EFFECT OF TWO-PION-EXCHANGE THREE-NUCLEON FORCES ON TRINUCLEON BOUND SYSTEMS ☆

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We have calculated the contributions of the two-pion-exchange three-body force mediated by the Δ(1232 MeV) resonance to the trinucleon ground state. The hyperspherical harmonic method has been used. Results show sensitivity to the assumed short-range part of the force.

A better understanding of the nuclear interaction requires the study of the many-body forces and in particular of the effect of the three-body force (3BF) in the trinucleon systems. The realistic two-body forces (2BF) which fit the two-nucleon data fail to reproduce experimental data for H^3 and He^3. For instance, the experimental binding energies (BE) for H^3 (8.482 MeV) and He^3 (7.718 MeV) fail to be reproduced by about 1.5 MeV. The charge from factors, $|F_{ch}(q)|$, show a striking disagreement with experiment: the theoretical momentum transfer at the first minimum is too high and the height of the second maximum too low [1–4]. Sick and collaborators [3] showed that the pointlike proton density distribution, $\rho(r)$, in He^3 [calculated from $F_{ch}(q)$] could present a central depression, in contrast with most calculations using realistic 2BF. Recent works [5–9] conjecture that the inclusion of three-body forces (3BF) should account for some of the above discrepancy in the data.

In this work we perform an accurate calculation where the hyperspherical harmonic (HH) method is used [10]. The Coulomb force and 3BF are taken into account. The Afnan–Tang S3 central potential [11] has been used to represent the two-body force.

A survey of the various versions of two-pion-exchange (TPE) three-nucleon force is given in ref. [12]. These 3BF consist in two terms W_s and W_p generated by the s and p waves of virtual pions, respectively. The W_s component which could be drastically reduced by the σ-meson exchange [13] is not considered here. We use the main contribution to W_p which refers to a Feynman diagram in which the intermediate state of the k-nucleon (the others are i and j nucleons) is the (1232 MeV) Δ-resonance [14]. The effect of 3BF due to more than TPE [15], corresponding to still shorter ranges which should be, to some extent, shadowed by the two-body repulsive core is neglected. We parametrize the unknown short-range part of the force by introducing either a cut-off parameter, x_0 [6,7] or a πNN form factor [12] (this last case is referred to in tables 1,2 and the figures as $x_0 = 0$). Only the space completely symmetrical S state of the trinucleon bound state will be used in this calculation. The effective potential acting on the S state has the form [13]

$$W \equiv \sum_{k=1,2,3} W_p(k)$$

$$= \sum_{k=1,2,3} C_p\,(3\cos^2\theta_k - 1)\,U_{(2)}(x_i)U_{(2)}(x_j)\,, \quad (1)$$

where

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$$U_{(2)}(x) = \left[-\frac{e^{-\mu x}}{\mu x}\left(1+\frac{3}{\mu x}+\frac{3}{(\mu x)^2}\right)\right]$$
$$+\left[\frac{\Lambda^2}{\mu^2}\,\frac{e^{-\Lambda x}}{\mu x}\left(1+\frac{3}{\Lambda x}+\frac{3}{(\Lambda x)^2}\right)\right.$$
$$\left.+\frac{1}{2}\frac{\Lambda}{\mu}\left(\frac{\Lambda^2}{\mu^2}-1\right)e^{-\Lambda x}\left(1+\frac{1}{\Lambda x}\right)\right], \qquad (2)$$

r_i are the nucleon coordinates, $x_k = |x_k| = |r_i - r_j|$, θ_k is the angle between the directions x_j and x_i, C_p is a coupling coefficient, μ is the pion mass and Λ is a constant which parametrizes the πNN form factor [12], ranging from 4 to 5 fm^{-1}. The estimated value of C_p ranges [7] from 0.46 in the earliest version [14] to about 1.3 MeV according to the πNΔ coupling constant used [12]. In the earliest version of the 3BF [13] the form factor has been omitted ($\Lambda = \infty$), and since at very short distances the effect of πN vertex factor, ρ-exchange, etc., become important [16], we have also studied the effect of introducing a purely phenomenological cut-off parameter [6,7], x_0, in eq. (2) for $\Lambda = \infty$:

$$U_{(2)}(x) = \bar{U}_{(2)}(x_0) \quad \text{for } x < x_0\,,$$
$$= \bar{U}_{(2)}(x) \quad \text{for } x > x_0\,. \qquad (3)$$

To differentiate the two forces we use a bar over the operators when $\Lambda = \infty$. W is negative for the equilateral triangle configuration and is positive for the aligned configuration of the nucleons [5]. Thus the triangle configuration which is favoured compared to the aligned one could lead to a central depression in the point-like proton density, $\rho(r)$.

The HH [10] method is particularly suited for investigating the effect of W (or $\bar{W}$) on the BE and $F_{ch}(q)$ of trinucleon systems, since in this method the Schrödinger equation has the same structure with or without 3BF. The space functions are expanded in a complete set of hyperangular functions, the so-called HH [10]. The partial waves $\phi_K(r)$ of the wave function, where r is the hyperradial variable, satisfy an infinite set of coupled differential equations [10]:

$$[-d^2/dr^2 + (2K+3/2)(2K+5/2)/r^2 + k^2]\,\phi_K(r)$$
$$+\sum_{K'}\langle K|V(r,\Omega)+W(r,\Omega)|K'\rangle\phi_{K'}(r) = 0\,, \qquad (4)$$

Table 1
Binding energy (BE), charge form factor for a momentum transfer 1 fm^{-1} [$F_{ch}(1)$], position of the first zero (q_0^2) and magnitude of the maximum (F_{max}) of $F_{ch}(q^2)$ using 2BF alone

	BE	$F_{ch}(1)$	q_0^2 (fm^{-2})	$F_{max} \cdot 10^3$
^{3}H(2BF)	−6.49	0.590	16.0	1.50
^{3}He(2BF)	−5.79	0.565	15.9	1.06

where $2K$ is the grand orbital quantum number. The 2BF, $V(r, \Omega)$, is the sum of three pairwise interactions. The 3BF, $W(r, \Omega)$, also contains three terms, each representing a Δ-resonance on the k-th nucleon line ($k = 1,2,3$).

For practical purposes, the set of equations (4) is truncated to a finite number of partial wave (the upper value of K is estimated according to the convergence of the solution [10]). The coupled equations (4) have been solved using the adiabatic approximation [17].

In the calculations we have taken 12 hyperspherical harmonics ($K = 0, 2, 3, \ldots, 12$) and 5 multipoles of 3BF have been included, leading to a well converged solution [10]. Results show that both BE and $F_{ch}(q)$ depend strongly on the cut-off parameter, x_0, in eq. (3). If we set as criterion for the value of x_0 that it should be around the hard-core radius of the Reid hard-core potential, then x_0 cannot be taken as a free phenomenological parameter enabling one to fit experimental data. Table 1 refers to the values obtained with the 2BF(S_3) alone for the binding energy, the charge form factor $F_{ch}(q)$ for the momentum transfer $q = 1$ fm^{-1} (related by $a \simeq \{6[1 - F_{ch}(1)]\}^{1/2}$ to the rms charge radius), the position q_0^2 of the first minimum and the magnitude F_{max} of the first maximum of $|F_{ch}(q)|$.

Table 2 exhibits the change of these observables induced by the introduction of the 3BF in the interaction in terms of the parameters Λ, C_p and x_0. The last line (called Needed) refers to an average over the difference between the results obtained with several realistic 2BF alone and the experimental data. In fig. 1, we plot $|F_{ch}(q)|$ for He^3 for the sets of parameters retained in table 2. In fig. 2 the He^3 proton density is drawn for the same sets of parameters. There is controversy about the existence of a hole in the proton density

Table 2
Increases of binding energy [ΔE = BE(2BF) − BE(2BF + 3BF)], $F_{ch}(1)$, q_0^2 and F_{max} generated by the introduction of 3BF for the parameters indicated in the first columns. The last row shows an average over the difference between experimental data and the results obtained using realistic two-body potentials.

	Λ^2 (fm^{-2})	C_p (MeV)	x_0 (fm)	ΔE (MeV)	$-\Delta F_{ch}(1)$	Δq_0^2 (fm^{-2})	$\Delta F_{max} \cdot 10^3$
^{3}H(2BF + 3BF)	25	1.32	0	0.27	0.023	0.07	0.15
	17			0.10	0.019	0.17	0.10
	∞	0.9	0.340	1.17	0.027	−0.49	0.44
^{3}He(2BF + 3BF)	25	1.32	0	0.27	0.043	0.06	0.54
	17			0.10	0.039	0.14	0.50
	∞	0.9	0.340	1.13	0.027	0.38	0.33
		0.46	0.277	0.70	0.016	0.37	0.52
Needed				1.3 ± 0.2	0.05 ± 0.01	4 ± 1	4.5 ± 0.5

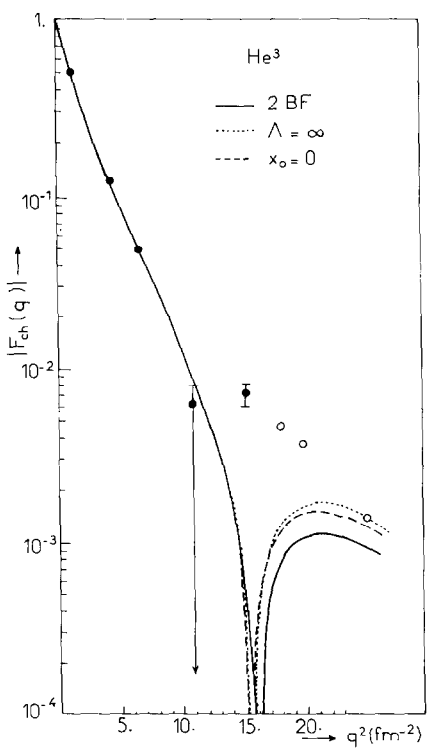

Fig. 1. Calculated charge form factor for ^{3}He using for $\Lambda = \infty$: C_p = 0.46 MeV and x_0 = 0.277 fm. For x_0 = 0 and C_p = 1.32 MeV the curves for Λ^2 = 17 and 25 fm^{-2} are too close to be drawn separately. For both forces μ = 0.7 fm^{-1}. The experimental data were taken from refs. [3,18] (closed circles) and from ref. [19] (open circles).

[3] able to explain the large amplitude of F_{max} which cannot be reached using realistic two-body nuclear potentials alone. In principle the 3BF mediated by the Δ-resonance could generate a hole [5], but we observed (see fig. 2) that the realistic 3BF is not strong enough to produce this hole in the proton density.

Nevertheless, it appears from the comparison between the data of the last line of table 2 and the contribution brought by the 3BF that, in agreement with results obtained in ref. [8], the trend is in the right direction, but only too small improvements are obtained. It means

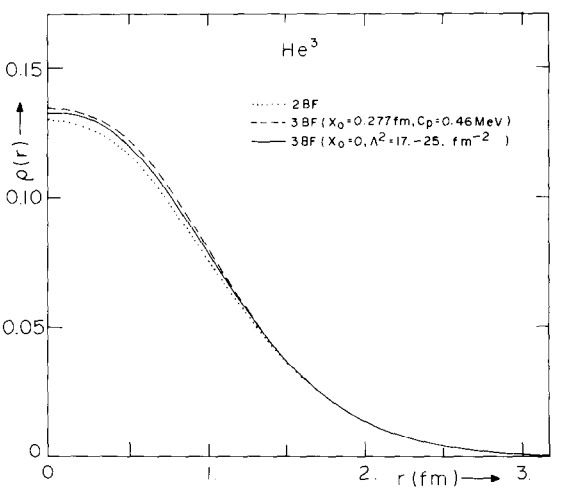

Fig. 2. Proton density for ^{3}He using the same parameters as for the form factors.

that the effect on the space fully symmetric S state of the 3BF W_p alone cannot explain all the discrepancy between the experimental data and the results obtained using exclusively the 2BF. The occurence of other 3BF such as W_s and 3BF arising from $\pi\rho$ and $\rho\rho$ exchange [16] are needed to explain the binding energy and meson-exchange currents should contribute much to $F_{ch}(q)$ [20].

Of course our results should be checked by a more realistic calculation using realistic 2BF and introducing the contribution of the D state in the wave function. But this last improvement should not change the results much, at least for the charge form factor which is only slightly sensitive to the D state because there is no cross term between the main S state and the D state in the form factor.

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