Effect of secondary electron emission on the propagation of dust acoustic waves in a dusty plasma

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Effect of secondary electron emission on the propagation of dust acoustic waves in a dusty plasma

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The effect of secondary electron emission on dust acoustic wave (DA) propagation has been investigated based on orbit motion limited theory of dust grain charging. The emitted secondaries are assumed to have the same temperature as that of the ambient plasma electrons so that the plasma effectively consists of three components: the ions, electrons, and the variable charge dusts. Together with the effect of secondary emission, the effect of ion and electron capture and ionization of neutral atoms and recombination have been included in the ion and electron fluid equations. Small amplitude perturbation is considered about a charge neutral steady state. It is seen that if the dust charge is positive there may occur under certain conditions zero frequency exponentially growing perturbation about the equilibrium. The possibility of the occurrence of such purely growing mode in a dusty plasma was not noted earlier. The frequency and damping decrement of DA waves in dusty plasmas with negatively charged dust and also of DA waves with positively charged dust, when they exist, are determined. Comparisons with corresponding results of DA waves in the absence of secondary emission are exhibited graphically.
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I. INTRODUCTION

The dust grains immersed in a plasma become highly charged due to collision with ambient electrons and ions; the presence of such highly electrically charged heavy mass dust particles causes the sustenance of very low frequency modes known as dust acoustic (DA) modes[1] and also affects the ion acoustic mode—this new mode being called the dust ion acoustic mode.[2] Both linear and nonlinear aspects of the propagation of these modes including the effect of dust charge variation have been studied for the last several years.[3–13] The impact with the incident plasma ions and electrons are the two principal mechanisms of dust charging, the magnitude of the ion and electron current flowing to the dust grain surface being in each case calculated on the basis of orbital motion limited (OML) theory.[14] Though the above-mentioned processes are indeed the more important ones, the dust particle may also get charged due to thermionic or photoelectric emission and their effects on very low frequency wave propagation in a dusty plasma have recently been studied.[15,16]

If the plasma electrons have sufficiently high energy, then there occurs another possible mechanism of dust charging. Electrons hitting a single dust grain may ionize the dust material resulting in ejection of electrons producing the secondary electron current. This is equivalent to the flow of a positive current to the dust surface. The secondary electron yield depends on the nature of the dust material and the kinetic energy of the incident electrons. Draine and Salpeter[17] gave corrections for dust size. Chow *et al.*[18] and Nemeck *et al.*[19] have investigated secondary electron production using different models for primary electron-dust impact geometry for a variety of dust size and compositions for a dust grain bombarded by monoenergetic electron beam. Fair agreement with the experimental diagnostics of Pavlu *et al.*[20] and Zilavy *et al.*[21] are obseved such that secondary electrons have small energies and cannot produce new secondaries.

However for analytical treatment of the problem at hand, namely the effect of secondary electron emission on dust acoustic waves, it is imperative that we use an analytic expression for secondary electron current from the dust grain as well as for the ambient ion and electron current to the dust grain surface. As a consequence our choice becomes restricted to the orbital motion limited (OML) theory based expression for the above currents. According to OML theory the dust charging current due to the flow of ions and electrons of the ambient plasma to the dust grain surface are, respectively,

$$I_i = eJ_i = \pi a^2 e \sqrt{\frac{8T_i}{\pi m_i}} n_i \begin{cases} \exp\left(\frac{-eq_d}{aT_i}\right), & q_d>0 \\ \left(1-\frac{eq_d}{aT_i}\right), & q_d<0, \end{cases} \tag{1}$$

$$I_e = -eJ_e = -\pi a^2 e \sqrt{\frac{8T_e}{\pi m_e}} n_e \begin{cases} \left(1+\frac{eq_d}{aT_e}\right), & q_d>0 \\ \exp\left(\frac{eq_d}{aT_e}\right), & q_d<0, \end{cases}$$

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where q_d is the charge on the dust grain surface. The current resulting from secondary electron emission[22] is (secondary electron temperature $T_s = T_e$ assumed)

$$I_e^s = eJ_e^s = 3.7\delta_M J \begin{cases} \exp\left(\frac{eq_d}{aT_e}\right) F_5(E_M/4T_e), & q_d<0 \\ \left(1+\frac{eq_d}{aT_e}\right) F_{5,B}(E_M/4T_e), & q_d>0, \end{cases} \tag{2}$$

where

$$J = \pi a^2 e \sqrt{\frac{8T_e}{\pi m_e}} n_e \tag{3}$$

and the secondary yield function $\delta(E)$ is approximated[23] by the relation

$$\delta(E) = 7.4\delta_M \frac{E}{E_M} \exp\left[-2\sqrt{\frac{E}{E_M}}\right] \tag{4}$$

so that δ_M is the maximum yield which occurs at $E = E_M$. Both δ_M and E_M are constants and are dependent on the dust material. There exists good agreement between Sternglass formula and the findings of Nemeck *et al.*[19] T_s is the secondary electron temperature. The functions $F_5(x)$ and $F_{5,B}(x)$ are given by

$$F_5(x) = x^2 \int_0^\infty u^5 \exp[-(xu^2+u)]du,$$

$$F_{5,B}(x) = x^2 \int_B^\infty u^5 \exp[-(xu^2+u)]du, \tag{5}$$

where

$$x = \frac{E_M}{4T_e}, \quad B = \sqrt{\frac{\frac{eq_d}{aT_e}}{x}}.$$

Since the electron thermal velocity is much higher than the ion thermal velocity, the equilibrium state dust surface charge acquires a negative value when I_i and I_e are the only charging currents. The inclusion of the secondary electron into the current balancing equation may shift the equlibrium state dust charge from a negative to a positive value depending on the values of the dust parameters δ_M, E_M, etc.

In this paper we have studied the effect of secondary electron emission by charged dust grain on dust acoustic wave propagation. In general the temperature of the emitted secondary electrons is different from the temperature of the ambient plasma electrons. Here we have assumed that they have the same temperature so that we have a simpler system: the dusty plasma remains a three component system with variable dust charge for otherwise we would have to consider a fourth component, namely secondary electrons which would be distinct from the ambient plasma electrons by having a different temperature. The dust charge may be positive or negative depending on the rate of electron emission from the dust grain. Besides the possibility of the shift in the algebric sign of the dust charge it also necessitates the inclusion of the effect of the increase in electron number density—in the equations of motion. To maintain uniformity it becomes imperative to include the effect of the decrease in the number density of the ambient plasma ions and electrons resulting from the charging of the dust grains through ion and electron capture. A consistent system of fluid equations for the ions and electrons incorporating the above-noted effects was formulated by Varma.[24] The effect of ions and electron capture/release by dust grains on dust acoustic[25] and dust ion acoustic waves[26] were also considered, though in a different setting, by including appropriate terms accounting for particle loss/gain as well as momentum loss in the fluid equations. But these did not take into account the effect of secondary electrons or the effect of the change in the electron and ion capture rate by the dust grains due to the dust density and dust charge variations which are always associated with propagation of DA waves.

In Sec. II we study the different possible equlibrium charge states when secondary electron emission takes place but these are assumed to have the same temperature as that of the ambient plasma electrons and so are not distinguishable from the latter. Section III is devoted to the fluid description of plasma ions and electrons taking into account the effect of ion and electron capture by the dust grains and secondary emission from them. Section IV presents the dispersion relation. If the dust charge is positive then it is seen that the DA wave disappears under certain conditions and in its place there occurs a zero frequency purely growing mode. Results and discussions are presented in Sec. V.

II. MULTIPLE DUST CHARGE STATES

The governing equation for the variation of charge q_d on the dust grain surface is

$$\frac{\partial q_d}{\partial t} + v_d \frac{\partial q_d}{\partial x} = I_i + I_e + I_e^s = I_{\text{tot}}. \tag{6}$$

Here we have assumed that the secondary electrons emitted by the dust have the same temperature as that of the ambient electrons ($T_s = T_e$) and therefore are not distinguishable from them with the consequence that the right-hand side (rhs) of Eq. (6) gives the total current to the dust grain surface.

The equilibrium dust charge state $q_d = q_{d0}$, with $n_i = n_{i0}$, $n_e = n_{e0}$ satisfies

$$I_{\text{tot}}(q_d = q_{d0}, n_i = n_{i0}, n_e = n_{e0}) = 0. \tag{7}$$

When $I_e^s = 0$, the dust surface is negatively charged. When $I_e^s \neq 0$, depending on the values of the different plasma and dust parameters, Eq. (7) may possess multiple roots defining different charge states with varying stability properties.[27] The equilibrium state $q_d = q_{d0}$, $n_e = n_{e0}$, $n_i = _{i0}$ is stable provided $(\partial I_{\text{tot}}/\partial q_d)_{\text{eq}} < 0$ and unstable when the same is >0. For sufficiently weak or strong secondary electron emission there exists only one equilibrium charge state with $q_d < 0$ or $q_d > 0$. For secondary electron emission of intermediate strength, three equilibrium charge states may exist.[27–30] The lowest charge state is negative while for the highest one the dust charge is positive, and both are stable states. The intermediate state dust charge is also positive but it is unstable as

$(\partial I_{\rm tot}/\partial q_d)_{\rm eq}>0$. Any small change in $I_{\rm tot}$ shifts this dust charge state to the neighboring stable one: $q_{d0}=-Z_{d0}e$ or the higher positive charge state $q_{d0}=+Z_{d0}e$. In this paper, we consider small amplitude perturbation about these two stable states and the resulting very low frequency behavior.

A. Case A

First we consider the positive charge state

$$(q_d)_{\rm eq}=q_{d0}^{(+)}=+Z_{d0}e. \tag{8}$$

Setting $n_i=n_{i0}$, $n_e=n_{e0}$ and q_{d0} as given above in Eq. (7) and using the expressions for $(I_i)_{\rm eq}$, $(I_e)_{\rm eq}$, and $(I_e^s)_{\rm eq}$ as given by (1) and (2) for positively charged dust grains, with $T_s=T_e$ we obtain (superscript +/− corresponds to positive/negative dust charge)

$$\left(\frac{n_{i0}}{n_{e0}}\right)^{(+)}=\sqrt{\frac{m_i}{m_e}}(1+z_0)\frac{1}{\sqrt{\sigma}}\exp\left(\frac{z_0}{\sigma}\right)\alpha_{1s}^{(+)}, \tag{9}$$

where

$$\alpha_{1s}^{(+)}=1-3.7\delta_M F_{5,B0}(x) \tag{10}$$

with

$$z_0=\frac{Z_{d0}e^2}{aT_e},\quad \sigma=\frac{T_i}{T_e},\quad x=\frac{E_M}{4T_e},\quad B_0=\sqrt{\frac{z_0}{x}}. \tag{11}$$

The positively charged state for dust grain is stable provided

$$\left(\frac{\partial I_{\rm tot}}{\partial q_d}\right)_{\rm eq}^{(+)}=-\nu_{D1}^{(+)}<0. \tag{12}$$

Spliting up

$$J_{\rm tot}^{(\pm)}=\frac{I_{\rm tot}^{(\pm)}}{e}=J_i^{(\pm)}-\bar{J}_e^{(\pm)}\quad \text{with}\ \ \bar{J}_e^{(\pm)}=J_e^{(\pm)}-J_e^{s(\pm)} \tag{13}$$

we have the following expressions for the equilibrium values ($q_d=Z_{d0}e+Z_{d0}eq_1$):

$$\frac{\partial J_{i0}^{(+)}}{\partial q_1^{(+)}}=-Z_{d0}\nu_{D1}^{i(+)};\quad \frac{\partial \bar{J}_{e0}^{(+)}}{\partial q_1^{(+)}}=Z_{d0}\nu_{D1}^{e(+)}, \tag{14}$$

where

$$(\nu_{D1}^{i(+)},\nu_{D1}^{e(+)})=\frac{a/\lambda_{Di}}{\sqrt{2\pi}}\omega_{pi}\exp\left(\frac{-z_0}{\sigma}\right)\left(1,\frac{\sigma\alpha_{2s}^{(+)}}{(1+z_0)\alpha_{1s}^{(+)}}\right) \tag{15}$$

(henceforth all equilibrium values will be denoted by the suffix 0) so that

$$\nu_{D1}^{(+)}=\frac{\frac{a}{\lambda_{Di}}}{\sqrt{2\pi}}\omega_{pi}\exp\left(\frac{-z_0}{\sigma}\right)\left[1+\frac{\sigma\alpha_{2s}^{(+)}}{(1+z_0)\alpha_{1s}^{(+)}}\right] =\nu_{D1}^{i(+)}+\nu_{D1}^{e(+)} \tag{16}$$

with

$$\alpha_{2s}^{(+)}=1-3.7\delta_M F_{5,B_0}(x)\left[1-\frac{1}{2}(1+z_0)\times\frac{z_0^2}{x}\frac{\exp[-z_0-\sqrt{z_0/x}]}{F_{5,B_0}(x)}\right]. \tag{17}$$

B. Case B

An equilibrium state with negatively charged dust is also possible:

$$(q_d)_{\rm eq}=q_{d0}^{(-)}=-Z_{d0}e. \tag{18}$$

Substitution of I_e, I_i and I_e^s from Eqs. (1) and (2) with $q_{d0}=-Z_{d0}e$, $n_e=n_{e0}$, $n_i=n_{i0}$ yields the following expression for the equilibrium state plasma ion-electron density ratio:

$$\left(\frac{n_{i0}}{n_{e0}}\right)^{(-)}=\sqrt{\frac{m_i}{m_e}}\frac{\sqrt{\sigma}}{\sigma+z_0}\exp[-z_0]\alpha_{1s}^{(-)} \tag{19}$$

with

$$\alpha_{1s}^{(-)}=1-3.7\delta_M F_5(x). \tag{20}$$

Clearly no steady state can exist with negatively charged dust if secondary emission is strong enough to make $\alpha_{1s}^{(-)}<0$. The dust charge state given by (18) and (19) is always stable as

$$\left(\frac{\partial I_{\rm tot}}{\partial q_d}\right)_{\rm eq}^{(-)}=-\nu_{D1}^{(-)}<0, \tag{21}$$

where

$$\nu_{D1}^{(-)}=\frac{\left(\frac{a}{\lambda_{Di}}\right)}{\sqrt{2\pi}}\omega_{pi}(1+\sigma+z_0). \tag{22}$$

Here also it is convenient to split up $\partial J_{\rm tot}^{(-)}/\partial q_1$ as the sum of contributions from ion part $J_i^{(-)}$ and electron part $\bar{J}_e^{(-)}$,

$$\frac{\partial J_{i0}^{(-)}}{\partial q_1}=-Z_{d0}\nu_{D1}^{i(-)},\quad \frac{\partial J_{e0}^{(-)}}{\partial q_1}=Z_{d0}\nu_{D1}^{e(-)}, \tag{23}$$

where

$$(\nu_{D1}^{i(-)},\nu_{D1}^{e(-)})=\frac{a/\lambda_{Di}}{\sqrt{2\pi}}\omega_{pi}(1,\sigma+z_0) \tag{24}$$

so that

$$\nu_{D1}^{(-)}=\nu_{D1}^{i(-)}+\nu_{D1}^{e(-)}. \tag{25}$$

The linearized equation for small charge variation $q_{d1}^{\pm}=Z_{d0}eq_1^{\pm}$ about the equilibrium state $q_{d0}^{\pm}=\pm Z_{d0}e$ is given by

$$\frac{\partial q_1^{(\pm)}}{\partial t}=\frac{1}{Z_{d0}}\left(\frac{\partial J_{\rm tot}}{\partial q_1^{\pm}}\right)_{\rm eq} q_1^{\pm}+\frac{1}{Z_{d0}}\left(\frac{\partial J_{i0}}{\partial n_i}\delta n_i^{(\pm)} - \frac{\partial \bar{J}_{e0}}{\partial n_e}\delta n_e^{(\pm)}\right) = -\nu_{D1}^{(\pm)} q_1^{(\pm)}+\nu_D^{(\pm)}\left(\frac{\delta n_i^{(\pm)}}{n_{i0}}-\frac{\delta n_e^{(\pm)}}{n_{e0}}\right), \tag{26}$$

where

$$\nu_D^{(+)}=\frac{\left(\frac{a}{\lambda_{Di}}\right)}{\sqrt{2\pi}}\omega_{pi}\exp\left[-\frac{z_0}{\sigma}\right]\left(\frac{1+\sigma}{z_0}\right),\quad \nu_D^{(-)}=\frac{\left(\frac{a}{\lambda_{Di}}\right)}{\sqrt{2\pi}}\omega_{pi}\frac{(\sigma+z_0)}{z_0}. \tag{27}$$

For convenience of subsequent calculation note the following relations:

$$J_{i0}^{(\pm)}=n_{i0}\frac{\partial J_{i0}^{(\pm)}}{\partial n_i}=\bar{J}_{e0}^{(\pm)}=n_{e0}\frac{\partial \bar{J}_{e0}^{(\pm)}}{\partial n_e}=Z_{d0}\nu_D^{(\pm)}. \tag{28}$$

III. ION AND ELECTRON FLUID EQUATIONS

Since we have taken $T_s=T_e$ the population of secondary electron is no longer distinguishable from the population of ambient plasma electrons. Inclusion of the effects of dust charging and secondary emission needs introduction of appropriate terms in the fluid equation for both electrons and ions. This was done in a general setting incorporating modifications consequent to both charging and possible discharging mechanism.

In addition to the charging collision effects we also include that of ionization of neutral atoms and recombination,[25,26,31]

$$A+e^-\rightleftharpoons_{k_{-1}}^{k_1} A^+ + e^- + e^-,$$

where k_1 and k_{-1} are the chemical kinetic rate constants of forward and backward reactions. The forward rate, i.e., the rate at which electrons/ions are created through ionization of neutral atoms is $k_1 n_A n_e$. For recombination to occur an electron must have sufficiently close collision (small impact parameter) with an ion. The number of such collisions per unit volume per unit time is proportional to $n_i n_e$. But a close collision alone cannot lead to a recombination for the occurrence of which the colliding electron has to pass from a free state (positive energy state) to a bound state (negative energy state). So to make the transition energetically possible the excess energy must be absorbed by a second electron in the neighborhood of the ion by interaction with the colliding electron. The probability of this later event is again proportional to n_e. Thus the recombination process considered here occurs through three body interaction, the recombination rate being proportional to $n_i n_e^2$. There may also occur a different type of recombination, i.e., photorecombination, where the excess energy is emitted as bremsstrahlung but this need not be considered here (at least not unless the electron number density is low enough to make the cross section of the two body photo-recombination process proportional to $n_i n_e$ significant in comparison with that of the three body interaction). Hence in our case the rate of change of electron and ion number densities due to ionization—(three body) recombination process is the same and is given by

$$\left(\frac{\delta n_e}{\delta t}\right)_{\text{ionization-recombination}}=\left(\frac{\delta n_i}{\delta t}\right)_{\text{ionization-recombination}} = k_1 n_A n_e - k_{-1} n_i n_e^2 = \nu_{\rm ion} n_e - r n_i n_e^2, \tag{29}$$

where n_A is the neutral atom number density and we have set $\nu_{\rm ion}=k_1 n_A$ and $k_{-1}=r$.

Ionization of a neutral atom due to impact with an ion ($n_A+i^+\rightleftharpoons n_A^+ + e^- + i^+$) is also possible but once again the cross section for such a process is much smaller[31] than that of the electron-impact ionization considered here.

Apart from change in number densities, momentum loss due to various kinds of elastic and inelastic collisions as detailed in Ref. 24 are to be accounted for in the equations of motion of the electron and ion fluid. At least in the linear theory the momentum loss rate of the electron fluid can be approximated as $\nu_e^{\rm eff} v_e$, where $\nu_e^{\rm eff}$ denotes the effective collision frequency

$$\nu_e^{\rm eff}=\nu_{ei}^{\rm el}+\nu_{en}^{\rm el}+\nu_{ed}^{\rm el}+\nu_{en}^{\rm ion}+\nu_{ed}^{\rm sec}+\nu_{ed}^{\rm ch}.$$

The superscript "el" stands for elastic collision (between electrons on the one hand and ions/neutrals/ dust grains on the other) while $\nu_{en}^{\rm ion}$, $\nu_{ed}^{\rm sec}$, $\nu_{ed}^{\rm ch}$ are, respectively, the frequencies of collisions with neutrals leading to ionization, of collision with dust grains resulting in secondary electron emission, and grain charging collisions. However in this connection two comments are in order. The total rate of loss of electron fluid momentum due to charging collision with dust is

$$(\delta/\delta t)(m_e n_e v_e)_{\rm ch}=m_e(\delta n_e/\delta t)_{\rm ch} v_e + m_e n_e(\delta v_e/\delta t)_{\rm ch}.$$

The first term which accounts for the rate of change of momentum consequent to change in electron number density resulting from capture by dust grain cancels out when the equation of continuity is used to bring the equation of motion to its standard form. The second term contributes only when a velocity change of the electron fluid also occurs during the charging process. Only in this case is the expression for $\nu_e^{\rm eff}$ to be supplemented by the addition of the last term $\nu_{ed}^{\rm ch}$. Moreover, $\nu_{ed}^{\rm sec}$ is assumed to contain an extra numerical factor to take into consideration the average number of electrons emitted per secondary electron generating collision. These points were first emphasized by Varma.[24] Similarly we use the effective collision frequency

$$\nu_i^{\rm eff}=\nu_{ie}^{\rm el}+\nu_{in}^{\rm el}+\nu_{id}^{\rm el}+\nu_{id}^{\rm ch},$$

where the same comments apply to $\nu_{id}^{\rm ch}$ as in case of $\nu_{ed}^{\rm ch}$.

Finally noting that $J_i^{(\pm)}/J_e^{(\pm)}$ are the rate of ion/electron capture by and $J_e^{\sec(\pm)}$ is the rate of electron emission from a single dust grain we have the following set of equations of motion for the electron and ion fluids:

$$\frac{\partial n_e^{(\pm)}}{\partial t}+\frac{\partial}{\partial x}(n_e^{(\pm)}v_e^{(\pm)})=-\bar{J}_e^{(\pm)}n_d^{(\pm)}+\nu_{\mathrm{ion}}n_e^{(\pm)}-rn_i^{(\pm)}n_e^{(\pm)2}, \tag{30}$$

$$\frac{\partial v_e^{(\pm)}}{\partial t}+v_e^{(\pm)}\frac{\partial v_e^{(\pm)}}{\partial x}=\frac{T_e}{m_e}\frac{\partial\Phi}{\partial x}-\frac{T_e}{m_e n_e^{\pm}}\frac{\partial n_e^{(\pm)}}{\partial x}-\nu_e^{\mathrm{eff}}v_e^{(\pm)}, \tag{31}$$

$$\frac{\partial n_i^{(\pm)}}{\partial t}+\frac{\partial}{\partial x}(n_i^{(\pm)}v_i^{(\pm)})=-J_i^{(\pm)}n_d^{(\pm)}+\nu_{\mathrm{ion}}n_e^{(\pm)}-rn_i^{(\pm)}n_e^{(\pm)2}, \tag{32}$$

$$\frac{\partial v_i^{(\pm)}}{\partial t}+v_i^{(\pm)}\frac{\partial v_i^{(\pm)}}{\partial x}=-\frac{T_i}{m_i}\frac{\partial\Phi}{\partial x}-\frac{T_i}{m_i n_i^{\pm}}\frac{\partial n_i^{(\pm)}}{\partial x}-\nu_i^{\mathrm{eff}}v_i^{(\pm)}, \tag{33}$$

where $\Phi=e\phi/T_e$ and ϕ is the electrostatic potential.

The number densities of the equilibrium state will satisfy the following:

$$-\bar{J}_{e0}^{(\pm)}n_{d0}^{(\pm)}+\nu_{\mathrm{ion}}n_{e0}^{(\pm)}-rn_{i0}^{(\pm)}n_{e0}^{(\pm)2}=0, \tag{34}$$

$$-\bar{J}_{i0}^{(\pm)}n_{d0}^{(\pm)}+\nu_{\mathrm{ion}}n_{e0}^{(\pm)}-rn_{i0}^{(\pm)}n_{e0}^{(\pm)2}=0. \tag{35}$$

The equilibrium state dust charge number Z_{d0} will satisfy the balance equation for the net current to the dust surface

$$e(J_{i0}^{(\pm)}-J_{e0}^{(\pm)}+J_{e0}^{\mathrm{sec}(\pm)})=0 \tag{36}$$

and the charge neutrality equation

$$n_{i0}-n_{e0}\pm Z_{d0}n_{d0}^{(\pm)}=0. \tag{37}$$

Equations (34)–(36) are consistent; only two of them are independent. Thus there are altogether three independent equations in the set (34)–(37) and four unknowns n_{i0}, n_{e0}, $n_{d0}^{(\pm)}$, Z_{d0} (and a large number of parameters including the neutral number density n_A) to satisfy them so that an infinite number of equilibrium states may exist.

From (34) and (28),

$$\nu_{\mathrm{ion}}=rn_{i0}n_{e0}+\frac{Z_{d0}n_{d0}^{(\pm)}}{n_{e0}}\nu_D^{(\pm)}. \tag{38}$$

Consider perturbation $(\delta n_e^{(\pm)},\delta n_i^{(\pm)},\delta n_d^{(\pm)},Z_{d0}eq_1^{(\pm)})e^{i(kx-\omega t)}$ about the equilibrium state $(n_{e0},n_{i0},n_{d0}^{(\pm)},\pm Z_{d0}e)$ defined above. Equation (26) yields

$$\delta q_1^{(\pm)}=\frac{\nu_D^{(\pm)}}{\nu_{D1}^{(\pm)}-i\omega}\left(\frac{\delta n_i^{(\pm)}}{n_{i0}}-\frac{\delta n_e^{(\pm)}}{n_{e0}}\right) \tag{39}$$

and with its help Eqs. (30)–(33) after elimination of $v_i^{(\pm)}$ and $v_e^{(\pm)}$ and some straightforward but lengthy algebra finally lead to the following system of equations for $\delta n_e^{(\pm)}/n_{e0}$ and $\delta n_i^{(\pm)}/n_{i0}$ under the approximation $|\omega/\nu_\alpha^{\mathrm{eff}}|\ll 1$:

$$A^{(\pm)}\frac{\delta n_e^{(\pm)}}{n_{e0}}+B^{(\pm)}\frac{\delta n_i^{(\pm)}}{n_{i0}}=-\mu_D^{(\pm)}\frac{\delta n_d^{(\pm)}}{n_{d0}^{\pm}}+\frac{k^2T_e}{m_e\nu_e^{\mathrm{eff}}}\Phi, \tag{40}$$

$$C^{(\pm)}\frac{\delta n_e^{(\pm)}}{n_{e0}}+D^{(\pm)}\frac{\delta n_i^{(\pm)}}{n_{i0}}=-\frac{n_{e0}}{n_{i0}}\mu_D^{(\pm)}\frac{\delta n_d^{(\pm)}}{n_{d0}^{\pm}}-\frac{k^2T_i}{m_i\nu_i^{\mathrm{eff}}}\Phi, \tag{41}$$

where

$$A^{(\pm)}=-i\omega+\mu_D^{(\pm)}a_e^{(\pm)}+\beta_e^{(\pm)}+\frac{k^2T_e}{m_e\nu_e^{\mathrm{eff}}}, \tag{42}$$

$$B^{(\pm)}=\mu_D^{(\pm)}(1-a_e^{(\pm)})+\beta_i^{\pm}, \tag{43}$$

$$C^{(\pm)}=\frac{n_{e0}}{n_{i0}}[\mu_D^{(\pm)}(1-a_i^{(\pm)})+\beta_e^{\pm}], \tag{44}$$

$$D^{(\pm)}=-i\omega+\frac{n_{e0}}{n_{i0}}[\mu_D^{(\pm)}a_i^{(\pm)}+\beta_i^{\pm}]+\frac{\sigma k^2T_e}{m_i\nu_i^{\mathrm{eff}}}. \tag{45}$$

The constants $\mu_D^{(\pm)}$, $a_e^{(\pm)}$, $a_i^{(\pm)}$, $\beta_e^{\pm}$, $\beta_i^{\pm}$ are expressed in terms of $\nu_D^{(\pm)}$, $\nu_{D1}^{(\pm)}$, $\nu_{D1}^{e(\pm)}$, $\nu_{D1}^{i(\pm)}$ given by Eqs. (15), (16), (23), (24), and (27) $(\alpha=e,i)$:

$$\mu_D^{(\pm)}=\frac{Z_{d0}n_{d0}^{(\pm)}}{n_{e0}}\nu_D^{(\pm)},\quad a_\alpha^{(\pm)}=1-\frac{\nu_{D1}^{\alpha(\pm)}}{\nu_{D1}^{(\pm)}-i\omega}, \tag{46}$$

$$\beta_i^{\pm}=rn_{e0}n_{i0}=\nu_{\mathrm{ion}}-\mu_D^{(\pm)},\quad \beta_e^{\pm}=\nu_{\mathrm{ion}}-2\mu_D^{(\pm)},$$

$$\mathrm{Re}\,a_e^{\pm}+\mathrm{Re}\,a_i^{\pm}=1. \tag{47}$$

In obtaining the equations for the density perturbations $\delta n_e^{(\pm)}$, $\delta n_i^{(\pm)}$ we have ignored the variation of the ionization frequency $\nu_{\mathrm{ion}}=k_1n_A$; this is justifiable if $\delta n_A/n_A\ll\delta n_i/n_{i0}$, $\delta n_e/n_{e0}$, i.e., if the dusty plasma is weakly ionized.

$\delta n_e^{(\pm)}/n_{e0}$ and $\delta n_i^{(\pm)}/n_{i0}$ are next determined from Eqs. (40) and (41):

$$(A^{\pm}D^{\pm}-B^{\pm}C^{\pm})\frac{\delta n_e^{(\pm)}}{n_{e0}}=\mu_D^{(\pm)}\left(\frac{n_{e0}}{n_{i0}}B^{\pm}-D^{\pm}\right)\frac{\delta n_d^{(\pm)}}{n_{d0}^{\pm}}+\frac{k^2T_e}{m_e\nu_e^{\mathrm{eff}}}\left(D^{\pm}+\sigma\frac{m_e}{m_i}\frac{\nu_e^{\mathrm{eff}}}{\nu_i^{\mathrm{eff}}}B^{\pm}\right)\Phi, \tag{48}$$

$$(A^{\pm}D^{\pm}-B^{\pm}C^{\pm})\frac{\delta n_i^{(\pm)}}{n_{i0}}=\mu_D^{(\pm)}\left(C^{\pm}-\frac{n_{e0}}{n_{i0}}A^{\pm}\right)\frac{\delta n_d^{(\pm)}}{n_{d0}^{\pm}}-\frac{k^2T_e}{m_e\nu_e^{\mathrm{eff}}}\left(C^{\pm}+\sigma\frac{m_e}{m_i}\frac{\nu_e^{\mathrm{eff}}}{\nu_i^{\mathrm{eff}}}A^{\pm}\right)\Phi. \tag{49}$$

The explicit expressions for $(A^{\pm}D^{\pm}-B^{\pm}C^{\pm})$ and the coefficient of $\delta n_d^{\pm}/n_{d0}^{\pm}$ and Φ appearing in Eqs. (48) and (49) are given in Appendix A.

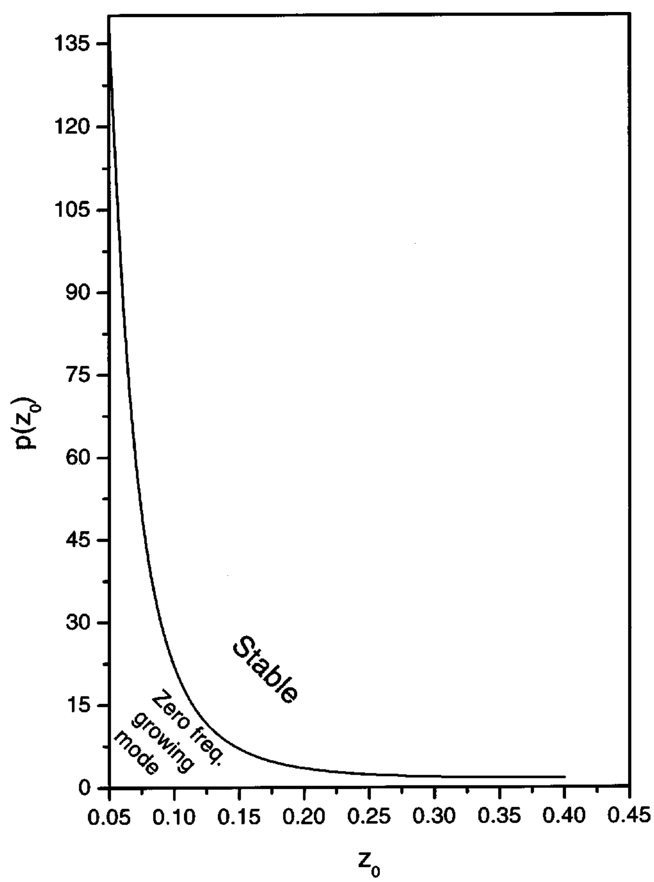

FIG. 1. The equation to the curve is given by the function $p(z_0)$ on the rhs of the inequality (59). For $k \rightarrow 0$ zero frequency growing mode exists for positively charged dust when $\nu_{\mathrm{ion}}/(a/\lambda_{Di}/\sqrt{2\pi})\omega_{pi}$ is below the curve.

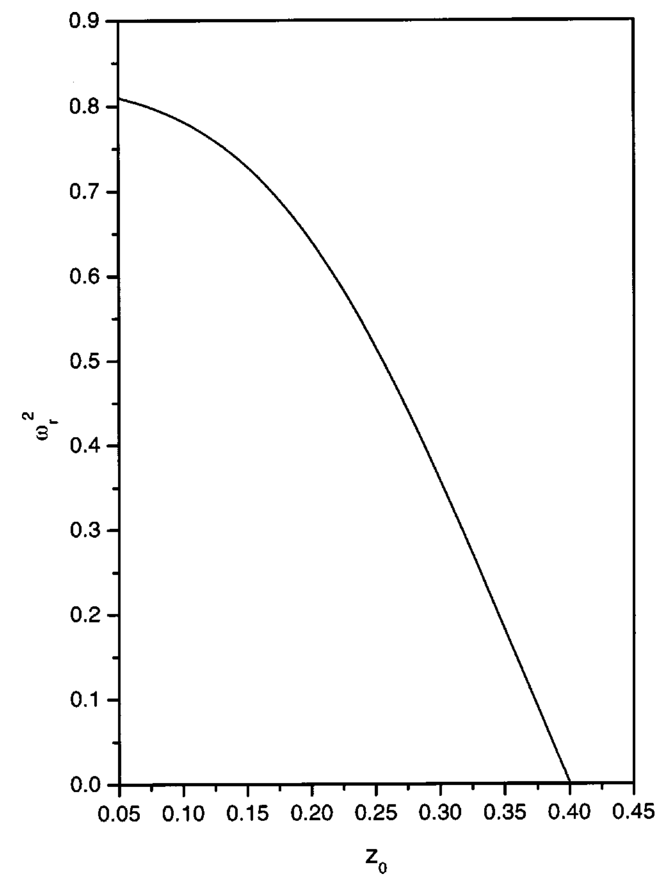

FIG. 2. Plot of the square of the normalized real frequency $\omega_r^2/(k^2 Z_{d0} T_e/m_d)$ (for dust charge positive) as given by (58) against z_0 of DA wave for positively charged dust grains for $\sigma=1.0$, $E_M=400$, $\delta_M=22$, and $y=10$.

IV. DISPERSION RELATION FOR DA WAVES

For the dust fluid the dust density perturbation is related to Φ by

$$\frac{\delta n_d^{\pm}}{n_{d0}^{\pm}} = \pm \frac{k^2}{\omega^2} \frac{Z_{d0} T_e}{m_d} \Phi. \tag{50}$$

Also with the help of Eq. (39) the long wavelength approximation of Poisson's equation reads

$$\left[\frac{n_{i0}}{n_{e0}} \frac{\delta n_i^{\pm}}{n_{i0}} - \frac{\delta n_e^{\pm}}{n_{e0}}\right] + \frac{Z_{d0} n_{d0}^{\pm}}{n_{e0}} \frac{\nu_D^{\pm}}{\nu_{D1}^{\pm} - i\omega}\left[\frac{\delta n_i^{\pm}}{n_{i0}} - \frac{\delta n_e^{\pm}}{n_{e0}}\right] \pm \frac{Z_{d0} n_{d0}^{\pm}}{n_{e0}} \frac{\delta n_d^{\pm}}{n_{d0}} = 0. \tag{51}$$

Substitution for $\delta n_e^{\pm}$ and $\delta n_i^{\pm}$ from Eqs. (48) and (49) into (51) yields

$$\Phi = \pm \frac{N^{\pm}}{M^{\pm}} \frac{\delta n_d^{\pm}}{n_{d0}^{\pm}}. \tag{52}$$

$M^{\pm}$ and $N^{\pm}$ are split up into their real and imaginary parts:

$$M^{\pm} = M_r^{\pm} + i\omega M_i^{\pm}, \quad N^{\pm} = N_r^{\pm} + i\omega N_i^{\pm}, \tag{53}$$

where

$$M_r^{\pm} = \left(\frac{n_{i0}}{n_{e0}} + \frac{Z_{d0} n_{d0}^{\pm}}{n_{e0}} \frac{\nu_D^{\pm}}{\nu_{D1}^{\pm}}\right) \times \left[\frac{n_{e0}}{n_{i0}}(\mu_D^{\pm} \operatorname{Re} a_e^{\pm} + \beta_e) + \frac{k^2 T_e}{m_e \nu_e^{\mathrm{eff}}}\right] + \left(1 + \frac{Z_{d0} n_{d0}^{\pm}}{n_{e0}} \frac{\nu_D^{\pm}}{\nu_{D1}^{\pm}}\right) \times \left[\frac{n_{e0}}{n_{i0}}(\mu_D^{\pm} \operatorname{Re} a_i^{\pm} + \beta_i) + \frac{k^2 T_i}{m_i \nu_i^{\mathrm{eff}}}\right], \tag{54}$$

$$N_r^{\pm} = \frac{Z_{d0} n_{d0}^{\pm}}{n_{e0}}\left[\frac{n_{e0}}{n_{i0}}(\mu_D^{\pm} \operatorname{Re} a_i^{\pm} + \beta_i) + \frac{k^2 T_i}{m_i \nu_i^{\mathrm{eff}}}\right] \mp \mu_D^{\pm}\left(1 + \frac{Z_{d0} n_{d0}^{\pm}}{n_{e0}} \frac{\nu_D^{\pm}}{\nu_{D1}^{\pm}}\right). \tag{55}$$

In obtaining the above expressions for $M_r^{\pm}$ and $N_r^{\pm}$ we have neglected terms $0(m_i \nu_i^{\mathrm{eff}}/m_e \nu_e^{\mathrm{eff}})$. $M_i^{\pm}$ and $N_i^{\pm}$ are given in Appendix B.

The dispersion relation is obtained by eliminating $\delta n_d^{\pm}/n_{d0}^{\pm}$ by using Eq. (50):

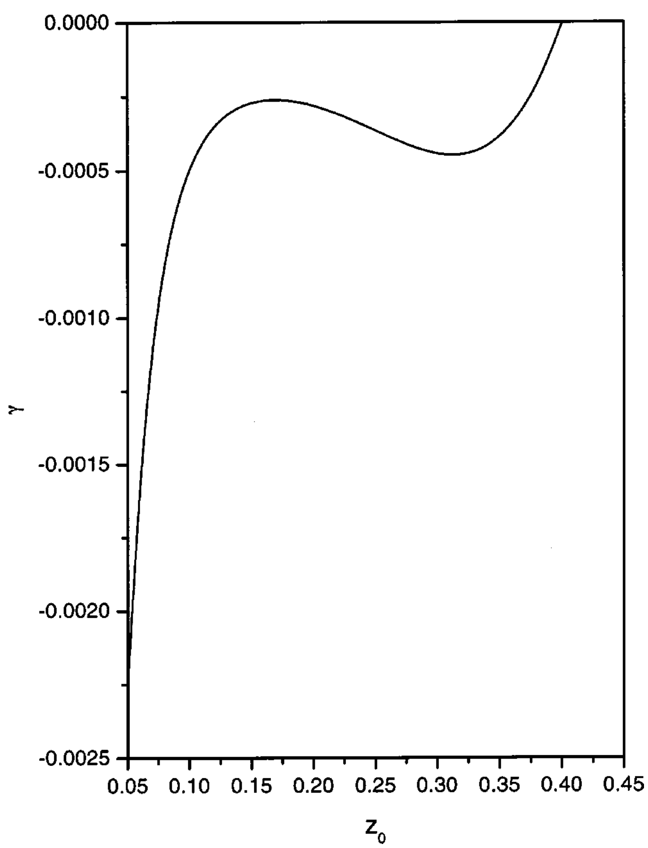

FIG. 3. Plot of normalized imaginary frequency γ (for dust charge positive) as given by Eq. (63) against z_0 of DA wave for positively charged dust grains for $\sigma=1.0$, $E_M=400$, $\delta_M=22$, and $y=10$.

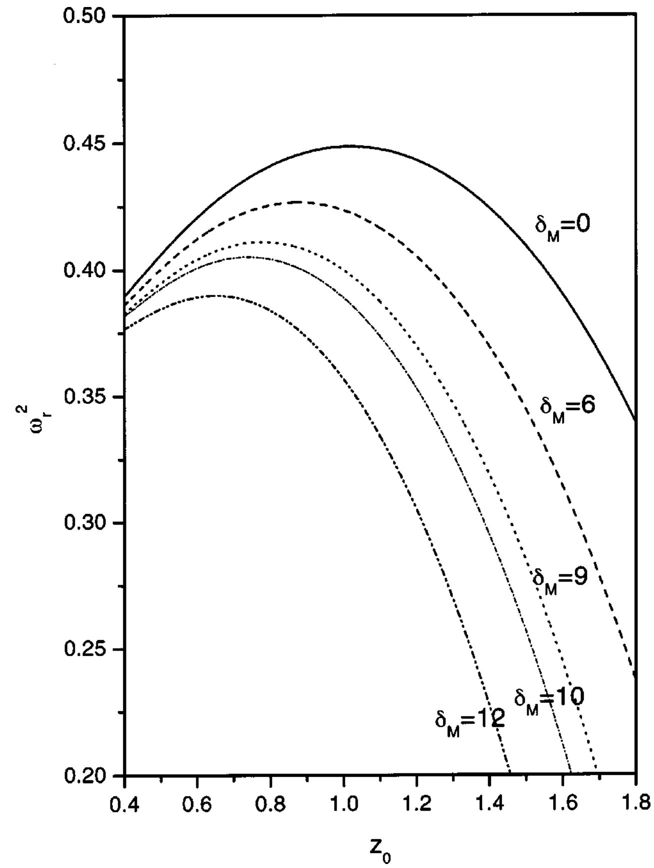

FIG. 4. Plot of the square of the normalized real frequency $\omega_r^2/(k^2 Z_{d0} T_e/m_d)$ against z_0 of DA wave for negatively charged dust grains for different δ_M and $E_M=350$, $\sigma=1$, $y=10$.

$$\epsilon(\omega,k)=\omega^2(M_r^{\pm}+i\omega M_i)-\frac{k^2 Z_{d0}T_e}{m_d}(N_r^{\pm}+i\omega N_i^{\pm})=0. \tag{56}$$

For $|\omega/\nu_{D1}^{\pm}|\ll 1$, which is true for frequencies ω close to dust accoustic frequency, we have $|\omega M_i|$, $|\omega N_i|\ll|M_r|$. In this case the real part of the frequency ω_r satisfying $\mathrm{Re}\,\epsilon(\omega,k)=0$ is given by

$$\omega_r^2=k^2\frac{Z_{d0}T_e}{m_d}\frac{N_r^{\pm}}{M_r^{\pm}}. \tag{57}$$

For $|\mu_D^{\pm}\,\mathrm{Re}\,a_\alpha^{\pm}+\beta_\alpha|\ll \mathrm{k}^2\mathrm{T_e}/\mathrm{m}_i\nu_i^{\mathrm{eff}}/$; $(\alpha=e,i)$ Eq. (57) yields the standard expression for DA wave frequency as expected. On the other hand when the above inequalities hold true in the opposite direction $(k^2T_e/m_i\nu_i^{\mathrm{eff}}\ll|\mu_D^{\pm}\,\mathrm{Re}\,a_\alpha^{\pm}+\beta_\alpha|)$ we have

$$\frac{\omega_r^{\pm 2}}{k^2}=\frac{Z_{d0}T_e}{m_d}\times\frac{\dfrac{Z_{d0}n_{d0}^{\pm}}{n_{e0}}\mp\dfrac{n_{i0}}{n_{e0}}\mu_D^{\pm}\dfrac{\left(1+\dfrac{Z_{d0}n_{d0}^{\pm}}{n_{e0}}\dfrac{\nu_D^{\pm}}{\nu_{D1}^{\pm}}\right)}{(\mu_D^{\pm}\,\mathrm{Re}\,a_i^{\pm}+\beta_i)}}{\left(\dfrac{n_{i0}}{n_{e0}}+\dfrac{Z_{d0}n_{d0}^{\pm}}{n_{e0}}\right)\dfrac{\mu_D^{\pm}\,\mathrm{Re}\,a_e^{\pm}+\beta_\mathrm{e}}{\mu_D^{\pm}\,\mathrm{Re}\,a_i^{\pm}+\beta_\mathrm{i}}+\left(1+\dfrac{Z_{d0}n_{d0}^{\pm}}{n_{e0}}\right)}. \tag{58}$$

For positively charged dust $\omega_r^{+2}<0$, i.e., there exists a zero frequency exponentially growing mode when the numerator N_r^+ of the ratio on the rhs of (54) is negative. The condition for the occurrence of such a mode works out to be

$$\frac{\left(\nu_{ion}+\dfrac{n_{i0}}{n_{e0}}\dfrac{\sigma k^2 T_e}{m_i\nu_i^{\mathrm{eff}}}\right)}{\left(\dfrac{a/\lambda_{Di}}{\sqrt{2\pi}}\omega_{pi}\right)}<\left[\frac{n_{i0}}{n_{e0}}+\left(1-\frac{n_{i0}}{n_{e0}}\right)\frac{1}{z_0}\frac{(\sigma+z_0)(1+z_0)\alpha_{1s}^{(+)}}{(1+z_0)\alpha_{1s}^{(+)}+\sigma\alpha_{2s}^{(+)}}\right]\frac{\sigma}{z_0}\times\exp\left[\frac{-z_0}{\sigma}\right]=p(z_0),\ \ (\mathrm{say}). \tag{59}$$

For negatively charged dust no such zero frequency exponentially growing mode exists as we always have $N_r^{(-)}>0$.

The following observation will help to give a physical picture of the connection between the effect of the occurrence of zero frequency growing perturbation and the behavior of the dust density. We make the following two assumptions:

(i) The dust charge variation is adiabatic which makes $q_1^{\pm}=\nu_D^{\pm}/\nu_{D1}^{\pm}(\delta n_i^{\pm}/n_{i0}-\delta n_e^{\pm}/n_{e0})$, i.e., we neglect terms of $0(\omega/\nu_{D1}^{\pm})$.

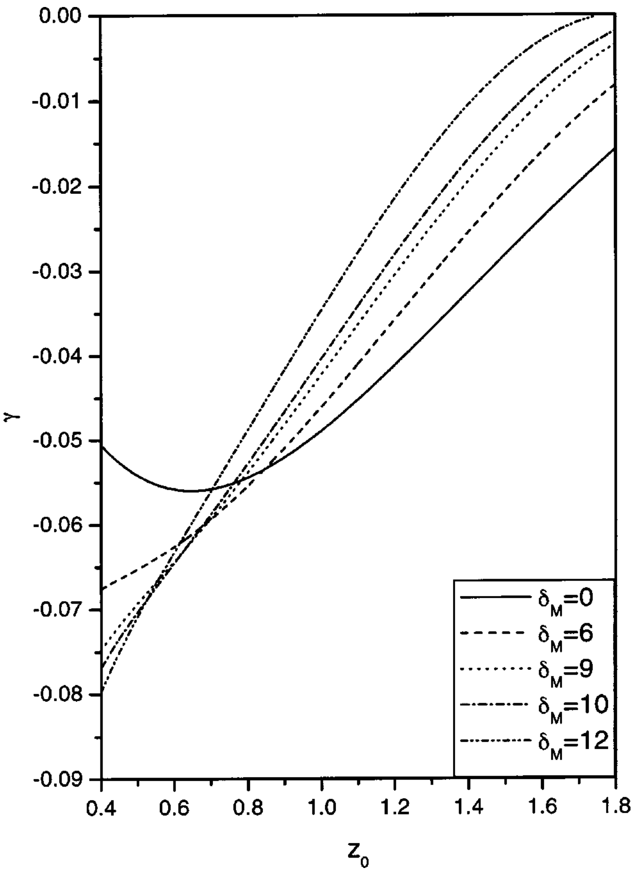

FIG. 5. Plot of normalized imaginary frequency γ against z_0 of the DA wave for negatively charged dust grains for different δ_M and $E_M=350$, $\sigma=1$, $y=10$.

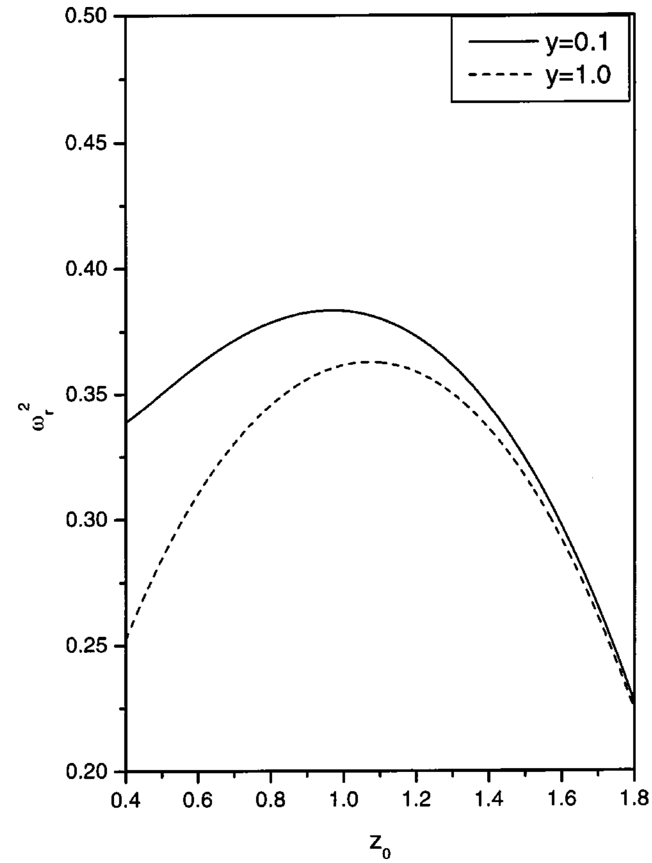

FIG. 6. Plot of the square of the normalized real frequency $\omega_r^2/(k^2 Z_{d0} T_e/m_d)$ against z_0 of DA wave for negatively charged dust grains for $y=0.1$, 1 and δ_M, $E_M=350$, $\sigma=1$.

(ii) Neglect the terms $\sim 0(\omega/(k^2 T_e/m_e \nu_e^{\text{eff}})$. This is justified for $\omega\sim$dust acoustic frequency.

Under the foregoing assumptions one can neglect $\omega M_i^{\pm}$ and $\omega N_i^{\pm}$ compared with $M_r^{\pm}$ and $N_r^{\pm}$ so that Eq. (49) simplifies to

$$\Phi = \pm \frac{N_r^{\pm}}{M_r^{\pm}} \frac{\delta n_d^{+}}{n_{d0}}. \tag{60}$$

We can now write using Eq. (60)

$$\frac{\partial}{\partial t}\delta v_d^{\pm} = \mp \frac{Z_{d0}T_e}{m_d}\frac{\partial}{\partial x}\Phi = -\frac{Z_{d0}T_e}{m_d}\frac{N_r^{\pm}}{M_r^{\pm}}\frac{\partial}{\partial x}\left(\frac{\delta n_d^{\pm}}{n_{d0}^{\pm}}\right). \tag{61}$$

For positively charged dust $N_r^{+}<0$; hence the dust fluid acceleration is positive/negative in the direction of increasing/decreasing dust density. The absence of restoring effect results in a zero frequency purely growing mode. The dust density tends to build up in the direction of higher density and the dust moves away from the region where the dust density is lower. On the other hand for negatively charged dust the acceleration is always directed against the density gradient ($N_r^{(-)}>0$); the effect is restorative and leads to sustained density oscillation for negatively charged dust.

For $N_r^{\pm}>0$, the real part $\omega_r^{\pm 2}$ is given by Eq. (54) while the imaginary part is

$$-\frac{1}{2}\frac{\operatorname{Im}\epsilon(\omega_r,k)}{\frac{\partial}{\partial\omega}\operatorname{Re}\epsilon(\omega_r,k)}. \tag{62}$$

This gives the damping rate $\omega_i^{\pm}$,

$$\frac{\omega_i^{\pm}}{k^2 Z_{d0}T_e/m_d} = -\frac{1}{2}\frac{(N_r^{\pm}/M_r^{\pm})M_i^{\pm}-N_i^{\pm}}{M_r^{\pm}}. \tag{63}$$

V. RESULTS

In Fig. 1 is shown the boundary curve between the region of instability as given by the rhs of the inequality (59) against z_0. For very large wavelength perturbations ($k^2 \to 0$) purely growing dust mode exists for positively charged dust for values of $\nu_{\text{ion}}/(a/\lambda_{Di}/\sqrt{2\pi})\omega_{pi}$ lying below the curve. For values of the same parameter lying above the boundary $\omega_r^2>0$, and $\gamma = \omega_i^{\pm}((a/\lambda_{Di}/\sqrt{2\pi})\omega_{pi})/(k^2 Z_{d0}T_e/m_d)$ as given by (63) is negative. Figures 2 and 3 give the graphs for $\omega_r^2>0$ and γ against z_0.

For negatively charged dust the corresponding quantites are also shown graphically. Figure 4 give $\omega_r^2(>0)$ against z_0 keeping $y=[(k^2T_e/m_i\nu_i^{\text{eff}})/(a/\lambda_{Di}/\sqrt{2\pi})]\omega_{pi}$ fixed but varying δ_M. It is found that in each case as δ_M increases from zero value (corresponding to no secondary emission) ω_r^2 increases with increasing z_0 initially but falls off subsequently.

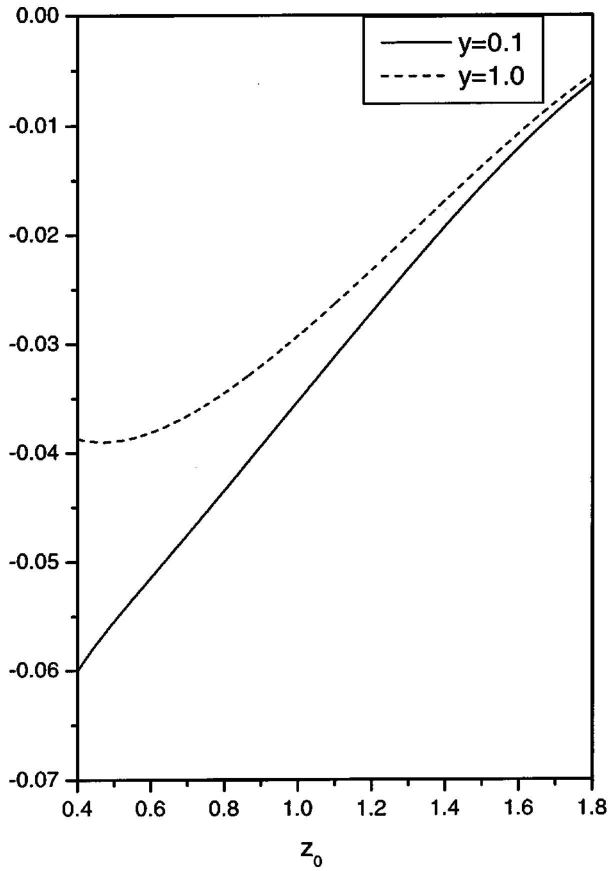

FIG. 7. Plot of normalized imaginary frequency γ against z_0 of the DA wave for negatively charged dust grains for $y=0.1$, 1 and δ_M, $E_M=350$, $\sigma=1$.

Moreover for each fixed z_0, ω_r^2 decreases with increasing δ_M. The damping rate as shown in Fig. 5 behaves somewhat differently. For $\delta_M=0$, γ initially decreases slowly with z_0 but after passing a minimum it increases with z_0. For higher values of δ_M however, γ increases monotonically with z_0. Also for lower values of z_0, γ decreases for small z_0 with increasing δ_M but increases with δ_M for large z_0. Figures 6 and 7 give ω_r^2 and γ with $\delta_M=6$ and $y=[(k^2T_e/m_i\nu_i^{\text{eff}})/(a/\lambda_{Di}/\sqrt{2\pi})]\omega_{pi}=0.1$, 1.0.

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APPENDIX A

The following are the expressions for $(A^{\pm}D^{\pm}-B^{\pm}C^{\pm})$ and the coefficients of $\delta n_d^{\pm}/n_{d0}^{\pm}$ and Φ appearing in Eqs. (45) and (46):

$$A^{\pm}D^{\pm}-B^{\pm}C^{\pm}$$
$$=\frac{k^2T_e}{m_e\nu_e^{\text{eff}}}\left[\left(\frac{n_{e0}}{n_{i0}}(\mu_D^{\pm}\,\text{Re}\,a_i^{\pm}+\beta_i)+\frac{\sigma k^2T_e}{m_i\nu_i^{\text{eff}}}\right)-i\omega\left(1+\frac{n_{e0}}{n_{i0}}\frac{\mu_D^{\pm}\nu_{D1}^{i(\pm)}}{\nu_{D1}^{(\pm)2}}\right)\right]+O\left(\frac{m_e\nu_e^{\text{eff}}}{m_i\nu_i^{\text{eff}}}\right), \quad \text{(A1)}$$

$$C^{\pm}+A^{\pm}\frac{m_e\nu_e^{\text{eff}}}{m_i\nu_i^{\text{eff}}}=\left(\frac{n_{e0}}{n_{i0}}(\mu_D^{\pm}\,\text{Re}\,a_e^{\pm}+\beta_e)+\frac{k^2T_e}{m_i\nu_i^{\text{eff}}}\right)+i\omega\frac{n_{e0}}{n_{i0}}\frac{\mu_D^{\pm}\nu_{D1}^{i(\pm)}}{\nu_{D1}^{(\pm)2}}, \quad \text{(A2)}$$

$$D^{\pm}+B^{\pm}\frac{m_e\nu_e^{\text{eff}}}{m_i\nu_i^{\text{eff}}}=\left(\frac{n_{e0}}{n_{i0}}(\mu_D^{\pm}\,\text{Re}\,a_i^{\pm}+\beta_i)+\frac{\sigma k^2T_e}{m_i\nu_i^{\text{eff}}}\right)-i\omega\left(1+\frac{n_{e0}}{n_{i0}}\frac{\mu_D^{\pm}\nu_{D1}^{i(\pm)}}{\nu_{D1}^{(\pm)2}}\right), \quad \text{(A3)}$$

$$C^{\pm}-\frac{n_{e0}}{n_{i0}}A^{\pm}=-\frac{n_{e0}}{n_{i0}}\frac{k^2T_e}{m_e\nu_e^{\text{eff}}}+i\omega\left(1+\frac{n_{e0}}{n_{i0}}\frac{\mu_D^{\pm}}{\nu_{D1}^{(\pm)}}\right), \quad \text{(A4)}$$

$$\frac{n_{e0}}{n_{i0}}B^{\pm}-D^{\pm}=-\sigma\frac{k^2T_e}{m_e\nu_e^{\text{eff}}}\frac{m_e\nu_e^{\text{eff}}}{m_i\nu_i^{\text{eff}}}+i\omega\left(1+\frac{n_{e0}}{n_{i0}}\frac{\mu_D^{\pm}}{\nu_{D1}^{(\pm)}}\right). \quad \text{(A5)}$$

APPENDIX B

$$M_i^{\pm}=\frac{Z_{d0}n_{d0}^{\pm}}{n_{e0}}\frac{\nu_D^{\pm}}{\nu_{D1}^{(\pm)2}}\left[\left(\frac{n_{e0}}{n_{i0}}(\mu_D^{\pm}\,\text{Re}\,a_e^{\pm}+\beta_e)+\frac{k^2T_e}{m_i\nu_i^{\text{eff}}}\right)+\left(\frac{n_{e0}}{n_{i0}}(\mu_D^{\pm}\,\text{Re}\,a_i^{\pm}+\beta_i)+\frac{\sigma k^2T_e}{m_i\nu_i^{\text{eff}}}\right)\right]+\left(1-\frac{n_{e0}}{n_{i0}}\right)\frac{\mu_D^{\pm}\nu_{D1}^{i(\pm)}}{\nu_{D1}^{(\pm)2}}-\left(1+\frac{Z_{d0}n_{d0}^{\pm}}{n_{e0}}\frac{\mu_D^{\pm}\nu_{D1}^{i(\pm)}}{\nu_{D1}^{(\pm)2}}\right), \quad \text{(B1)}$$

$$N_i^{\pm}=-\frac{Z_{d0}n_{d0}^{\pm}}{n_{e0}}\left(1+\frac{n_{e0}}{n_{i0}}\frac{\mu_D^{\pm}\nu_{D1}^{i(\pm)}}{\nu_{D1}^{(\pm)2}}\right)\mp\frac{Z_{d0}n_{d0}^{\pm}}{n_{e0}}\frac{\mu_D^{\pm}\nu_D^{\pm}}{\nu_{D1}^{(\pm)2}}. \quad \text{(B2)}$$

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