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Effect of Landau damping on kinetic Alfvén and ion-acoustic solitary waves in a magnetized nonthermal plasma with warm ions

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The evolution equations describing both kinetic Alfvén wave and ion-acoustic wave in a nonthermal magnetized plasma with warm ions including weak nonlinearity and weak dispersion with the effect of Landau damping have been derived. These equations reduce to two coupled equations constituting the KdV–ZK (Korteweg–de Vries–Zakharov–Kuznetsov) equation for both kinetic Alfvén wave and ion-acoustic wave, including an extra term accounting for the effect of Landau damping. When the coefficient of the nonlinear term of the evolution equation for ion-acoustic wave vanishes, the nonlinear behavior of ion-acoustic wave, including the effect of Landau damping, is described by two coupled equations constituting the modified KdV–ZK (MKdV–ZK) equation, including an extra term accounting for the effect of Landau damping. It is found that there is no effect of Landau damping on the solitary structures of the kinetic Alfvén wave. Both the macroscopic evolution equations for the ion-acoustic wave admits solitary wave solutions, the former having a sech^2 profile and the latter having a sech profile. In either case, it is found that the amplitude of the ion-acoustic solitary wave decreases slowly with time. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1490132]

I. INTRODUCTION

The existence and stability of solitary waves in a magnetized plasma consisting of nonthermal electrons and cold ions or adiabatic warm ions have been investigated by Cairns *et al.*,[1–3] Mamun and Cairns,[4] and Bandyopadhyay and Das.[5–8] These investigations were motivated by observations of solitary structures with density depletion made by the Freja Satellite (Dovner *et al.*[9]). Recently Bandyopadhyay and Das[10] have derived two macroscopic evolution equations that describe the ion-acoustic wave in a nonthermal magnetized plasma with warm ions, including weak nonlinearity and weak dispersion with the effect of Landau damping. These evolution equations are same as the KdV–ZK (Korteweg–de Vries–Zakharov–Kuznetsov) and MKdV–ZK (modified KdV–ZK) equations (10) and (14) of Bandyopadhyay and Das[5] in their paper, except for an extra term accounting for the effect of Landau damping. These evolution equations admit solitary wave solutions, the former having a sech^2 profile and the latter having a sech profile, but the amplitude of the solitary waves in either case decreases slowly with time.

In the investigations mentioned in the above paragraph for magnetized nonthermal plasma, only electrostatic perturbations have been considered. If magnetic field perturbations are also taken into account, then, in addition to the ion-acoustic mode, the Alfvén mode will also appear, which becomes dispersive if kinetic effects like ion drift velocity are included in the ion continuity equation and in the ion fluid equation of motion. Bandyopadhyay and Das[11] have extended the analysis of their previous paper[5] by including magnetic field perturbations and by including kinetic effects mentioned above. The linear dispersion relation (1) of Bandyopadhyay and Das[11] in their paper shows a coupling between the kinetic Alfvén wave and the ion-acoustic wave. In this paper[11] they have derived two coupled equations, constituting the KdV–ZK equation for both the kinetic Alfvén wave and the ion-acoustic wave. When the coefficient of the nonlinear term of the KdV–ZK equation for the ion-acoustic wave vanishes, Bandyopadhyay and Das[11] have derived two coupled equations constituting a modified KdV–ZK (MKdV–ZK) equation to describe the nonlinear behavior of an ion-acoustic wave. In the derivation of these equations the effect of Landau damping has not been considered. The evolution equations, for weakly nonlinear and a weakly dispersive kinetic Alfvén and ion-acoustic wave, derived in the paper, by Bandyopadhyay and Das[11] are extended in the present paper to include the effect of Landau damping. We start from the same set of governing equations as given in the paper of Bandyopadhyay and Das.[11] But their equation (11b) for the number density of electrons is replaced by the Vlasov–Boltzmann equation for electrons with equilibrium nonthermal electron distribution. The extended evolution equations derived here reduced to the equations derived by Bandyopadhyay and Das[11] when extra terms responsible for Landau damping effect are dropped. But for the kinetic Alfvén wave there is no extra term responsible for Landau damping. It is shown that the KdV–ZK equation derived here for the ion-acoustic wave admits a solitary wave solution propagating obliquely to the external magnetic field having a sech^2 profile, but its amplitude decreases slowly with time. The modified KdV–ZK equation, derived here for the ion-acoustic wave for the case when the coefficient of a nonlinear term of the corresponding KdV–ZK equation vanishes, also admits a solitary wave solution having a sech profile. But its amplitude also decreases slowly with time.

This paper is organized as follows. In Sec. II basic equations are given. In Sec. III evolution equations for an ion-acoustic wave and a kinetic Alfvén wave are derived. The solitary wave solutions with time-dependent amplitude are given in Sec. IV. Finally, brief conclusions are given in Sec. V.

II. GOVERNING EQUATIONS

We consider a plasma consisting of warm adiabatic ions and nonthermal electrons immersed in a uniform external magnetic field B_0 directed along the z axis. We assume that the ratio of the particle pressure to the magnetic pressure is small and the characteristic frequency is much smaller than the ion-cyclotron frequency. The nonlinear behavior of kinetic Alfvén waves and ion-acoustic waves may be described by the following set of equations, which consist of the ion continuity equation, the parallel component of the ion fluid equation of motion, the electron continuity equation, the quasineutrality condition, Ampère's law in the parallel direction, and the adiabatic pressure law for ions, in which we express the number density of electrons in terms of its velocity distribution function, where the velocity distribution function of electrons must satisfy the Vlasov–Boltzmann equation. These equations are to be supplemented by the equilibrium nonthermal velocity distribution function of electrons. In the expression for ion-drift velocity both $\vec{E}\times\vec{B}$ drift and polarization drift terms are retained and the electric field intensity is represented by two potentials according to Kadomtsev:[12]

$$\frac{\partial n_i}{\partial t}+\vec{\nabla}_\perp\cdot(n_i\vec{\nu}_{\mathrm{id}})+\frac{\partial}{\partial z}(n_i\nu_{iz})=0, \tag{1}$$

$$\frac{\partial \nu_{iz}}{\partial t}+(\vec{\nu}_{\mathrm{id}}\cdot\vec{\nabla}_\perp)\nu_{iz}+\nu_{iz}\frac{\partial \nu_{iz}}{\partial z}=E_z-\frac{\sigma}{n_i}\frac{\partial P}{\partial z}, \tag{2}$$

$$\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial z}(n_e\nu_{ez})=0, \tag{3}$$

$$n_i=n_e, \tag{4}$$

$$\frac{\partial}{\partial z}\nabla_\perp^2(\varphi-\psi)=\frac{1}{2}\beta\frac{\partial j_z}{\partial t}, \tag{5}$$

$$P=n_i^\gamma, \tag{6}$$

where the electron number density n_e, ion-drift velocity $\vec{\nu}_{\mathrm{id}}$, and parallel current density j_z are given by

$$n_e=\int_{-\infty}^{\infty} f\,d\nu_{11}, \tag{7}$$

$$\vec{\nu}_{\mathrm{id}}=\vec{E}_\perp\times\hat{z}+\frac{d\vec{E}_\perp}{dt}, \tag{8}$$

$$j_z=n_i\nu_{iz}-n_e\nu_{ez}. \tag{9}$$

The velocity distribution function f of electrons appearing in Eq. (7) satisfies the following Vlasov–Boltzmann equation:

$$\sqrt{\frac{m_e}{m_i}}\frac{\partial f}{\partial t}+\nu_{11}\frac{\partial f}{\partial z}+\frac{\partial \psi}{\partial z}\frac{\partial f}{\partial \nu_{11}}=0. \tag{10}$$

We assume that the equilibrium velocity distribution f_0 is nonthermal and is given by[1–4]

$$f_0(\nu_{11})=\frac{1}{\sqrt{2\pi}(1+3\alpha')}[1+\alpha'\nu_{11}^4]\exp\left(-\frac{1}{2}\nu_{11}^2\right), \tag{11}$$

where α' is a non-negative real number that determines the proportion of fast electrons. The electric field intensity components are determined from the two potentials φ and ψ, according to

$$\vec{E}_\perp=-\vec{\nabla}_\perp\varphi, \quad E_z=-\frac{\partial\psi}{\partial z} \tag{12}$$

(Kadomtsev[12]), where $\perp$ and z indicate components perpendicular and parallel to the ambient magnetic field. In the above equations, n_e and n_i are the electron and ion number densities, ν_{iz} and ν_{ez} are the parallel ion and electron fluid velocities, E_z and $\vec{E}_\perp$ are the parallel and perpendicular components of electric field intensity vector, β is the ratio of particle and magnetic pressure, and $\gamma\,(=\frac{5}{3})$ is the ratio of two specific heats. The above equations have been written in dimensionless form by normalizing the space coordinates (x,y,z), time (t), velocities $(\nu_{iz},\nu_{ez},\vec{\nu}_{\mathrm{id}})$, pressure (P), electric potentials (φ,ψ), parallel current density j_z, electron and ion number densities (n_i,n_e) by ρ_s, ω_{ci}^{-1}, c_s, $n_0K_{\mathrm{B}}T_i$, $K_{\mathrm{B}}T_e/e$, en_0c_s, and n_0, respectively, where $\rho_s=c_s/\omega_{ci}$ is the equivalent ion gyroradius, K_{B} is the Boltzmann's constant, T_e and T_i are the electron's and ion's temperature, n_0 is the unperturbed number density of electrons and ions, and e is the electronic charge. The phase space velocity (ν_{11}) of electrons is normalized by $\nu_e=\sqrt{K_{\mathrm{B}}T_e/m_e}$.

In this paper our main interest is to introduce the effect of Landau damping on weakly nonlinear and weakly dispersive ion-acoustic wave and kinetic Alfvén wave. Following the prescription of Ott and Sudan,[13] we assume that $\sqrt{m_e/m_i}\sim O(\epsilon)$, where ϵ is a small parameter indicating the weakness of dispersion as well as the weakness of Landau damping. So replacing $(m_e/m_i)^{1/2}$ by $(m_e/m_i)^{1/2}\epsilon$ in Eq. (10), this equation can be rewritten as follows:

$$\alpha_1\epsilon\frac{\partial f}{\partial t}+\nu_{11}\frac{\partial f}{\partial z}+\frac{\partial\psi}{\partial z}\frac{\partial f}{\partial \nu_{11}}=0. \tag{13}$$

Therefore Eqs. (1)–(7) and (13) are our basic equations.

III. DERIVATION OF EVOLUTION EQUATIONS

A. Derivation of macroscopic equations

To derive the macroscopic equations governing the development of weakly nonlinear and weakly dispersive ion-acoustic and kinetic Alfvén waves in a magnetized nonthermal plasma with warm ions, we make the following stretchings of space coordinates and time:

$$\xi=\epsilon^{1/2}x, \quad \eta=\epsilon^{1/2}y, \quad \zeta=\epsilon^{1/2}(z-Vt), \quad \tau=\epsilon^{3/2}t, \tag{14}$$

where V is a constant and ϵ is a small parameter, which is a measure of weakness of dispersion and is also a measure of weakness Landau damping. We also make the following perturbation expansions of dependent variables:

$$\begin{aligned} n&=1+\epsilon n^{(1)}+\epsilon^2 n^{(2)}+\cdots,\\ \varphi&=\varphi^{(1)}+\epsilon\varphi^{(2)}+\cdots,\\ \psi&=\epsilon\psi^{(1)}+\epsilon^2\psi^{(2)}+\cdots,\\ \nu_{iz}&=\epsilon\nu_{iz}^{(1)}+\epsilon^2\nu_{iz}^{(2)}+\cdots,\\ \nu_{ez}&=\epsilon\nu_{ez}^{(1)}+\epsilon^2\nu_{ez}^{(2)}+\cdots,\\ f&=f_0+\epsilon f^{(1)}+\epsilon^2 f^{(2)}+\cdots, \end{aligned} \tag{15}$$

where $n=n_i=n_e$ according to the quasineutrality condition (4).

Substituting the stretchings (14) and the perturbation expansions (15) in the governing equation (1)–(3), (5), (7), and (13), and then equating the different powers of ϵ on both sides of each equation, we get a sequence of equations for the perturbed quantities.

From the lowest-order equations obtained from (1)–(3), (5), and (7), we get

$$\begin{aligned} &\nu_{ez}^{(1)}=Vn^{(1)}, \quad \nu_{iz}^{(1)}=\frac{1}{V}\left(\psi^{(1)}+\frac{5}{3}\sigma n^{(1)}\right),\\ &\nabla_{\perp\xi}^2\varphi^{(1)}=\frac{1}{V^2}\left[\left(V^2-\frac{5}{3}\sigma\right)n^{(1)}-\psi^{(1)}\right],\\ &D_A(V)\left[\left(V^2-\frac{5}{3}\sigma\right)n^{(1)}-\psi^{(1)}\right]=0,\\ &n^{(1)}=\int_{-\infty}^{\infty}f^{(1)}d\nu_{11}, \end{aligned} \tag{16}$$

where

$$\nabla_{\perp\xi}^2=\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}, \quad D_A(V)=\frac{1}{2}\beta V^2-1. \tag{17}$$

From Eq. (13) at the order $\epsilon^{3/2}$, we get

$$\nu_{11}\frac{\partial f^{(1)}}{\partial\zeta}+\frac{\partial\psi^{(1)}}{\partial\zeta}\frac{\partial f_0}{\partial\nu_{11}}=0. \tag{18}$$

As Eq. (18) does not have a unique solution, we include, following Ott and Sudan,[13] an extra higher-order term $\epsilon^{7/2}\alpha_1(\partial f^{(1)}/\partial\tau)$ originating from the term $\alpha_1\epsilon(\partial f/\partial\tau)$ of (13) and write this equation as

$$\alpha_1\epsilon^2\frac{\partial f_\epsilon^{(1)}}{\partial\tau}+\nu_{11}\frac{\partial f_\epsilon^{(1)}}{\partial\zeta}+\frac{\partial f_0}{\partial\nu_{11}}\frac{\partial\psi^{(1)}}{\partial\zeta}=0. \tag{19}$$

Then $f^{(1)}$ is obtained from the unique solution of Eq. (19) by the relation

$$f^{(1)}=\lim_{\epsilon\to 0+}f_\epsilon^{(1)}. \tag{20}$$

Equation (19) is the same as Eq. (19) of our previous paper,[10] with a slight modification in notations. So following the method of Ref. 10 we arrive at the following solution for $f^{(1)}$:

$$f^{(1)}=-2\frac{\partial f_0}{\partial\nu_{11}^2}\psi^{(1)}. \tag{21}$$

Substituting (21) into the last equation of (16) and then performing integration, we get

$$n^{(1)}=(1-\beta')\psi^{(1)}, \tag{22}$$

where

$$\beta'=\frac{4\alpha'}{1+3\alpha'}.$$

Equation (22) and the fourth equation of (16) give the following linear dispersion relation determining V:

$$D_A(V)D_I(V)=0, \tag{23}$$

where

$$\begin{aligned} D_I(V)&=1-\frac{1}{V^2(1-\beta')}\left(1+\frac{5}{3}\sigma(1-\beta')\right),\\ D_A(V)&=\frac{1}{2}\beta V^2-1. \end{aligned} \tag{24}$$

Equation (23) is the same as Eq. (17) of Bandyopadhyay and Das.[11] Equation (24) gives two values of V:

$$V=[(5/3)\sigma+(1-\beta')^{-1}]^{1/2}, \quad V=[2/\beta]^{1/2}, \tag{25}$$

and these are obtained from $D_I(V)=0$ and $D_A(V)=0$. These values of V correspond, respectively, to an ion-acoustic wave and a kinetic Alfvén wave.

Therefore the first-order perturbed quantities in terms of $n^{(1)}$ can be expressed as

$$\begin{aligned} &\nu_{iz}^{(1)}=V[1-D_I(V)]n^{(1)}, \quad \nu_{ez}^{(1)}=Vn^{(1)},\\ &\psi^{(1)}=(1-\beta')^{-1}n^{(1)},\\ &\nabla_{\perp\xi}^2\varphi^{(1)}=D_I(V)n^{(1)}, \quad f^{(1)}=-2(1-\beta')^{-1}\frac{\partial f_0}{\partial\nu_{11}^2}n^{(1)}. \end{aligned} \tag{26}$$

Equations (1), (2), and (3) at the order $\epsilon^{5/2}$ are now solved for $\partial\nu_{iz}^{(2)}/\partial\zeta$, $\partial\nu_{ez}^{(2)}/\partial\zeta$, and $\partial(\nabla_{\perp\xi}^2\varphi^{(2)})/\partial\zeta$ to express them in terms of $\partial n^{(2)}/\partial\zeta$, $\partial\psi^{(2)}/\partial\zeta$, and first-order perturbed quantities. Substituting these solutions in Eq. (5) at the order $\epsilon^{5/2}$, we get the following equation:

$$D_A(V)\left[\left(V^2-\frac{5}{3}\sigma\right)\frac{\partial n^{(2)}}{\partial\zeta}-\frac{\partial\psi^{(2)}}{\partial\zeta}\right]$$
$$=2V[D_I(V)+D_A(V)]\frac{\partial n^{(1)}}{\partial\tau}+\left[D_A(V)\left(3V^2-\frac{5}{9}\sigma\right)\right.$$
$$\left.-V^2D_I(V)\right]n^{(1)}\frac{\partial n^{(1)}}{\partial\zeta}+V^2[D_A(V)-1]$$
$$\times\left(\frac{\partial^2\varphi^{(1)}}{\partial\xi\partial\zeta}\frac{\partial n^{(1)}}{\partial\xi}+\frac{\partial^2\varphi^{(1)}}{\partial\eta\partial\zeta}\frac{\partial n^{(1)}}{\partial\eta}\right)$$
$$-V^2(1-\beta')\frac{\partial}{\partial\zeta}(\nabla^2_{\perp\xi}n^{(1)}). \tag{27}$$

Equation (13) at the order $\epsilon^{5/2}$ is the following, in which, as in the lowest-order case, an extra higher-order term $\epsilon^{9/2}\alpha_1(\partial f^{(2)}/\partial\tau)$ has been included and $f^{(2)}$ has been replaced by $f^{(2)}_\epsilon$. Also, we have substituted for $f^{(1)}$ the expression given by Eq. (21):

$$\alpha_1\epsilon^2\frac{\partial f^{(2)}_\epsilon}{\partial\tau}+\nu_{11}\frac{\partial f^{(2)}_\epsilon}{\partial\zeta}+2\nu_{11}\frac{\partial f_0}{\partial\nu_{11}^2}\frac{\partial\psi^{(2)}}{\partial\zeta}$$
$$=-2V\alpha_1(1-\beta')^{-1}\frac{\partial f_0}{\partial\nu_{11}^2}x_2$$
$$+4(1-\beta')^{-2}\nu_{11}\frac{\partial^2 f_0}{\partial(\nu_{11}^2)^2}y_2, \tag{28}$$

where

$$x_2=\frac{\partial n^{(1)}}{\partial\zeta},\quad y_2=n^{(1)}\frac{\partial n^{(1)}}{\partial\zeta}, \tag{29}$$

and as in the lowest-order case, $f^{(2)}$ can be obtained form the unique solution of Eq. (28) by the following relation:

$$f^{(2)}=\lim_{\epsilon\to 0+}f^{(2)}_\epsilon. \tag{30}$$

Equation (28) is the same as Eq. (31) of our previous paper,[10] with a slight modification in notations. So following the method of Ref. 10 we get the following solution for $\hat{f}^{(2)}$, which is the Fourier Transform of $f^{(2)}$ with respect to the variable ζ, defined in the same way as in our previous paper:[10]

$$\hat{f}^{(2)}=-2\frac{\partial f_0}{\partial\nu_{11}^2}\hat{\psi}^{(2)}-\frac{4i}{k}(1-\beta')^{-2}\frac{\partial^2 f_0}{\partial(\nu_{11}^2)^2}\hat{y}_2$$
$$+2iV\alpha_1(1-\beta')^{-1}\hat{x}_2\left[P\left(\frac{1}{k\nu_{11}}\right)\right.$$
$$\left.+i\pi\delta(k\nu_{11})\right]\frac{\partial f_0}{\partial\nu_{11}^2}. \tag{31}$$

Integrating the above equation over the entire range of ν_{11} (i.e., from $-\infty$ to $+\infty$) and using the equation

$$n^{(2)}=\int_{-\infty}^{\infty}f^{(2)}d\nu_{11}, \tag{32}$$

obtained from Eq. (7) at the order ϵ^2, we get the following equation:

$$ik[\hat{n}^{(2)}-(1-\beta')\hat{\psi}^{(2)}]$$
$$=(1-\beta')^{-2}\hat{y}_2+\frac{1}{4}\sqrt{\frac{\pi}{2}}iV\alpha_1(1-\beta')^{-1}$$
$$\times(4-3\beta')\operatorname{sgn}(k)\hat{x}_2. \tag{33}$$

Taking Fourier inversion of the above equation, we obtain

$$\frac{\partial\psi^{(2)}}{\partial\zeta}=(1-\beta')^{-1}\frac{\partial n^{(2)}}{\partial\zeta}-(1-\beta')^{-3}n^{(1)}\frac{\partial n^{(1)}}{\partial\zeta}$$
$$+\frac{1}{4\sqrt{2\pi}}V\alpha_1(1-\beta')^{-2}(4-3\beta')$$
$$\times P\int_{-\infty}^{\infty}\frac{\partial n^{(1)}}{\partial\zeta'}\frac{d\zeta'}{\zeta-\zeta'}, \tag{34}$$

in which the convolution theorem has been used to find the inverse Fourier transform of $\operatorname{sgn}(k)\hat{x}_2$.

Eliminating $\partial\psi^{(2)}/\partial\zeta$ from Eqs. (27) and (34), we get the following equation, in which the term containing $\partial n^{(2)}/\partial\zeta$ gets eliminated due to Eq. (23):

$$2V[D_I(V)+D_A(V)]\frac{\partial n^{(1)}}{\partial\tau}+\left[-V^2D_I(V)\right.$$
$$\left.+D_A(V)\left(3V^2-\frac{5}{9}\sigma(1-\beta')^{-3}\right)\right]n^{(1)}\frac{\partial n^{(1)}}{\partial\zeta}$$
$$+V^2[D_A(V)-1]\left(\frac{\partial^2\varphi^{(1)}}{\partial\xi\partial\zeta}\frac{\partial n^{(1)}}{\partial\xi}+\frac{\partial^2\varphi^{(1)}}{\partial\eta\partial\zeta}\frac{\partial n^{(1)}}{\partial\eta}\right)$$
$$-V^2(1-\beta')^{-1}\frac{\partial}{\partial\zeta}(\nabla^2_{\perp\xi}n^{(1)})+\frac{1}{4\sqrt{2\pi}}V\alpha_1D_A(V)$$
$$\times(4-3\beta')(1-\beta')^{-2}P\int_{-\infty}^{\infty}\frac{\partial n^{(1)}}{\partial\zeta'}\frac{d\zeta'}{\zeta-\zeta'}=0. \tag{35}$$

Equation (35) together with the fourth equation of (26) constitute the macroscopic evolution equations for the kinetic Alfvén wave or ion-acoustic wave according to whether the constant V appearing in the equation is determined from $D_A(V)=0$ or $D_I(V)=0$. The last term of Eq. (35) accounts for the effect of Landau damping and this term vanishes for kinetic Alfvén wave, since $D_A(V)=0$. In this case, i.e., in the case of kinetic Alfvén wave the coupled equations (35) and the fourth equation of (26) reduce to Eqs. (19) and (20), respectively, of Bandyopadhyay and Das.[11] Consequently, there is no effect of Landau damping on the kinetic Alfvén wave. Since for the ion-acoustic wave $D_I(V)=0$, we have $\varphi^{(1)}\equiv 0$ according to the fourth equation of (26), and therefore (35) assumes the following form:

$$\frac{\partial n^{(1)}}{\partial \tau} - A'B'n^{(1)}\frac{\partial n^{(1)}}{\partial \zeta} - A'C'\frac{\partial}{\partial \zeta}(\nabla^2_{\perp\xi}n^{(1)}) + A'E'\alpha_1 P\int_{-\infty}^{\infty}\frac{\partial n^{(1)}}{\partial \zeta'}\frac{d\zeta'}{\zeta-\zeta'} = 0, \quad (36)$$

where

$$A' = [2VD_A(V)]^{-1},$$
$$B' = D_A(V)[(1-\beta')^{-3} - 3(1-\beta')^{-1} - (40/9)\sigma],$$
$$C' = V^2(1-\beta')^{-1}, \quad E' = \frac{1}{4\sqrt{2\pi}}VD_A(V)\frac{4-3\beta'}{(1-\beta')^2}. \quad (37)$$

The last term in (36) accounts for the effect of Landau damping, and if this term is neglected the equation reduces to Eq. (22) of Bandyopadhyay and Das,[11] which describes the long-time evolution of a weakly nonlinear and a weakly dispersive ion-acoustic wave.

From the expression of B' as given in Eqs. (37), we find that the coefficient of the nonlinear term of Eq. (36) vanishes along a particular curve in the $\beta'\sigma$ plane (Fig. 1 of Bandyopadhyay and Das[11]). Therefore, for the values of β' and σ lying on this curve, it is not possible to study the nonlinear behavior of the ion-acoustic wave, together with the effect of Landau damping. In the next section, we shall derive the modified macroscopic evolution equations to discuss the nonlinear behavior of ion-acoustic waves when $B'=0$.

B. Derivation of modified macroscopic equations

When $B'=0$, we give the same stretchings of space coordinates and time as given by Eq. (14) but make the following perturbation expansions of the dependent variables:[14]

$$n = 1 + \epsilon^{1/2}n^{(1)} + \epsilon n^{(2)} + \epsilon^{3/2}n^{(3)} + \cdots,$$
$$\varphi = \varphi^{(1)} + \epsilon^{1/2}\varphi^{(2)} + \epsilon\varphi^{(3)} + \cdots,$$
$$\psi = \epsilon^{1/2}\psi^{(1)} + \epsilon\psi^{(2)} + \epsilon^{3/2}\psi^{(3)} + \cdots,$$
$$\nu_{iz} = \epsilon^{1/2}\nu_{iz}^{(1)} + \epsilon\nu_{iz}^{(2)} + \epsilon^{3/2}\nu_{iz}^{(3)} + \cdots, \quad (38)$$
$$\nu_{ez} = \epsilon^{1/2}\nu_{ez}^{(1)} + \epsilon\nu_{ez}^{(2)} + \epsilon^{3/2}\nu_{ez}^{(3)} + \cdots,$$
$$f = f_0 + \epsilon^{1/2}f^{(1)} + \epsilon f^{(2)} + \epsilon^{3/2}f^{(3)} + \cdots,$$

where $n_i = n_e = n$ according to the quasineutrality condition (4).

Substituting the stretching (14) and the perturbation expansions (38) into our basic equations (1)–(3), (5), (7), and (13) and then equating coefficients of different powers of ϵ on both sides of each equation, we get a sequence of equations for the perturbed quantities. With the help of this sequence of equations, proceeding in the same way as given in Sec. IV of our previous paper[10] with a slight modification for coupled equations, as has been done in Sec. III A, we get the following evolution equations:

$$\frac{\partial}{\partial \zeta}\nabla^2_{\perp\xi}\left(A'C'n^{(1)} + \frac{V}{2}\varphi^{(2)}\right) - \frac{1+D_A(V)}{2D_A(V)}L_{12} = 0, \quad (39)$$

$$\frac{\partial n^{(1)}}{\partial \tau} - A'B''[n^{(1)}]^2\frac{\partial n^{(1)}}{\partial \zeta} - A'C'\frac{\partial}{\partial \zeta}(\nabla^2_{\perp\xi}n^{(1)}) + \frac{1-D_A(V)}{2D_A(V)}L_{12} + A'E'\alpha_1 P\int_{-\infty}^{\infty}\frac{\partial n^{(1)}}{\partial \zeta'}\frac{d\zeta'}{\zeta-\zeta'} = 0, \quad (40)$$

where

$$L_{12} = \frac{\partial n^{(1)}}{\partial \xi}\frac{\partial \varphi^{(2)}}{\partial \eta} - \frac{\partial n^{(1)}}{\partial \eta}\frac{\partial \varphi^{(2)}}{\partial \xi},$$
$$B'' = -\frac{1}{(1-\beta')^4}D_A(V)\left\{(1-\beta')^4\left[\frac{10}{27}\sigma - 6V^2 + \frac{3}{2}(1-\beta')\left(3V^2 - \frac{5}{9}\sigma\right)^2\right] - \frac{1}{2}(1+3\beta')\right\},$$

and the constant V is determined from the linear dispersion relation $D_I(V)=0$, which was obtained earlier for the case of an ion-acoustic wave.

In the derivation of the above evolution equations, we have used the linear dispersion relation $D_I(V)=0$ and the critical condition $B'=0$. Again in this case, also it is found that $\varphi^{(1)}\equiv 0$. Therefore when $B'=0$, Eq. (40) together with Eq. (39) constitutes the modified macroscopic evolution equations, which describe the nonlinear behavior of an ion-acoustic wave, including effect of Landau damping. The last term in Eq. (40) accounts for the effect of Landau damping. If this term is neglected, then Eqs. (40) and (39) reduce to Eqs. (26) and (27) of Bandyopadhyay and Das.[11] Now the critical condition $B'=0$ and the linear dispersion relation $D_I(V)=0$ give the values of V and σ as a function of β', and these are given by

$$V = \{[8(1-\beta')^3]^{-1}[3-(1-\beta')^2]\}^{1/2},$$
$$\sigma = (9/40)(1-\beta')^{-3}[1-3(1-\beta')^2]. \quad (41)$$

Again, $\sigma \geqslant 0$ imposes a restriction on β', and this restriction is $0.423 \leqslant \beta' < 1$. As there is no effect of Landau damping on the kinetic Alfvén wave, the soliton solution for this mode, which has been extensively investigated by Bandyopadhyay and Das,[11] remains unchanged, therefore, we shall only consider the soliton solution for the ion-acoustic wave described by Eq. (36) for the case when $B'\neq 0$ and the coupled equations (40) and (39) for the case when $B'=0$.

IV. SOLITARY WAVE SOLUTIONS OF THE EVOLUTION EQUATIONS

A. For the case $B'\neq 0$

The appropriate evolution equation in this case is Eq. (36), which is the usual KdV–ZK equation, except for a term accounting for the effect of Landau damping. The solitary wave solution of this equation with the neglect of this term has been investigated extensively by Bandyopadhyay and Das.[11] In fact, the solitary wave solution of Eq. (36) for $\alpha_1 = 0$, i.e., with the neglect of the term accounting for Landau damping, propagating at an arbitrary angle α to the external magnetic field lying in the $\xi\zeta$ plane is given by the following equation:

$$n^{(1)}=N(z)=a\,\mathrm{sech}^2\,pz, \tag{42}$$

where

$$z=\xi\sin\alpha+\zeta\cos\alpha-U\tau,$$

$$a\equiv\frac{3U}{d_1}=\frac{6U\sec\alpha}{\frac{40}{9}\sigma-\frac{1-3(1-\beta')^2}{(1-\beta')^3}}\sqrt{\frac{1}{1-\beta'}\left\{1+\frac{5}{3}\sigma(1-\beta')\right\}},$$

$$p\equiv\sqrt{\frac{U}{4d_3}}=\left\{\frac{1}{2}U\,\mathrm{cosec}^2\alpha\sec\alpha\sqrt{\frac{1-\beta'}{1+\frac{5}{3}\sigma(1-\beta')}}\left[1-\beta'-\frac{1}{2}\beta\left(1+\frac{5}{3}\sigma(1-\beta')\right)\right]\right\}^{1/2}, \tag{43}$$

$$d_1=-A'B'\cos\alpha,\quad d_3=-A'C'\cos\alpha\sin^2\alpha.$$

Using (43), Eq. (42) can be written as follows:

$$n^{(1)}=N(z)=a\,\mathrm{sech}^2\left[\sqrt{a}\sqrt{\frac{d_1}{12d_3}}\left(\xi\sin\alpha+\zeta\cos\alpha-\frac{1}{3}d_1a\tau\right)\right]. \tag{44}$$

When $\alpha_1\neq 0$, following Ott and Sudan[13] and using the macroscopic evolution equation (36), it can easily be verified that

$$\frac{\partial}{\partial\tau}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(n^{(1)})^2 d\xi\, d\eta\, d\zeta\leqslant 0, \tag{45}$$

which states that the initial perturbation in the form of (44) will decay to zero. Therefore the wave amplitude a is no longer a constant but decreases slowly with time. So following Ott and Sudan,[13] we introduce new space coordinates in a frame moving with solitary waves and normalized to its width as

$$z=\sqrt{a}\sqrt{\frac{d_1}{12d_3}}\left(\xi\sin\alpha+\zeta\cos\alpha-\frac{1}{3}d_1\int_0^\tau a\,d\tau\right). \tag{46}$$

Under this change of variable, the macroscopic evolution equation (36) assumes the following form:

$$\frac{\partial n^{(1)}}{\partial\tau}+\left(-\frac{1}{3}d_1pa+\frac{z}{2a}\frac{da}{d\tau}\right)\frac{\partial n^{(1)}}{\partial z}+d_1pn^{(1)}\frac{\partial n^{(1)}}{\partial z}+p^3d_3\frac{\partial^3 n^{(1)}}{\partial z^3}+A'E'\alpha_1p\cos\delta P\int_{-\infty}^{\infty}\frac{\partial n^{(1)}}{\partial z'}\frac{dz'}{z-z'}=0, \tag{47}$$

where we have used the following notation:

$$\frac{\partial n^{(1)}}{\partial z'}=\frac{\partial n^{(1)}}{\partial z},\quad \text{at } z=z'.$$

Now, to investigate the solution of Eq. (47), we follow Ott and Sudan[13] and generalizing the multiple-time scale analysis with respect to α_1, by setting

$$n^{(1)}=n^{(1)}(z,\tau)=q^{(0)}+\alpha_1q^{(1)}+\alpha_1^2q^{(2)}+\alpha_1^3q^{(3)}+\cdots, \tag{48}$$

where each $q^{(j)}(j=0,1,2,3,...,)$ are the functions of $\tau=\tau_0$, τ_1, $\tau_2,\ldots$. Here τ_j is given by

$$\tau_j=\alpha_1^j\tau,\quad j=0,1,2,3,\ldots. \tag{49}$$

Substituting (48) into Eq. (47) and then equating the coefficient of different power of α_1 on each side of Eq. (47), we get a sequence of equations. The first two members of this sequence can be written as follows:

$$\rho\left(\frac{\partial}{\partial\tau}+\frac{z}{2a}\frac{\partial a}{\partial\tau}\frac{\partial}{\partial z}\right)q^{(0)}+L\partial_z q^{(0)}=0, \tag{50}$$

$$\rho\left(\frac{\partial}{\partial\tau}+\frac{z}{2a}\frac{\partial a}{\partial\tau}\frac{\partial}{\partial z}\right)q^{(1)}+\partial_z Lq^{(1)}=\rho Mq^{(0)}, \tag{51}$$

where

$$L=\frac{\partial^2}{\partial z^2}+4\left(3\frac{q^{(0)}}{a}-1\right),\quad \partial_z=\frac{\partial}{\partial z},\quad \rho=(p^3d_3)^{-1},$$
$$Mq^{(0)}=-\left(\frac{\partial q^{(0)}}{\partial\tau_1}+\frac{z}{2a}\frac{\partial a}{\partial\tau_1}\frac{\partial q^{(0)}}{\partial z}+A'E'p\cos\alpha P\int_{-\infty}^{\infty}\frac{\partial q^{(0)}}{\partial z'}\frac{dz'}{z-z'}\right). \tag{52}$$

Now it can easily be verified that $q^{(0)}=a\,\mathrm{sech}^2 z$ is the soliton solution of the equation $L\partial_z q^{(0)}=0$ and, consequently, $q^{(0)}=a\,\mathrm{sech}^2 z$ will be the soliton solution of (50) if and only if

$$\frac{\partial a}{\partial\tau}=0. \tag{53}$$

Using (53), we can write Eq. (51) as follows:

$$\rho\frac{\partial q^{(1)}}{\partial\tau}+\partial_z Lq^{(1)}=\rho Mq^{(0)}. \tag{54}$$

Now, for the solution of Eq. (54) to exist, its right-hand side must be perpendicular to the kernel of the operator adjoint to the operator $\partial_z L$; this kernel, which must tend to zero as $|z|\to\infty$ is $\mathrm{sech}^2 z$. Thus, we get the following consistency condition for the existence of the solution of Eq. (54):

$$\int_{-\infty}^{\infty}(\mathrm{sech}^2 z)Mq^{(0)}dz=0. \tag{55}$$

The consistency condition (55) gives the following differential equation for the evolution of the solitary wave amplitude a:

$$a^{-3/2}\frac{\partial a}{\partial \tau_1}+A'E'\cos\alpha\sqrt{\frac{d_1}{12d_3}}P\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\operatorname{sech}^2 z \times\frac{\partial}{\partial z'}(\operatorname{sech}^2 z')\frac{dz'dz}{z-z'}=0. \tag{56}$$

Solving Eq. (56), we get

$$a=a_0\left(1+\frac{\tau}{\tau'}\right)^{-2}, \tag{57}$$

where a_0 is the initial value of a, i.e., the value of a when $\tau=0$ and τ' is given by the following equation:

$$\tau'=\left(\frac{1}{4}\alpha_1 A'E'\sqrt{\frac{B'a_0}{3C'}}\cos\alpha|\operatorname{cosec}\alpha| \times P\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\operatorname{sech}^2 z\frac{\partial}{\partial z'}(\operatorname{sech}^2 z')\frac{dz'dz}{z-z'}\right)^{-1}. \tag{58}$$

B. For the case $B'=0$

In this case, if we set $\alpha_1=0$, i.e., if we neglect the Landau damping effect, the coupled equations (39) and (40) reduce to a modified KdV–ZK equation derived by Bandyopadhyay and Das,[11] the solitary wave solution of which has also been investigated by them. In fact, the solitary wave solution of the coupled equations (40) and (39) for $\alpha_1=0$, propagating at an arbitrary angle α to the external magnetic field lying in the $\xi\zeta$ plane is given by the following equations:

$$n^{(1)}\equiv N_0(z)=\pm\bar{a}\operatorname{sech}\bar{p}z,\quad \varphi^{(2)}\equiv\varphi_0(z). \tag{59}$$

Here $\varphi_0(z)$ is given by the following equation:

$$d_3N_0+\bar{d}_3\varphi_0(z)=0, \tag{60}$$

where

$$\begin{aligned}
&z=\zeta\cos\alpha+\xi\sin\alpha-U\tau,\\
&\bar{a}=\sqrt{\frac{6U}{\bar{d}_1}}=\left(\frac{6UV(1-\beta')^4\sec\alpha}{\frac{1}{2}\left[(1-\beta')^4\left\{\frac{10\sigma}{27}-6V^2+\frac{3}{2}(1-\beta')\left(3V^2-\frac{5\sigma}{9}\right)^2\right\}-\frac{1}{2}(1+3\beta')\right]}\right)^{1/2},\\
&\bar{p}=\sqrt{\frac{U}{d_3}}=\left\{\frac{U\operatorname{cosec}^2\alpha\sec\alpha}{V}\left[1-\beta'-\frac{1}{2}\left(1+\frac{5}{3}\sigma(1-\beta')\right)\right]\right\}^{1/2},\\
&\bar{d}_1=-AB''\cos\alpha,\quad (d_3,\bar{d}_3)=-\left(A'C',\frac{V}{2}\right)\cos\alpha\sin^2\alpha.
\end{aligned} \tag{61}$$

Using (61), the first equation of (59) can be rewritten as

$$n^{(1)}=N_0(z)=\pm\bar{a}\operatorname{sech}\left[\bar{a}\sqrt{\frac{\bar{d}_1}{6d_3}}\left(\xi\sin\alpha+\zeta\cos\alpha-\frac{1}{6}\bar{d}_1\bar{a}^2\tau\right)\right]. \tag{62}$$

Now if $\alpha_1\neq 0$, then using the coupled equations (40) and (39) it can easily be verified that the inequality (45) holds and, consequently, the wave amplitude $\bar{a}$ is no longer a constant but decreases slowly with time. Therefore, according to the solitary structure as given in Eq. (62), we have introduced a new space coordinate in a frame moving with a solitary wave and normalized to its width, defined by

$$z=\bar{a}\sqrt{\frac{\bar{d}_1}{6d_3}}\left(\xi\sin\alpha+\zeta\cos\alpha-\frac{1}{6}\bar{d}_1\int_0^{\tau}\bar{a}^2d\tau\right), \tag{63}$$

in which $\bar{a}$ is a slowly varying function of time. Obviously this coordinate z varies only along the direction of propagation of the solitary wave.

Under the above change of variables and assuming that $n^{(1)}$ is a function of z, τ only, Eq. (40) assumes the following forms:

$$\frac{\partial n^{(1)}}{\partial\tau}+\left(-\frac{1}{6}\bar{d}_1\bar{a}^2\bar{p}+\frac{z}{\bar{a}}\frac{d\bar{a}}{d\tau}\right)\frac{\partial n^{(1)}}{\partial z}+\bar{d}_1\bar{p}(n^{(1)})^2\frac{\partial n^{(1)}}{\partial z} +\bar{p}^3\bar{d}_3\frac{\partial^3 n^{(1)}}{\partial z^3}+A'E'\alpha_1 p\cos\delta P\int_{-\infty}^{\infty}\frac{\partial n^{(1)}}{\partial z'}\frac{dz'}{z-z'}=0. \tag{64}$$

Now, according to multiple-time scale analysis with respect to the small parameter α_1, we take the same perturbation equation of $n^{(1)}(z,\tau)$ as given by Eq. (48). Substituting (48) into Eq. (64) and then equating the coefficients of different powers of α_1 on both sides of Eq. (64), we get a sequence of equations. With the help of this sequence of equations, proceeding in the same way as given in Sec. V B of our previous paper,[10] we get the following evolution equation for $\bar{a}$:

$$\frac{\partial\bar{a}}{\partial\tau_1}+\bar{a}^2A'E'\sqrt{\frac{\bar{d}_1}{6d_3}}\cos\alpha \times P\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\operatorname{sech} z\frac{\partial}{\partial z'}(\operatorname{sech} z')\frac{dzdz'}{z-z'}=0. \tag{65}$$

Solving the above equation we get

$$\bar{a}=\bar{a}_0(1+\tau/\bar{\tau})^{-1}, \tag{66}$$

where $\bar{a}_0$ is the initial value of $\bar{a}$, i.e., the value of $\bar{a}$ when $\tau=0$ and $\bar{\tau}$ is given by the following equation:

$$\bar{\tau}=\left(A'E'\alpha_1\bar{a}_0\sqrt{\frac{B''}{6C'}}|\cos ec\,\alpha|\cos\alpha \times P\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\operatorname{sech} z\frac{\partial}{\partial z'}(\operatorname{sech} z')\frac{dzdz'}{z-z'}\right)^{-1}. \tag{67}$$

V. CONCLUSIONS

The evolution equations describing both a kinetic Alfvén wave and an ion-acoustic wave in a nonthermal magnetized plasma with warm ions, including weak nonlinearity and weak dispersion and the effect of Landau damping, have been derived. These equations reduce to two coupled equations constituting the KdV–ZK equation of Bandyopadhyay and Das[11] if the electron to ion mass ratio is set equal to zero, i.e., if the effect of Landau damping is neglected. It has been shown that there is no effect of Landau damping on the solitary structures of the kinetic Alfvén wave. The solitary wave solutions of the macroscopic evolution equation for the ion acoustic wave have been obtained by generalizing the multiple-time scale method of Ott and Sudan.[13] It is found that the amplitude of the ion-acoustic solitary wave is no longer constant but decreases slowly with time τ as $(1+\tau/\tau')^{-2}$, where τ' is a constant depending on the initial amplitude of the solitary wave and the angle between the direction of propagation of the solitary wave and the external magnetic field. When the coefficient of the nonlinear term of the evolution equation for an ion-acoustic wave vanishes, then the nonlinear behavior of an ion-acoustic wave, including the effect of Landau damping, is described by two coupled equations constituting the MKdV–ZK equation if the electron to ion mass ratio is set equal to zero, i.e., if the effect of Landau damping is neglected. The multiple time-scale method of Ott and Sudan[13] has also been generalized here to solve these evolution equations. It is found that the evolution equations admit a solitary wave solution having a sech profile, but the amplitude of the solitary wave slowly decreases with time τ as $(1+\tau/\bar{\tau})^{-1}$, where $\bar{\tau}$ is a constant depending on the initial amplitude of the solitary wave and the angle between the direction of propagation of the solitary wave and the external magnetic field.

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