Electroweak Precision Data
and a Heavy Z'

Anirban Kundu [1]

Theory Group, Saha Institute of Nuclear Physics,

1/AF Bidhannagar, Calcutta - 700064, India.

Electronic Address: *akundu@theory.tifr.res.in*

Abstract

We consider the physics of an extra $U(1)$ gauge boson Z', which can mix with Z through intermediate fermion loops. The loop contribution due to the heavy top quark significantly affects the low-energy observables, and for $m_{Z'} > m_Z$, one can always adjust the shifts in these observables to be in the right direction suggested by experiments, when we impose the anomaly cancellation conditions for Z'.

---

[1]Present address: Theoretical Physics Group, Tata Institute of Fundamental Research, Bombay - 400 005

With the ever-increasing precision of the electroweak experiments, some disturbing signatures about the validity of the Standard Model (SM) are coming into view. Most notable among them are (i) $R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to hadrons)$, (ii) the left-right asymmetry A_{LR} measured at SLAC, and (iii) the τ-polarization asymmetry, $\mathcal{P}_\tau$. At the same time, observables such as the total Z-width, Γ_Z, and the hadronic cross section at the Z-peak, σ_{had}, are so well measured that arbitrary extensions of the SM are severely constrained. Among the non-supersymmetric extensions, technicolor is struggling to make itself compatible with the oblique electroweak parameters, R_b, and the FCNC data, and is not yet convincingly successful; extra fermion generations do not seem to resolve the discrepancies in the measured values of the abovementioned quantities, and are also restricted by the oblique parameters S and T. It has been shown [1] that addition of any number of arbitrary scalar representations, satisfying the constraints on ρ and on asymptotic unitarity, invariably worsens the discrepancy in R_b, and is totally insensitive to A_{LR}.

The only physically interesting choice that remains is the addition of one or more extra gauge bosons. Holdom [2] and Caravaglios and Ross [3] have already discussed that possibility in the literature. Both of these references add an extra neutral gauge boson Z' to the SM particle spectra. While Holdom has considered a tree-level mixing between Z and Z', Caravaglios and Ross have focussed on the Born graph of $e^+e^- \to f\bar{f}$ mediated by Z'. However, the $Z'f\bar{f}$ couplings derived from the experimentally measured parameters are not free from anomaly, and thus one has to add extra fermions to the model. These fermions not only contribute to the oblique parameters,

but may also introduce significant loop corrections to the observables, thus making the whole pattern of the new couplings somewhat confusing, and at the worst case, untraceable. The oblique parameters are also affected by a tree-level $Z - Z'$ mixing.

The important point stressed by Caravaglios and Ross is that one needs an imaginary amplitude coming from new physics effects to give a nonzero interference with the SM amplitude. In other words, the real part of the new physics amplitude does not contribute to physical observables if $|\mathcal{M}_{new}|^2 \ll |\mathcal{M}_{SM}|^2$. To satisfy this property, the authors in ref. [3] have considered a Z' nearly degenerate with Z so that both Z and Z' propagators are imaginary (apart from a factor of $-ig_{\mu\nu}$). However, the Z lineshape and Γ_Z, as measured at LEP, are in such conformity with the SM that the $Z'e^+e^-$ coupling has to be unreasonably small compared to the $Z'b\bar{b}$ coupling, whose value is fixed from the measurement of R_b. Unless there is some strong logic (as suggested in ref. [2]) which forbids Z' to couple with the first two fermion generations (in the weak eigenbasis), such a model, according to our view, seems to be quite artificial.

In this letter we consider what we think to be a much more realistic scenario. We assume that there is only one neutral $U(1)$ gauge boson Z'. There exists a number of models which predict such a Z', though their properties vary with the models chosen. We want to make an analysis which is sufficiently model-independent, except the existence of a Z', which is the common factor among these variety of models. As we do not confine ourselves within a particular model, our results are more qualitative than quantitative and

2

to be taken as trends. However, in nearly all the cases, the trends are in conformity with the experimental data.

Even in performing a general analysis, one requires some sort of a guideline, and fortunately, the Z'-physics is so well-studied that we have quite a few of them. For example, Langacker and Luo [4] have shown that a $Z - Z'$ mixing at tree-level, if exists, is bound to be very small (less than 1%). Thus one does not make any great error in neglecting the tree-level $Z - Z'$ mixing altogether; moreover, it keeps the oblique parameters unaffected by Z'. Another guideline is the condition that Z'-current is to be anomaly-free, and if one does not want to extend the fermion spectrum, it imposes some restriction on the $Z' f \bar{f}$ couplings. Thus, our study will be a general one except the imposition of these two constraints. There also exists a mass bound on Z': for a Z' with SM couplings to the fermions, the mass limit (at 95% CL) is 412 GeV (from direct search in $p\bar{p}$ colliders) and 779 GeV (from electroweak fit to the LEP data) [5]. If the $Z' f \bar{f}$ couplings do not mimic the SM ones, these limits may not be valid (*e.g.*, Z' which couples only to the third generation fermions). However, there is no reason for Z' to be nearly degenerate with Z, and we will drop this assumption made in ref. [3].

One notes that if $m_{Z'} \neq m_Z$, the only way to have a non-vanishing interference term is to consider a $Z - Z'$ mixing mediated by fermion loops, as shown in fig. 1. This is similar to the well-studied $\gamma - Z$ mixing; while the latter effects are subtracted from experimental measurements, the former effects are not, and so the concerned amplitude is a coherent sum of two amplitudes: pure SM electroweak, and that arising from new physics. As

3

the loop contribution is proportional to m_f^2, only the top loop is considered. Note that the two-loop $Z - Z' - Z$ amplitude is real and hence does not affect the interference term.

First, let us consider a toy model in which Z' couples only to the third generation. This will help us to understand the trend. The SM amplitude of $e^+ e^- \to f\bar{f}$ is

$$\mathcal{M}_Z = ir_1 [\bar{e}(p_1)\gamma^\mu (g_V^e - g_A^e \gamma_5)e(p_2)][\bar{f}(p_3)\gamma_\mu (g_V^f - g_A^f \gamma_5)f(p_4)] \qquad (1)$$

and the new physics amplitude is

$$\mathcal{M}_{new} = ir_2 [\bar{e}(p_1)\gamma^\mu (g_V^e - g_A^e \gamma_5)e(p_2)][\bar{q}(p_3)\gamma_\mu (g_V^{q\,'} - g_A^{q\,'} \gamma_5)q(p_4)] \qquad (2)$$

where the conventional $Zf\bar{f}$ vector and axialvector couplings are denoted by g_V^f and g_A^f respectively, and analogous quantities for the $Z'q\bar{q}$ vertex (we will always use q to denote a third generation fermion) are denoted by $g_V^{q\,'}$ and $g_A^{q\,'}$ (thus, the $Z'q\bar{q}$ vertex factor is given by $(g/2\cos\theta_W)\gamma^\mu (g_V^{q\,'} - g_A^{q\,'}\gamma_5)$. We neglect the QED terms in the amplitudes. At the Z-peak, one has

$$r_1 = \frac{\sqrt{2}Gm_Z}{\Gamma_Z}, \qquad (3)$$

$$r_2 = \frac{2G^2 m_Z}{(1 - \zeta^2)\Gamma_Z} f, \qquad (4)$$

where $\zeta = m_{Z'}/m_Z$ (as we are not on the Z'-peak, $\Gamma_{Z'}$ can be neglected), and f is the two-point loop integral given in Appendix 1. With $m_t = 175$ GeV [6] and taking the QCD corrections into account, we get

$$f = 2.90(0.018 g_V^{t\,'} - g_A^{t\,'}) \times 10^3. \qquad (5)$$

4

With $g_V^{t\,\prime}$, $g_A^{t\,\prime}$ and ζ of the order of unity, $|r_2/r_1|$ is of the order of 0.1, so it is justifiable to neglect the $|r_2|^2$ contributions. We have also neglected the QCD and the electroweak corrections to the internal top loop, as well as the threshold effects of $\mathcal{O}(\alpha\alpha_s^2 m_t^2)$, and have only taken the corrections to the external fermions into account. This introduces an error of at most two to three per cent and as we mainly concentrate on the qualitative features, the approximation is a good one. Anyway, the quantitative results are hardly affected. We note that it is the massive top quark that makes the interference amplitude non-negligible.

The cross-section with initially polarized electron beam comes out to be

$$\sigma_L(\theta) = A r_1^2 (g_L^e)^2 [(1 + \cos\theta)^2 T_1 + (1 - \cos\theta)^2 T_2], \tag{6}$$

$$\sigma_R(\theta) = A r_1^2 (g_R^e)^2 [(1 + \cos\theta)^2 T_2 + (1 - \cos\theta)^2 T_1], \tag{7}$$

where A is a numerical constant $(= m_Z^2/64\pi^2)$, and T_1, T_2 are given by

$$T_1 = N_c [r_1 (g_L^f)^2 + 2 r_2 (g_L^f g_L^{q\,\prime})], \tag{8}$$

$$T_2 = N_c [r_1 (g_R^f)^2 + 2 r_2 (g_R^f g_R^{q\,\prime})]. \tag{9}$$

In the above formulae, N_c is the relevant color factor, which is 1 for leptons and $3(1 + \alpha_s(m_Z^2)\pi^{-1} + 1.409\alpha_s^2(m_Z^2)\pi^{-2} - 12.77\alpha_s^3(m_Z^2)\pi^{-3})$ for quarks. The right- and the left-handed fermion couplings are related to the vector and axialvector couplings in the conventional way:

$$g_V = \frac{1}{2}(g_L + g_R), \quad g_A = \frac{1}{2}(g_L - g_R). \tag{10}$$

From eqs. (6) and (7), it is clear that only those observables which involve third generation fermions in the final state will be modified. Thus,

the forward-backward electron asymmetry A^e_{FB} or the partial width $\Gamma(Z \to e^+e^-)$ retain their SM values, while observables like Γ_Z, A^b_{FB}, $\mathcal{P}_\tau$, R_b (and other partial widths) will have contributions coming from the $Z - Z'$ mixing. Low-energy observables are not sensitive to this mixing as the Z-propagator, apart from $-ig_{\mu\nu}$, is real, and the interference term vanishes. Lepton universality is also not respected in this model. The expressions for the modified observables follow immediately from eqs. (1) and (2); however, they do not throw much light on the nature of the modification, as one has to take account of seven arbitrary $Z'q\bar{q}$ couplings (three in the lepton sector and four in the quark sector). Here we impose the condition that the Z' current has to be anomaly free. This assures that no new fermions are required in the model and eq. (5) remains unchanged. A simple way to do that is to take the new couplings proportional to the hypercharge Y of the corresponding fermions (this is, by no means, the only choice). Denoting this proportionality constant by a, we obtain

$$(g_L^{\nu_\tau\,\prime}, g_L^{\tau\,\prime}, g_R^{\tau\,\prime}, g_L^{t\,\prime}, g_R^{t\,\prime}, g_L^{b\,\prime}, g_R^{b\,\prime}) = (-a, -a, -2a, a/3, 4a/3, a/3, -2a/3). \quad (11)$$

The total e^+e^- annihilation cross-section at $s = m_Z^2$ changes by an amount $\delta\sigma$, which is also a measure of the change in Γ_Z. With the couplings given in eq. (11), this change comes out to be

$$\frac{\delta\sigma}{\sigma} = \frac{\delta\Gamma_Z}{\Gamma_Z} = -8.76 \times 10^{-3} \frac{a^2}{1 - \zeta^2} \quad (12)$$

where we have taken $G = 1.16639 \times 10^{-5}$ GeV^{-2}, $m_Z = 91.189$ GeV and $\Gamma_Z = \Gamma_Z^{SM} = 2.497$ GeV. Note that eq. (12) is independent of the sign of

6

a; this is because $Z'q\bar{q}$ couplings always come in pair, one being the internal $Z't\bar{t}$ coupling. It depends on the sign of ζ, and for $m_{Z'} > m_Z$, the deviation is positive. From the experimental bound

$$\frac{\delta\Gamma_Z}{\Gamma_Z} \leq 3 \times 10^{-3}, \tag{13}$$

one gets

$$-0.34 \leq \frac{a^2}{1 - \zeta^2}, \tag{14}$$

which, for $a = 1$, yields $m_{Z'} \geq 181$ GeV. The change in the hadronic cross-section is

$$\frac{\delta\sigma_{had}}{\sigma_{had}} = -5.8 \times 10^{-3} \frac{a^2}{1 - \zeta^2} \tag{15}$$

which is well within the allowed limit, and can be used to find the change in R_b:

$$\begin{aligned} R_b &= R_b^{SM} + (1 - R_b^{SM}) \frac{\delta\Gamma(Z \to b\bar{b})}{\Gamma(Z \to hadrons)} \\ &\leq 0.2172. \end{aligned} \tag{16}$$

The SM value of R_b, 0.2156, is for $m_t = 175$ GeV and takes the two-loop corrections induced by the heavy top quark into account [7]. Branching fraction for charm, R_c, is reduced, but not very significantly:

$$\frac{\delta R_c}{R_c} \geq -0.0020. \tag{17}$$

The change in forward-backward b asymmetry is small, and negative:

$$\frac{\delta A_{FB}^b}{A_{FB}^b} = 0.0130 \frac{a^2}{1 - \zeta^2} \tag{18}$$

whereas for the τ-lepton, the fractional change in the left-right asymmetry $\delta A_{LR}^{\tau}/A_{LR}^{\tau}$ is negative, and thus more than resolves the discrepancy of the experimental value with the SM prediction:

$$\frac{\delta A_{LR}^{\tau}}{A_{LR}^{\tau}} = -0.3637. \tag{19}$$

We note that in all these cases, the changes are in the right direction, and more often than not, are in the right ballpark. However, the lepton-universality breaking ratio, $\Gamma(Z \to \tau^+\tau^-)/(Z \to e^+e^-)$, does not allow such a high value of $a^2/(1-\zeta^2)$:

$$\frac{\Gamma(Z \to \tau^+\tau^-)}{(Z \to e^+e^-)} = 1 - 0.0387\frac{a^2}{1-\zeta^2} \leq 1.013. \tag{20}$$

Also, the effective number of light neutrino species is enhanced, but within the allowed limit:

$$\delta N_{\nu} = -0.0493\frac{a^2}{1-\zeta^2} \geq +0.016. \tag{21}$$

Thus, the upper bound of $a^2/(1-\zeta^2)$ is one order of magnitude smaller than that allowed by Γ_Z. As Holdom has pointed out [2], if the $Z'\tau^+\tau^-$ coupling is dominantly vectorial in nature, the bounds obtained from the last two equations can be evaded.

From eqs. (6) and (7), it is evident that A_{LR} does not change. This motivates us to move to our second model, where Z' couples to all the known fermions. The condition of anomaly cancellation hints to a coupling pattern as shown in eq. (11), but the a's may be different for different generations. Thus, we are introducing three new parameters in this case compared to one

in the earlier case. Evidently, it will be easier to match the experimental data by adjusting these parameters; on the other hand, predictive power of the model will be somewhat lost. However, there are certain model-independent facts which one should take into account.

First, the Born graph, $e^+e^- \to f\bar{f}$ mediated by Z', will not contribute to the interference, and therefore the new physics contribution to the tree-level amplitude will be suppressed by a factor of $1/\zeta^2$. Second, if all the a_i's ($i = 1, 2, 3$) are same, there will be no lepton non-universality, and it is possible to tune the a_i's in such a way that the non-universality remains within the allowed limit, while keeping other predictions more or less intact. Third, even for $\zeta > 1$, the shift in the total cross-section at the Z-peak, $\delta\sigma_{tot}$, can be either positive or negative.

Eqs. (6) and (7) are now modified to

$$
\begin{aligned}
\sigma_L(\theta) &= Ar_1[(1 + \cos\theta)^2\{(g_L^e)^2 T_1 + (g_L^e g_L^{e\,\prime}) T_2\} \\
&\quad + (1 - \cos\theta)^2\{(g_L^e)^2 T_3 + (g_L^e g_L^{e\,\prime}) T_4\}],
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
\sigma_R(\theta) &= Ar_1[(1 - \cos\theta)^2\{(g_R^e)^2 T_1 + (g_R^e g_R^{e\,\prime}) T_2\} \\
&\quad + (1 + \cos\theta)^2\{(g_R^e)^2 T_3 + (g_R^e g_R^{e\,\prime}) T_4\}],
\end{aligned}
\tag{23}
$$

where

$$
\begin{aligned}
T_1 &= N_c r_1 (g_L^f)^2 + 2r_2 (g_L^f g_L^{f\,\prime}), \tag{24} \\
T_2 &= 2N_c r_2 (g_L^f)^2, \tag{25} \\
T_3 &= N_c r_1 (g_R^f)^2 + 2r_2 (g_R^f g_R^{f\,\prime}), \tag{26} \\
T_4 &= 2N_c r_2 (g_R^f)^2. \tag{27}
\end{aligned}
$$

First let us assume, for simplicity, $a_1 = a_2 = a_3 = a$. The limiting value of $a^2/(1 - \zeta^2)$, as obtained from $\delta\Gamma_Z/\Gamma_Z$, is more constrained compared to model 1:

$$\frac{a^2}{1 - \zeta^2} \geq -0.069 \tag{28}$$

leading to $m_{Z'} \geq 446$ GeV for $a = 1$. Unfortunately, δA_{LR} is negative ($= -0.0065$), and so this choice fails to be the desired one. However, if one puts $-a_1 = a_2 = a_3 = a$, the total cross-section decreases (for $\zeta > 1$), and from the experimental bound, one obtains

$$\delta A_{LR} = 0.015 \tag{29}$$

which explains the trend of the SLAC result perfectly.

One must comment about the other observables, none of which are much affected, due to the highly constrained value of $a^2/1 - \zeta^2$. The change in R_b, for the latter choice of a's, is positive, and the result is in agreement with the experimental data.

Thus, both these models allow FCNC processes, forbidden in the SM. For the second model, one needs different a_i's (and thus the maximum splitting between the a_i's can be restricted). The processes now allowed include GIM-violating Z-decays, and tree-level $B_d - \bar{B}_d$ (and $B_s - \bar{B}_s$) mixing. However, the inherent uncertainties limit the usefulness of such processes in detecting a new gauge boson indirectly.

In this letter, we show that the trend of some of the present experimental data, which may indicate a deviation from the SM, can be explained by

considering a heavy neutral gauge boson Z'. A crucial role is played by the heavy top quark which ensures a significant contribution from the interference term in the $e^+e^- \to f\bar{f}$ amplitude. Two models are considered; one in which Z' couples only to the third generation fermions and another in which it couples to all the three generations. The first model allows a lower value of $m_{Z'}$. Guided by the anomaly cancellation conditions of the new gauge boson, we find that the shifts in the measured observables are *always* in the right direction. We expect that these results may motivate a search, direct or indirect, for Z' in the future colliders.

Appendix 1

The two-point function (fig. 2), $i\Pi_{\mu\nu}$, can be written as

$$i\Pi_{\mu\nu}(m_1, m_2, \lambda, \lambda') = \frac{i}{4\pi^2} \int_0^1 dx[\Delta + \ln(\mu^2/M^2)]$$

$$\times [2(1 + \lambda\lambda')x(1-x)q_\mu q_\nu + (1 + \lambda\lambda')(-2x(1-x)q^2 + m_1^2 x + m_2^2(1-x))g_{\mu\nu}$$

$$-(1 - \lambda\lambda')m_1 m_2 g_{\mu\nu}], \tag{A.1}$$

where

$$\Delta = \frac{1}{\epsilon} - \gamma + \ln 4\pi, \tag{A.2}$$

and

$$M^2 = -q^2 x(1-x) + m_1^2 x + m_2^2(1-x). \tag{A.3}$$

The vertex factors are $\gamma_\mu(1 - \lambda\gamma_5)$ and $\gamma_\nu(1 - \lambda'\gamma_5)$ respectively.

Neglecting $q_\mu q_\nu$ terms (they vanish if external fermions are massless), and putting $m_1 = m_2 = m$, we get

$$f(m, \lambda, \lambda') = -\frac{1}{2\pi^2}\left[(1 + \lambda\lambda')\left\{(\Delta + \ln\mu^2)(\frac{1}{6}q^2 - \frac{1}{2}m^2) + q^2(I_1 - I_2) + \frac{1}{2}m^2 I_3\right\}\right.$$

$$\left. + (1 - \lambda\lambda')\left\{(\Delta + \ln\mu^2)\frac{m^2}{2} - \frac{1}{2}m^2 I_3\right\}\right] \tag{A.4}$$

where

$$I_1, I_2, I_3 = \int_0^1 dx(x^2, x, 1)\ln M^2, \tag{A.5}$$

and

$$\Pi_{\mu\nu} = if g_{\mu\nu}. \tag{A.6}$$

For $m \geq q/2$, the expressions for the I's are

$$I_1 = \frac{\ln m^2}{3} - \frac{2}{3}\left[\frac{13}{12} - \frac{m^2}{q^2} - \left(\frac{5}{4}\frac{m^2}{q^2} - \frac{m^4}{q^4} - \frac{1}{4}\right)\frac{2}{\eta}\tan^{-1}\frac{1}{2\eta}\right], \tag{A.7}$$

12

$$I_2 = \frac{\ln m^2}{2} - \left[1 - 2\eta \tan^{-1} \frac{1}{2\eta}\right], \tag{A.8}$$

$$I_3 = \ln m^2 - 2 + 4\eta \tan^{-1} \frac{1}{2\eta}, \tag{A.9}$$

where

$$\eta = (m^2/q^2 - 1/4)^{1/2}. \tag{A.10}$$

In the text, we use the $\overline{MS}$ scheme and take the subtraction point $\mu = m_Z$ to obtain the numerical values.

References

[1] A. Kundu and B. Mukhopadhyaya, hep-ph/9507305

[2] B. Holdom, Phys. Lett. **B339** (1994), 114

[3] F. Caravaglios and G.G. Ross, Phys. Lett. **B346** (1995), 159

[4] P. Langacker and M. Luo, Phys. Rev. **D45** (1992), 278

[5] Review of Particle Properties, Phys. Rev. **D50** (1994), 1173

[6] F. Abe *et al* (CDF Collaboration), Phys. Rev. Lett. **74** (1995) 2626; S. Abachi *et al* (D0 Collaboration), Phys. Rev. Lett. **74** (1995) 2632

[7] J. Erler and P. Langacker, Phys. Rev. **D52** (1995) 441

Figure Captions

1. $Z - Z'$ mixing mediated by t loop.

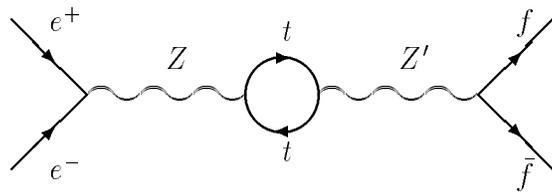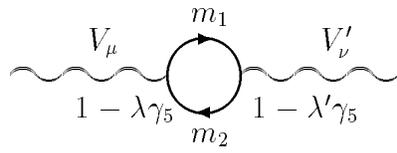2. The two-point gauge boson vacuum polarization diagram.

Fig. 1

Fig. 2