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Electrical tuning of intersubband transition in a semiconductor quantum ring

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In this paper, the intersubband transition energy in an *n*-type semiconductor quantum ring has been investigated in the presence of an electric field perpendicular to the plane of the ring. The analysis has been done considering the effect of band nonparabolicity of the semiconductor. The results show that at high electric field energy varies nonlinearly with field and the optical transition between the two lowest quantized subbands can be controlled by the electric field. It has also been shown how this fine wavelength tuning by electric field depends on the band gap of the semiconductor. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3087480]

I. INTRODUCTION

In recent years, there has been a great deal of interest among researchers in the study of semiconductor nanostructures including quantum dots or boxes and quantum rings mainly because of the rapid advancement in the nanofabrication technology.[1–3] In nanostructures, dimensions (one or more) are comparable to or less than the de Bröglie wavelength[4,5] (few nanometers) and the electron motion along the reduced dimensions becomes restricted resulting in the quantization of energy states.[6,7] The restricted motion of the carriers leads to quantization of the energy levels and produces significant changes in both microscopic and macroscopic properties of the structure. A quantum dot structure with a "hole" at the middle is called quantum ring, and this particular class of nanostructures is being intensively investigated due to the possibility of experimental observations of the Aharonov–Bohm effect in such structures. In quantum ring structures, electrons are confined in all directions resulting in the quantization of their energy states. A number of theoretical and experimental investigations of their electrical and optical properties have already been reported.[8–12] The strong optical transition from valance band to conduction band is the basis for applications of quantum dot (ring) as laser emitters, detectors, storage devices, and fluorescent markers.[12–15]

Thanks to the advanced growth and lithography techniques, quantum rings of controllable thickness and predefined lateral dimensions can be formed on the epitaxial layer. The all-important energy states are functions of the size and shape of the nanostructure. In addition, an externally applied electric field alters the potential energy profile of the structure, and accordingly the positions of the energy levels also change. These affect the tunneling probability of electrons, and have important roles in the functioning of resonant tunneling devices made of quantum rings. Optical transition between two subbands has recently got much importance because of its numerous as well as specialized applications (such as medical diagnosis, environmental pollution monitoring, and food quality control) in the medium wavelength and long wavelength infrared regions of electromagnetic spectra. The characteristic wavelength is very important in these cases, e.g., it may carry the signature of a particular virus. The transition energy (and, hence, the wavelength) can be changed by choosing different sizes of the device, but once the device is made, then this transition energy is fixed. The only possibility is by adopting external means to change this transition energy. In this paper, we have investigated how an external electric field can be utilized in controlling the transition energy between the two subbands in a semiconductor quantum ring. A very preliminary result of the study has been reported by the authors elsewhere.[16] Semiconductor quantum dots/rings fabricated are, in many cases, made of narrow-gap semiconductor, where band nonparabolicity is an important issue. This paper also shows how the intersubband tuning by electric field depends on the band gap of the semiconductor.

The remaining sections of this paper are organized as follows. In Sec. II, a theoretical background is described. The results and discussion are given in Sec. III. Finally, in Sec. IV, a conclusion is given.

II. THEORETICAL BACKGROUND

Let us consider the schematic structure of a cylindrical quantum ring shown in Fig. 1. We assume that an electric field (F) is applied along the axis of the ring. Using the cylindrical coordinate system (ρ, θ, z), the time-independent Schrödinger equation for the electron wave function Ψ can be written as

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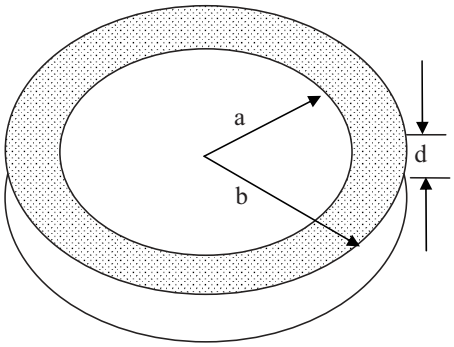

FIG. 1. Schematic structure of a cylindrical quantum ring. The inner and outer radii are a and b, respectively. The thickness of the ring is d.

$$-\frac{\hbar^2}{2m^*}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Psi}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2\Psi}{\partial\rho^2}+\frac{\partial^2\Psi}{\partial z^2}\right] - \alpha\left[-\frac{\hbar^2}{2m^*}\left\{\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Psi}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2\Psi}{\partial\rho^2}+\frac{\partial^2\Psi}{\partial z^2}\right\}\right]^2 - eF\Psi = E\Psi, \tag{1}$$

for $a \le \rho \le b$ and $z \le d$, where a is the inner diameter, b is the outer diameter, and d is the thickness of the ring. In the above equation, E is the energy eigenvalue, m^* is the effective mass of electron, e is the electronic charge, $\hbar$ is the Dirac constant, and α is the nonparabolicity factor.

To solve the above equation, we assume that the nonparabolicity term perturbs the Hamiltonian of the system. So, we first calculate the energy eigenvalues (E_0) of the unperturbed system (with $\alpha=0$) and then apply the perturbation technique to obtain the total energy of the system.

The solution for the unperturbed wave function is done using the method of separation of variables. Applying the boundary conditions that z-dependent wave function vanishes at $z=0$ and $z=d$, the energy eigenvalues for motion along the z-direction (E_z) can be obtained by solving the following determinant equation:

$$\begin{vmatrix} J_{1/3}\left(\frac{2}{3}\zeta_1^{3/2}\right) & J_{-1/3}\left(\frac{2}{3}\zeta_1^{3/2}\right) \\ J_{1/3}\left(\frac{2}{3}\zeta_2^{3/2}\right) & J_{-1/3}\left(\frac{2}{3}\zeta_2^{3/2}\right) \end{vmatrix} = 0, \tag{2}$$

where

$$\zeta = \left(\frac{2m^*eF}{\hbar^2}\right)^{1/3}\left(z+\frac{E_z}{eF}\right), \tag{3}$$

$\zeta_1(\zeta_2)$ being the value of ζ at $z=0(z=d)$, and $J_{1/3}(J_{-1/3})$ is the Bessel function of first kind[17,18] of order 1/3 (−1/3). Solution of Eq. (2) exists for some discrete values of ζ. So, a quantum number n is associated with E_z, and, henceforth, E_z will be symbolized by E_{zn}.

The ρ-dependent component of the wave function (R) can be determined by solving the following differential equation:

$$\rho^2\frac{\partial^2 R}{\partial\rho^2}+\frac{\partial R}{\partial\rho}+(\lambda^2\rho^2-m^2)R=0 \quad \text{for} \quad a \le \rho \le b, \tag{4}$$

where

$$\lambda^2 = \frac{2m^*(E_0-E_{zn})}{\hbar^2}. \tag{5}$$

Equation (4) is a Bessel differential equation of order m, the general solution of which is given by

$$R = AJ_m(\lambda\rho)+BY_m(\lambda\rho). \tag{6}$$

However, with the application of boundary conditions, $R=0$ at $\rho=a$ ($\rho=b$), the nontrivial solution of R can be obtained when the following determinant relation is satisfied:

$$\begin{vmatrix} J_m(\lambda a) & Y_m(\lambda a) \\ J_m(\lambda b) & Y_m(\lambda b) \end{vmatrix} = 0. \tag{7}$$

The above equation has solution for some discrete values of λ for a particular m and, so, it requires two quantum numbers, m and l (say), which indicates the lth solution of λ for the mth order Bessel function. Thus, the unperturbed energy eigenvalue E_0 contains three quantum numbers (l,m,n) and is given by

$$E_0 = E_{lmn} = \frac{\hbar^2}{2m^*}\lambda_{ml}^2 + E_{zn}. \tag{8}$$

E_{zn} can be determined by solving Eq. (2) numerically using the values of Bessel function for a wide range of arguments. For very small values of electric field, however, the expression for E_{zn} can be simplified as

$$E_{lmn} = \frac{\hbar^2}{2m^*}\lambda_{ml}^2 + \frac{\hbar^2}{2m^*}\left(\frac{n\pi}{d}\right)^2 - \frac{eFd}{2}, \tag{9}$$

which shows the linear variation of energy with field.

Now, we use the perturbation technique to find the energy eigenvalue due to the effect of band nonparabolicity. For this we need the perturbing Hamiltonian operator $\hat{H}'$, which is the α-dependent term on the left hand side of Eq. (1). Assuming only the first order correction due to the perturbation, the corrected eigenvalue can be written as

$$E'_{lmn} = E_{lmn} + H'_{ss}, \tag{10}$$

where

$$H'_{ss} = \frac{\int_V \Psi_{0s}^* \hat{H}' \Psi_{0s} dV}{\int_V \Psi_{0s}^* \Psi_{0s} dV}, \tag{11}$$

Ψ_{0s}^* being the complex conjugate of the unperturbed wave function Ψ_{0s} of state s (lmn). The above integration is taken over the volume of the ring. After calculation of H'_{ss} and using Eq. (10), the energy eigenvalue of the quantum ring of a nonparabolic semiconductor can be written as

$$E'_{lmn} = E_{lmn} - \alpha E_{lmn}^2. \tag{12}$$

We now calculate the energy required for intersubband transition. We assume that the transition occurs between two quantized states arising out of the restricted motion along the z-direction (which is also the direction of the applied electric field). For such intersubband transition, $\Delta n=1$ for $\Delta l=\Delta m=0$. The transition energy between two subbands n and n

+1 has been calculated. As the transition between the lowest two subbands are important in many practical cases, the transition energy for such intersubband transition has been derived and is obtained as

$$\Delta E_1 = 3\frac{\hbar^2}{2m^*}\left(\frac{\pi}{d}\right)^2\left\{1+\frac{2m^*q^2F^2d^4}{32\hbar^2\pi^2}\right\} \times\left\{1-\alpha\left(5\frac{\hbar^2}{2m^*}\left(\frac{\pi}{d}\right)^2-qFd\right)\right\}. \quad (13)$$

The above equation shows the effect of field and the nonparabolicity on the transition energy and is prominent when the field (F) is high. The range of energy over which this transition can be tuned by electric field is given by

$$\delta(\Delta E_1) = 3\frac{\hbar^2}{2m^*}\left(\frac{\pi}{d}\right)^2\left[\frac{2m^*q^2F^2d^4}{32\hbar^2\pi^2} + \alpha\left\{qFd\left(1+\frac{2m^*q^2F^2d^4}{32\hbar^2\pi^2}\right) - 5\frac{\hbar^2}{2m^*}\left(\frac{\pi}{d}\right)^2\frac{2m^*q^2F^2d^4}{32\hbar^2\pi^2}\right\}\right]. \quad (14)$$

This relation shows that this tuning range depends also on the nonparabolicity factor.

III. RESULTS AND DISCUSSION

Using Eq. (12), the energy eigenvalues of electrons in a quantum ring of an n-type semiconductor are computed and plotted in Fig. 2(a) for the lowest three subbands as a function of inner diameter (a) of the ring in the presence and absence of electric field for a fixed width ($w=b-a=10$ nm) of the ring. In general, as the inner diameter of the ring increases, the energy decreases except at the lowest state for which the energy increases with inner diameter. The first three states in this plot correspond to the first three zeros (i.e., solutions for argument $\alpha_{ml}=\lambda_{ml}a$) of Eq. (7) for Bessel function of order zero (i.e., $m=0$). The values of the first zero satisfying Eq. (7) are such that λ_{ml} increases as a increases, and so the energy E'_{101} increases with a [Eq. (8)]. For the higher order zeros of Eq. (7), λ_{ml} decreases with increases in a. This special behavior is arising out of the cylindrical symmetry of the structure. In Fig. 2(b), the lowest three subband energies are shown as function of width (w). The energy decreases as the width w increases. In both Figs. 2(a) and 2(b), we see that the effect of electric field is to lower the energy states but the lowering is independent of a and w. This is because the electric field is applied along the z-direction and decrement in energy with field is determined by the thickness (d) of the ring. So, the energy lowering due to the applied field becomes a function of the thickness of the ring, as shown in Fig. 2(c), where the variation in subband energy states with d is shown in Fig. 2(c). The more the thickness, the more is the lowering of energy due to the electric field.

The variation in the position of energy subbands with electric field in a semiconductor quantum ring is shown in Fig. 3. It can be seen that the energy decreases with electric field linearly at low field and nonlinearly at high field. The

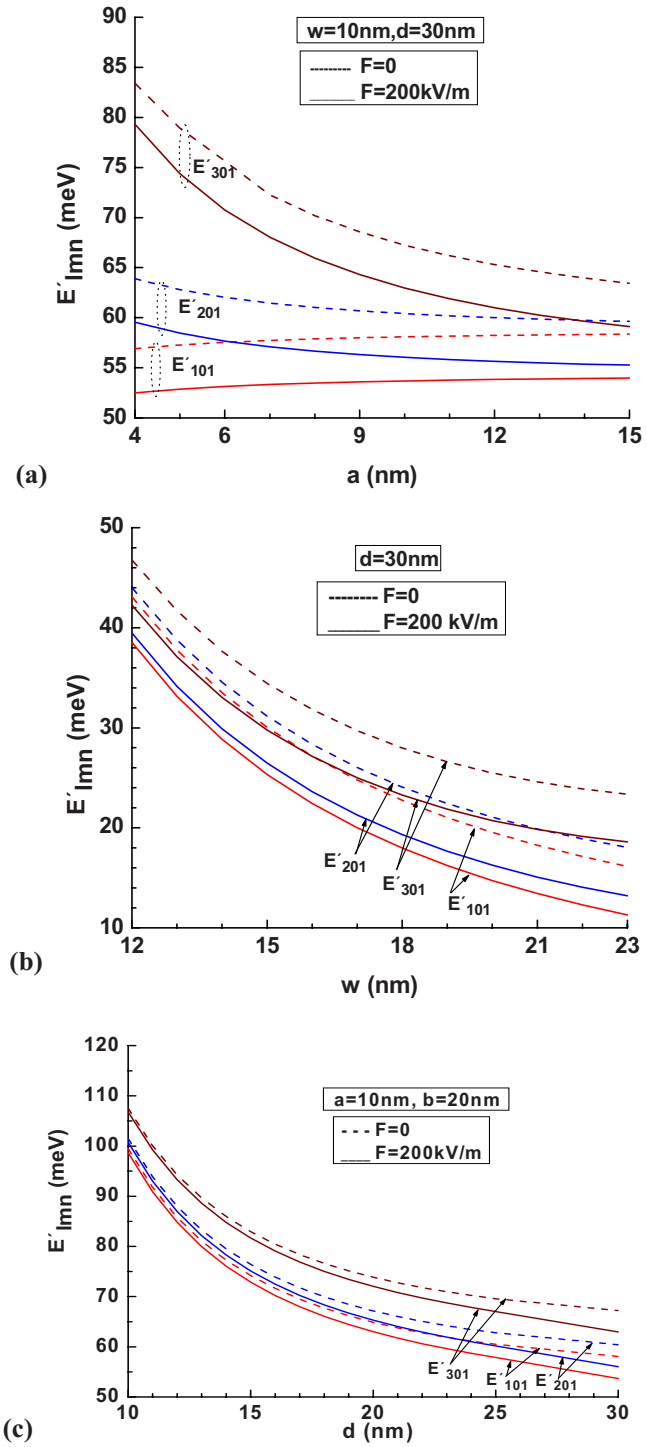

FIG. 2. (Color online) Plot of the three lowest electron energy states (E'_{lmn}) in a cylindrical quantum ring of an n-type semiconductor, (a) as a function of inner radius a for ring thickness d of 30 nm and width w ($=b-a$) of 10 nm, (b) as a function of the width w of the ring for $d=30$ nm, and (c) as a function of the thickness of the ring, where the inner and outer radii are $a=10$ nm and $b=20$ nm, respectively. The electric field (F) is taken to be 200 kV/m (solid lines) and the absence of field is shown by dashed lines.

decrement with field is nonlinear and rapid when the field is high. This nonlinearity is the basis of tuning of intersubband transition energy by electric field. The rate of decrement in energy with electric field is more for larger d.

The transition energy between the lowest subband and the next higher subband satisfying $\Delta n=1$ for $\Delta l=\Delta m=0$ is

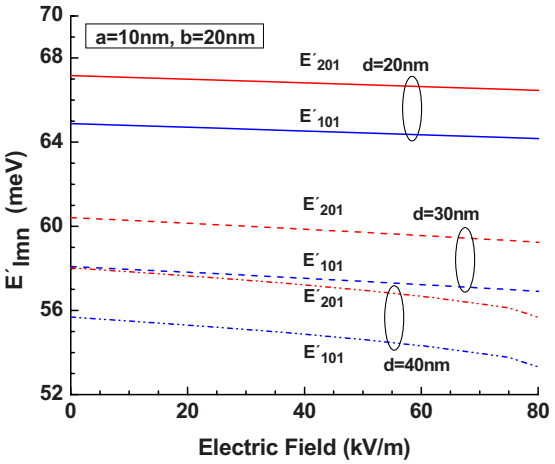

FIG. 3. (Color online) Two lowest energy states of electrons in a cylindrical quantum ring as a function of electric field for $a=10$ nm, $b=20$ nm, and three different values of thickness, $d=20$, 30, and 40 nm.

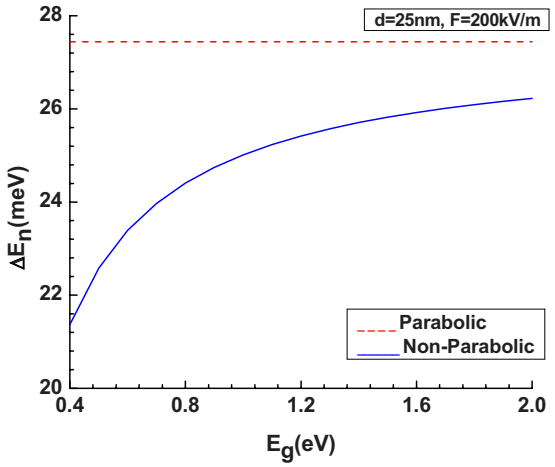

FIG. 5. (Color online) Transition energy between two lowest subbands ΔE_n (i.e., $\Delta E_1=E'_{102}-E'_{101}$) plotted as a function of band gap, E_g, for $F=200$ kV/m and $d=25$ nm. The dashed straight line indicates the result obtained using assumption of parabolicity of band.

plotted in Fig. 4 as a function of electric field. The results are shown for both parabolic and nonparabolic band semiconductors. It can be seen that as electric field increases, the transition energy increases, i.e., blueshift occurs. In general, this change in energy is small, however, more for higher d. This small change in ΔE_n with high electric field indicates that fine-tuning of intersubband transition is possible with electric field. The band nonparabolicity causes a decrease in the transition energy. So, it can be seen that the calculations with assumption of band nonparabolicity may largely overestimate the transition energy.

The effect of band nonparabolicity on the transition energy is shown in Fig. 5, where the transition energy is plotted as a function of the band gap. Nonparabolicity has nearly an inverse relation with band gap. Here, we see that the transition energy increases as the band gap increases, and thus, the overestimation with the parabolic assumption of the band structure is more as the band gap is smaller. So, the calculation using parabolic assumption is nearly valid only when the band gap is large.

The range of intersubband tuning by electric field is shown in Fig. 6. The sensitivity of the tuning range with field increases in the high field region. It can also be seen that wide range of tuning is possible with the choice of higher thickness of the quantum ring. The nonparabolic nature of the band also enhances the tuning range for intersubband transition.

IV. CONCLUSION

Quantized electron states for different dimensions of a quantum ring of an n-type semiconductor have been calculated and plotted for a wide range of electric fields. The subband energy decreases with an increase in the dimensions of the ring. From our study, we see that subband energy decreases with an increase in the electric field showing more pronounced effect at higher thicknesses of the ring, while the effect is independent of the radius of the ring. It has been seen that electric field can be used for fine-tuning of wave-

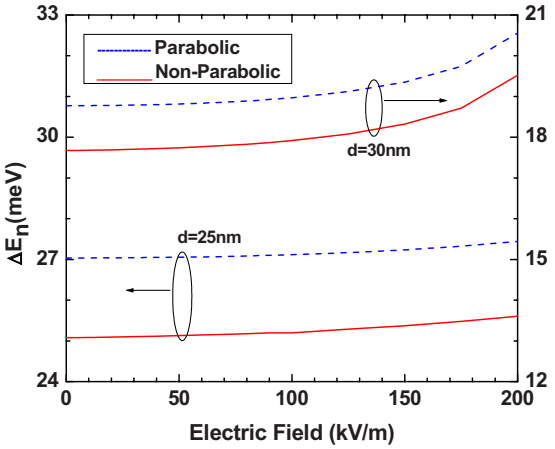

FIG. 4. (Color online) Transition energy between two lowest subbands ΔE_n (i.e., $\Delta E_1=E'_{102}-E'_{101}$) plotted as a function of electric field ranging from $F=0$ kV/m to 200 kV/m, taking $E_g=1.0$ eV for two values of thickness, $d=25$ and 30 nm. The dashed lines show the results with parabolic assumption of the band structure.

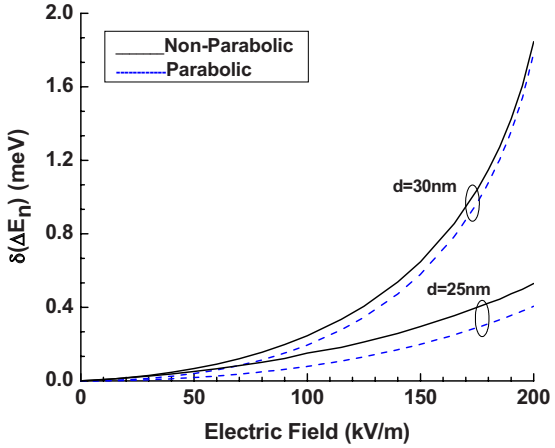

FIG. 6. (Color online) The range of tuning for intersubband transition, $\delta(\Delta E_1)$, between two lowest subbands for transition $(E'_{102}\langle-\rangle E'_{101})$ as a function of electric field taking $E_g=1.0$ eV for two different values of ring thicknesses ($d=25$ and 30 nm). The dashed lines show the results with the assumption of band parabolicity.

length in intersubband optical transitions. The range of tuning can be controlled by choosing suitable thickness of the ring and the applied electric field.

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