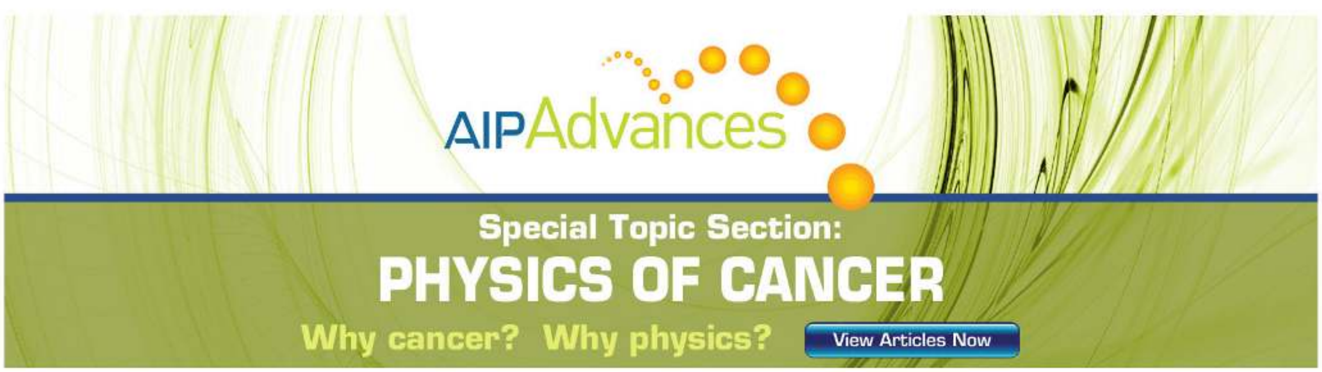
AIP Physics of Plasmas

Effect of nonadiabatic dust charge variations on nonlinear dust acoustic waves with nonisothermal ions

Samiran Ghosh, Susmita Sarkar, Manoranjan Khan, and M. R. Gupta

Citation: Phys. Plasmas 9, 1150 (2002); doi: 10.1063/1.1455627
View online: http://dx.doi.org/10.1063/1.1455627
View Table of Contents: http://pop.aip.org/resource/1/PHPAEN/v9/i4
Published by the American Institute of Physics.



Effect of nonadiabatic dust charge variations on nonlinear dust acoustic waves with nonisothermal ions

Samiran Ghosh,[a)] Susmita Sarkar,[b)] Manoranjan Khan,[c)] and M. R. Gupta
Centre for Plasma Studies, Faculty of Science, Jadavpur University, Calcutta—700 032, India

(Received 5 October 2001; accepted 14 December 2002)

The effect of nonadiabatic dust charge variation on the nonlinear dust acoustic wave in a dusty plasma consisting of warm adiabatic dust grains, isothermal electrons and nonisothermal ions have been investigated by reductive perturbation technique. It has been shown that due to the nonadiabatic dust charge variation and also for the presence of nonisothermal ions, nonlinear dust acoustic wave is governed by a modified Korteweg–de Vries Burger (mKdVB) equation. The Burger term arises due to the nonadiabatic dust charge variation, whereas the nonlinear terms are modified due the different effects of nonisothermal ions. Numerical integration of mKdVB equation shows that the nonlinear dust acoustic wave admits negative potentials with the oscillatory (dispersion dominant) or monotonic (dissipation dominant) shock transition and exhibits compressional shock wave. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1455627]

I. INTRODUCTION

Recently, experimental[1–3] and theoretical[4,5] investigations on the propagation characteristics of "dust acoustic" and "dust ion acoustic" collective oscillations in a weakly coupled unmagnetized dusty plasma have gathered momentum because of their relevance in space physics, astrophysics, and also in laboratory plasmas.[6]

The dust grains immersed in a plasma are highly charged, and the charge on the dust grains also fluctuates due to the fluctuating electrostatic force. This is an important characteristic of dusty plasma, which distinguishes it from a three-component plasma. The charging processes and grain charge fluctuations are also interesting phenomena and have been recently investigated.[7–10]

Recently, the nonlinear regime of dust acoustic and dust ion acoustic waves has also been observed in laboratory plasma.[11–14] In this regime, several authors have theoretically studied the nonlinear properties of both the dust acoustic and dust ion acoustic wave, considering adiabatic (under the assumption $\tau_{ch}/\tau_d \approx 0$)[15–17] and nonadiabatic (under the assumption that τ_{ch}/τ_d is small but finite)[18,19] dust charge variations considering isothermal electrons and ions. Schekinov[20] analytically studied the nonlinear properties of dust acoustic waves in a dusty plasma consisting of fixed charged cold dust grains and hot nonisothermal ions. Kakati and Goswami[21] studied nonlinear dust acoustic waves considering nonisothermal ions and adiabatic dust charge variations by reductive perturbation technique.

In this paper, the effects of both nonadiabatic dust charge variation and the reflected ions on nonlinear dust acoustic waves have been studied by reductive perturbation technique. The adiabatic pressure variations of the dust grains are also incorporated here. As proposed by Schamel[22–24] the shifted Maxwellian distribution for the background ions have been considered. Employing the reductive perturbation technique with the scaling $\tau_{ch}/\tau_d = O\ (\epsilon^{1/4})$ for $b[=(1-\sigma)/\sqrt{\pi}] \gg O(\sqrt{\epsilon})$, where, $\sigma = T_{if}/T_{it}$ and T_{if} (T_{it}) is the temperature of free (trapped) ions and ϵ is the usual expansion parameter, it is seen that the nonadiabatic dust charge variation plays a dissipative role. Under this condition the nonlinear propagation of small but finite amplitude dust acoustic waves is seen to be governed by a modified Korteweg–de Vries (KdV) Burger equation. But with the scaling $\tau_{ch}/\tau_d = O\ (\epsilon^{1/2})$ for $b(=(1-\sigma)/\sqrt{\pi}) \approx O\ (\sqrt{\epsilon})$, the nonlinear equation is governed by a mKdV Burger equation with an extra nonlinear term $\Phi^{(1)}(\partial\Phi^{(1)}/\partial\xi)$. The presence of the Burger term prevents any disturbance from developing into solitons and leads to the formation of a shock-like structure, whereas the nonisothermal ions modify the nonlinear term. Numerical investigations show that the dust acoustic wave possesses a compressional shock wave, which provides sufficient dust density enhancement—a prerequisite for star formation through subsequent gravitational contraction. Numerical investigations show that, in the case of the nonisothermal parameter $b \gg O(\sqrt{\epsilon})$, the shock strength increases (decreases) according to σ (the ratio of free to trapped ion temperature) increases (decreases), whereas in the case of $b \approx O(\sqrt{\epsilon})$, the shock strength decreases (increases) according to σ increases (decreases). It is interesting to note that the dust temperature is enhanced due to the adiabatic compression of the dust fluid caused by the shock.

The paper is organized in the following manner. Section II contains the basic equations. The modified KdV Burger equation describing the propagation is derived in Sec. III. The results obtained from the mKdV Burger description are given in section IV. In Sec. V we describe the numerical

[a)]Also at Netaji Subhas Engineering College, Techno City, Panchpotha, Police Para, Garia, Calcutta, India. Electronic mail: sran_g@yahoo.com and sran@jufs.ernet.in
[b)]Also at the Department of Applied Mathematics, University Of Calcutta, 92, A. P. C. Road, Calcutta—700 009, India.
[c)]Electronic mail: mk@jufs.ernet.in

results with their graphical explanations and a summary of the results.

II. BASIC EQUATIONS

A collisionless, nonrelativistic three-component dusty plasma consisting of Boltzmann distributed electrons, nonisothermal ions, and high negatively charged warm adiabatic dust grains have been considered. The nonadiabatic dust charge variations are also considered. In equilibrium, the charge neutrality condition is

$$n_{j0}+z_j n_{d0}=z_j n_{i0}, \tag{1}$$

where n_{j0} $(j=e,i,d)$ is the equilibrium number density of the jth species and z_d is the equilibrium number of electrons residing on dust grains.

The dynamics of low-frequency dust acoustic oscillations in a three-component dusty plasma is governed by the following continuity and momentum fluid equations for the dust grains and Poisson's equations, which in normalized form reads as

$$\frac{\partial N_d}{\partial T}+\frac{\partial(N_d V_d)}{\partial X}=0, \tag{2}$$

$$\frac{\partial V_d}{\partial T}+V_d\frac{\partial V_d}{\partial X}=-(\Delta Q-1)\frac{\partial \Phi}{\partial X}-\frac{\sigma_d}{N_d}\frac{\partial P_d}{\partial X}, \tag{3}$$

$$\frac{\partial^2 \Phi}{\partial X^2}=\frac{1}{z_d n_{d0}}[n_e-n_i-z_d n_{d0}N_d(\Delta Q-1)], \tag{4}$$

$$n_e=n_{e0}\exp(\Phi), \tag{5}$$

$$P_d=cN_d^{\gamma}. \tag{6}$$

To consider the effect of the nonisothermal ions, the following ion number densities have been considered.[22–24] In normalized form in the small-amplitude limit, it reads as

$$n_i=n_{i0}\left[1-\frac{\Phi}{\sigma_i}-\frac{4}{3}b\left(-\frac{\Phi}{\sigma_i}\right)^{3/2}+\frac{1}{2}\left(\frac{\Phi}{\sigma_i}\right)^2\right]. \tag{7}$$

The nonisothermal parameter $b=(1-\sigma)/\sqrt{\pi}$, $\sigma=T_{if}/T_{it}$, where T_{if} and T_{it} are the temperatures of free and trapped ions. The above expression (7) is true for both $\sigma<0$ and $\sigma>0$. The negative value of σ describes a situation in which there is a deficit of trapped ions.

In the above equations (2)–(7), $N_d=n_d/n_{d0}$; $\delta=n_{i0}/n_{e0}$, $\sigma_d=T_d/z_d T_e$, and $\sigma_i=T_{if}/T_e$, T_e is the electron temperature. P_d is the dust pressure normalized to $n_{d0}T_d$, c is a constant, and γ is the ratio of specific heat at constant constant pressure (C_p) to specific heat at constant volume (C_v). The dust charge $Q_d=-z_d e+\Delta Q_d$, where ΔQ_d is the fluctuating dust charge, becomes $(\Delta Q-1)$ $[\Delta Q=\Delta Q/z_d e]$, normalized in units of the equilibrium dust charge $z_d e$. The electrostatic potential Φ is normalized by T_e/e. The velocity V_d of the dust grain is normalized in units of the dust acoustic velocity $c_d=\sqrt{z_d T_e/m_d}$. The time scale T and space scale X are normalized by ω_{pd}^{-1} and λ_{D}, respectively. Here $\omega_{pd}=\sqrt{z_d^2 e^2 n_{d0}/\epsilon_0 m_d}$ is the dust plasma frequency and $\lambda_{\mathrm{D}}=\sqrt{\epsilon_0 T_e/z_d n_{d0}e^2}$ is the dust Debye length.

Using Eq. (6), the last term in Eq. (3) can be expressed as

$$\frac{\sigma_d}{N_d}\frac{\partial P_d}{\partial X}=\gamma\sigma_d N_d^{\gamma-2}\frac{\partial N_d}{\partial X}. \tag{8}$$

Therefore Eq. (3) can be rewritten as

$$\frac{\partial V_d}{\partial T}+V_d\frac{\partial V_d}{\partial X}=-(\Delta Q-1)\frac{\partial \Phi}{\partial X}-\gamma\sigma_d N_d^{\gamma-2}\frac{\partial N_d}{\partial X}. \tag{9}$$

To determine the normalized charge variable ΔQ, we consider the following orbital motion limited[25] charge current balance equation. In normalized form it reads as

$$\frac{\tau_{\mathrm{ch}}}{\tau_d}\left(\frac{\partial \Delta Q}{\partial T}+V_d\frac{\partial \Delta Q}{\partial X}\right)=\frac{\tau_{\mathrm{ch}}}{z_d e}(I_e+I_i), \tag{10}$$

where I_e and I_i are the electron and ion current, respectively. The normalized expressions for the electron and ion currents for spherical dust grains with radius a reads as

$$I_e=-\pi a^2 e\sqrt{\frac{8T_e}{\pi m_e}}\,n_{e0}\exp(\Phi)\exp[z(\Delta Q-1)], \tag{11}$$

$$I_i=\pi a^2 e\sqrt{\frac{8T_{if}}{\pi m_i}}\,n_{i0}\left[1-\frac{\Phi}{\sigma_i}-\frac{4}{3}b\left(-\frac{\Phi}{\sigma_i}\right)^{3/2}+\frac{1}{2}\left(\frac{\Phi}{\sigma_i}\right)^2\right]\times\left[\left(1+\frac{z}{\sigma_i}\right)-\frac{z}{\sigma_i}\Delta Q\right], \tag{12}$$

where $z=z_d e^2/4\pi\epsilon_0 a T_e$, $4\pi\epsilon_0 a$ is the capacitance of the spherical dust grain of radius a. Here $\tau_d(\approx\omega_{pd}^{-1})$ is the dust oscillation time scale and τ_{ch} is the charging time scale. The charging time is given by

$$\tau_{\mathrm{ch}}=\left(\frac{a}{\sqrt{2\pi}}\frac{\omega_{pi}^2}{V_{\mathrm{th}i}}(1+z+\sigma_i)\right)^{-1}, \tag{13}$$

where ω_{pi} and $V_{\mathrm{th}i}$ are the ion plasma frequency and the ion thermal velocity, respectively.

III. NONLINEAR WAVE PROPAGATION EQUATIONS

To describe the nonlinear evolution equations, the following two cases have been considered.

A. The nonisothermal parameter $b \gg O(\sqrt{\epsilon})$

In this case, in order to study the nonlinear propagation of small-amplitude dust acoustic waves in an unmagnetized dusty plasma by the reductive perturbation technique, the independent variables are scaled[22–24] according to

$$\xi=\epsilon^{1/4}(X-\lambda T);\quad \tau=\epsilon^{3/4}T, \tag{14}$$

where λ is the phase velocity of the linear dust acoustic wave normalized by the dust acoustic speed, and ϵ is a small parameter characterizing the strength of the nonlinearity.

The dependent variables are expressed as

$$f=f^{(0)}+\epsilon f^{(1)}+\epsilon^{3/2}f^{(2)}+\cdots, \tag{15}$$

where $f^{(0)}=0$ for $f=V_d$, Φ, and ΔQ, while $f^{(0)}=1$ for $f=N_d$. To consider the effect of nonadiabatic dust charge variations, τ_{ch}/τ_d is assumed to be small but finite.[18,19] To make the nonlinear perturbation consistent, it is assumed that τ_{ch}/τ_d is proportional to $\epsilon^{1/4}$. Thus, we take

$$\frac{\tau_{ch}}{\tau_d} = \nu\epsilon^{1/4}. \tag{16}$$

Introducing the transformation (14) and expansions (15) in the set of equations (2), (4), and (10), equating the terms of order $\epsilon^{3/4}$, and finally using boundary conditions $V_d^{(1)}$, $\Phi^{(1)}$, $\Delta Q^{(1)} \to 0$ as $X \to -\infty$, we get

$$\lambda N_d^{(1)} = V_d^{(1)}, \tag{17}$$

$$\lambda V_d^{(1)} = \gamma\sigma_d N_d^{(1)} - \Phi^{(1)}, \tag{18}$$

$$\Delta Q^{(1)} = N_d^{(1)} + \frac{(\delta+\sigma_i)}{\sigma_i(\delta-1)}\Phi^{(1)}. \tag{19}$$

Due to the assumption (16), Eq. (10) with the help of Eqs. (11) and (12) reduces to

$$\epsilon^{3/2}\nu\lambda\frac{\partial \Delta Q^{(1)}}{\partial \xi} = \epsilon[\beta_d\Phi^{(1)} + \Delta Q^{(1)}] + \epsilon^{3/2}\left[\beta_d\Phi^{(2)} + \Delta Q^{(2)} + \frac{4}{3}\frac{\sigma_i}{(\sigma_i+1)}\beta_d b\left(-\frac{\Phi^{(1)}}{\sigma_i}\right)^{3/2}\right], \tag{20}$$

where

$$\beta_d = \frac{(z+\sigma_i)(\sigma_i+1)}{z\sigma_i(1+\sigma_i+z)}. \tag{21}$$

Equating the terms containing ϵ, from Eq. (20) we get the following relation for $\Delta Q^{(1)}$:

$$\Delta Q^{(1)} = -\beta_d\Phi^{(1)}. \tag{22}$$

From Eqs. (17) and (18), we get

$$N_d^{(1)} = -\frac{\Phi^{(1)}}{(\lambda^2-\gamma\sigma_d)}. \tag{23}$$

The phase velocity λ of the dust acoustic wave can be determined self-consistently from Eqs. (19), (22), and (23) in the following form:

$$\lambda = \sqrt{\gamma\sigma_d + \frac{1}{\beta_d + \frac{(\delta+\sigma_i)}{\sigma_i(\delta-1)}}}. \tag{24}$$

The term β_d arises here due to the dust charge variation and $\gamma\sigma_d$ due to the warm adiabatic dust grains.

Equating the terms containing next higher order in ϵ, we get

$$\frac{\partial N_d^{(1)}}{\partial \tau} = \lambda\frac{\partial N_d^{(2)}}{\partial \xi} - \frac{\partial V_d^{(2)}}{\partial \xi}, \tag{25}$$

$$\frac{\partial V_d^{(1)}}{\partial \tau} = \lambda\frac{\partial V_d^{(2)}}{\partial \xi} + \frac{\partial \Phi^{(2)}}{\partial \xi} - \gamma\sigma_d\frac{\partial N_d^{(2)}}{\partial \xi}, \tag{26}$$

$$\frac{\partial^2 \Phi^{(1)}}{\partial \xi^2} = \left[N_d^{(2)} - \Delta Q^{(2)} + \frac{(\delta+\sigma_i)}{\sigma_i(\delta-1)}\Phi^{(2)} + \frac{4}{3}\frac{\delta}{(\delta-1)}b\left(-\frac{\Phi^{(1)}}{\sigma_i}\right)^{3/2}\right], \tag{27}$$

$$\Delta Q^{(2)} = \nu\lambda\frac{\partial \Delta Q^{(1)}}{\partial \xi} - \beta_d\Phi^{(2)} - \frac{4}{3}\frac{\sigma_i}{(\sigma_i+1)}\beta_d b\left(-\frac{\Phi^{(1)}}{\sigma_i}\right)^{3/2}. \tag{28}$$

Eliminating $\partial V_d^{(2)}/\partial \xi$ from Eqs. (25) and (26), we get

$$\frac{\partial N_d^{(2)}}{\partial \xi} = -\frac{1}{(\lambda^2-\gamma\sigma_d)}\frac{\partial \Phi^{(2)}}{\partial \xi} - \frac{2\lambda}{(\lambda^2-\gamma\sigma_d)}\frac{\partial \Phi^{(1)}}{\partial \tau}. \tag{29}$$

Differentiating Eq. (27) with respect to ξ, and using Eqs. (28) and (22), we get the following modified KdV Burger (mKdVB) equation:

$$\frac{\partial \Phi^{(1)}}{\partial \tau} + \alpha\sqrt{-\Phi^{(1)}}\frac{\partial \Phi^{(1)}}{\partial \xi} + \beta\frac{\partial^3 \Phi^{(1)}}{\partial \xi^3} = \mu\frac{\partial^2 \Phi^{(1)}}{\partial \xi^2}, \tag{30}$$

where

$$\alpha = b\frac{(\lambda^2-\gamma\sigma_d)^2}{\lambda\sqrt{\sigma_i}}\left(\frac{\beta_d\sigma_i(\delta-1)+\delta(\sigma_i+1)}{\sigma_i(\delta-1)(\sigma_i+1)}\right), \tag{31}$$

$$\beta = \frac{(\lambda^2-\gamma\sigma_d)^2}{2\lambda}, \tag{32}$$

$$\mu = \nu\beta_d\frac{(\lambda^2-\gamma\sigma_d)^2}{2}. \tag{33}$$

The expression (31) for α shows that $\alpha > or < o$ according as $b > or < 0$, i.e., $T_{it} > or < T_{if}$.

B. The nonisothermal parameter $b = O(\sqrt{\epsilon})$

In this case, the independent variables are scaled[26] according to

$$\xi = \epsilon^{1/2}(X-\lambda T); \quad \tau = \epsilon^{3/2}T. \tag{34}$$

The dependent variables are expressed as

$$f = f^{(0)} + \epsilon f^{(1)} + \epsilon^2 f^{(2)} + \cdots, \tag{35}$$

where $f^{(0)} = 0$ for $f = V_d$, Φ, and ΔQ, while $f^{(0)} = 1$ for $f = N_d$. To obtain a consistent perturbation expansion incorporating the effect of nonadiabatic dust charge variation, we set τ_{ch}/τ_d proportional to $\epsilon^{1/2}$.[18,19] Thus

$$\frac{\tau_{ch}}{\tau_d} = \nu\epsilon^{1/2}. \tag{36}$$

The results of the linear analysis are the same as in Sec. III A, but the nonlinear part is different. With τ_{ch}/τ_d given by (36), Eq. (10), with the help of Eqs. (11) and (12), reduces to

$$\epsilon^2 \nu\lambda \frac{\partial \Delta Q^{(1)}}{\partial \xi}$$

$$= \epsilon[\beta_d \Phi^{(1)} + \Delta Q^{(1)}] + \epsilon^2 \left[\beta_d \Phi^{(2)} + \Delta Q^{(2)} + \frac{4}{3} \frac{\sigma_i}{(\sigma_i+1)} \times \beta_d b \left(-\frac{\Phi^{(1)}}{\sigma_i} \right)^{3/2} + \frac{\beta_d(\sigma_i-1)}{2\sigma_i} \Phi^{(1)^2} + \frac{z^2 \beta_d \sigma_i}{2(\sigma_i+1)} \times \Delta Q^{(1)^2} + \frac{(\sigma_i+z)}{(1+\sigma_i+z)} \left(1 - \frac{1}{\sigma_i(\sigma_i+z)} \right) \Delta Q^{(1)} \Phi^{(1)} \right]. \tag{37}$$

Introducing (35), (36) into (2), (4), and (10) and then equating the terms in highest powers of ϵ, we obtain

$$\frac{\partial N_d^{(1)}}{\partial \tau} = \lambda \frac{\partial N_d^{(2)}}{\partial \xi} - \frac{\partial V_d^{(2)}}{\partial \xi}, \tag{38}$$

$$\frac{\partial V_d^{(1)}}{\partial \tau} + V_d^{(1)} \frac{\partial V_d^{(1)}}{\partial \xi} + \Delta Q^{(1)} \frac{\partial \Phi^{(1)}}{\partial \xi} + \gamma(\gamma-2)\sigma_d N_d^{(1)} \frac{\partial N_d^{(1)}}{\partial \xi} = \lambda \frac{\partial V_d^{(2)}}{\partial \xi} + \frac{\partial \Phi^{(2)}}{\partial \xi} - \gamma\sigma_d \frac{\partial N_d^{(2)}}{\partial \xi}, \tag{39}$$

$$\frac{\partial^2 \Phi^{(1)}}{\partial \xi^2} = N_d^{(2)} - \Delta Q^{(2)} + \frac{(\delta+\sigma_i)}{\sigma_i(\delta-1)} \Phi^{(2)} + \frac{4}{3} \frac{b\delta}{(\delta-1)} \times \left(-\frac{\Phi^{(1)}}{\sigma_i} \right)^{3/2} + \frac{(\sigma_i^2-\delta)}{2\sigma_i^2(\delta-1)} \Phi^{(1)^2} - \Delta Q^{(1)} N_d^{(1)}, \tag{40}$$

$$\Delta Q^{(2)} = \nu\lambda \frac{\partial \Delta Q^{(1)}}{\partial \xi} - \beta_d \Phi^{(2)} - \frac{z\beta_d^2}{2(\sigma_i+z)(1+\sigma_i+z)} \Phi^{(1)^2} - \frac{4}{3} \frac{\sigma_i}{(\sigma_i+1)} \beta_d b \left(-\frac{\Phi^{(1)}}{\sigma_i} \right)^{3/2}. \tag{41}$$

Eliminating $\partial V_d^{(2)}/\partial \xi$ from Eqs. (38) and (39), we get

$$\frac{\partial N_d^{(2)}}{\partial \xi} = -\frac{1}{(\lambda^2-\gamma\sigma_d)} \left[\frac{\partial \Phi^{(2)}}{\partial \xi} + 2\lambda \frac{\partial \Phi^{(1)}}{\partial \tau} - \frac{[3\lambda^2 + \gamma(\gamma-2)\sigma_d - \beta_d(\lambda^2-\gamma\sigma_d)^2]}{(\lambda^2-\gamma\sigma_d)^2} \Phi^{(1)} \frac{\partial \Phi^{(1)}}{\partial \xi} \right]. \tag{42}$$

Differentiating Eq. (40) with respect to ξ, and using Eqs. (41) and (42), we get the following modified KdV Burger (mKdVB) equation with both nonlinearities:

$$\frac{\partial \Phi^{(1)}}{\partial \tau} + \alpha\sqrt{-\Phi^{(1)}} \frac{\partial \Phi^{(1)}}{\partial \xi} - l\Phi^{(1)} \frac{\partial \Phi^{(1)}}{\partial \xi} + \beta \frac{\partial^3 \Phi^{(1)}}{\partial \xi^3} = \mu \frac{\partial^2 \Phi^{(1)}}{\partial \xi^2}, \tag{43}$$

where

$$l = \beta \left(\frac{(\sigma_i^2-\delta)}{\sigma_i^2(\delta-1)} + \frac{3\lambda^2 + \gamma(\gamma-2)\sigma_d + \beta_d(\lambda^2-\gamma\sigma_d)^2}{(\lambda^2-\gamma\sigma_d)^3} + \frac{z\beta_d^2}{(\sigma_i+z)(1+z+\sigma_i)} \right). \tag{44}$$

Here α, β, and μ are the same as given by Eqs. (31), (32), and (33). From the expression of α, l, it is seen that the coefficient of nonlinear terms is significantly modified by the dust charge variations and the warm adiabatic dust grains. The expression (33), shows that the Burger term μ present here is due to the nonadiabatic dust charge variation and is proportional to the term β_d.

IV. SHOCK STRUCTURE

A. For $b \gg O(\sqrt{\epsilon})$

1. $\alpha > 0$

In this section we consider $\sigma < 1$, i.e., the temperature of free ions T_{if} is less than the temperature of trapped ions T_{it}.

The Burger term in (30) implies the possibility of the existence of a shock wave. On transforming to the wave frame,

$$\eta = V\tau - \xi = \epsilon^{1/4} \frac{[c_d(\lambda + \sqrt{\epsilon}V)t - x]}{\lambda_D}, \tag{45}$$

the mKdV Burger equation (30) reduces to

$$\frac{d^2 \Phi^{(1)}}{d\eta^2} = \frac{V}{\beta} \Phi^{(1)} + \frac{2\alpha}{3\beta} (\Phi^{(1)})^{3/2} - \frac{\mu}{\beta} \frac{d\Phi^{(1)}}{d\eta}. \tag{46}$$

Equation (46) has two fixed points $\Phi^{(1)}=0$, $(d\Phi^{(1)}/d\eta = 0)$ and $\Phi^{(1)} = -(3V/2\alpha)^2$, $(d\Phi^{(1)}/d\eta = 0)$. The first one $\Phi^{(1)}=0$ is a saddle point, while the second one $\Phi^{(1)} = -(3V/2\alpha)^2$ is a stable focus or a stable node according to

$$\left(\frac{\tau_{\rm ch}}{\tau_d} \right)^2 < \text{or} > \frac{4(M-1)}{\beta_d^2(\lambda^2-\gamma\sigma_d)^2}, \tag{47}$$

where M is defined by the ratio of the nonlinear wave velocity to the linear dust acoustic wave velocity $c_d\lambda$,

$$M = 1 + \sqrt{\epsilon} \frac{V}{\lambda}. \tag{48}$$

Thus, $\Phi^{(1)}(\eta)$ is obtained by numerical integration of (46) subject to the boundary condition $\Phi^{(1)} \to 0$ as $\eta \to -\infty$. Thus, at any x the potential $\Phi^{(1)}$ builds up from a near zero value at long past $t \to -\infty$ $(\eta \to -\infty)$ to a steady value,

$$\Phi^{(1)} = -\left(\frac{3(M-1)\lambda}{2\alpha} \right)^2, \tag{49}$$

as $t \to \infty$, showing an oscillatory and quasimonotone shock structure, as illustrated in Figs. 1 and 2, when $(\tau_{\rm ch}/\tau_d)^2$ satisfies the first or second restriction given in Eq. (47). Since the potential is negative, the negatively charged dust grains with energies less than

$$E = -z_d e\phi = z_d T_e \left(\frac{3(M-1)\lambda}{2\alpha} \right)^2 \tag{50}$$

are reflected by the shock wave.

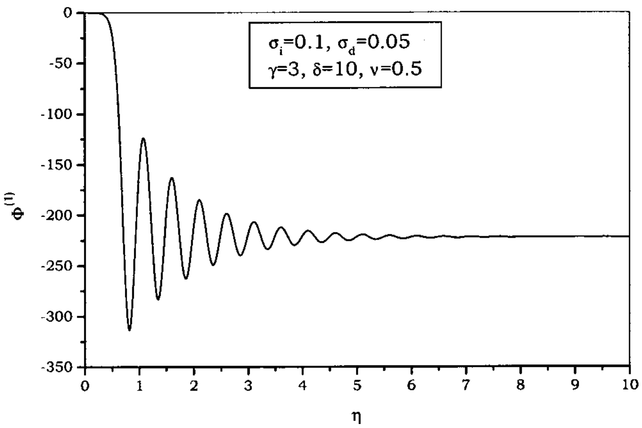


FIG. 1. Variation of $\Phi^{(1)}$ with η [given in Eq. (46)] for weak dissipation with $\nu=0.5$ [given in Eq. (16)]: Oscillatory shock structure.

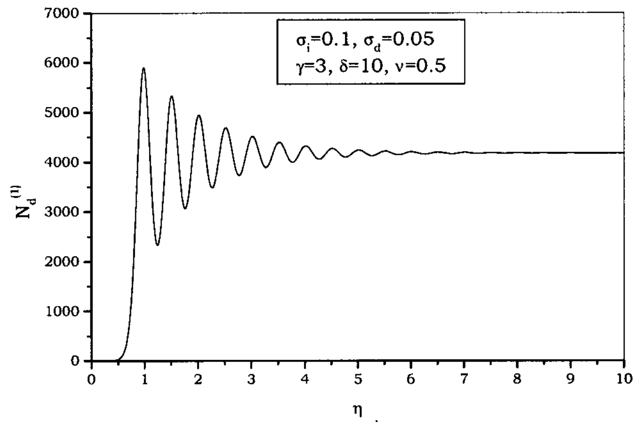


FIG. 3. Compression of dust number density, as given by Eq. (51).

The dust number density $N_d^{(1)}$ is related to the negative potential $\Phi^{(1)}$ by the relation (23). Substituting (23) in Eq. (46), the following equation for the dust number density $N_d^{(1)}$ can be obtained:

$$\frac{d^2 N_d^{(1)}}{d\eta^2}=\frac{V}{\beta}N_d^{(1)}-\frac{2\alpha}{3\beta}\sqrt{(\lambda^2-\gamma\sigma_d)}N_d^{(1)^{3/2}}-\frac{\mu}{\beta}\frac{dN_d^{(1)}}{d\eta}. \tag{51}$$

Numerical integration of (51) for $N_d^{(1)}(\eta)$ shows that the dust number density is compressed due to the dust acoustic shock wave, i.e., it exhibits a compressional shock wave, as illustrated in Fig. 3.

Again, due to the adiabatic pressure variation of the dust grains, the dust temperature T_d is related to the dust number density by the relation, where normalized dust temperature T_d is

$$T_d=\frac{T_d^{(1)}}{T_0}=(\gamma-1)N_d^{(1)}, \tag{52}$$

where $T_d=T_0$ at $\eta=-\infty$, i.e., $N_d=1$. Using the equation (52) in equation (51), the differential equation for T_d can be determined as

$$\frac{d^2T_d}{d\eta^2}=\frac{V}{\beta}T_d-\frac{2\alpha}{3\beta}\sqrt{\frac{(\lambda^2-\gamma\sigma_d)}{\gamma-1}}T_d^{3/2}-\frac{\mu}{\beta}\frac{dT_d}{d\eta}. \tag{53}$$

Numerical integration of (53) shows that as the shock propagates the dust temperature increases due to adiabatic compression, leading to a dust density increase.

2. α<0

In this section we consider the case $\sigma>1$, i.e., the temperature of free ions T_{if} is greater than the temperature of trapped ions T_{it}.

In this case the differential equation (46) for $\Phi^{(1)}$ becomes

$$\frac{d^2\Phi^{(1)}}{d\eta^2}=\frac{V}{\beta}\Phi^{(1)}-\frac{2|\alpha|}{3\beta}(-\Phi^{(1)})^{3/2}-\frac{\mu}{\beta}\frac{d\Phi^{(1)}}{d\eta}. \tag{54}$$

Equation (54) has only one fixed point $\Phi^{(1)}=0$, $d\Phi^{(1)}/d\eta=0$. The possibility of the existence of the other zero is ruled out as $\Phi^{(1)}<0$ and $\sqrt{-\Phi^{(1)}}>0$. Thus, in this case no shock generation is possible.

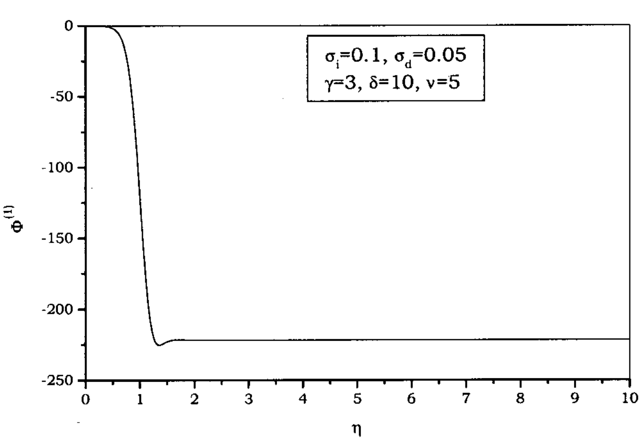


FIG. 2. Variation of $\Phi^{(1)}$ with η [given in Eq. (46)] for strong dissipation with $\nu=5$ [given in Eq. (16)]: Monotone shock structure.

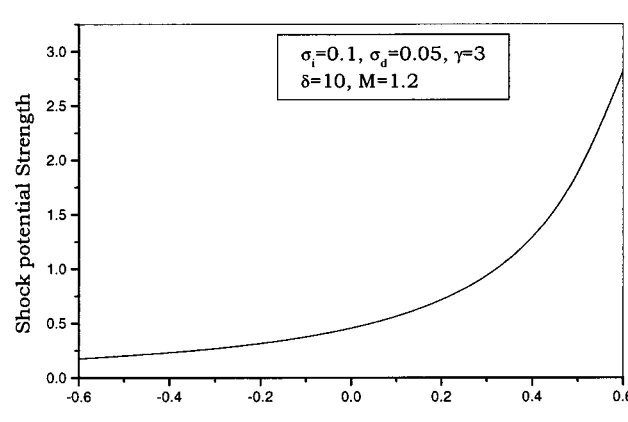


FIG. 4. Variation of shock potential strength with $\sigma=T_{if}/T_{it}$ for Mach number [given in Eq. (48)] $M=1.2$.

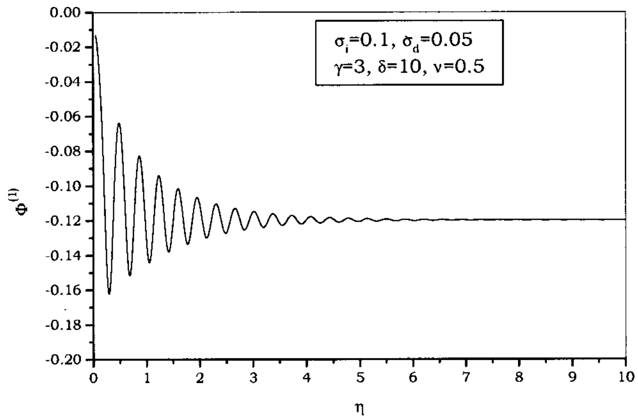


FIG. 5. Variation of $\Phi^{(1)}$ with η as given in Eq. (57) for weak dissipation with $\nu=0.5$: Oscillatory shock structure.

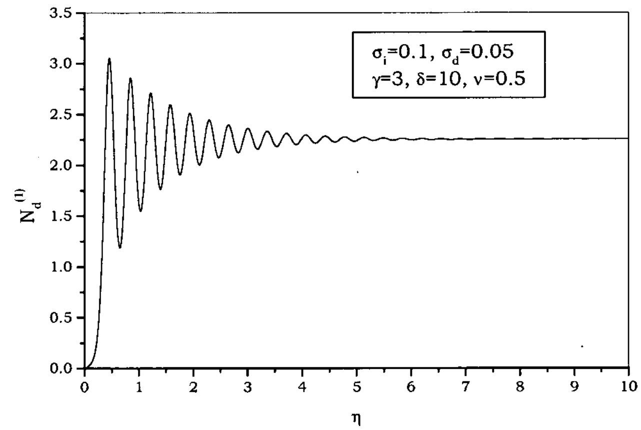


FIG. 7. Compression of dust number density as given in Eq. (61).

B. For $b=O(\sqrt{\epsilon})$

The Burger term in (43) also implies the possibility of the existence of a shock structure. Transforming to the wave frame,

$$\eta=V\tau-\xi=\epsilon^{1/2}\frac{[c_d(\lambda+\epsilon V)t-x]}{\lambda_{\mathrm{D}}}, \tag{55}$$

the mKdV Burger equation (43) reduces to

$$\frac{d^2\Phi^{(1)}}{d\eta^2}=\frac{V}{\beta}\Phi^{(1)}+\frac{2\alpha}{3\beta}(-\Phi^{(1)})^{3/2}+\frac{l}{2\beta}\Phi^{(1)^2}-\frac{\mu}{\beta}\frac{d\Phi^{(1)}}{d\eta}. \tag{56}$$

Clearly we must have $\Phi^{(1)}<0$ and $\sqrt{-\Phi^{(1)}}>0$. This condition restricts the number of zeros of

$$\frac{1}{\beta}\left[V\Phi^{(1)}+\frac{2\alpha}{3}(-\Phi^{(1)})^{3/2}+\frac{l}{2}\Phi^{(1)^2}\right], \tag{57}$$

to two, namely $\Phi^{(1)}=0$ and

$$\sqrt{-\Phi^{(1)}}=-\frac{2\alpha}{3l}+\sqrt{\frac{4\alpha^2}{9l^2}+\frac{2V}{l}} \quad \text{if } \alpha>0,$$

$$=\frac{2|\alpha|}{3l}-\sqrt{\frac{4\alpha^2}{9l^2}+\frac{2V}{l}} \quad \text{if } \alpha<0. \tag{58}$$

In either case,

$$\Phi^{(1)}=-r^2=-\left(\frac{2|\alpha|}{3l}-\sqrt{\frac{4\alpha^2}{9l^2}+\frac{2V}{l}}\right)^2. \tag{59}$$

The fixed points of (56) are at $(\Phi^{(1)}=0, d\Phi^{(1)}/d\eta=0)$ and $(\Phi^{(1)}=-r^2, d\Phi^{(1)}/d\eta=0)$. See Fig. 4. As in Sec. IV A 1, the first point is a saddle point, while the second one is a stable focus [oscillatory shock structure Fig. 5) or a stable node (monotonic structure, Fig. 6) according to $\lambda(M-1)+(|\alpha|/3l)(-2|\alpha|/3l+\sqrt{4\alpha^2/9l^2+2V/l})>$ or $<\mu/\beta$, where M is the Mach number defined by $M=1+\epsilon(V/\lambda)$. Hence, also the negatively charged dust grains with energies less than

$$E=-z_d e\phi=z_d T_e\left(\frac{2|\alpha|}{3l}-\sqrt{\frac{4\alpha^2}{9l^2}+\frac{2(M-1)\lambda}{l}}\right)^2, \tag{60}$$

suffer reflection by the negative potential of the shock wave.

In this case the dust number density $N_d^{(1)}$ and dust temperature T_d satisfy the following differential equations:

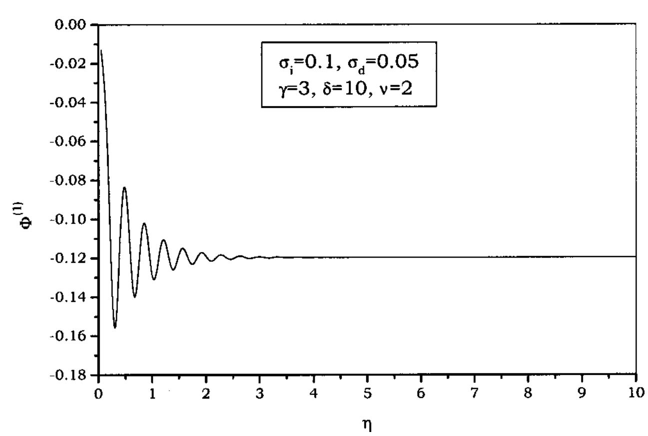


FIG. 6. Variation of $\Phi^{(1)}$ with η as given in Eq. (56) for strong dissipation $\nu=2$: Monotone shock transition.

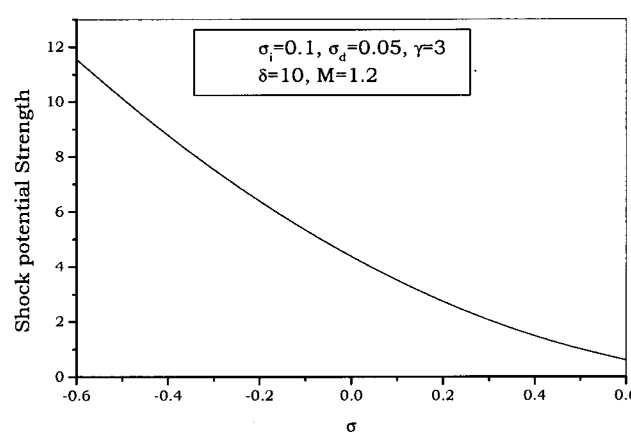


FIG. 8. Variation of shock potential strength with $\sigma=T_{if}/T_{it}$ for Mach number $M=1.2$.

$$\frac{d^2 N_d^{(1)}}{d\eta^2} = \frac{V}{\beta} N_d^{(1)} - \frac{2\alpha}{3\beta}\sqrt{(\lambda^2 - \gamma\sigma_d)} N_d^{(1)^{3/2}} - \frac{l}{2\beta}(\lambda^2 - \gamma\sigma_d) N_d^{(1)^2} - \frac{\mu}{\beta}\frac{dN_d^{(1)}}{d\eta}, \quad (61)$$

$$\frac{d^2 T_d}{d\eta^2} = \frac{V}{\beta} T_d - \frac{2\alpha}{3\beta}\sqrt{\frac{\lambda^2 - \gamma\sigma_d}{\gamma - 1}} T_d^{3/2} - \frac{l}{2\beta}\left(\frac{\lambda^2 - \gamma\sigma_d}{\gamma - 1}\right) T_d^2 - \frac{\mu}{\beta}\frac{dT_d}{d\eta}. \quad (62)$$

V. DISCUSSION

The results that have been found in this investigation may be summarized as follows.

(1) It is seen that due to the effect of the nonadiabatic dust charge variations and the nonisothermal ions, the nonlinear dust acoustic wave is governed by a modified KdV Burger equation (30). The dissipation coefficient μ (proportional to τ_{ch}/τ_d) vanishes in the adiabatic limit, i.e., the oscillation time scale $\tau_d \gg \tau_{ch}$, the charging time. The modification of the KdV Burger equation is due to the presence of the nonisothermal ions in the dusty plasma. It is interesting to note that, for the stronger nonisothermal effect, the nonlinear dust acoustic wave is governed by a modified KdV Burger equation with a single nonlinear term $\sqrt{-\Phi^{(1)}} \times(\partial\Phi^{(1)}/\partial\xi)$ [Eq. (30)], whereas for the lower nonisothermal effect, it is governed by a modified KdV Burger equation with both nonlinearities $\sqrt{-\Phi^{(1)}}(\partial\Phi^{(1)}/\partial\xi)$ and $\Phi^{(1)} \times(\partial\Phi^{(1)}/\partial\xi)$ [Eq. (43)].

(2) It is interesting to note that the dust acoustic shock wave propagation is not possible if the temperature of the free ions is greater than that of the trapped ions.

(3) Figures 1–8 are drawn for the parameter $\sigma_i = 0.1$, $\sigma_d = 0.05$, $\delta = 10, 2, 5$, and $\gamma = 3$. Of these, the Figs. 1–4 are drawn for the case $b \gg O(\sqrt{\epsilon})$, whereas the other figures are drawn for the case $b \approx (\sqrt{\epsilon})$.

The structure of steady small-amplitude shocks [given by Eq. (46) and Eq. (56)] are shown in Figs. 1, 2, 5, and 6. The transition from the upstream to the far downstream state changes from a oscillatory to monotonic nature, depending on the magnitude of the dissipation coefficient μ. It is also seen that the dust acoustic wave admits a negative potential (Fig. 1, Fig. 2, Fig. 6, and Fig. 7).

(4) Since the dust acoustic wave admits negative potentials (Fig. 1, Fig. 2, Fig. 5, and Fig. 6), the dust number density increases as given by Eqs. (23). Figures 3 and 7 show that it exhibits compressional shock wave (dust density increases).

(5) It is seen that the temperature of the dust cloud increases due to adiabatic compression of the dust cloud, as given by Eq. (52).

(6) It is also seen that the shock potential strength increases (Fig. 4) or decreases (Fig. 8) with the increase of the ratio of free to trapped ion temperature depending on the nonisothermal effects of ions.

(7) From the present study it is also seen that nonlinear wave amplitude decreases in case of the smaller effects of the nonisothermal ions more than that of the higher. This happened, because when $b \approx O(\sqrt{\epsilon})$, the nonlinearities of the dust acoustic wave increases more than that of $b \gg O(\sqrt{\epsilon})$.

[1] J. H. Chu, J. B. Du, and I. J. Lin, J. Phys. D **27**, 296 (1994).
[2] N. D'Angelo, J. Phys. D **28**, 1009 (1995).
[3] H. Prabhakara and V. L. Thana, Phys. Plasmas **3**, 3176 (1996).
[4] N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. **38**, 543 (1990).
[5] P. K. Shukla and V. P. Silin, Phys. Scr. **45**, 508 (1992).
[6] C. K. Goertz, Rev. Geophys. **27**, 271 (1989).
[7] R. K. Varma, P. K. Shukla, and V. Krishan, Phys. Rev. E **47**, 3612 (1993).
[8] M. R. Jana, A. Sen, and P. K. Kaw, Phys. Rev. E **48**, 3930 (1993).
[9] F. Melandso, T. Aslaksen, and O. Havnes, Planet. Space Sci. **41**, 321 (1993).
[10] S. V. Singh and N. N. Rao, J. Plasma Phys. **60**, 541 (1998).
[11] A. Barkan, R. Merlino, and N. D'Angelo, Phys. Plasmas **2**, 3563 (1995).
[12] A. Barkan, R. Merlino, and N. D'Angelo, Phys. Lett. A **222**, 329 (1996).
[13] Q. Z. Luo, N. D'Angelo, and R. L. Merlino, Phys. Plasmas **6**, 3345 (1999).
[14] Y. Nakamura, H. Bailung, and P. K. Shukla, Phys. Rev. Lett. **83**, 1602 (1999).
[15] J. X. Ma and X. Liu, Phys. Plasmas **4**, 253 (1997).
[16] B. Xie, K. He, and Z. Huang, Phys. Lett. A **247**, 403 (1998).
[17] S. Ghosh, S. Sarkar, M. Khan, and M. R. Gupta, Phys. Plasmas **7**, 3594 (2000).
[18] M. R. Gupta, S. Sarkar, S. Ghosh, M. Debnath, and M. Khan, Phys. Rev. E **63**, 046406 (2001).
[19] S. Ghosh, S. Sarkar, M. Khan, and M. R. Gupta, Phys. Lett. A **274**, 162 (2000).
[20] Yu. A. Schekinov, Phys. Lett. A **225**, 117 (1997).
[21] M. Kakati and K. S. Goswami, Phys. Plasmas **5**, 4508 (1998).
[22] H. Schamel, J. Plasma Phys. **14**, 905 (1972).
[23] H. Schamel, J. Plasma Phys. **9**, 377 (1973).
[24] H. Schamel, J. Plasma Phys. **13**, 139 (1975).
[25] J. E. Allen, Phys. Scr. **45**, 497 (1992).
[26] H. Washimi and T. Tanuiti, Phys. Rev. Lett. **17**, 996 (1966).