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Evaluation of Modal Spot Size in Single-mode Graded Index Fibers by a Simple Technique

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Summary

Based on the recently proposed simple functional form for the fundamental mode of single-mode graded index fibers derived by Chebyshev technique, we report analytical expressions of normalised Petermann I and II spot sizes and their evaluations for step and parabolic index fibers. The concerned calculations require much less computations. The predictions of our analysis agree excellently with the exact numerical results.

1 Introduction

The modal spot size is an important parameter for a single-mode fiber. Of various recently proposed definitions of spot size, Petermann I spot size is suitable for predicting microbending and small angular misalignment losses [1] while Petermann II spot size is specially convenient to estimate group delay, modal dipersion and coupling losses in a fiber joint having a transverse offset less than the said spot size [1–3]. In order to obtain the fundamental modal field for a graded index profile by solving the scalar wave equation, one has to apply numerical techniques or approximate methods excepting the case of step index fiber where analytic expressions are easily found. Two parameter trial functions [4] approximating the fundamental mode of single-mode fiber estimate the propagation characteristics more accorately than the single parameter trial functions [5]. All these analyses, however, involve cumbersome computations. Therefore, one should employ a simple functional form for the fundamental mode of graded index flbers. The approximate but very useful form of the fundamental mode of graded index fibers has already been reported [6]. This method retains only four terms in the Chebyshev power series approximation for the fundamental mode and also incorporates formulation of a linear relation of $K_1(W)/K_0(W)$ with 1/W over a iong and practical range of W values appropriate for single moded guidance for such fibers. The coefficients of the power series are evaluated by solving some algebraic equations framed at appropriate Chebyshev points [6].

In this communication, we report the applicablity of the fundamental modal field obtained by the said Chebyshev technique to prescribe analytical expressions for normalised Petermann I and II spot sizes and their calculations involving very less computations.

2 Theory

The refractive index profile for a weakly guiding fiber can be given by

$$n^2(R) = n_1^2[1-2\delta f(R)], \qquad R \le 1$$
$$= n_2^2, \qquad R > 1 \quad (1)$$

where R=r/a, ‘a’ is the radius of the core and $\delta = (n_1^2 - n_2^2)/2n_1^2$, n_1 and n_2 being the axial and cladding refractive indices. Here, f(R) defines the shape of the profile and for graded index fiber it is given by

$$f(R) = R^q, \qquad R \le 1 \quad (2)$$

‘q’ is the profile exponent and its values are ∞ and 2 for step and parabolic index fibers respectively.

The fundamental modal field ψ (R) inside the core is expressed by the following scalar wave equation

$$\frac{d^2\psi}{dR^2} + \frac{1}{R}\frac{d\psi}{dR} + \left(V^2(1-f(R)) - W^2\right)\psi = 0, \qquad R \le 1 \quad (3)$$

together with the boundary condition

$$\left(\frac{1}{\psi}\frac{d\psi}{dR}\right)_{R=1} = -\frac{WK_1(W)}{K_0(W)}. \quad (4)$$

The fundamental modal field in the cladding can be given by

$$\psi(R) \sim K_0(WR), \qquad R > 1 \quad (5)$$

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We have elucidated in the appendix how one can obtain the fundamental modal field $\psi(R)$ by the simple method involving Chebyshev technique and the formulated linear relationship of $K_1(W)/K_0(W)$ with $1/W$ over a long and practical values of W appropriate for single-moded guidance in such fibers [6].

The normalised Petermann I spot size, w_e is defined as [1]

$$w_e^2 = 2\frac{\int_0^{\infty}\psi^2(R)R^3 dR}{\int_0^{\infty}\psi^2(R)R dR}. \qquad (6)$$

Employing the expression of $\psi(R)$ given by (A8) in the Appendix, we obtain from (6) [7, 8]

$$w_e^2 = 2\frac{3(T_1+T_2)+2T_3\left[\left(0.5+1/W_c^2\right)T_6^2-0.5+T_6/W_c\right]}{3\left[T_4+2T_5+T_3\left(T_6^2-1\right)\right]} \qquad (7)$$

where W_c is the value of W evaluated by the present Chebyshev method and

$$\begin{aligned}
T_1 &= 1/2+A_2^2/4+A_4^2/6+A_6^2/8\\
T_2 &= 2\left(A_2/3+A_4/4+A_6/5\right.\\
&\quad \left.+A_2A_4/5+A_2A_6/6+A_4A_6/7\right)\\
T_3 &= \left(1+A_2+A_4+A_6\right)^2\\
T_4 &= 1+A_2^2/3+A_4^2/5+A_6^2/7\\
T_5 &= A_2/2+A_4/3+A_6/4+A_2A_4/4\\
&\quad +A_2A_6/5+A_4A_6/6\\
T_6 &= 1.034623+0.3890323/W_c. \qquad (8)
\end{aligned}$$

Again, the normalised Petermann II spot size, w_d is expressed as [1]

$$w_d^2 = 2\frac{\int_0^{\infty}\psi^2(R)R\ dR}{\int_0^{\infty}\left[\psi'(R)\right]^2 R\ dR} \qquad (9)$$

Using the analytic expression for fundamental modal field $\psi(R)$ given by (A8) in (9) we get [7, 8]

$$w_d^2 = 2\frac{T_4+2T_5+T_3\left(T_6^2-1\right)}{4T_7+T_3\left(W_c^2+2W_cT_6-W_c^2T_6^2\right)} \qquad (10)$$

where

$$\begin{aligned}
T_7 &= A_2^2/2+A_4^2+3A_6^2/2+4A_2A_4/3\\
&\quad +3A_2A_6/2+12A_4A_6/5. \qquad (11)
\end{aligned}$$

3 Results and discussions

In order to verify the validity of our analytic formulations for normalised Petermann I and II spot sizes for single-mode region, we compare our results with the analytical results in case of step index fiber and also with the available exact numerical results for parabolic index fibers. In Table 1, we present the values of w_e obtained by our technique along with the available exact results for a step index fiber. It is found that for V = 1.6 the error maximum with a value of 0.41% while for V ≥ 2.0 the values found by our method match identically with the exact ones. In Table 2, we also present our values of w_d together with the exact values [1] for a step index fiber. It is seen that maximum and minimum errors in our predictions for w_d are 0.44% and 0.09% respectively corresponing to respective V values of 1.8 and 2.4. In Fig. 1, we present the variation of normalised Petermann I and II spot sizes with effective normalised frequency V_e for a parabolic index fiber where V_e = 2.405 (V/V_c), V_c being the corresponding cutoff value. Further, in this figure our values and the exact numerical ones [1] are represented by crosses and solid lines respectively. It is found that the agreement between crosses and solid lines is excellent. Therefore, our simple formulations estimate the fundamental modal spot sizes excellently both for step and parabolic index fibers.

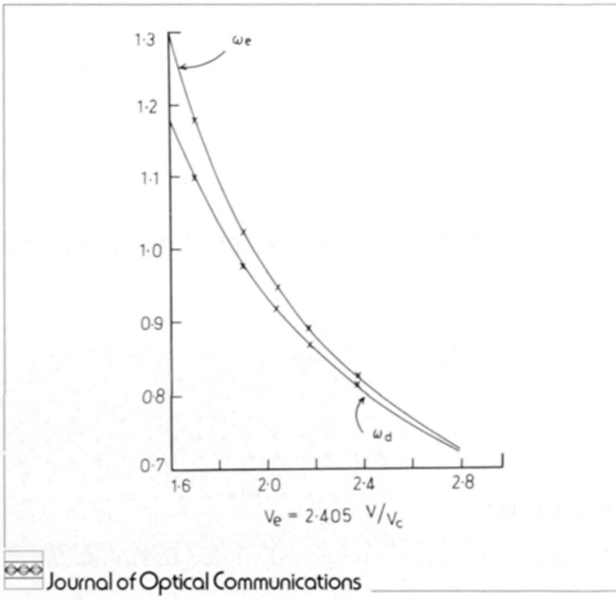

Fig. 1: Variation of normalised Petermann I and II spot sizes w_e and w_d, respectively, with effective normalised frequency $V_e = 2.405(V/V_c)$ for single-mode parabolic index fiber (x our theoretical results based on Chebyshev technique; —— exact numerical results [1].)

Table 1: Values of normalised Petermann I spot size w_e for single-mode step index fiber.

V	w_e (Found)	w_e (Exact) [1]	Error
1.4	2.189	2.184	0.23%
1.6	1.715	1.722	0.41%
1.8	1.453	1.454	0.07%
2.0	1.291	1.291	0.0%
2.2	1.181	1.181	0.0%
2.4	1.103	1.103	0.0%

Table 2: Values of normalised Petermann II spot size w_d for single-mode step index fiber.

V	w_d (Found)	w_d (Exact) [1]	Error
1.4	1.879	1.871	0.43%
1.6	1.555	1.559	0.26%
1.8	1.363	1.369	0.44%
2.0	1.237	1.241	0.32%
2.2	1.148	1.150	0.17%
2.4	1.081	1.082	0.09%

4 Conclusions

We present analytic expressions for normalised Petermann I and II spot sizes in case of graded index single-mode fibers. The concerned calculations require much less computations. The theory is based on a recent approximation of the fundamental modal field by a suitable Chebyshev power series and also formulation of a linear relation of $K_1(W)/K_0(W)$ with $1/W$ over a long and practical range of W values, appropriate for single moded guidance in such fibers. It is shown in case of step and parabolic index fibers that our estimations agree excellently with the exact results and thus the correctness of our simple technique is verified.

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6 Appendix

As $\psi(R)$ in case of fundamental mode is an even function of R, $\psi(0)$ is nonzero and $\psi'(0)$ is zero, we can express $\psi(R)$ in terms of a Chebyshev power series as [9, 10]

$$\psi(R) = \sum_{j=0}^{j=M-1} a_{2j} R^{2j}. \qquad \text{(A1)}$$

The Chebyshev points are given by [10]

$$R_m = \cos\left(\frac{2m-1}{2M-1}\frac{\pi}{2}\right); \quad m = 1,2,\ldots(M-1). \qquad \text{(A2)}$$

In order to simplify calculations we take M = 4 [6] when we obtain

$$\psi(R) = \sum_{j=0}^{j=3} a_{2j} R^{2j} \qquad \text{(A3)}$$

and the corresponding Chebyshev points for M = 4 are found from (A2) as

$$R_1 = 0.4338, \; R_2 = 0.7818 \; \text{and} \; R_3 = 0.9749. \qquad \text{(A4)}$$

These three values of R in (A4) and $\psi(R)$ given by (A3) are used in (3) to obtain three equations as [6]

$$\begin{aligned} & a_0\left[V^2\left(1-f(R_i)\right)-W^2\right] \\ & +a_2\left[4+R_i^2\left(V^2\left(1-f(R_i)\right)-W^2\right)\right] \\ & +a_4\left[16R_i^2+R_i^4\left(V^2\left(1-f(R_i)\right)-W^2\right)\right] \\ & +a_6\left[36R_i^4+R_i^6\left(V^2\left(1-f(R_i)\right)-W^2\right)\right]=0 \end{aligned} \qquad \text{(A5)}$$

i= 1, 2 and 3 imply the three equations.

Applying the least square fitting procedure over the interval $0.6 \le W \le 2.5$ we get [6]

$$\frac{K_1(W)}{K_0(W)} = 1.034623 + 0.3890323/W \qquad \text{(A6)}$$

Further, (A3) and (A6) can be employed in (4) to obtain

$$\begin{aligned} & a_0(1.034623W+0.3890323) \\ & +a_2(1.034623W+2.3890323) \\ & +a_4(1.034623W+4.3890323) \\ & +a_6(1.034623W+6.3890323)=0 \end{aligned} \qquad \text{(A7)}$$

W for a given value of V is given by the condition of the three equations in (A5) along with (A7) being conformable for solution [6]. Again, for a given value of V we can get the constants a_{2j}(j = 1, 2, 3) in terms of a_0 from any three of the four equations given by (A5) and (A7). Thus we find the fundamental field for the single-mode graded index fiber as

$$\begin{aligned} \psi(R) &= a_0\left(1+A_2R^2+A_4R^4+A_6R^6\right), && R \le 1 \\ &= a_0\left(1+A_2+A_4+A_6\right)\frac{K_0(W_cR)}{K_0(W_c)} && R > 1 \end{aligned} \qquad \text{(A8)}$$

where $A_{2j} = a_{2j}/a_0$; j = 1,2 and 3; W_c is the value of normalised propagation constant W found by the present Chebyshev technique.

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