AIP Journal of Applied Physics

Energy and momentumloss rates of onedimensional hot electrons in semiconductor quantumwell wires

A. Kabasi, D. Chattopadhyay, and C. K. Sarkar

Citation: Journal of Applied Physics **65**, 1598 (1989); doi: 10.1063/1.342951
View online: http://dx.doi.org/10.1063/1.342951
View Table of Contents: http://scitation.aip.org/content/aip/journal/jap/65/4?ver=pdfcov
Published by the AIP Publishing

Energy- and momentum-loss rates of one-dimensional hot electrons in semiconductor quantum-well wires

A. Kabasi and D. Chattopadhyay
Institute of Radio Physics and Electronics, 92, Acharya Prafulla Chandra Road, Calcutta 700 009, India
C. K. Sarkar
Department of Electronics and Telecommunication Engineering, Jadavpur University, Calcutta 700 032, India

(Received 8 August 1988; accepted for publication 13 October 1988)

Energy- and momentum-loss rates of degenerate hot electrons moving one-dimensionally in a quantum-well wire structure of square cross section of side length L are calculated theoretically considering polar coupling to longitudinal-optic (LO) phonons and deformation potential and piezoelectric couplings to acoustic phonons. Piezoelectric scattering contributes the least, while the effect of deformation potential scattering in energy loss is unimportant compared to that of LO phonon scattering for electron temperatures (T_e) above about 50 K. In momentum-loss rate, however, acoustic scattering continues to be important at higher values of T_e. The nonequilibrium distribution of LO phonons is found to reduce both the energy- and momentum-loss rates. The reduction factor of the total energy-loss rate is 4.4 for $L = 10$ nm and 6.2 for $L = 6$ nm at $T_e = 100$ K. The corresponding values for the net momentum-loss rate are 2.3 and 2.5, respectively.

I. INTRODUCTION

Transport properties of low-dimensional quantized semiconducting systems are attracting attention since they help in understanding the basic physics and in realization of devices. Although most of the studies refer to two-dimensional (2D) systems involving GaAs/AlGaAs quantum wells, one-dimensional (1D) transport in quantum-well wires is also of considerable interest.[1–3]

Shah *et al.*[4] have measured the energy-loss rates of hot electrons in 2D systems. These measurements and their analyses throw light on the electron-phonon interaction and reveal the importance of nonequilibrium longitudinal optic (LO) phonons in slowing down the cooling rate.[5] Similar studies in a 1D system will be important since the density of states and the scattering rates here differ from those of 2D systems and the bulk. With this motivation we theoretically study here the influence of nonequilibrium LO phonons on the energy- and momentum-loss rates of 1D hot electrons in quantum-well wires. The loss rates for acoustic phonons via deformation potential and piezoelectric couplings are also investigated. The size effects are explicitly included in our analysis.

II. MODEL AND METHOD

A quantum-well wire of square cross section having a side length L is considered. We take x and y directions along the two adjacent sides of the cross section of the quantum-well wire and the z direction along the longitudinal direction of motion in which an electric field ϵ is applied. For a typical one-dimensional carrier density of 7×10^7 m^{-1} the Fermi level is located about 9 meV above the lowest subband edge. The carrier distribution function is therefore degenerate. As the electron-electron interactions are quite strong here, the distribution function in the presence of the field is taken to be shifted Fermi–Dirac type:

$$f(K_z) = f_0 - \frac{\hbar^2 d}{m} K_z \frac{\partial f_0}{\partial E}, \tag{1}$$

where f_0 is the Fermi–Dirac function characterized by an electron temperature T_e, K_z is the wave vector of energy E, $\hbar$ is Planck's constant divided by 2π, d is the drift wave vector, and m is the electron effective mass.

We assume here that the carriers are confined to the lowest subband. This assumption is reasonable since the energy separation between the lowest and the next higher subband is more than an order of magnitude greater than the Fermi energy.

The cooling rate of 2D electron gas has been found to be affected more by the nonequilibrium LO phonons than by free-carrier screening.[4] In this work we therefore neglect the effect of screening. Furthermore, the phonon modes are assumed to be bulklike. The phonon spectrum may be modified in heterostructures causing a diminution in the electron-phonon interaction.[6,7] Similar effects are likely to occur in 1D structures, but the problem has not yet been studied in detail. However, the weakening of the electron-phonon interaction is moderated by the long-range nature of the polar interaction which would cause remote interaction with phonons in the neighboring materials. Therefore, the assumption of bulklike phonons is not likely to cause any serious error in the study of the effect on energy- and momentum-loss rates. In fact, the observations in cooling rate of the 2D system have been interpreted with bulklike phonons.[4] Other recent investigations in heterostructures have employed the same phonons.[1–3,8]

The square of the matrix element for polar interaction with LO phonons is[2,9]

$$M_{p0}^2 = \frac{e^2\hbar\omega}{2Vq^2}\left(\frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0}\right) \times F^2(q_x,q_y)\left(N(q) + \frac{1}{2} \mp \frac{1}{2}\right), \tag{2}$$

 0021-8979/89/041598-04$02.40

where the minus and the plus signs hold, respectively, for the absorption and the emission of phonons, $N(q)$ is the occupation number of LO phonons of the wave vector $\mathbf{q}$ and angular frequency ω, V is the crystal volume, e is the electronic charge, ϵ_∞ and ϵ_0 are the optic and the static permittivities, q_x and q_y are the x and the y components of $\mathbf{q}$, and

$$F(q_x,q_y) = \frac{\pi^4 \sin[q_x(L/2)]\sin[q_y(L/2)]}{\{\pi^2 - [q_x(L/2)]^2\}\{\pi^2 - [q_y(L/2)]^2\}[q_x q_y (L^2/4)]}. \tag{3}$$

The rate of generation of LO phonons for the 1D electrons is derived following the procedure[9] for three-dimensional (3D) electrons using the matrix element (2). The quantity $N(q)$ is determined by equating the generation rate to the decay rate of LO phonons, i.e., $[N(q) - N_0]/\tau_{\rm ph}$, where N_0 is the phonon occupation number at the lattice temperature and $\tau_{\rm ph}$ is the phonon lifetime. With this $N(q)$ the scattering rate out of the state K_z due to the absorption $[\tau_a^{-1}(E)]$ and that due to emission $[\tau_e(E)^{-1}]$ of LO phonons is calculated, including the effect of degeneracy.[10] The average rate of energy loss for one electron is then determined from the relationship

$$-\left\langle \frac{dE}{dt} \right\rangle = \frac{\int_0^\infty \hbar\omega[\tau_e(E)^{-1} - \tau_a(E)^{-1}]f_0(E)E^{-1/2}\,dE}{\int_0^\infty f_0(E)E^{-1/2}\,dE}. \tag{4}$$

The momentum-loss rate is calculated by noting that the change in momentum in a scattering event is $\hbar q_z$. Using Eqs. (1) and (2) and following the approach for the 3D electrons,[9] we obtain for the average rate of loss of momentum per electron due to LO phonons

$$\begin{aligned}-\left\langle \frac{dp}{dt} \right\rangle = \int_0^\infty C^2 d \Bigg\{ &\frac{(N_0+1)}{\sqrt{E-\hbar\omega}}\Bigg[\Bigg(\sqrt{\frac{E-\hbar\omega}{E}}+1\Bigg)I_{\rm 1D}[q_{ze}^+(E)] + \Bigg(\sqrt{\frac{E-\hbar\omega}{E}}-1\Bigg)I_{\rm 1D}[q_{ze}^-(E)]\Bigg] \\ &- \frac{N_0}{\sqrt{E+\hbar\omega}}\Bigg[\Bigg(\sqrt{\frac{E+\hbar\omega}{E}}-1\Bigg)I_{\rm 1D}[q_{za}^+(E)] - \Bigg(\sqrt{\frac{E+\hbar\omega}{E}}+1\Bigg)I_{\rm 1D}[q_{za}^-(E)]\Bigg]\Bigg\} \\ &\times\left(-\frac{\partial f_0}{\partial E}\right)E^{1/2}\,dE\Big/\int_0^\infty f_0(E)E^{-1/2}\,dE,\end{aligned} \tag{5}$$

where

$$C^2 = \frac{e^2\omega\sqrt{m}}{2\sqrt{2}\pi^2\hbar}\left(\frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0}\right), \tag{6}$$

$$I_{\rm 1D}[q_z(E)] = 4\int_0^\infty \int_0^\infty \frac{F^2(q_x,q_y)\,dq_x\,dq_y}{q_x^2+q_y^2+q_z^2}, \tag{7}$$

$$q_{za}^\pm = \frac{\sqrt{2m}}{\hbar}(\sqrt{E+\hbar\omega} \mp \sqrt{E}), \tag{8}$$

and

$$q_{ze}^\pm = \frac{\sqrt{2m}}{\hbar}(\sqrt{E} \pm \sqrt{E-\hbar\omega}). \tag{9}$$

The energy- and momentum-loss rates for interaction with deformation potential acoustic phonons are determined using the square of the matrix element[1]

$$|M|_a^2 = \frac{E_1^2\hbar q}{2V\rho u}\left(N_a(q) + \frac{1}{2} \mp \frac{1}{2}\right)F^2(q_x,q_y), \tag{10}$$

where E_1 is the acoustic deformation potential constant, $N_a(q)$ is the acoustic phonon occupation number, ρ is the mass density, and u is the longitudinal acoustic velocity. The positive and the negative signs in Eq. (10) signify the absorption and the emission of acoustic phonons, respectively.

The piezoelectric scattering is included[11] by replacing E_1^2 by $[(eh_{14})^2/q^2]A(q)$ and $N_a(q)$ by $N_p(q)$ in Eq. (10) where h_{14} is the piezoelectric tensor component, $A(q)$ is the anisotropy factor, and $N_p(q)$ is the occupation number of piezoelectric phonons.

Since the decay time for the acoustic phonons is much larger than that for the LO phonons,[12] contrary to the LO phonons the acoustic phonons escape out of the quantum-well structure without significantly affecting the values of $N_a(q)$ and $N_p(q)$ interacting with the 1D electrons. The nonequilibrium acoustic phonons are therefore neglected and $N_a(q)$ and $N_p(q)$ are assumed to retain their equilibrium lattice-temperature values. As acoustic phonon energy is a function of q, the average rates of loss of energy and momentum of an electron due to acoustic phonons via deformation potential and piezoelectric couplings are calculated numerically, incorporating the degeneracy of the distribution function.

The energy and momentum balance equations relating T_e to ϵ and d are

$$-\left\langle \frac{dE}{dt} \right\rangle = \frac{e\epsilon\hbar d}{m} \tag{11}$$

and

$$-\left\langle \frac{dp}{dt} \right\rangle = e\epsilon. \tag{12}$$

III. CALCULATED RESULTS

Numerical results are obtained for a lattice temperature of 1.8 K using the parameter values of GaAs: effective mass $= 0.6103\times10^{-31}$ kg, LO phonon angular frequency $= 5.49\times10^{13}$ rad s^{-1}, static permittivity $(\epsilon_0) = 1.133\times10^{-10}$ F m^{-1}, optic permittivity $(\epsilon_\infty) = 0.9651\times10^{-10}$ F m^{-1},

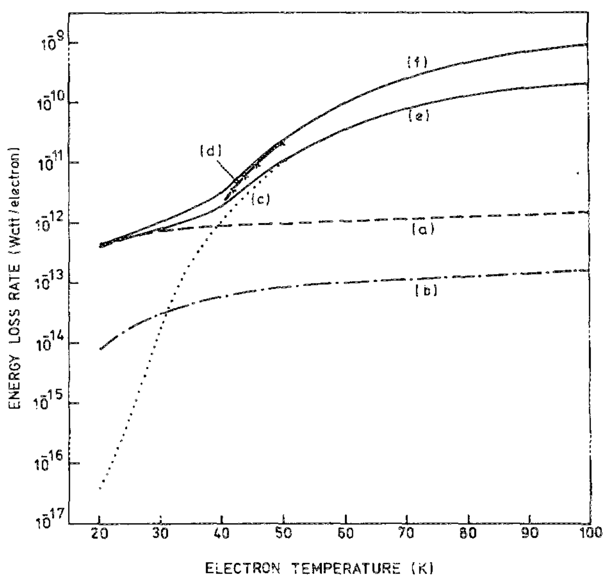

FIG. 1. Plot of the average energy-loss rate of an electron vs electron temperature for $L = 10$ nm. (a) Deformation potential scattering; (b) piezoelectric scattering; (c) scattering via nonequilibrium LO phonons; (d) scattering via thermal-equilibrium LO phonons; (e) total scattering with nonequilibrium LO phonons; (f) total scattering with thermal-equilibrium LO phonons.

LO phonon lifetime $(\tau_{ph}) = 7$ ps, mass density $(\rho) = 5.15 \times 10^3$ kg m^{-3}, longitudinal acoustic velocity $(u) = 5.22 \times 10^3$ m s^{-1}, piezoelectric tensor component $(h_{14}) = 1.44 \times 10^9$ V m^{-1}, and anisotropy factor $[A(q)] = \frac{4}{3}$.

The acoustic deformation potential is taken to be 11 eV; this value is higher than the commonly accepted value of 7 eV, but agrees with the recent analyses of 2D systems.[13]

It has previously been found[10] that the LO phonon occupation number differs significantly from the lattice-temperature value, which is practically zero here. The carrier transport is thus expected to be affected by the nonequilibrium phonon distribution. The energy-loss rates for $L = 10$ and 6 nm are shown in Figs. 1 and 2, respectively. The nonequilibrium LO phonons slow down the cooling rate by a factor which rises with T_e due to greater phonon disturbance. The reduction of the energy-loss rate at $T_e = 100$ K due to the LO phonon disturbance is about a factor of 4.4 at $L = 10$ nm and 6.2 at $L = 6$ nm. The effect is larger for the lower value of L due to the stronger scattering rate.[2] Deformation potential acoustic scattering is found to dominate the energy loss for values of T_e less than 30 K, while LO phonon scattering dominates for T_e above about 50 K.

The momentum-loss rates for $L = 10$ and 6 nm are shown in Figs. 3 and 4, respectively. Note that the expression for $\langle dE/dt \rangle$ given by Eq. (4) does not involve ϵ or d, whereas that for $\langle dp/dt \rangle$ given by Eq. (5) involves d. Thus, while the total energy-loss rate is obtained by summing up the individual loss rates, the total momentum-loss rate is not obtained in the same manner; the momentum-loss rate in each case is determined by solving coupled Eqs. (11) and (12).

The nonequilibrium LO phonons also affect the momentum-loss rate. While this scattering mechanism acting alone can stabilize the carrier system for $L = 6$ nm, the stabilization is not obtained for $L = 10$ nm. This is because of the weaker scattering rate for larger L: interactions with the nonequilibrium LO phonons here lead to a gain rather than a loss of average momentum. However, acoustic scattering is

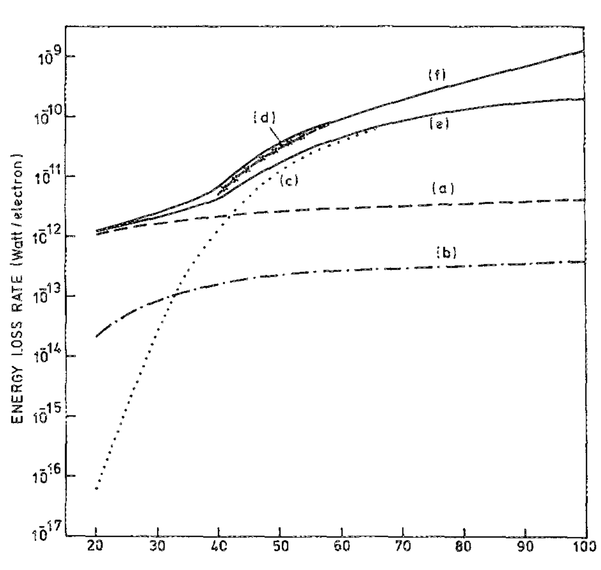

FIG. 2. The same as Fig. 1 for $L = 6$ nm.

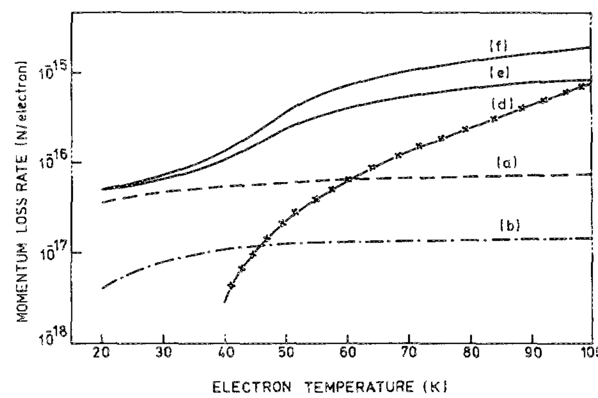

FIG. 3. Plot of the average momentum loss of an electron vs electron temperature for $L = 10$ nm. The labels (a)–(f) have the same meaning as in Fig. 1. Curve (c) is not present as the carrier system does not stabilize.

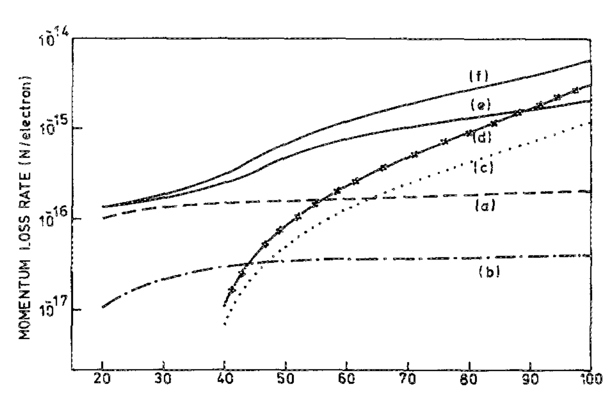

FIG. 4. The same as Fig. 3 for $L = 6$ nm.

TABLE I. Electric field-electron temperature data.

T_e (K)	ϵ(kV/m)			
	$(L = 10\text{ nm})^a$	$(L = 10\text{ nm})^b$	$(L = 6\text{ nm})^a$	$(L = 6\text{ nm})^b$
20	0.307	0.307	0.853	0.853
40	0.855	0.664	1.94	1.56
50	2.24	1.60	4.65	3.28
100	12.2	5.21	33.4	13.3

[a] Values for interactions with acoustic and thermal-equilibrium LO phonons.

[b] Values for interactions with acoustic and nonequilibrium LO phonons.

always present, so that the net momentum-loss rate is always positive and the system stabilizes. Note that although the nonequilibrium LO phonons alone do not stabilize the system for $L = 10$ nm, equilibrium LO phonons can do so because in the latter case only phonon emission takes place, yielding a positive value for $-\langle dp/dt \rangle$. It is found that at $T_e = 100$ K the nonequilibrium LO phonons reduce the net momentum-loss rate by a factor of 2.3 for $L = 10$ nm and by a factor of 2.5 for $L = 6$ nm.

Note that piezoelectric scattering contributes much less than the deformation potential scattering both in energy- and momentum-loss rates. As the nonequilibrium LO phonons slow down the loss rates, the electric field ϵ corresponding to a given T_e is less. The ϵ-T_e data, when all the scattering processes are operative, depicted in Table I, clearly illustrate this point for $L = 10$ and 6 nm.

The nonequilibrium LO phonons increase the energy relaxation time for the hot electrons. Thus, their presence will deteriorate the performance of high-speed devices. Also, the device will be more noisy. The influence of the nonequilibrium LO phonons on the energy-loss rates can be experimentally observed using the techniques for 2D systems.[4]

[1] J. Lee and M. O. Vassell, J. Phys. C **17**, 2525 (1984).
[2] J. P. Leburton, J. Appl. Phys. **56**, 2850 (1984).
[3] A. Ghosal, D. Chattopadhyay, and A. Bhattacharyya, J. Appl. Phys. **59**, 2511 (1986).
[4] J. Shah, A. Pinczuk, A. C. Gossard, and W. Wiegmann, Phys. Rev. Lett. **54**, 2045 (1985).
[5] P. J. Price, Superlattices and Microstructures **1**, 255 (1985).
[6] B. K. Ridley, in *Proceedings of the 17th International Conference on Physics of Semiconductors*, San Francisco (Springer, New York, 1984), p. 401.
[7] A. K. Sood, J. Menendez, M. Cardona, and K. Moog, Phys. Rev. Lett. **54**, 2111 (1985).
[8] K. Yokoyama and K. Hess, Phys. Rev. B **33**, 5595 (1986).
[9] E. M. Conwell, *Solid State Physics Suppl. 9*, edited by H. Ehrenreech, F. Seitz, and D. Turnbull (Academic, New York, 1967), pp. 120 and 129.
[10] A. Kabasi, D. Chattopadhyay, and C. K. Sarkar, Semicond. Sci. Technol. **3**, 1025 (1988).
[11] P. J. Price, Ann. Phys. **133**, 217 (1981).
[12] M. Lax and V. Naryanamurti, Phys. Rev. B **24**, 4692 (1981).
[13] K. Hirakawa and H. Sakaki, Appl. Phys. Lett. **49**, 889 (1986).