EXCITATION AND KINETIC ENERGIES OF PROMPT FISSION FRAGMENTS

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Excitation and kinetic energies of prompt fragments from the thermal neutron induced fission of ^{233}U and spontaneous fission of ^{252}Cf are estimated by using a renormalised gas model and the potential energy surface concept. The predictions agree fairly well with experiments.

Our theoretical understanding of the energy balance and energy partition in prompt fission phenomena is very incomplete. Measurements exist for the fragment kinetic energies T_F [1, 2] and their excitation energies E_F [3, 4]. Attempts have been made to ascertain the fission energy partitions from the liquid drop model viewpoint [5] and from the free Fermi gas model description [6], respectively.

In the present work, we consider the microscopic potential energy surface concept of Mosel and Greiner [7] to (i) evaluate the different energies relevant to the fission phenomena and to (ii) estimate the total energy release E_R. We also introduce and use an independent microscopic model [8] of interacting fermions and refer to this formulation as the renormalised gas model; it treats the structural effects in the nuclear ground states specifically as suitable energy corrections on the Fermi surface of the free Fermi gas reference nucleus. The renormalised gas model essentially extracts the structurally sensitive parts of the potential energy surface concept and evaluates a part U_F^{RGM} of the fragment excitation U_F. This U_F^{RGM} fluctuates with the fragment mass A_F. The remaining part of the excitation is obtained from the deformation-dependent part U_F^{PES} of the potential energy surface concept. The kinetic energy T_F is obtained from the remaining parts of E_R. The entire fission kinetics is treated in a self-consistent way at the scission point without the help of a conventional mass formula [9, 10]. This procedure is tested here for the thermal neutron fission of ^{233}U and spontaneous fission of ^{252}Cf and is compared with the available experimental data.

In the potential energy surface concept description, the total energy at the ground state deformation $\beta\,(\beta \ll 1)$ of a nucleus A is expressed in the BCS formalism [11] as

$$E_A(\beta) = \sum_{Z,N}\left(\sum_i e_i v_i^2 + \Delta^2/G\right) + E_{C_A}(\beta), \quad (1)$$

where $e_i v_i^2$ are the quasiparticle energies summed over the total number of nucleons Z and N, Δ^2/G is the pairing interaction, and $E_{C_A}(\beta)$ is the Coulomb energy of the nucleus A.

Assuming axial symmetry of the potential energy surface concept, we may expand the nuclear energy part of the potential energy surface concept [7] for an arbitrary deformation α as

$$E_A(\alpha_A) = E_A(\beta_A) + \tfrac{1}{2}C_{0_A}(\alpha_A - \beta_A)^2 + C'_A(\alpha_A - \beta_A)^3 + \dots \quad (2)$$

where C_{0_A} and C'_A are the collective stiffness coefficients resisting a deformation from the β- to the α-shape. The Coulomb energy may be similarly expanded in terms of that of the spherical nucleus as

$$E_{C_A}(\alpha_A) = E_{C_A}(0)\left[1 - (\alpha_A^2 - \beta_A^2)/4\pi + (5/\pi)^{\frac{1}{2}}(19/1680)(\alpha_A^3 - \beta_A^3)\right], \quad (3)$$

neglecting terms higher than α^3.

The energy balance equation for a fissioning nucleus I may now be written down by equating the total energies in the initial and final states. We fix the deformations α to be those at the scission point; we also assume that the two-step expansion (up to $\beta \approx 0.35$, $\alpha - \beta \approx 0.35$, i.e., $\alpha \approx 0.7$) is valid:

$$E_I(\alpha_I) = \sum_{F=L,H} \{E_F(\beta_F) + \tfrac{1}{2}C_{o_F}(\alpha_F - \beta_F)^2 + C'_F(\alpha_F - \beta_F)^3 + E_{C_F}(\alpha_F) - E_{C_F}(\beta_F)\} + \frac{Z_L Z_H e^2}{\sum_F R_{o_F}}\left[1 - \left(\frac{5}{4\pi}\right)^{\frac{1}{2}} \frac{\sum R_{o_F}\alpha_F}{\sum_F R_{o_F}} + \frac{3}{5}\left(\frac{5}{4\pi}\right)^{\frac{1}{2}} \frac{\sum R_{o_F}^2 \alpha_F}{(\sum_F R_{o_F})^2}\right]. \quad (4)$$

The terms inside the curly brackets represent the energy balance of the fragment 'self energies' (nuclear and Coulomb parts) as functions of deformations; they appear from the energy changes of the fragments from the static stable β_F-shapes at infinite mutual separation to the scission point α_F-shapes. The terms inside the square brackets have been discussed by Wilets [12]; they originate from the mutual Coulomb repulsion of the fragments and may be abbreviated as

$$E_r = Z_L Z_H e^2/(R_{o_L} + R_{o_H}), \quad (5a)$$

$$E_d = -(5/4\pi)^{\frac{1}{2}} Z_L Z_H e^2 (R_{o_L}\alpha_L + R_{o_H}\alpha_H)/(R_{o_L} + R_{o_H})^2, \quad (5b)$$

$$E_q = \tfrac{3}{5}(5/4\pi)^{\frac{1}{2}} Z_L Z_H e^2 (R_{o_L}^2 \alpha_L + R_{o_H}^2 \alpha_H)/(R_{o_L} + R_{o_H})^3. \quad (5c)$$

We note that E_r is due to the mutual Coulomb repulsion ('recoil energy') measured from the centre of the deformed charge of the initial nucleus I, E_d originates from the finite separation between the centres of the fragment charges ('displacement energy') and E_q comes from the quadrupole interaction of one deformed charge with respect to the monopole centre of charge of the other ('quadrupole energy').

The total available energy (the energy release) in fission is then

$$E_R = E_I(\alpha_I) - \sum_F E_F(\beta_F) \quad (6)$$

which is partitioned into the total excitation energy U (= $\sum_F U_F$) and the total kinetic energy T (= $\sum_F T_F$). Our problem now is to estimate these partitions into different conjugate pairs and to distribute them in the fragments themselves. We divide the excitation energy in two parts U^{PES} and U^{RGM} for convenience. The static part U^{RGM} is evaluated from the renormalised gas model. A system, conservative in energy and in the total number of particles, going from an initial state I to a final state F in a positive Q-reaction, releases U^{RGM} amount of energy. It appears from the static structural part; if the static corrections are $\partial\epsilon_I$ and $\partial\epsilon_F$, the structural parts $U^{RGM} = \sum_F \partial\epsilon_F - \partial\epsilon_I$ and $U_F^{RGM} = \partial\epsilon_F - \frac{1}{2}\partial\epsilon_I$ appear as a part of the excitation energy. The part U^{PES} comes from the change of the deformation-dependent parts of the potential energy surface concept alone in eq. (4), i.e.,

$$U_F^{PES} = \tfrac{1}{2}C_{o_F}(\alpha_F - \beta_F)^2 + C'_F(\alpha_F - \beta_F)^3 + \Delta E_{C_F} + E_{q_F}, \quad (7)$$

where ΔE_{C_F} is the Coulomb energy difference in the α- and β-shapes.

In our renormalised gas model picture, a one-fermion spherical system of ν particles has a free gas Fermi surface $\epsilon_{o\nu}$. If the system obeys a shell structure, the closed shell system has a Fermi surface at $\epsilon'_{o\nu}$. The unfilled shell system with n extracore nucleons has a Fermi surface shifted relative to $\epsilon_{o\nu}$ due to the pres-

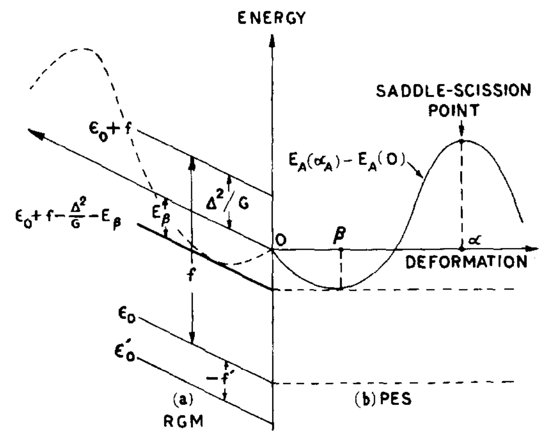

Fig. 1. Schematic diagram of the renormalised gas model and the deformation-dependent part of the potential energy surface concept, of a nucleus A, shown in a three-dimensional representation. See text for the descriptions of the models. The renormalised gas model ground state at a deformation β coincides with $E_A(\beta) - E_A(0)$ of the potential energy surface concept. In the renormalised gas model all energies are measured from the free Fermi gas surface ϵ_0. The interaction corrections are shown as energy shifts f, Δ^2/G, and E_β. The ground state of a nearby doubly magic nucleus is at ϵ'_0 below ϵ_0 by the magnitude of $-f'$.

ence of long range parts of the residual nuclear interactions and the coherent collective short range interactions. These residual interactions introduce energy corrections f_n, $-\Delta_n^2/G_\nu$ and $-E_{\beta_n}$ on $\epsilon_{0\nu}$. An even system is additionally bound by $\sim \Delta_n$ relative to its nearest odd neighbours. The free Fermi system is thus converted into an interacting gas Fermi system through the normalisation

$$\partial\epsilon_n = f_n - \Delta_n^2/G_\nu - E_{\beta_n}. \tag{8}$$

These corrections, assuming sufficient adiabaticity between single particle and collective motions, may be estimated from the Rosenzweig combinatorial model [13] and the BCS formalism [11]. A small correction is introduced by altering the BCS shell boundaries through an adjustment of the free Fermi gas spacing d_ν to the normalised gas spacing d'_ν $(= d_\nu \cdot \epsilon_{0\nu}/(f_n + \epsilon_{0\nu})$ due to the single particle correction f_n.

In the nuclear system with n and p extracore neutrons and protons, all structural corrections f, $-\Delta^2/G$ and $-E_\beta$ over the free nuclear Fermi surface ϵ_0 are additive:

$$\partial\epsilon_A = \partial\epsilon_n + \partial\epsilon_p. \tag{9}$$

Fig. 1 gives a schematic description of the

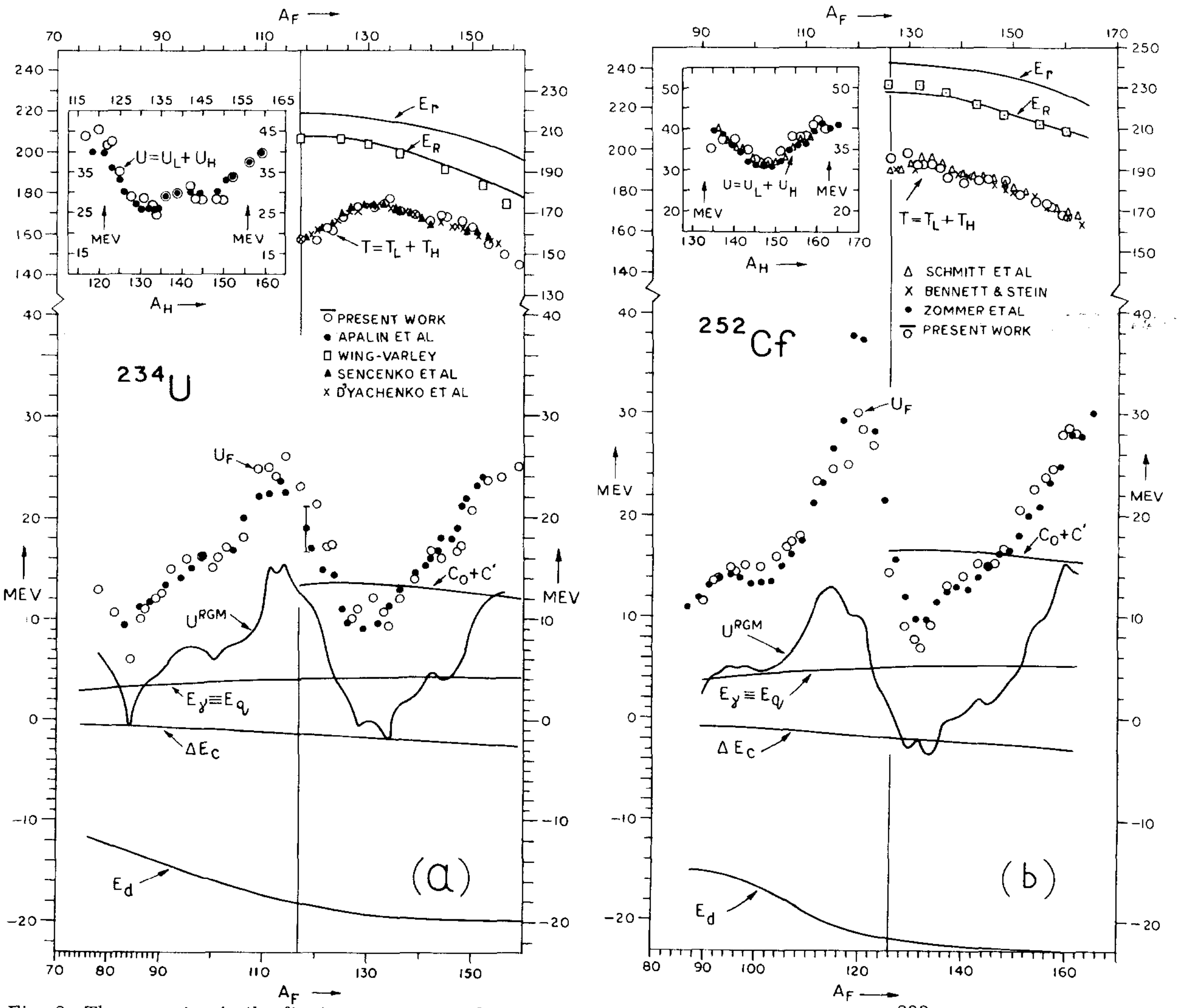

Fig. 2. The energies in the fission processes of a) the thermal neutron induced fission of ^{233}U and b) the spontaneous fission of ^{252}Cf. See text for the origin of various energy terms. Note that the total energy release E_R, the fragment excitation U_F, the total excitation $U = U_L + U_H$ in the inserts at left, and the total kinetic energy T agree with observations fairly satisfactorily in both cases.

model and connects the renormalised gas model with the potential energy surface concept. The renormalised gas model is free of any arbitrary parameters and depends only on the choice of the fundamental quantities ϵ_0 and G. We have taken ϵ_0 = 31.0 MeV following Cindro [14] and $G = 23/A$ following Lane [15].

In our attempt to reproduce a) the total energy release E_R, b) the excitation energy U_F and c) the kinetic energy T_F, we have used $R_0 = 1.20A^{\frac{1}{3}}$ and $C_{0I} = C_{0L} = C_{0H}$ = 160 MeV and $C'_I = C'_L = C'_H$ = 80 MeV as average values. The fragment mass regions with $C_{0F} \approx 0$ and $C'_F \approx 0$ usually have the configuration of one hole or one extra particle in a well defined large angular momentum state (usually a proton); in these special regions, the corresponding conjugate fragments are expected to appear with much higher excitation from the energy conservation principle (the complimentary fragments will receive about twice the energy calculated with the average C-values).

Typical calculated maximum (midshell) values of $f, \Delta^2/G$ and E_β are about 6.0, -1.0 and -1.5 MeV in the fragment mass range of A_L = 70 to A_H = 170; the minimum of $f\ (= \epsilon' - \epsilon_0)$ is about -3.0 MeV. Approximate solutions of eq. (4) by minimising the total fragment energy at the scission point (by setting $\partial E_R/\partial\alpha_F = 0$) gives the α_F-values which agree with the LDM estimates [5]. This procedure gives the correct barrier height for the thermal neutron induced fission of ^{233}U as 5.6 MeV (the measured threshold is 5.49 MeV); it also reproduces the α_I-values estimated from the LDM ($\alpha_I \approx 0.7$).

Our detailed calculations require the nuclear composition of the fragments. We have taken this from [10] assuming that the most probable compositions (with minimum total energy) occur with unit probability.

The total fission energy release E_R is correctly estimated from (4) and (6) and agrees with the predictions of the mass formula [9]. The different energies in (5) and (7), viz., E_r, E_d, E_q, ΔE_c and stiffness dependent terms, are shown in fig. 2(a) for the thermal neutron fission of ^{233}U and in fig. 2(b) for the spontaneous fission of ^{252}Cf. The part U^{RGM} predicts peaks at $A_F \sim 113$ in $^{233}U + n$ and at $A_F \sim 115$ in the ^{252}Cf fission. Using the principles stated above, a fragment excitation peak appears when two conditions are simultaneously satisfied: (i) the fragment composition corresponds to the midshell regions of neutrons and protons (U_F^{RGM} is near maximum in this region) and (ii) the respective conjugate fragment has one extracore proton (or a proton hole) in a large angular momentum state. In the thermal neutron fission of ^{233}U, both conditions fix the maximum of U_F at $A_F \sim 113$; in our preliminary calculations [8] we nave noted that these conditions also apply to the thermal neutron fission of ^{235}U and ^{239}Pu. In the spontaneous fission of ^{252}Cf, condition (ii) shifts the peak from $A_F \sim 115$ to fix it at $A_F \approx$ ≈ 120.

A detailed point by point comparison in figs. 2(a) and 2(b) shows that the agreement is satisfactory in both cases within the reported experimental error. Some discrepancy in the absolute magnitudes of the peaks of U_F, however, exists.

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