0) = 0$ for all $r = 0, 1, 2, \ldots$. Therefore, Eq. (6) assumes the following form:

$$\begin{aligned}\frac{n_e}{n_{e0}} &= 1 + (1-\beta_e)\phi + \sum_{r=2}^{\infty}\left(1 - 2\beta_e r + \beta_e r^2\right)\frac{\phi^r}{r!} \\ &\quad + \frac{1}{n_{e0}}\sum_{r=1}^{\infty}\left[\frac{2}{(2r)!}\left\{f_{et}^{(2r)}(0) - (-1)^{r} f_{e0}^{(2r)}(0)\right\}\right. \\ &\quad \left.\times\left\{\int_0^{\sqrt{2\phi}}\left(2\phi - v^2\right)^{r} dv\right\}\right].\end{aligned} \tag{9}$$

It is simple to check that the series $\sum_{r=2}^{\infty}(1 - 2\beta_e r + \beta_e r^2)\frac{\phi^r}{r!}$ is a convergent series for any real value of ϕ, whereas we have assumed that the series $\sum_{r=1}^{\infty}\left[\frac{2}{(2r)!}\left\{f_{et}^{(2r)}(0) -(-1)^r f_{e0}^{(2r)}(0)\right\}\int_0^{\sqrt{2\phi}}(2\phi - v^2)^r dv\right]$ is convergent for small positive values of ϕ. Therefore, for a small amplitude limit, keeping the terms up to $\phi^{\frac{5}{2}}$, from Eq. (9), we get

$$\frac{n_e}{n_{e0}} = 1 + (1-\beta_e)\phi + \frac{2^{5/2}}{3n_{e0}}\left[f''_{et}(0) + f''_{e0}(0)\right]\phi^{3/2} + \frac{1}{2}\phi^2 + \frac{2^{7/2}}{45 n_{e0}}\left[f_{et}^{(4)}(0) - f_{e0}^{(4)}(0)\right]\phi^{5/2}. \tag{10}$$

For $\beta_e = 0 (\Longleftrightarrow \alpha_e = 0)$, the form of this equation is similar to Eq. (15) of Schamel.[33] Now, using Eqs. (4) and (5), from Eq. (10), we get

$$\frac{n_e}{n_{e0}} = 1 + (1-\beta_e)\phi - \frac{(1-\beta)(4-3\beta_e)}{3\sqrt{\pi}}\phi^{\frac{3}{2}} + \frac{1}{2}\phi^2 - \frac{2(1-\beta^2)(4+5\beta_e)}{15\sqrt{\pi}}\phi^{\frac{5}{2}}. \tag{11}$$

For $\beta_e = 0$, i.e., for the usual Maxwellian distribution, Eq. (11) assumes the following form:

$$\frac{n_e}{n_{e0}} = 1 + \phi - \frac{4}{3\sqrt{\pi}}(1-\beta)\phi^{\frac{3}{2}} + \frac{1}{2}\phi^2 - \frac{8}{15\sqrt{\pi}}(1-\beta^2)\phi^{\frac{5}{2}}, \tag{12}$$

which is the Schamel's standard expression and this expression has been used in several papers.[32,34–36]

For $\beta = 1$, from the analytic expressions of $f_{e0}(\xi)$ and $f_{et}(\xi)$, it is simple to check that $f_{et}^{(2r)}(0) = (-1)^r f_{e0}^{(2r)}(0)$ for all $r = 0, 1, 2, \ldots$ and for this case, Eq. (9) can be put in the following form:

$$\frac{n_e}{n_{e0}} = \left(1 - \beta_e\phi + \beta_e\phi^2\right)e^{\phi}. \tag{13}$$

This is the usual number density of the nonthermal electrons as prescribed by Cairns *et al.*[23] So, for the small amplitude limit, Eq. (11) can be taken as the density of the nonthermal electrons having vortex-like velocity distribution.

Particle trapping occurs not only in space plasmas[35,37] but also in laboratory plasmas.[38,39] Several authors[40–57] investigated small or arbitrary amplitude IA/DIA waves in different plasma systems including trapped electrons. Schamel *et al.*[58] reported the existence of a slower dust acoustic wave due to dust trapping in the potential trough. Another interesting problem of trapping related issues on nonlinear shielding investigation was considered by Das and Schamel.[59] Moslem *et al.*[60] studied the combined effects of trapped electrons, transverse perturbation, ion streaming velocity, and dust charge fluctuations on the propagation of DIA solitons in dusty plasmas. Moslem and El-Taibany[61] considered the propagation of nonlinear DIA solitons in a dusty plasma consisting of warm positive ions, negatively charged dust particles with charge fluctuations, and two temperature trapped electrons. Recently, Hafez *et al.*[62] investigated the three dimensional nonlinear dynamics of ion acoustic waves in a collisionless, unmagnetized plasmas consisting of trapped electrons and isothermal positrons. They derived a modified KP (mKP) equation which is exactly the same as the Schamel's modified Kadomtsev Petviashvili (SKP) equation. They transformed this mKP equation into an energy integral and discussed IA solitary waves with respect to different plasma parameters with the help of the Sagdeev pseudopotential method.

The present paper is a further extension of the recently published paper of Sardar *et al.*[19] in the following directions:

(i) Following Das *et al.*,[47] two different concepts of non-isothermality of electrons have been combined to investigate the existence and stability of DIA solitary waves. The first one was introduced by Cairns *et al.*[23] and introduces non-isothermality by the background electron distribution, whereas the second one was introduced by Schamel[31,32,35] and uses the incomplete trapping of electrons in the potential trough of the perturbation as a possible deviation from isothermality.

(ii) Giving appropriate stretching of the coordinates and time, and appropriate perturbation expansions of the dependent variables, a three-dimensional Schamel's modified Kadomtsev Petviashvili (SKP) equation is derived in a collisionless unmagnetized dusty plasma consisting of warm adiabatic ions, static negatively charged dust grains, nonthermal electrons having vortex-like velocity distribution, and isothermal positrons when the weak dependence of the spatial coordinates perpendicular to the direction of propagation of the wave is taken into account. The solitary wave solution of this SKP equation and its lowest order stability can be discussed with the help of Eqs. (31) and (47) of Sardar *et al.*[19] for $r = \frac{1}{2}$.

(iii) It is found that the coefficient of the nonlinear term of the SKP equation vanishes when the temperature of the free electrons is the same as that of the trapped electrons. But if the temperature of the trapped electrons is the same as that of the free electrons, the nonthermal vortex-like distribution of electrons simply becomes Cairns distributed nonthermal electrons and the nonlinear dynamics of the same DIA wave is described by the same KP equation (16) of Sardar *et al.*[19]

(iv) If the temperature of the trapped electrons approaches the temperature of the free electrons, i.e., if $\beta \to 1$, the coefficient of the nonlinear term of the SKP equation is not equal to zero, but it is close to zero and it is found that the SKP equation fails to describe the nonlinear behaviour of the DIA wave. In this situation, i.e., when $B \approx O(\epsilon)$, the evolution equation is a combined SKP-KP equation, where ϵ is a small parameter. This equation admits solitary wave solutions having profile different from both sech^4 and sech^2. The condition for the existence of the alternative solitary wave solution of the combined SKP-KP equation has been investigated.

(v) The small-k perturbation expansion method of Rowlands and Infeld[24–28] has been used to analyse the lowest order stability of the alternative solitary wave solution of the combined SKP-KP equation. The problem of stability of solitary wave solutions of the combined SKP-KP equation has not been considered by any author in any plasma system.

Kadomtsev and Petviashvili[63] have made an attempt to investigate the stability of Korteweg-de Vries (KdV) solitons in a collisionless unmagnetized plasma. Kako and Rowlands[64] derived the Kadomtsev Petviashvili (KP) equation for ion acoustic waves in a collisionless unmagnetized plasma. Infeld *et al.*[65] used this KP equation to study the stability of ion acoustic KdV solitons and reported that ion acoustic solitary wave solutions are stable with respect to the transverse long-wavelength plane-wave perturbation.

In fact, KP and different modified KP equations have been used to investigate the transverse stabilities of Korteweg-de Vries (KdV) and different modified Korteweg-de Vries (mKdV) equations describing the nonlinear behaviour of dust ion acoustic waves in different regions of parameter space when the weak dependence of the spatial coordinates perpendicular to the direction of propagation of the wave is taken into account. This is the only reason why the non-uniformity of ion species is taken into account and not for the other species. There is no other physical explanation of taking the non-uniformity of only ion species and not for the other species.

It is important to mention here that basically the problem is one dimensional, i.e., we want to find the solitary wave solution of different modified KdV equations, such as SKdV equation, combined SKdV-KdV equation. In the present paper, we want to investigate the stability of the solitary wave solution of the combined SKdV-KdV equation when the weak dependence of the spatial coordinates perpendicular to the direction of propagation of the wave is taken into account, and for this reason, the solution of the modified KP equation (SKP equation or combined SKP-KP equation) has been taken along the x-axis only. Our aim is to consider the transverse stability of the solitary wave solution propagating along the x-axis only. So, we have considered very small perturbation along y and z directions.

This paper is organized as follows: the basic equations are given in Sec. II. The SKP and the KP equations are given in Sec. III. In Sec. IV, we have derived a combined SKP-KP equation. The alternative solitary wave solution of the combined SKP-KP equation has been investigated in Sec. V. The stability of the alternative solitary wave solution of the combined SKP-KP equation has been considered in Sec. VI. Finally, conclusions are given in Sec. VII.

II. BASIC EQUATIONS

We consider a collisionless unmagnetized unbounded dusty plasma consisting of warm adiabatic ions, static negatively charged dust grains, nonthermal electrons having vortex-like velocity distribution, and isothermal positrons. The nonlinear behaviour of DIA waves in this plasma may be described by the following set of fluid equations, which consist of the equation of continuity of ions, the equation of motion of ion fluid, the pressure equation for ion fluid, and the Poisson equation

$$\frac{\partial n_i}{\partial t} + \nabla.(n_i\mathbf{u}) = 0, \tag{14}$$

$$M_s^2\left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u}\right] + \frac{(1-p)\sigma_{ie}}{n_i}\nabla P + \nabla\phi = 0, \tag{15}$$

$$\frac{\partial P}{\partial t} + (\mathbf{u}.\nabla)P + \gamma P(\nabla.\mathbf{u}) = 0, \tag{16}$$

$$C\nabla^2\phi = n_e - n_i - n_p + \frac{Z_d n_{d0}}{N_0}, \tag{17}$$

where

$$C = \frac{1-p}{M_s^2 - \gamma\sigma_{ie}}, \tag{18}$$

and we have used the following notations:

$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

Here n_s ($s = i$, e, and p stand for ion, electron, and positron), $\mathbf{u} = (u, v, w)$, P, ϕ, (x, y, z), and t are, respectively, the number density of the s-th species, the ion fluid velocity, the ion fluid pressure, the electrostatic potential, the spatial variables and time, and these quantities have been normalized, respectively, by $N_0(= n_{i0} + n_{p0} = n_{e0} + Z_d n_{d0})$, C_D (linearized velocity of the DIA wave in the present plasma system for long wavelength plane wave perturbation), $n_{i0}K_BT_i$, $\frac{K_BT_{ef}}{e}$, λ_D (Debye length of the present plasma system) and λ_D/C_D, where n_{s0}, n_{d0}, T_i, T_{ef}, and Z_d are, respectively, the unperturbed number density of the s-th species, the constant dust number density, the average unperturbed temperature of ions, the average temperature of free electrons, and the number of electrons residing on the dust grain surface, and $\gamma(=3)$ is the adiabatic index.

The expression of M_s and the four basic parameters p, μ, σ_{ie}, and, σ_{pe} is given by

$$M_s = \sqrt{\gamma\sigma_{ie} + \frac{(1-p)\sigma_{pe}}{p + \mu(1-\beta_e)\sigma_{pe}}}, \tag{19}$$

$$p = \frac{n_{p0}}{N_0}, \quad \mu = \frac{n_{e0}}{N_0}, \quad \sigma_{ie} = \frac{T_i}{T_{ef}}, \quad \sigma_{pe} = \frac{T_p}{T_{ef}}, \tag{20}$$

where T_p is the average temperature of isothermal positrons.

Based on the above-mentioned normalization of the independent and dependent variables, from Equation (11), the number density of electrons can be written as

$$n_e = \mu\left[1 + (1-\beta_e)\phi - \frac{(1-\beta)(4-3\beta_e)}{3\sqrt{\pi}}\phi^{\frac{3}{2}} + \frac{1}{2}\phi^2 - \frac{2(1-\beta^2)(4+5\beta_e)}{15\sqrt{\pi}}\phi^{\frac{5}{2}}\right]. \tag{21}$$

It can be easily checked that $0 \le \beta_e < 4/3$. Here, as β_e increases with α_e for $\alpha_e \ge 0$, β_e can also be taken as a nonthermal parameter that determines the proportion of the fast energetic electrons. However, we cannot take the whole interval of β_e. In fact, for increasing β_e, the velocity distribution function of nonthermal electrons develops wings, which become stronger as β_e increases and, at the same time, the center density in phase space drops; consequently, we should not take values of $\beta_e > 4/7$ as that stage might stretch the credibility of the Cairns model too far.[66] So, we take $0 \leq \beta_e \leq 4/7 = 0.6$ (approximately).

The normalized number density of isothermal positrons can be written as

$$n_p = pe^{-\frac{\phi}{\sigma_{pe}}}. \tag{22}$$

The above system of equations is supplemented by the unperturbed charge neutrality condition

$$\frac{n_{i0}}{N_0} = 1 - p \quad \text{and} \quad \frac{Z_d n_{d0}}{N_0} = 1 - \mu. \tag{23}$$

Expanding n_e and n_p as given by (21) and (22) up to $\phi^{\frac{5}{2}}$, the Poisson equation (17) can be written as

$$C\nabla^2\phi = 1 - p + \sum_{i=1}^{4} Q_i \phi^{\frac{i+1}{2}} - n_i, \tag{24}$$

where Q_1, Q_2, Q_3, and Q_4 are given in Appendix A.

We have used Eqs. (14)–(16) and (24) to derive the different evolution equations.

III. SKP AND KP EQUATIONS

Following Sardar *et al.*,[19] we have used the following stretchings of space coordinates and time to derive the different evolution equations:

$$\xi = \epsilon(x - Vt), \quad \eta = \epsilon^2 y, \quad \zeta = \epsilon^2 z, \quad \tau = \epsilon^3 t, \tag{25}$$

where ϵ is a small parameter measuring the weakness of dispersion and weakness of nonlinearity, and V is a constant being independent of space coordinates and time. These stretchings of space coordinate (x coordinate) and time are exactly similar to the stretchings of space coordinate and time of Schamel[35] if we define $\epsilon = \psi^{1/4}$.

A. SKP equation

To derive the SKP equation describing the nonlinear behaviour of DIA waves in the present plasma system, we have used the following perturbation expansions of the dependent variables along with the stretchings (25):

$$f = f^{(0)} + \sum_{j=1}^{\infty} \epsilon^{2(j+1)} f^{(j)}, \quad g = g^{(0)} + \sum_{j=1}^{\infty} \epsilon^{j+4} g^{(j)}, \tag{26}$$

where $f = n_i, P, \phi, u$ with $n_i^{(0)} = 1 - p, P^{(0)} = 1, \phi^{(0)} = 0, u^{(0)} = 0$, and $g = v, w$ with $v^{(0)} = w^{(0)} = 0$. Substituting the stretchings (25) and the perturbation expansions (26) in Eqs. (14)–(16) and (24), and equating the coefficient of different powers of ϵ on each side of every equation, one can get a sequence of equations. From these sequences of equations, we get the following SKP equation:

$$\frac{\partial}{\partial \xi}\left[\phi_\tau^{(1)} + AB\sqrt{\phi^{(1)}}\phi_\xi^{(1)} + \frac{1}{2}AC\phi_{\xi\xi\xi}^{(1)}\right] + \frac{1}{2}AD\left(\phi_{\eta\eta}^{(1)} + \phi_{\zeta\zeta}^{(1)}\right) = 0, \tag{27}$$

where the coefficients A, B, and D are given by

$$A = \frac{1}{1-p}\frac{\left(M_s^2V^2 - \gamma\sigma_{ie}\right)^2}{M_s^2 V}, \tag{28}$$

$$B = \mu\frac{(1-\beta)(4-3\beta_e)}{4\sqrt{\pi}}, \tag{29}$$

$$D = (1-p)\frac{M_s^2V^2}{\left(M_s^2V^2 - \gamma\sigma_{ie}\right)^2}, \tag{30}$$

and the constant V is determined by

$$V^2 = 1. \tag{31}$$

Equation (27) describes the propagation of long wavelength weakly nonlinear and weakly dispersive DIA waves if the coefficient AB of the nonlinear term of (27) is not equal to zero. If we ignore the weak dependence of the spatial coordinates perpendicular to the direction of propagation of the wave, then this SKP equation is nothing but an SKdV equation. In fact, the weak dependence of the spatial coordinates perpendicular to the direction of propagation of the wave is taken into account to investigate the stability of the solitary wave solution of the SKdV equation. For the first time, the solitary wave solution of the SKdV equation was presented by Schamel.[32] When $AB \neq 0$, the solitary wave solution of the SKdV equation corresponding to the SKP equation (27) and its lowest order stability can be discussed with the help of Eqs. (31) and (47) of Sardar *et al.*[19] for $r = \frac{1}{2}$.

The SKP equation (27) cannot describe the nonlinear dynamics of DIA waves when the coefficient AB of the nonlinear term of (27) vanishes, i.e., $AB = 0$. As $A \neq 0$ for any set of physically admissible values of the parameters of the system, $AB = 0$ implies $B = 0$. Since $\mu > 0$ and $0 \leq \beta_e < \frac{4}{3}$ give $4 - 3\beta_e > 0$, $B = 0 \Longleftrightarrow \beta = 1$. Therefore, $B = 0$ if and only if the temperature of the free electrons is the same as that of the trapped electrons. But for $\beta = 1$, the number density of the electrons having vortex-like distribution simply reduces to the usual number density of the nonthermal electrons as prescribed by Cairns *et al.*,[23] and Sardar *et al.*[19] have derived the following KP equation for the Cairns distributed nonthermal electrons.

B. KP equation

When $B = 0$, we have used the following perturbation expansions of the dependent variables:

$$f = f^{(0)} + \sum_{j=1}^{\infty} \epsilon^{2j} f^{(j)}, \quad g = g^{(0)} + \sum_{j=1}^{\infty} \epsilon^{2j+1} g^{(j)}, \tag{32}$$

where $f = n_i, P, \phi, u$ with $n_i^{(0)} = 1 - p, P^{(0)} = 1, \phi^{(0)} = 0, u^{(0)} = 0$, and $g = v, w$ with $v^{(0)} = w^{(0)} = 0$. Substituting the stretchings (25) and the perturbation expansions (32) in Eqs. (14)–(16) and (24), and then, equating the coefficient of different powers of ϵ on each side of every equation, one can get a sequence of equations. From these sequences of equations, Sardar *et al.*[19] have derived the following KP equation:

$$\frac{\partial}{\partial \xi}\left[\phi_{\tau}^{(1)} + AB_1\phi^{(1)}\phi_{\xi}^{(1)} + \frac{1}{2}AC\phi_{\xi\xi\xi}^{(1)}\right] + \frac{1}{2}AD\left(\phi_{\eta\eta}^{(1)} + \phi_{\zeta\zeta}^{(1)}\right) = 0, \tag{33}$$

where

$$B_1 = \frac{1}{2}\left[(1-p)\frac{3M_s^2V^2 + \gamma(\gamma-2)\sigma_{ie}}{\left(M_s^2V^2 - \gamma\sigma_{ie}\right)^3} - \left(\mu - \frac{p}{\sigma_{pe}^2}\right)\right], \tag{34}$$

A and D are given by Eqs. (28) and (30), respectively, and V is determined by (31).

This KP equation (33) describes the nonlinear behaviour of DIA waves only when $B=0$. When $B \neq 0$ but B is close to zero, then the SKP equation (27) cannot describe the nonlinear behaviour of DIA waves because the amplitude of the solitary wave solution defined by the SKP equation (27) assumes a very large numerical value when B is close to zero. In fact, in Sec. III A, we have mentioned that the SKP equation (27) reduces to the SKdV equation if we ignore the weak dependence of the spatial coordinates perpendicular to the direction of propagation of the wave and therefore, the solitary wave solution of the SKP equation (27) propagating along the ξ axis with a velocity U (=dimensionless velocity (normalized by C_D) of the solitary wave) can be written as

$$\phi^{(1)} = \phi_0(X) = \bar{a}\,\mathrm{sech}^4\left(\frac{X}{W}\right), \quad X = \xi - U\tau, \tag{35}$$

where

$$\bar{a} = \frac{225U^2}{64A^2B^2} \quad \text{and} \quad W^2 = \frac{8AC}{U}. \tag{36}$$

The solitary wave solution (35) was indeed first presented by Schamel[32] [Eq. (39) of Schamel[32]]. This solution clearly shows that the SKP equation (27) fails to describe the nonlinear behaviour of DIA waves when $B \neq 0$ but B is close to zero. Again, as $4-3\beta_e > 0$ for the entire physically admissible values of β_e, from the expression of B as given in Eq. (29), one can easily find that $B \to 0 \Longleftrightarrow \beta \to 1$. Consequently, neither SKP nor KP equation can describe the nonlinear behaviour of DIA waves when temperature of the free electrons is close to that of the trapped electrons. So, a further modification of the SKP equation (27) is necessary. In Sec. IV, we have derived a combined SKP-KP equation to describe the nonlinear behaviour of DIA waves when B is close to zero, i.e., $B \approx O(\epsilon)$.

IV. COMBINED SKP-KP EQUATION

To derive the combined SKP-KP equation, we have used the same stretchings (25) and the same perturbation expansions of the dependent variables as given by (32). But instead of considering $B=0$, we assume that $B \approx O(\epsilon)$ (Nejoh[67]). Substituting the stretchings (25) and perturbation expansions (32) in Eqs. (14)–(16) and (24) and finally, equating the coefficient of different powers of ϵ on each side of every equation, we get a sequence of equations. At the lowest order ($O(\epsilon)=3$), from the equation of continuity of ions, x-component of the equation of motion of ion fluid, and the pressure equation of ion fluid, we get the following equations:

$$\left(n_i^{(1)}, u^{(1)}, P^{(1)}\right) = \frac{\phi^{(1)}}{M_s^2V^2 - \gamma\sigma_{ie}}(1-p, V, \gamma). \tag{37}$$

From the expression of $n_i^{(1)}$ as given in the first equation of (37) and the Poisson equation (24) at the order ϵ^2, we get the dispersion relation (31) and this dispersion relation determines the constant V.

At the next order ($O(\epsilon)=4$), the perpendicular components of equation of motion for ions give the following expression for $\frac{\partial}{\partial\xi}\left(v_\eta^{(1)} + w_\zeta^{(1)}\right)$:

$$\frac{\partial}{\partial \xi}\left(v_\eta^{(1)} + w_\zeta^{(1)}\right) = \frac{V}{M_s^2V^2 - \gamma\sigma_{ie}}\left(\phi_{\eta\eta}^{(1)} + \phi_{\zeta\zeta}^{(1)}\right). \tag{38}$$

It is simple to check that the Poisson equation (24) at the order ϵ^3 is identically satisfied since the only nonvanishing term, $-\frac{4}{3}B(\phi^{(1)})^{\frac{3}{2}}$, is of the order ϵ^4 as $B \approx O(\epsilon)$ and therefore, this term has to be included in the next higher order Poisson equation.

At the order ϵ^5, differentiating the continuity equation of ions, the x-component of equation of motion for ions, and the pressure equation for ion fluid with respect to ξ, we get three equations for the unknowns $n_{i\xi\xi}^{(2)}\left(=\frac{\partial^2 n_i^{(2)}}{\partial\xi^2}\right)$, $u_{\xi\xi}^{(2)}\left(=\frac{\partial^2 u^{(2)}}{\partial\xi^2}\right)$, and $P_{\xi\xi}^{(2)}\left(=\frac{\partial^2 P^{(2)}}{\partial\xi^2}\right)$. Solving the resulting equations for the unknowns $n_{i\xi\xi}^{(2)}$, $u_{\xi\xi}^{(2)}$, and $P_{\xi\xi}^{(2)}$, we can express $n_{i\xi\xi}^{(2)}$ (as well as $u_{\xi\xi}^{(2)}$ and $P_{\xi\xi}^{(2)}$) as a function of $\phi^{(2)}$, $\phi^{(1)}$ and their different derivatives. The final expression of $n_{i\xi\xi}^{(2)}$ can be written as

$$n_{i\xi\xi}^{(2)} = \frac{1-p}{M_s^2V^2 - \gamma\sigma_{ie}}\phi_{\xi\xi}^{(2)} + \frac{2(1-p)M_s^2V}{\left(M_s^2V^2 - \gamma\sigma_{ie}\right)^2}\phi_{\xi\tau}^{(1)} + (1-p)\frac{3M_s^2V^2 + \gamma(\gamma-2)\sigma_{ie}}{\left(M_s^2V^2 - \gamma\sigma_{ie}\right)^3}\left(\phi^{(1)}\phi_{\xi}^{(1)}\right)_{\xi} + \frac{(1-p)M_s^2V^2}{\left(M_s^2V^2 - \gamma\sigma_{ie}\right)^2}\left(\phi_{\eta\eta}^{(1)} + \phi_{\zeta\zeta}^{(1)}\right), \tag{39}$$

where we have used Eqs. (37) and (38) to simplify Eq. (39). At the order ϵ^4, the Poisson equation including the term, $-\frac{4}{3}B(\phi^{(1)})^{\frac{3}{2}}$, can be written as

$$C\phi_{\xi\xi}^{(1)} = Q_1\phi^{(2)} - n_i^{(2)} + Q_2\left(\phi^{(1)}\right)^2 - \frac{4}{3}B\left(\phi^{(1)}\right)^{\frac{3}{2}}. \tag{40}$$

It is important to note that the term, $-\frac{4}{3}B(\phi^{(1)})^{\frac{3}{2}}$, which was omitted from its previous order equation, is a term of order ϵ^4, and therefore, this term has been included in the Poisson equation at the order ϵ^4. Now, differentiating Eq. (40) with respect to ξ twice, we get

$$C\phi^{(1)}_{\xi\xi\xi\xi} = Q_1\phi^{(2)}_{\xi\xi} - n^{(2)}_{i\xi\xi} + \frac{\partial^2}{\partial\xi^2}\left[Q_2\left(\phi^{(1)}\right)^2 - \frac{4}{3}B\left(\phi^{(1)}\right)^{\frac{3}{2}}\right]. \tag{41}$$

Eliminating $n^{(2)}_{i\xi\xi}$ from Eqs. (39) and (41), we get the following combined SKP-KP equation:

$$\frac{\partial}{\partial\xi}\left[\phi^{(1)}_\tau + AB\sqrt{\phi^{(1)}}\phi^{(1)}_\xi + AB_1\phi^{(1)}\phi^{(1)}_\xi + \frac{1}{2}AC\phi^{(1)}_{\xi\xi\xi}\right] + \frac{1}{2}AD\left(\phi^{(1)}_{\eta\eta} + \phi^{(1)}_{\zeta\zeta}\right) = 0. \tag{42}$$

Here A, B, D, V, and B_1 are the same as those given, respectively, by Eqs. (28)–(31) and (34).

V. SOLITARY WAVE SOLUTION OF THE COMBINED SKP-KP EQUATION

For a solitary wave solution of the combined SKP-KP equation (42), we take following transformation of the independent variables:

$$X = \xi - U\tau, \quad \eta' = \eta, \quad \zeta' = \zeta, \quad \tau' = \tau. \tag{43}$$

Here, U is the dimensionless velocity (normalized by C_D) of the travelling wave moving along ξ-axis, i.e., U is the dimensionless velocity of the wave frame.

Based on the above changes of the independent variables, the combined SKP-KP equation (42) assumes the following form (in which we drop the primes on the independent variables η, ζ, and τ to simplify the notations):

$$\frac{\partial}{\partial X}\left[-U\phi^{(1)}_X + \phi^{(1)}_\tau + AB\sqrt{\phi^{(1)}}\phi^{(1)}_X + AB_1\phi^{(1)}\phi^{(1)}_X + \frac{1}{2}AC\phi^{(1)}_{XXX}\right] + \frac{1}{2}AD\left(\phi^{(1)}_{\eta\eta} + \phi^{(1)}_{\zeta\zeta}\right) = 0. \tag{44}$$

Now, for the travelling wave solitons of (44), we set

$$\phi^{(1)} = \phi_0(X). \tag{45}$$

Substituting (45) in (44), we get

$$\frac{d^2}{dX^2}\left[-U\phi_0 + \frac{2}{3}AB(\phi_0)^{\frac{3}{2}} + \frac{1}{2}AB_1(\phi_0)^2 + \frac{1}{2}AC\frac{d^2\phi_0}{dX^2}\right] = 0. \tag{46}$$

To get the solitary wave solution of Eq. (46), we use the following boundary conditions:[68]

$$\frac{d^n\phi_0}{dX^n} \to 0 \text{ as } |X| \to \infty \text{ for all } n = 1, 2, 3, \ldots \tag{47}$$

together with the condition that the electrostatic potential ϕ_0 vanishes at infinity, i.e.,

$$\lim_{|X|\to\infty} \phi_0(X) = 0. \tag{48}$$

Using the boundary conditions (47) and (48), we can write Eq. (46) as

$$-U\phi_0 + \frac{2}{3}AB(\phi_0)^{\frac{3}{2}} + \frac{1}{2}AB_1(\phi_0)^2 + \frac{1}{2}AC\frac{d^2\phi_0}{dX^2} = 0. \tag{49}$$

According to Das *et al.*,[47] we take

$$\phi_0 = \left\{a_0\frac{\text{sech}^2\frac{X}{W_1}}{b_0 + c_0\text{sech}^2\frac{X}{W_1}}\right\}^2 = \left\{a_0\frac{\text{sech}^2 p_1X}{b_0 + c_0\text{sech}^2 p_1X}\right\}^2,$$
$$p_1 = \frac{1}{W_1} \tag{50}$$

as a solution of Eq. (49). In fact, Eq. (50) can be put in the following form:

$$\phi_0 = \psi_1\frac{\text{sech}^4 Y_1}{\left(1 + \frac{\tanh^2 Y_1}{1+Q}\right)^2}, \tag{51}$$

where

$$\psi_1 = \left(\frac{a_0}{b_0 + c_0}\right)^2, \quad Q = -2 - \frac{b_0}{c_0}, \quad Y_1 = \frac{X}{W_1}. \tag{52}$$

For the first time, Eq. (51) was given by Schamel[32] [Eq. (48) of Schamel[32]], and consequently, Eq. (50) is equivalent to Eq. (48) of Schamel.[32]

Substituting (50) into (49) and following the same method as given in Das *et al.*,[47] the alternative solitary wave solution (50) of (49) can be put in the form

$$\phi_0 = a_1\frac{S^2}{\Psi_1^2}, \tag{53}$$

where

$$S = \text{sech}[2p_1X], \quad \Psi_1 = BS + \lambda\sqrt{L}, \tag{54}$$

$$a_1 = 900C^2p_1^4, \quad L = B^2 + \frac{75}{2}B_1Cp_1^2, \tag{55}$$

and $\lambda = \pm 1$. The solution (53) exists if and only if

$$L = B^2 + \frac{75}{2}B_1Cp_1^2 > 0, \tag{56}$$

and if this condition holds good, then U is given by

$$U = 8ACp_1^2. \tag{57}$$

As $B \approx O(\epsilon)$, the numerical value of B is close to zero, and consequently, one can take a small numerical value of B. From the expression of L as given in the second equation of (55), we see that the conditions $B \approx O(\epsilon)$, $C > 0$, and $B_1 > 0$ always give $L > 0$. Therefore, from Fig. 2(a) of Sardar *et al.*,[19] we find that $L > 0$ for the entire physically admissible intervals of β_e and μ whenever $p > 0.08$ and from Figs. 2(a) and 2(b) of Sardar *et al.*,[19] we find that there exists a domain of μ (may depend on the nonthermal parameter β_e) such that $L > 0$ for $0 \le p \le 0.08$. For definiteness, let us take $B = 0.0001$. In Fig. 1, L is plotted against μ for different values of β_e with $\gamma = 3$, $\sigma_{ie} = \sigma_{pe} = 0.9$ when $B = 0.0001$. The values of p for the Figs. 1(a)–1(c) are 0.068, 0.069, and 0.2, respectively. In each figure of Fig. 1, we see that L decreases with increasing values of β_e and for $\beta_e = 0$, L is always positive. Again, from Fig. 1(a), we see that there exists a small interval of μ in which L is negative for $p = 0.068$ only when $\beta_e = 0.6$ whereas for $p = 0.069$, L is positive even when $\beta_e = 0.6$. Therefore, we can conclude that there exists a critical value p_{cr} of p such that L is positive for the entire physically admissible intervals of β_e and μ whenever $p > p_{cr}$. But, in the neighbourhood of $p = p_{cr}$, L may be close to zero and in this case the denominator $\Psi_1 = BS + \lambda\sqrt{L}$ of the alternative solitary wave solution (53) is also close to zero, and consequently, this solution cannot describe the nonlinear behaviour of DIA waves because the amplitude of the alternative solitary wave solution defined by the combined SKP-KP equation assumes a very large numerical value. So, if $L = 0$ or $L \approx O(\epsilon)$, then this alternative solitary wave solution (53) fails to describe the nonlinear behaviour of DIA waves. Therefore, if $L > 0$ and L is of moderate numerical value, i.e., $L > 0$ and L is not very close to zero, then only one can use alternative solitary wave solution (53) to describe the nonlinear behaviour of DIA waves when $B \approx O(\epsilon)$. Further investigation is necessary when $L = 0$ or $L \approx O(\epsilon)$. Therefore, there exists a region in parameter space such that $L > 0$. Again, from the Figs. 1(a)–1(c), we see that L increases with the increasing values of p for any fixed values of μ and β_e

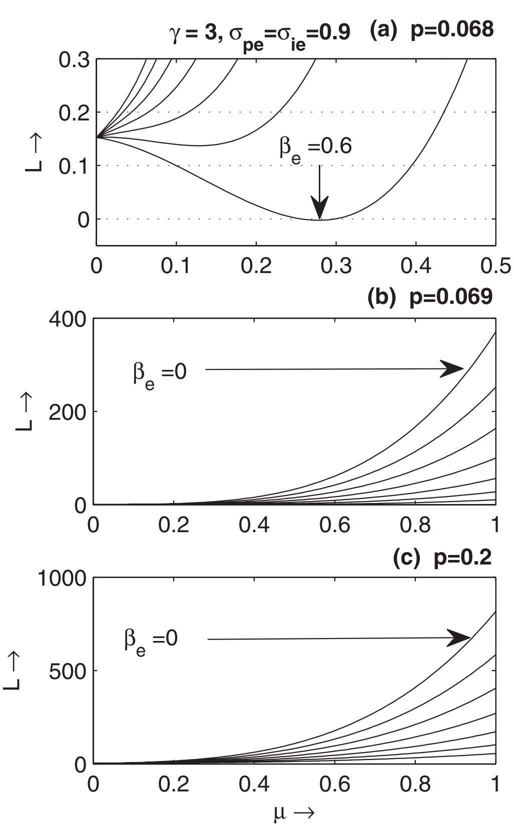

FIG. 1. L is plotted against μ for different values of β_e with $B = 0.0001$, $\gamma = 3$, $\sigma_{ie} = \sigma_{pe} = 0.9$ and (a) $p = 0.068$, (b) $p = 0.069$, and (c) $p = 0.2$.

and for a moderate value of p and μ, $L > 0$, and L is of moderate magnitude. Therefore, there exists a region in the parameter space where $L > 0$ and L is of moderate magnitude when B is close to zero. In this situation, we can use the alternative solitary wave solution (53) to describe the nonlinear behaviour of DIA waves.

As $B \approx O(\epsilon)$, the numerical value of B is close to zero, and consequently, for the limiting case where $B \to 0$, from the solution (53) of the combined SKP-KP equation, we get the following equation:

$$\lim_{B\to 0} \phi_0 = \frac{3U}{AB_1}\left(\operatorname{sech}\frac{2X}{W_1}\right)^2 \text{ with } W_1^2 = \frac{8AC}{U}. \tag{58}$$

Taking $W_1 = 2W$, the above equation can be written as

$$\lim_{B\to 0} \phi_0 = \frac{3U}{AB_1}\left(\operatorname{sech}\frac{X}{W}\right)^2 \text{ with } W^2 = \frac{2AC}{U}. \tag{59}$$

Again, it is simple to check that Eqs. (31) and (32) of Sardar *et al.*[19] can be put in the form of the above equation for $r = 1$, and consequently, the alternative solitary wave solution (53) simply reduces to the solitary wave solution of the KP equation (33) when $B \to 0$. This is expected because we use the same perturbation expansions of the dependent variables and the same stretching of space coordinates and time to derive both KP and combined SKP-KP equations. The only exception is that we use the critical condition $B = 0$ to derive the KP equation, whereas we use the condition $B \approx O(\epsilon)$ to derive the combined SKP-KP equation. On the other hand, if we take the limit $B \to 0$ on both sides of the combined SKP-KP equation (42), this equation simply reduces to the KP equation (33), and consequently, it is expected that the solitary wave solution of the combined SKP-KP equation (42) will converge to the solitary wave solution of the KP equation (33) when $B \to 0$. Therefore, we can conclude that under certain condition [inequality (56)], the solitary wave solution (53) of the combined SKP-KP equation (42) fills the gap between the solitary wave solution (sech^4—profile) of the SKP equation and the solitary wave solution (sech^2—profile) of the KP equation. In Sec. VI, we shall consider the stability of the alternative solitary wave solution (53) of the combined SKP-KP equation (42). To simplify the calculations, we have used the following notations:

$$a = \frac{a_1}{B^2}, \quad M = \frac{L}{B^2}, \quad \Psi = \frac{\Psi_1}{B}. \tag{60}$$

Based on the above-mentioned notations, the expression of ϕ_0 as given by the equation (53) can be written as

$$\phi_0 = a\frac{S^2}{\Psi^2}. \tag{61}$$

VI. STABILITY OF ALTERNATIVE SOLITARY WAVE

To analyze the stability of the alternative solitary wave solution (53) or equivalently (61) of Eq. (44) by the small-k perturbation expansion method of Rowlands and Infeld,[24–28] we decompose $\phi^{(1)}$ as

$$\phi^{(1)} = \phi_0(X) + q(X,\eta,\zeta,\tau), \tag{62}$$

where $\phi_0(X)$ is the steady state alternative solitary wave solution (53) of Eq. (44) and $q(X, \eta, \zeta, \tau)$ is the perturbed part of $\phi^{(1)}$. Now, for long-wavelength plane-wave perturbation along a direction having direction cosines l, m, n, we set

$$q(X,\eta,\zeta,\tau) = \bar{q}(X)e^{i\{k(lX+m\eta+n\zeta)-\omega\tau\}}, \tag{63}$$

where k is small and $l^2+m^2+n^2=1$.

According to the small-k perturbation expansion method of Rowlands and Infeld,[24–28] $\bar{q}(X)$ and ω can be expanded as follows:

$$\bar{q}(X) = \sum_{j=0}^{\infty} k^j q^{(j)}(X), \quad \omega = \sum_{j=0}^{\infty} k^j \omega^{(j)} \tag{64}$$

with $\omega^{(0)}=0$. Substituting (62) into (44) and then linearizing it with respect to q, we get a linear equation for q. Substituting (63) into this linear equation of q, we get an equation of $\bar{q}$. Finally, substituting (64) into the equation of $\bar{q}$ and then equating the coefficient of different powers of k on the both sides of the resulting equation, we get the following sequence of equations:

$$\frac{d}{dX}\left(M_1 q^{(j)}\right) = Q^{(j)}, \quad j=0,1,2,\ldots, \tag{65}$$

where

$$M_1 = -U + AB\phi_0^{\frac{1}{2}} + AB_1\phi_0 + \frac{1}{2}AC\frac{d^2}{dX^2}, \tag{66}$$

and

$$Q^{(j)} = \int_{-\infty}^{X} R^{(j)} dX, \tag{67}$$

and $R^{(j)}$ for $j=0, 1, 2$ are given in Appendix B.

Assuming that $q^{(j)}$ and its derivative up to third order vanish as $|X| \to \infty$, the general solution of (65) can be written as

$$q^{(j)} = A_1^{(j)} f + A_2^{(j)} g + A_3^{(j)} h + \chi^{(j)}, \tag{68}$$

where $A_1^{(j)}$, $A_2^{(j)}$, and $A_3^{(j)}$ are the integration constants and f, g, h, $\chi^{(j)}$ are given by

$$f = \frac{d\phi_0}{dX}, \quad g = f\int \frac{1}{f^2}dX, \quad h = f\int \frac{\phi_0}{f^2}dX, \tag{69}$$

$$\chi^{(j)} = \frac{2}{AC} f \int \frac{\int \left(f \int Q^{(j)} dX \right) dX}{f^2} dX. \tag{70}$$

From the expressions for f, g, and h as given by (69) and using MATHEMATICA,[69] we get

$$\lim_{|X|\to\infty}(f,g,h) = \left(0, \frac{-1}{sign[a]}\times\infty, -\frac{1}{16p_1^2}\right), \tag{71}$$

where

$$sign[a] = \begin{cases} 1 & \text{for } a > 0, \\ -1 & \text{for } a < 0. \end{cases} \tag{72}$$

Therefore, to make $q^{(j)}$ bounded, we must have

$$A_2^{(j)} = 0 \quad \text{for } j = 0, 1, 2, \ldots. \tag{73}$$

Consequently, Eq. (68) assumes the following form:

$$q^{(j)} = A_1^{(j)} f + A_3^{(j)} h + \chi^{(j)}. \tag{74}$$

As $R^{(0)}=0$, the solution (74) for $j=0$ can be written as

$$q^{(0)} = A_1^{(0)} f + A_3^{(0)} h. \tag{75}$$

To make $q^{(0)}$ consistent with the boundary condition, i.e., $q^{(0)} \to 0$ as $|X| \to \infty$, we must have $A_3^{(0)} = 0$. Therefore, Eq. (75) assumes the following form:

$$q^{(0)} = A_1^{(0)} f. \tag{76}$$

Using (76) and MATHEMATICA,[69] the bounded and consistent solution (74) for $j=1$ can be written as

$$q^{(1)} = A_1^{(1)} f + iA_1^{(0)}\left[s_1 X f + s_2\frac{S^2}{\Psi^3} + s_3\frac{S^3}{\Psi^3}\right], \tag{77}$$

where

$$s_1 = \frac{1}{2U}\{u_1 - 4lU\},$$
$$s_2 = \frac{a\lambda}{\sqrt{M}U}\left\{(M+1)u_1 - \frac{16}{15}lAB\sqrt{a} - \frac{2}{3}lAB_1 a\right\},$$
$$s_3 = \frac{2a}{U}\left\{u_1 - \frac{8}{15}lAB\sqrt{a}\right\}, \text{ with } u_1 = \omega^{(1)} + 2lU.$$

Now, for the solution of Eq. (65) to exist, the right hand side of this Eq. (65) must be perpendicular to the kernel of the operator adjoint to the operator $\frac{d}{dX}M_1$. This kernel, which must tend to zero as $|X| \to \infty$, is ϕ_0. Thus, we get the following consistency condition for the existence of the solution of Eq. (65):

$$\int_{-\infty}^{\infty} \phi_0 Q^{(j)} dX = 0. \tag{78}$$

It is simple to check that the consistency condition (78) is trivially satisfied for $j=0$ and for $j=1$. Using Eqs. (76) and (77), the consistency condition (78) for $j=2$ can be written as

$$Fu_1^2 + Gu_1 + H = 0, \tag{79}$$

where $u_1 = (\omega^{(1)} + 2lU)$ and F, G, H are given by the following equations:

$$F = \frac{1}{4lU}\left\{J_{4,4} - \frac{4(M+1)}{\lambda\sqrt{M}}J_{4,5} - 8J_{5,5}\right\}, \tag{80}$$

$$G = \left\{ -2J_{4,4} + \frac{2(M+1)}{\lambda\sqrt{M}} J_{4,5} + \frac{5}{2} J_{5,5} - MJ_{4,6} + \frac{15(M+1)}{2\lambda\sqrt{M}} J_{5,6} + 16J_{6,6} + \frac{4M(M+1)}{\lambda\sqrt{M}} J_{4,7} + (11M-7)J_{5,7} - \frac{(11M+5)}{\lambda\sqrt{M}} J_{6,7} - 6(M+1)J_{7,7} \right\}, \tag{81}$$

$$H = \left[lU \left\{ J_{4,4} + \frac{17}{2} J_{5,5} + 10MJ_{4,6} - \frac{3(6M+5)}{\lambda\sqrt{M}} J_{5,6} - \frac{1}{2}(15M+71)J_{6,6} - \frac{8M(M+1)}{\lambda\sqrt{M}} J_{4,7} - 2(11M-7)J_{5,7} + \frac{2(11M+5)}{\lambda\sqrt{M}} J_{6,7} + 12(M+1)J_{7,7} \right\} + \frac{1}{2} AD \frac{m^2+n^2}{l} J_{4,4} \right], \tag{82}$$

where we have used the following notation:

$$J_{m,n} = \int_{-\infty}^{\infty} \frac{\text{sech}^m Y}{\left(\text{sech} Y + \lambda\sqrt{M}\right)^n} dY. \tag{83}$$

Using Eqs. (80)–(83), Eq. (79) can be simplified as

$$(\omega^{(1)})^2 = \frac{1}{3} \frac{(11N+4)}{(1-N)(N+2)} UV(m^2+n^2)\chi, \tag{84}$$

where

$$N = \frac{1}{M} \quad \text{and} \quad \chi = \frac{1 - 6\pi_\lambda \left(\sqrt{\frac{N}{1-N}} \right) \frac{2N+3}{11N+4}}{1 - 6\pi_\lambda \left(\sqrt{\frac{N}{1-N}} \right) \frac{1}{N+2}}. \tag{85}$$

Here π_λ is given by

$$\pi_\lambda = \arctan\left(\frac{\lambda - \sqrt{N}}{\sqrt{1-N}} \right). \tag{86}$$

A. Stability analysis for $B \to 0$

Now as B is small enough, we can consider the limiting case where $B \to 0 \Longleftrightarrow M \to \infty \Longleftrightarrow N \to 0$. For this limiting case, the consistency condition (84) assumes the following form:

$$(\omega^{(1)})^2 = \frac{2}{3} UV(m^2+n^2). \tag{87}$$

This equation is exactly the same as Eq. (47) of Sardar *et al.*[19] for $r = 1$, and consequently, if $B \to 0 \Longleftrightarrow M \to \infty \Longleftrightarrow N \to 0$, the first order stability analysis of the solitary wave solution of the combined SKP-KP equation is exactly the same as that of the KP equation as presented in the paper of Sardar *et al.*[19] Therefore, the steady state solitary wave solution (53) of the combined SKP-KP equation (42) and its first order stability analysis are exactly the same as those of the solitary wave solution of the KP equation (33) if $B \to 0$.

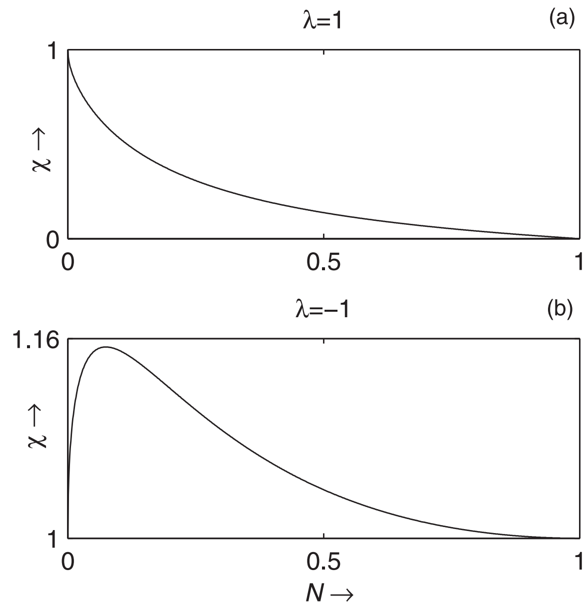

FIG. 2. χ is plotted against N in (a) $\lambda = 1$ and in (b) $\lambda = -1$.

B. Stability analysis for physically admissible values of the parameters of the system satisfying the conditions $B \approx O(\epsilon)$ and $L > 0$

From the definition of M as given by the second equation of (60) and from the definition of N as given by first equation of (85), it is simple to check that $0 \leq N < 1$. Again, for $0 \leq N < 1$, the right hand side of (84) is positive if and only if χ is positive. In Fig. 2(a), χ is plotted against N for $\lambda = 1$, whereas for $\lambda = -1$, χ is plotted against N in Fig. 2(b). These figures show that χ is positive for $\lambda = 1$ and also for $\lambda = -1$ for any N lying within the interval $0 \leq N < 1$. Therefore, Eq. (84) gives a real solution for $\omega^{(1)}$, and consequently, the solitary wave solution (53) of the combined SKP-KP equation (42) is always stable at the lowest order of k.

VII. CONCLUSIONS

A three-dimensional SKP equation describes the nonlinear behaviour of long wavelength weakly nonlinear and weakly dispersive DIA waves in a collisionless unmagnetized dusty plasma consisting of warm adiabatic ions and nonthermal electrons having a vortex-like velocity distribution and isothermal positrons when the weak dependence of the spatial coordinates perpendicular to the direction of propagation of the wave is taken into account. The analyses as given in Secs. IV and V of Sardar *et al.*[19] for $r = \frac{1}{2}$ clearly show that this SKP equation admits solitary wave solution having a profile sech^4 and this solitary wave solution is stable.

If the temperature of the trapped electrons is the same as that of the free electrons, the nonthermal vortex-like distribution of electrons simply becomes the Cairns distributed nonthermal electrons and the nonlinear dynamics of the same

DIA wave is described by the KP equation of Sardar *et al.*[19] According to Sardar *et al.*,[19] this equation admits solitary wave solution having a sech^2 profile, and the solitary wave solution of this equation is also stable.

If the temperature of the trapped electrons approaches the temperature of the free electrons, i.e., the coefficient of the nonlinear term of the SKP equation is not exactly equal to zero but it is close to zero, neither SKP equation nor KP equation can describe the nonlinear behaviour of DIA waves. In this case, a combined SKP-KP equation has been derived, which efficiently describes the nonlinear behaviour of the same DIA wave. This equation admits alternative solitary wave solution having a profile different from both sech^4 and sech^2. The condition for the existence of the alternative solitary wave solution of the combined SKP-KP equation has been derived.

The alternative solitary wave solution of the combined SKP-KP equation is exactly the same as the solitary wave solution (sech^2-profile) of the KP equation if the coefficient of the nonlinear term of the SKP equation tends to zero, i.e., when the contribution of trapped electrons tends to zero.

The stability analysis of the alternative solitary wave solution of the combined SKP-KP equation is exactly the same as that of the solitary wave solution of the KP equation when the coefficient of the nonlinear term of the SKP equation approaches to zero, i.e., when the temperature of the trapped electrons approaches the temperature of the free electrons.

The alternative solitary wave solution of the combined SKP-KP equation is always stable. In this connection, it is important to note that the solitary wave solutions of both SKP and KP equations are stable.

Sardar *et al.*[20] reported that the alternative solitary wave solution of the combined MKP-KP equation fills the gap between the sech^2-profile of KP equation and the sech-profile of the MKP equation, whereas from the present paper, we see that the alternative solitary wave solution of the combined SKP-KP equation fills the gap between the sech^4-profile of S-KP equation and the sech^2-profile of the KP equation. Consequently, combining the results of Sardar *et al.*[20] along with the results of the present paper, we have the following schematic diagram:

SKP (sech^4—profile) → combined SKP-KP (having a profile different from both sech^4 and sech^2) → KP (sech^2—profile) → combined MKP-KP (having a profile different from both sech^2 and sech) → MKP (sech—profile).

On the other hand, from the two papers of Das *et al.*,[47,70] we have seen the same schematic diagram for Korteweg-de Vries-Zakharov-Kuznetsov (KdV-ZK), modified KdV-ZK (MKdV-ZK), Schamel's modified KdV-ZK (S-ZK), combined MKdV-KdV-ZK, and combined S-KdV-ZK equations; i.e., here we have the following schematic diagram:

S-ZK (sech^4—profile) → combined S-KdV-ZK → KdV-ZK (sech^2—profile) → combined MKdV-KdV-ZK → MKdV-ZK (sech—profile).

For each ZK like equation as mentioned above, there exists a parameter region such that the solitary wave solution of the corresponding equation is not stable, whereas the solitary wave solutions of the SKP, combined SKP-KP, KP, combined MKP-KP, and MKP equations are stable for any physically admissible values of the parameters of the system.

ACKNOWLEDGMENTS

The authors are grateful to the referee for extremely helpful comments, without which this paper could not have been written in its present form.

APPENDIX A: THE COEFFICIENTS Q_l OF $\phi^{\frac{l+1}{2}}$ IN EQ. (24) FOR l = 1, 2, 3, AND 4

$$Q_1 = \mu(1-\beta_e) + \frac{p}{\sigma_{pe}}, \quad Q_2 = \mu\frac{(\beta-1)(4-3\beta_e)}{3\sqrt{\pi}}, \quad \text{(A1)}$$

$$Q_3 = \frac{1}{2}\left[\mu - \frac{p}{\sigma_{pe}^2}\right], \quad Q_4 = \mu\frac{2(\beta^2-1)(4+5\beta_e)}{15\sqrt{\pi}}, \quad \text{(A2)}$$

APPENDIX B: $R^{(J)}$—INTEGRAND OF THE INTEGRATION IN EQ. (67) FOR J = 0, 1, AND 2

$$R^{(0)} = 0, \quad R^{(1)} = i\frac{d}{dX}\left[-2lM_2q^{(0)} + \omega^{(1)}q^{(0)}\right], \quad \text{(B1)}$$

$$R^{(2)} = i\frac{d}{dX}\left[-2lM_2q^{(1)} + \omega^{(2)}q^{(0)} + \omega^{(1)}q^{(1)}\right] + l^2M_3q^{(0)} - l\omega^{(1)}q^{(0)}. \quad \text{(B2)}$$

where

$$M_2 = -U + AB\phi_0^{\frac{1}{2}} + AB_1\phi_0 + AC\frac{d^2}{dX^2}, \quad \text{(B3)}$$

$$M_3 = -U + AB\phi_0^{\frac{1}{2}} + AB_1\phi_0 + 3AC\frac{d^2}{dX^2} + \frac{1}{2}AD\frac{m^2+n^2}{l^2}. \quad \text{(B4)}$$

[1]W. H. Zurek, Astrophys. J. **289**, 603 (1985).
[2]J. C. Higdon, R. E. Lingenfelter, and R. E. Rothschild, Astrophys. J. **698**, 350 (2009).
[3]P. K. Shukla, Phys. Scr. **77**, 068201 (2008).
[4]A. E. Dubinov, D. Y. Kolotkov, and M. A. Sazonkin, Tech. Phys. **57**, 585 (2012).
[5]H. Alfvén, *Cosmic Plasma* (D. Reidel Publishing Company, 1981), Vol. 82.
[6]A. Gusev, U. Jayanthi, I. Martin, G. Pugacheva, and W. Spjeldik, Braz. J. Phys. **30**, 590 (2000).
[7]A. Gusev, U. Jayanthi, I. Martin, G. Pugacheva, and W. Spjeldvik, J. Geophys. Res. **106**, 26111, doi:10.1029/1999JA000443 (2001).
[8]R. L. Merlino, Plasma Phys. Appl. **81**, 73 (2006).
[9]M. Horányi, T. Hartquist, O. Havnes, D. Mendis, and G. Morfill, Rev. Geophys. **42**, RG4002, doi:10.1029/2004RG000151 (2004).
[10]P. K. Shukla and V. P. Silin, Phys. Scr. **45**, 508 (1992).
[11]S. Ghosh and R. Bharuthram, Astrophys. Space Sci. **314**, 121 (2008).
[12]S. A. El-Tantawy, N. A. El-Bedwehy, and W. M. Moslem, Phys. Plasmas **18**, 052113 (2011).
[13]S. A. El-Tantawy and W. M. Moslem, Phys. Plasmas **18**, 112105 (2011).

[14]N. S. Saini, B. S. Chahal, and A. S. Bains, Astrophys. Space Sci. **347**, 129 (2013).
[15]M. M. Masud, S. Sultana, and A. A. Mamun, Astrophys. Space Sci. **348**, 99 (2013).
[16]M. Bacha and M. Tribeche, J. Plasma Phys. **79**, 569 (2013).
[17]A. Paul and A. Bandyopadhyay, Astrophys. Space Sci. **361**, 172 (2016).
[18]G. Banerjee and S. Maitra, Phys. Plasmas **23**, 123701 (2016).
[19]S. Sardar, A. Bandyopadhyay, and K. P. Das, Phys. Plasmas **23**, 073703 (2016).
[20]S. Sardar, A. Bandyopadhyay, and K. P. Das, Phys. Plasmas **23**, 123706 (2016).
[21]A. Paul, A. Das, and A. Bandyopadhyay, Plasma Phys. Rep. **43**, 218 (2017).
[22]A. Paul, A. Bandyopadhyay, and K. P. Das, Phys. Plasmas **24**, 013707 (2017).
[23]R. A. Cairns, A. A. Mamum, R. Bingham, R. Boström, R. O. Dendy, C. M. C. Nairn, and P. K. Shukla, Geophys. Res. Lett. **22**, 2709, doi:10.1029/95GL02781 (1995).
[24]G. Rowlands, J. Plasma Phys. **3**, 567 (1969).
[25]E. Infeld, J. Plasma Phys. **8**, 105 (1972).
[26]E. Infeld and G. Rowlands, J. Plasma Phys. **10**, 293 (1973).
[27]K. P. Das and F. Verheest, J. Plasma Phys. **41**, 139 (1989).
[28]V. E. Zakharov and A. M. Rubenchik, Sov. Phys. -JETP **38**, 494 (1974).
[29]W. Malfliet and W. Hereman, Phys. Scr. **54**, 563 (1996).
[30]P. O. Dovner, A. I. Eriksson, R. Boström, and B. Holback, Geophys. Res. Lett. **21**, 1827, doi:10.1029/94GL00886 (1994).
[31]H. Schamel, Plasma Phys. **13**, 491 (1971).
[32]H. Schamel, Plasma Phys. **14**, 905 (1972).
[33]H. Schamel, J. Plasma Phys. **13**, 139 (1975).
[34]H. Schamel, Phys. Scr. **20**, 306 (1979).
[35]H. Schamel, J. Plasma Phys. **9**, 377 (1973).
[36]F. Bauer and H. Schamel, Phys. D **54**, 235 (1992).
[37]H. Schamel, Phys. Rep **140**, 161 (1986).
[38]K. Saeki, P. Michelsen, H. L. Pécseli, and J. J. Rasmussen, Phys. Rev. Lett. **42**, 501 (1979).
[39]P. Guio, S. Børve, L. K. S. Daldorff, J.-P. Lynov, P. Michelsen, H. L. Pécseli, J. J. Rasmussen, K. Saeki, and J. Trulsen, Nonlinear Processes Geophys. **10**, 75 (2003).
[40]M. W. Coffey, J. Phys. A: Math Gen. **24**, L1345 (1991).
[41]F. Verheest and W. Hereman, Phys. Scr. **50**, 611 (1994).
[42]A. A. Mamun, R. A. Cairns, and P. K. Shukla, Phys. Plasmas **3**, 2610 (1996).
[43]A. A. Mamun, Phys. Plasmas **5**, 322 (1998).
[44]T. S. Gill, H. Kaur, and N. S. Saini, Phys. Plasmas **10**, 3927 (2003).
[45]S. K. El-Labany, W. M. Moslem, and A. E. Mowafy, Phys. Plasmas **10**, 4217 (2003).
[46]H. Alinejad, S. Sobhanian, and J. Mahmoodi, Phys. Plasmas **13**, 012304 (2006).
[47]J. Das, A. Bandyopadhyay, and K. P. Das, J. Plasma Phys. **73**, 869 (2007).
[48]S. k A. Islam, A. Bandyopadhyay, and K. P. Das, Phys. Plasmas **16**, 022307 (2009).
[49]H. Alinejad, Astrophys. Space Sci. **325**, 209 (2010).
[50]M. Shalaby, S. K. El-Labany, E. F. El-Shamy, and M. A. Khaled, Phys. Plasmas **17**, 113709 (2010).
[51]H. Alinejad and M. Tribeche, Phys. Plasmas **17**, 123712 (2010).
[52]S. S. Duha, M. G. M. Anowar, and A. A. Mamun, Phys. Plasmas **17**, 103711 (2010).
[53]T. Singh Gill, C. Bedi, and N. S. Saini, Phys. Plasmas **18**, 043701 (2011).
[54]O. Rahman and A. A. Mamun, Phys. Plasmas **18**, 083703 (2011).
[55]H. K. Malik, R. Kumar, K. E. Lonngren, and Y. Nishida, Phys. Rev. E **92**, 063107 (2015).
[56]M. Toida and J. Inagaki, Phys. Plasmas **22**, 062305 (2015).
[57]S. Guo, L. Mei, Y.-L. He, H. Guo, and Y. Zhao, Eurpohys. Lett. **114**, 25002 (2016).
[58]H. Schamel, N. Das, and N. N. Rao, Phys. Plasmas **8**, 671 (2001).
[59]N. Das and H. Schamel, J. Plasma Phys. **71**, 769 (2005).
[60]W. M. Moslem, W. F. El-Taibany, E. K. El-Shewy, and E. F. El-Shamy, Phys. Plasmas **12**, 052318 (2005).
[61]W. M. Moslem and W. El-Taibany, Phys. Plasmas **12**, 122309 (2005).
[62]M. G. Hafez, N. C. Roy, M. R. Talukder, and M. Hossain Ali, Phys. Plasmas **23**, 082904 (2016).
[63]B. B. Kadomtsev and V. I. Petviashvili, Sov. Phys. Dokl. **15**, 539 (1970).
[64]M. Kako and G. Rowlands, Plasma Phys. **18**, 165 (1976).
[65]E. Infeld, G. Rowlands, and M. Hen, Acta Phys. Pol. **A54**, 131 (1978).
[66]F. Verheest and S. R. Pillay, Phys. Plasmas **15**, 013703 (2008).
[67]Y. Nejoh, Phys. Fluids B **4**, 2830 (1992).
[68]J. A. Pava, *Nonlinear Dispersive Equations: Existence and Stability of Solitary and Periodic Travelling Wave Solutions* (American Mathematical Society, 2009), Vol. 156, p. 26.
[69]S. Wolfram, *The MATHEMATICA® Book, Version 4* (Cambridge University Press, 1999).
[70]J. Das, A. Bandyopadhyay, and K. P. Das, Phys. Plasmas **14**, 092304 (2007).