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Effect of Landau damping on ion-acoustic solitary waves in a magnetized nonthermal plasma with warm ions

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An evolution equation describing weakly nonlinear and weakly dispersive ion-acoustic waves in magnetized nonthermal plasma with warm ions, including the effect of Landau damping, has been derived. It is found that the coefficient of the nonlinear term of this equation vanishes along a particular curve in $\beta\sigma$-plane, where β is a parameter that determines the proportion of fast electrons and σ is the ratio of electron to ion temperature. In this case, the modified evolution equation has also been derived. The solitary wave solutions of these equations propagating obliquely to the external magnetic field have been obtained by the multiple time scale method. It has been found that in either case the amplitude of the solitary waves slowly decreases with time. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1427022]

I. INTRODUCTION

Motivated by observation of solitary structures with density depletion made by the Freja Satellite (Dovner *et al.*[1]), Cairns *et al.*[2,3] have shown that the presence of nonthermal electrons change the properties of ion-sound solitary waves, and solitons with both positive and negative density perturbation can exist. The effects of ion temperature, external static magnetic field and oblique propagation on the structure of these solitary waves have also been investigated by Cairns *et al.*[4] The same problem for cold ions was considered by Mamun and Cairns,[5] in which they have not only shown the existence of compressive and rarefactive solitons but also investigated their stabilities. Bandyopadhyay and Das[6] have extended this paper of Mamun and Cairns[5] in the following two directions: (i) Instead of considering the ions as cold, the ion temperature has been included and (ii) the case has been considered when the coefficient of the nonlinear term of the KdV–ZK (Korteweg–de Vries–Zakharav–Kuzentsov) equation derived in the first case vanishes. A factor B of the coefficient of the nonlinear term in the KdV–ZK equation considering ions as warm becomes a function of β and σ, where β is a parameter which determines the proportion of fast electrons and σ is the ratio of ion and electron temperature. This factor B becomes zero along a curve in the $\beta\sigma$-plane. An MKdV-ZK (modified KdV-ZK) equation has been derived by Bandyopadhyay and Das[6] to investigate the nonlinear behavior of ion-acoustic wave for the values of the parameter β and σ lying on the curve along which $B=0$. Recently, Bandyopadhyay and Das[7] have derived a combined MKdV–KdV–ZK equation which remains valid at points near the curve along $\beta\sigma$-plane on which $B=0$. It has been found that this combined MKdV–KdV–ZK equation admits a double layer solution or solitary wave solution having a profile different from sech^2 or sech, propagating obliquely to the external magnetic field according to $B^2+12B'(\cos^2\delta+D\sin^2\delta)=0$, or >0, where B, B' are the coefficients (except for a common factor) of two nonlinear terms and D is the coefficient of the perpendicular dispersive term of the combined MKdV–KdV–ZK equation, and δ is the angle made by the direction of propagation of the solitary structures with the external magnetic field.

In all the above-mentioned investigations for magnetized nonthermal plasma with warm ions, the problem ultimately has been reduced to either a KdV–ZK equation or an MKdV–ZK equation or a combined MKdV–KdV–ZK equation. All these equations include both nonlinearity and dispersion without explicit inclusion of Landau damping. In the present paper, we have considered the nonlinear behavior of ion-acoustic waves including the effect of Landau damping. It has been shown that the amplitude of the solitary wave is no longer a constant but decreases with time. In fact, the amplitude of the solitary wave is proportional to $(1+\tau/\tau')^{-2}$ for KdV-solitons where τ' is a constant depending on initial amplitude and direction of propagating of the solitary waves. For MKdV-solitons, the amplitude of the solitary wave decreases slowly with time as $(1+\tau/\tau')^{-1}$ where τ' is a constant depending on initial amplitude and direction of propagating of the solitary waves.

II. BASIC EQUATIONS

We consider a fully ionized collisionless plasma consisting of nonthermal electrons and adiabatic warm ions immersed in a uniform external magnetic field $\boldsymbol{B}_0=B_0\hat{z}$. We assume that the characteristic frequency is much less than the ion cyclotron frequency and the particle pressure is much less than the magnetic pressure. The nonlinear behavior of ion-acoustic waves in this plasma may be described by the following set of fluid equations:[4]

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$$\frac{\partial n}{\partial t}+\nabla\cdot(n\boldsymbol{u})=0, \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t}+(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}=-\nabla\varphi+\omega_c(\boldsymbol{u}\times\hat{z})-\frac{\sigma}{n}\nabla P, \tag{2}$$

$$\nabla^2\varphi=n_e-n, \tag{3}$$

$$P=n^{\gamma}, \tag{4}$$

where

$$n_e=\int_{-\infty}^{\infty} f dv_{11}, \tag{5}$$

and the velocity distribution function of electrons (f) must satisfy the Vlasov–Boltzmann equation

$$\sqrt{\frac{m_e}{m_i}}\frac{\partial f}{\partial t}+v_{11}\frac{\partial f}{\partial z}+\frac{\partial \varphi}{\partial z}\frac{\partial f}{\partial v_{11}}=0. \tag{6}$$

In the above equations n, n_e, $\boldsymbol{u}$, P, φ, (x,y,z) and t are, respectively, the ion number density, electron number density, ion fluid velocity, ion pressure, electrostatic potential, spatial coordinates and time, and they have been normalized, respectively, by n_0 (unperturbed ion number density), n_0, $c_s=(K_BT_e/m_i)^{1/2}$ (ion-acoustic speed), $n_0K_BT_i$, K_BT_e/e, $\lambda_D=[K_BT_e/(4\pi n_0e^2)]^{1/2}$ (Debye length), ω_P^{-1} (ion plasma period), where $\sigma=T_i/T_e$, ω_c is the ion-cyclotron frequency normalized by $\omega_P=(4\pi n_0e^2/m_i)^{1/2}$ and $\gamma(=5/3)$ is the ratio of two specific heates. Here K_B is the Boltzmann's constant, T_e, T_i are, respectively, the electron and ion temperatures, m_e and m_i are, respectively, the mass of an electron and an ion, e is the electronic charge, v_{11} is the velocity of electrons in phase space normalized to $v_e=(K_BT_e/m_e)^{1/2}$.

Since the electrons are assumed to be nonthermally distributed, their unperturbed velocity distribution function can be taken as[2–8]

$$f_0(v_{11})=\frac{1}{\sqrt{2\pi}(1+3\alpha)}[1+\alpha v_{11}^4]\exp\left(-\frac{1}{2}v_{11}^2\right). \tag{7}$$

As in the present paper our main interest is to introduce the Landau damping effect on an ion-acoustic wave in a magnetized plasma including weak nonlinearity and weak dispersion, we follow the method of Ott and Sudan[9] and, therefore, we replace $(m_e/m_i)^{1/2}$ by $(m_e/m_i)^{1/2}\varepsilon$, where ε is a small parameter, and write Eq. (6) as follows:

$$\alpha_1\varepsilon\frac{\partial f}{\partial t}+v_{11}\frac{\partial f}{\partial z}+\frac{\partial \varphi}{\partial z}\frac{\partial f}{\partial v_{11}}=0, \tag{8}$$

where $\alpha_1=(m_e/m_i)^{1/2}$.

Again using Eq. (4), Eq. (2) can be written as follows:

$$\frac{\partial \boldsymbol{u}}{\partial t}+(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}=-\nabla\varphi+\omega_c(\boldsymbol{u}\times\hat{z})-\frac{5}{3}\sigma n^{-1/3}\nabla n. \tag{9}$$

Therefore, Eqs. (1), (9), (3), (5), and (8) are our basic equations.

III. DERIVATION OF MACROSCOPIC EQUATION

In order to derive the long time evolution of a long wave length nonlinear ion-acoustic wave in a magnetized nonthermal collissionless plasma with warm ions, we make the following stretching of coordinates and time:

$$\xi=\varepsilon^{1/2}x,\quad \eta=\varepsilon^{1/2}y,\quad \zeta=\varepsilon^{1/2}(z-Vt),\quad \tau=\varepsilon^{3/2}, \tag{10}$$

where V is a constant and ε is a small parameter, which is a measure of weakness of dispersion.

With the stretching given by (10) the governing equations (1), (9), (3), (5), and (8) become as follows:

$$-V\varepsilon^{1/2}\frac{\partial n}{\partial \zeta}+\varepsilon^{3/2}\frac{\partial n}{\partial \tau}+\varepsilon^{1/2}\nabla_{\xi}\cdot(n\boldsymbol{u})=0, \tag{11}$$

$$-V\varepsilon^{1/2}\frac{\partial \boldsymbol{u}}{\partial \zeta}+\varepsilon^{3/2}\frac{\partial \boldsymbol{u}}{\partial \tau}+\varepsilon^{1/2}(\boldsymbol{u}\cdot\nabla_{\xi})\boldsymbol{u}$$
$$=-\varepsilon^{1/2}\nabla_{\xi}\varphi+\omega_c(\boldsymbol{u}\times\hat{z})-\tfrac{5}{3}\varepsilon^{1/2}\sigma n^{-1/3}\nabla_{\xi}n, \tag{12}$$

$$\varepsilon\nabla_{\xi}^2\varphi=n_e-n, \tag{13}$$

$$n_e=\int_{-\infty}^{\infty} f dv_{11}, \tag{14}$$

$$-V\alpha_1\varepsilon^{3/2}\frac{\partial f}{\partial \zeta}+\alpha_1\varepsilon^{5/2}\frac{\partial f}{\partial \tau}+\varepsilon^{1/2}v_{11}\frac{\partial f}{\partial \zeta}+\varepsilon^{1/2}\frac{\partial \varphi}{\partial \zeta}\frac{\partial f}{\partial v_{11}}=0, \tag{15}$$

where

$$\nabla_{\xi}=\hat{x}\frac{\partial}{\partial \xi}+\hat{y}\frac{\partial}{\partial \eta}+\hat{z}\frac{\partial}{\partial \zeta}.$$

We now make the following perturbation expansion of the dependent variables:

$$\begin{aligned}
n&=1+\varepsilon n^{(1)}+\varepsilon^2 n^{(2)}+\cdots,\\
n_e&=1+\varepsilon n_e^{(1)}+\varepsilon^2 n_e^{(2)}+\cdots,\\
\varphi&=\varepsilon\varphi^{(1)}+\varepsilon^2\varphi^{(2)}+\cdots,\\
w&=\varepsilon w^{(1)}+\varepsilon^2 w^{(2)}+\cdots,\\
u&=\varepsilon^{3/2}u^{(1)}+\varepsilon^2 u^{(2)}+\cdots,\\
v&=\varepsilon^{3/2}v^{(1)}+\varepsilon^2 v^{(2)}+\cdots,\\
f&=f_0+\varepsilon f^{(1)}+\varepsilon^2 f^{(2)}+\cdots,
\end{aligned} \tag{16}$$

where $\boldsymbol{u}=(u,v,w)$.

Substituting the expressions (16) in Eqs. (11)–(15) and then equating different powers of ε on both sides of these equations, we get a sequence of equations.

From the lowest order equations obtained from (11)–(14) we get

$$n^{(1)}=(V^2-\tfrac{5}{3}\sigma)^{-1}\varphi^{(1)},\quad w^{(1)}=V(V^2-\tfrac{5}{3}\sigma)^{-1}\varphi^{(1)},$$

$$u^{(1)}=-\frac{V^2}{\omega_c}\left(V^2-\frac{5}{3}\sigma\right)^{-1}\frac{\partial \varphi^{(1)}}{\partial \eta},$$

$$v^{(1)}=\frac{V^2}{\omega_c}\left(V^2-\frac{5}{3}\sigma\right)^{-1}\frac{\partial\varphi^{(1)}}{\partial\xi}, \tag{17}$$

$$n_e^{(1)}=n^{(1)},\quad n_e^{(1)}=\int_{-\infty}^{\infty}f^{(1)}dv_{11}.$$

Equation (15) at the order $\varepsilon^{3/2}$ gives

$$v_{11}\frac{\partial f^{(1)}}{\partial\zeta}+\frac{\partial\varphi^{(1)}}{\partial\zeta}\frac{\partial f_0}{\partial v_{11}}=0. \tag{18}$$

As this equation does not have a unique solution, we include, following Ott and Sudan,[9] an extra higher order term $\varepsilon^{7/2}\alpha_1(\partial f^{(1)}/\partial\tau)$ originating from the term $\varepsilon^{5/2}\alpha_1(\partial f/\partial\tau)$ of Eq. (15) and write this equation as

$$\alpha_1\varepsilon^2\frac{\partial f_\varepsilon^{(1)}}{\partial\tau}+v_{11}\frac{\partial f_\varepsilon^{(1)}}{\partial\zeta}+\frac{\partial f_0}{\partial v_{11}}\frac{\partial\varphi^{(1)}}{\partial\zeta}=0. \tag{19}$$

Then $f^{(1)}$ is obtained from the unique solution of Eq. (19) by the relation

$$f^{(1)}=\lim_{\varepsilon\to 0+}f_\varepsilon^{(1)}. \tag{20}$$

Assuming τ-dependence of $f_\varepsilon^{(1)}$ and $\varphi^{(1)}$ to be of the form $\exp(i\omega\tau)$, Eq. (19) can be written as

$$i\alpha_1\omega\varepsilon^2 f_\varepsilon^{(1)}+v_{11}\frac{\partial f_\varepsilon^{(1)}}{\partial\zeta}+\frac{\partial f_0}{\partial v_{11}}\frac{\partial\varphi^{(1)}}{\partial\zeta}=0. \tag{21}$$

Now taking Fourier transform of this equation with respect to the variable ζ according to the definition

$$\hat{g}=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}g(\zeta)e^{-ik\zeta}d\zeta, \tag{22}$$

and following Landau prescription to resolve the singularities involved, the solution of Eq. (19) is obtained as

$$\hat{f}_\varepsilon^{(1)}=-P\frac{k\dfrac{\partial f_0}{\partial v_{11}}}{kv_{11}+\alpha_1\omega\varepsilon^2}\hat{\varphi}^{(1)} -i\pi k\hat{\varphi}^{(1)}\frac{\partial f_0}{\partial v_{11}}\delta(kv_{11}+\alpha_1\omega\varepsilon^2). \tag{23}$$

From this equation proceeding to the limit $\varepsilon\to 0$ and then taking Fourier inversion we get the following, where we use the relations $xP(1/x)=1$ and $x\delta(x)=0$:

$$f^{(1)}=-2\frac{\partial f_0}{\partial v_{11}^2}\varphi^{(1)}. \tag{24}$$

Substituting (24) in the last equation of (17) and then performing integration we get

$$n_e^{(1)}=(1-\beta)\varphi^{(1)}, \tag{25}$$

where

$$\beta=\frac{4\alpha}{1+3\alpha}. \tag{26}$$

Equation (25) and the first equation of (17) give the linear dispersion relation determining V

$$(1-\beta)\left(V^2-\frac{5\sigma}{3}\right)=1, \tag{27}$$

which is same as Eq. (12) of Bandyopadhyay and Das.[6]

From the x and y component of Eq. (12) both at the order ε^2, we can deduce the following by the use of the relation (17):

$$\frac{\partial u^{(2)}}{\partial\xi}+\frac{\partial v^{(2)}}{\partial\eta}=\frac{V^3}{\omega_c^2}\left(V^2-\frac{5\sigma}{3}\right)^{-1}\frac{\partial}{\partial\zeta}\left(\frac{\partial^2\varphi^{(1)}}{\partial\xi^2}+\frac{\partial^2\varphi^{(1)}}{\partial\eta^2}\right). \tag{28}$$

Equation (13) at the order ε^2 gives

$$n_e^{(2)}-n^{(2)}=\frac{\partial^2\varphi^{(1)}}{\partial\xi^2}+\frac{\partial^2\varphi^{(1)}}{\partial\eta^2}+\frac{\partial^2\varphi^{(1)}}{\partial\zeta^2}. \tag{29}$$

Solving Eq. (11) and the z-component of Eq. (12), both at the order $\varepsilon^{5/2}$, for $\partial n^{(2)}/\partial\zeta$ in terms of $\varphi^{(2)}$ and the first-order perturbed quantities, we get the following equation:

$$\frac{\partial n^{(2)}}{\partial\zeta}=\left(V^2-\frac{5}{3}\sigma\right)^{-1}\Bigg[\frac{\partial\varphi^{(2)}}{\partial\zeta}+2V\left(V^2-\frac{5\sigma}{3}\right)^{-1}\frac{\partial\varphi^{(1)}}{\partial\tau} +\frac{V^4}{\omega_c^2}\left(V^2-\frac{5\sigma}{3}\right)^{-1}\frac{\partial}{\partial\zeta}\left(\frac{\partial^2\varphi^{(1)}}{\partial\xi^2}+\frac{\partial^2\varphi^{(1)}}{\partial\eta^2}\right) +\left(3V^2-\frac{5\sigma}{9}\right)\left(V^2-\frac{5\sigma}{3}\right)^{-1}\varphi^{(1)}\frac{\partial\varphi^{(1)}}{\partial\zeta}\Bigg], \tag{30}$$

in which use has been made of Eq. (17).

Equation (15) at the order $\varepsilon^{5/2}$ is the following, in which as in the lowest order case an extra higher order term $\varepsilon^{9/2}\alpha_1(\partial f^{(2)}/\partial\tau)$ has been included and $f^{(2)}$ has been replaced by $f_\varepsilon^{(2)}$. Also we have substituted for $f^{(1)}$ the expression given by (24).

$$\alpha_1\varepsilon^2\frac{\partial f_\varepsilon^{(2)}}{\partial\tau}+v_{11}\frac{\partial f_\varepsilon^{(2)}}{\partial\zeta}+\frac{\partial f_0}{\partial v_{11}}\frac{\partial\varphi^{(2)}}{\partial\zeta} =-2V\alpha_1\frac{\partial f_0}{\partial v_{11}^2}\frac{\partial\varphi^{(1)}}{\partial\zeta}+4v_{11}\frac{\partial^2 f_0}{\partial(v_{11}^2)^2}\varphi^{(1)}\frac{\partial\varphi^{(1)}}{\partial\zeta}. \tag{31}$$

As in the lowest order case, $f^{(2)}$ can be obtained from the unique solution of Eq. (31) by the following relation:

$$f^{(2)}=\lim_{\varepsilon\to 0+}f_\varepsilon^{(2)}. \tag{32}$$

As in the first order case, assuming τ-dependence to be of the form $\exp(i\omega\tau)$ and taking Fourier transforms with respect to ζ, we get an expression for $\hat{f}_\varepsilon^{(2)}$ from (31). Now following Landau prescription to remove singularities involved and finally proceeding to the limit $\varepsilon\to 0+$ we get the following expression for $\hat{f}^{(2)}$:

$$\hat{f}^{(2)}=-2\frac{\partial f_0}{\partial v_{11}^2}\hat{\varphi}^{(2)}+2iV\alpha_1\left[\frac{\partial f_0}{\partial v_{11}^2}P\left(\frac{1}{kv_{11}}\right) +i\pi\frac{\partial f_0}{\partial v_{11}^2}\delta(kv_{11})\right]\hat{c}_2-\frac{4i}{k}\frac{\partial^2 f_0}{\partial(v_{11}^2)^2}\hat{d}_2, \tag{33}$$

where

$$c_2 = \frac{\partial\varphi^{(1)}}{\partial\zeta}, \quad d_2 = \varphi^{(1)}\frac{\partial\varphi^{(1)}}{\partial\zeta}.$$

Integrating Eq. (33) over the entire range of v_{11} (i.e., from $-\infty$ to $+\infty$), and using the equation

$$n_e^{(2)} = \int_{-\infty}^{\infty} f^{(2)} dv_{11}, \tag{34}$$

obtained from Eq. (14) at the order ε^2 we get the following equation:

$$ik[\hat{n}_e^{(2)} - (1-\beta)\hat{\varphi}^{(2)}] = -\frac{1}{4} iV\alpha_1(4-3\beta)\sqrt{\frac{\pi}{2}}\,\mathrm{sgn}(k)\hat{c}_2 + \hat{d}_2, \tag{35}$$

in which we have used the relation $k\delta(kv_{11}) = \mathrm{sgn}(k)\delta(v_{11})$ to simplify the equation.

Taking Fourier inversion of the above equation we get the following equation:

$$\frac{\partial n_e^{(2)}}{\partial\zeta} = (1-\beta)\frac{\partial\varphi^{(2)}}{\partial\zeta} + \varphi^{(1)}\frac{\partial\varphi^{(1)}}{\partial\zeta} - \frac{1}{4\sqrt{2\pi}} V\alpha_1(4-3\beta)P\int_{-\infty}^{\infty}\frac{\partial\varphi^{(1)}}{\partial\zeta'}\frac{d\zeta'}{\zeta-\zeta'}. \tag{36}$$

Eliminating $\partial n^{(2)}/\partial\zeta$ and $\partial n_e^{(2)}/\partial\zeta$ from Eqs. (30) and (36) with the help of Eq. (29), we get the following equation for $\varphi^{(1)}$, in which the linear dispersion relation (27) has been used to eliminate the term containing $\partial\varphi^{(2)}/\partial\zeta$.

$$\frac{\partial\varphi^{(1)}}{\partial\tau} + AB\varphi^{(1)}\frac{\partial\varphi^{(1)}}{\partial\zeta} + \frac{1}{2}A\frac{\partial^3\varphi^{(1)}}{\partial\zeta^3} + \frac{1}{2}AD\frac{\partial}{\partial\zeta} \times\left(\frac{\partial^2\varphi^{(1)}}{\partial\xi^2} + \frac{\partial^2\varphi^{(1)}}{\partial\eta^2}\right) + \frac{1}{2}AE\alpha_1 P\int_{-\infty}^{\infty}\frac{\partial\varphi^{(1)}}{\partial\zeta'}\frac{d\zeta'}{\zeta-\zeta'} = 0, \tag{37}$$

where

$$A = [V(1-\beta)^2]^{-1},$$
$$B = \frac{1}{2}\left[(1-\beta)^3\left(3V^2 - \frac{5\sigma}{9}\right) - 1\right],$$
$$D = 1 + \frac{V^4(1-\beta)^2}{\omega_c^2}, \tag{38}$$
$$E = \frac{V}{4\sqrt{2\pi}}(4-3\beta).$$

Equation (37) is the evolution equation describing the weakly nonlinear and weakly dispersive ion-acoustic wave in a magnetized nonthermal plasma including the effect of Landau damping.

Now if we neglect the electron to ion mass ratio, i.e., if we set $\alpha_1 = 0$ then Eq. (37) reduces to the KdV–ZK equation (10) of Bandyopadhyay and Das.[6] From Eq. (37) we see that the nonlinearity of the ion-acoustic waves is only due to the second term of (37). When $B=0$, it is not possible to discuss the nonlinear behavior of ion-acoustic wave including the effect of Landau damping with the help of Eq. (37). Figure 1 shows the variation of σ with β when $B=0$. In the next section, we shall derive the modified macroscopic evolution equation to discuss the nonlinearity of ion-acoustic wave with an explicit inclusion of Landau damping when $B=0$.

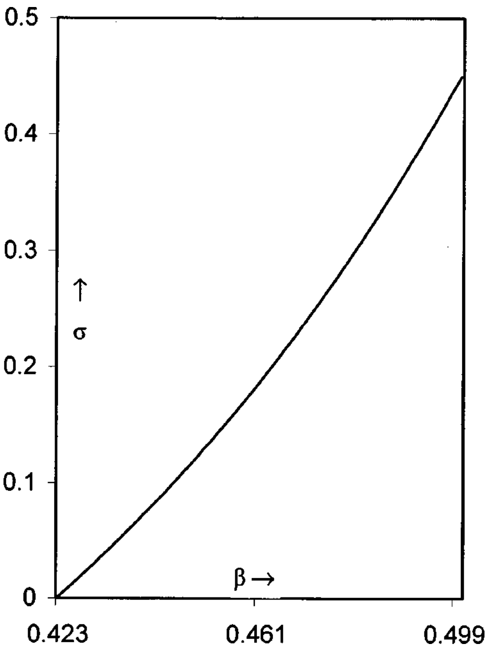

FIG. 1. σ is plotted against β when $B=0$.

IV. DERIVATION OF MODIFIED EVOLUTION EQUATION FOR $B=0$

When $B=0$, we take the same stretching of space coordinates and time as given by Eq. (10) and consequently Eqs. (11)–(15) remain unchanged. But in this case, we take the following perturbation expansion of the field quantities to make a balance between nonlinear and dispersive terms:[10]

$$\begin{aligned} n &= 1 + \varepsilon^{1/2}n^{(1)} + \varepsilon n^{(2)} + \varepsilon^{3/2}n^{(3)} + \cdots, \\ n_e &= 1 + \varepsilon^{1/2}n_e^{(1)} + \varepsilon n_e^{(2)} + \varepsilon^{3/2}n_e^{(3)} + \cdots, \\ \varphi &= \varepsilon^{1/2}\varphi^{(1)} + \varepsilon\varphi^{(2)} + \varepsilon^{3/2}\varphi^{(3)} + \cdots, \\ w &= \varepsilon^{1/2}w^{(1)} + \varepsilon w^{(2)} + \varepsilon^{3/2}w^{(3)} + \cdots, \\ u &= \varepsilon u^{(1)} + \varepsilon^{3/2}u^{(2)} + \varepsilon^2 u^{(3)} + \cdots, \\ v &= \varepsilon v^{(1)} + \varepsilon^{3/2}v^{(2)} + \varepsilon^2 v^{(3)} + \cdots, \\ f &= f_0 + \varepsilon^{1/2}f^{(1)} + \varepsilon^{3/2}f^{(3)} + \cdots. \end{aligned} \tag{39}$$

Substituting the perturbation expansion (39) into Eqs. (11)–(15) and then equating the coefficient of different powers of ε on both sides of each equation, we get a sequence of equations for the perturbed quantities. The lowest order analysis remains unchanged as in Sec. III and so we can frequently use Eqs. (17), (24)–(27).

In the next order, i.e., at the order $\varepsilon^{3/2}$, solving the ion continuity equation and the z-component of ion fluid equation of motion for $n^{(2)}$ and $w^{(2)}$ in terms of $\varphi^{(2)}$ and $\varphi^{(1)}$, we get the following equations

$$n^{(2)}=(1-\beta)\varphi^{(2)}+\tfrac{1}{2}(1-\beta)^3(3V^2-\tfrac{5}{9}\sigma)[\varphi^{(1)}]^2,$$
$$w^{(2)}=V(1-\beta)\varphi^{(2)}+\tfrac{1}{2}(1-\beta)^2\{(1-\beta)(3V^2-\tfrac{5}{9}\sigma)-2\}[\varphi^{(1)}]^2. \tag{40}$$

From the x and y component of ion fluid equation of motion at the order $\varepsilon^{3/2}$, we get the following equation:

$$\frac{\partial u^{(2)}}{\partial \xi}+\frac{\partial v^{(2)}}{\partial \eta}=\frac{V^3}{\omega_c^2}\frac{\partial}{\partial \zeta}\left(\frac{\partial^2\varphi^{(1)}}{\partial \xi^2}+\frac{\partial^2\varphi^{(1)}}{\partial \eta^2}\right). \tag{41}$$

Equation (15) at the order $\varepsilon^{3/2}$ is the following, in which as mentioned in the previous section an extra time derivative term $\varepsilon^{7/2}\alpha_1(\partial f^{(2)}/\partial\tau)$ has been included and $f^{(2)}$ has been replaced by $f_\varepsilon^{(2)}$. Also we have substituted for $f^{(1)}$ the expression given by (24).

$$\alpha_1\varepsilon^2\frac{\partial f_\varepsilon^{(2)}}{\partial \tau}+v_{11}\frac{\partial f_\varepsilon^{(2)}}{\partial \zeta}+\frac{\partial f_0}{\partial v_{11}}\frac{\partial \varphi^{(2)}}{\partial \zeta}=2v_{11}\frac{\partial^2 f_0}{\partial (v_{11}^2)^2}\frac{\partial}{\partial \zeta}(\varphi^{(1)})^2. \tag{42}$$

Now $f^{(2)}$ can be obtained uniquely from the solution of Eq. (42) by the following relation:

$$f^{(2)}=\lim_{\varepsilon\to 0+} f_\varepsilon^{(2)}. \tag{43}$$

Assuming τ-dependence to be of the form $\exp(i\omega\tau)$, taking Fourier transform with respect to ζ, following Landau prescription to remove the singularities involved and finally proceeding to the limit $\varepsilon\to 0+$, we get according to (43) the following expression for $\hat{f}^{(2)}$:

$$\hat{f}^{(2)}=-2\frac{\partial f_0}{\partial v_{11}^2}\hat{\varphi}^{(2)}+2\frac{\partial^2 f_0}{\partial (v_{11}^2)^2}\hat{d}_3, \tag{44}$$

in which we have used the results $x\delta(x)=0$ and $xP(1/x)=1$ to simplify the above equation and we have used the following notation for $[\varphi^{(1)}]^2$:

$$d_3=[\varphi^{(1)}]^2, \tag{45}$$

Taking Fourier inversion of Eq. (45), we get

$$f^{(2)}=-2\frac{\partial f_0}{\partial v_{11}^2}\varphi^{(2)}+2\frac{\partial^2 f_0}{\partial (v_{11}^2)^2}d_3. \tag{46}$$

Now from Eqs. (13) and (14) at the order ε, we get

$$-\int_{-\infty}^{\infty} f^{(2)}dv_{11}+n^{(2)}=0. \tag{47}$$

Substituting the values of $n^{(2)}$ and $f^{(2)}$ as given in Eqs. (40) and (46), respectively, we find that Eq. (47) is satisfied identically in view of the linear dispersion relation (27) and the critical condition $B=0$.

In the next order, i.e., at the order ε^2, solving the ion continuity equation and the z component of ion fluid equation of motion for $\partial n^{(3)}/\partial\zeta$ to express it in terms of $\varphi^{(3)}$, $\varphi^{(2)}$, $\varphi^{(1)}$, we get following equation:

$$\frac{\partial n^{(3)}}{\partial \zeta}=(1-\beta)\frac{\partial \varphi^{(3)}}{\partial \zeta}+2V(1-\beta)^2\frac{\partial \varphi^{(1)}}{\partial \tau}+\frac{V^4}{\omega_c^2}(1-\beta)^2\frac{\partial}{\partial \zeta}\left(\frac{\partial^2\varphi^{(1)}}{\partial \xi^2}+\frac{\partial^2\varphi^{(1)}}{\partial \eta^2}\right)+(1-\beta)^4\times\left\{\frac{10\sigma}{27}-6V^2+\frac{3}{2}(1-\beta)\left(3V^2-\frac{5}{9}\sigma\right)\right\}[\varphi^{(1)}]^2\times\frac{\partial \varphi^{(1)}}{\partial \zeta}+(1-\beta)^3\left(3V^2-\frac{5}{9}\sigma\right)\frac{\partial}{\partial \zeta}(\varphi^{(1)}\varphi^{(2)}), \tag{48}$$

where we have used Eqs. (17), (24)–(27), (40), and (41) to simplify the above equation. Now Eqs. (13) and (14) at the order $\varepsilon^{3/2}$ give the following equations:

$$n_e^{(3)}-n^{(3)}=\frac{\partial^2\varphi^{(1)}}{\partial \xi^2}+\frac{\partial^2\varphi^{(1)}}{\partial \eta^2}+\frac{\partial^2\varphi^{(1)}}{\partial \zeta^2}, \tag{49}$$

$$n_e^{(3)}=\int_{-\infty}^{\infty} f^{(3)}dv_{11}. \tag{50}$$

As in the lowest order case, Eq. (15) at the order ε^2 is the following, in which an extra higher order term $\varepsilon^4\alpha_1(\partial f^{(3)}/\partial\tau)$ has been included and $f^{(3)}$ has been replaced by $f_\varepsilon^{(3)}$. Also, we have substituted the expression for $f^{(1)}$ and $f^{(2)}$ as given by Eqs. (24) and (46), respectively.

$$\varepsilon^2\alpha_1\frac{\partial f_\varepsilon^{(3)}}{\partial \tau}+v_{11}\frac{\partial f_\varepsilon^{(3)}}{\partial \zeta}+\frac{\partial f_0}{\partial v_{11}}\frac{\partial \varphi^{(3)}}{\partial \zeta}=-2V\alpha_1\frac{\partial f_0}{\partial v_{11}^2}x_2+4v_{11}\frac{\partial^2 f_0}{\partial (v_{11}^2)^2}y_2-4v_{11}\frac{\partial^3 f_0}{\partial (v_{11}^2)^3}z_2, \tag{51}$$

where

$$x_2=\frac{\partial \varphi^{(1)}}{\partial \zeta},\quad y_2=\frac{\partial}{\partial \zeta}(\varphi^{(1)}\varphi^{(2)}),\quad z_2=[\varphi^{(1)}]^2\frac{\partial \varphi^{(1)}}{\partial \zeta}.$$

Therefore, $f^{(3)}$ is obtained from the unique solution of Eq. (51) by the relation

$$f^{(3)}=\lim_{\varepsilon\to 0+} f_\varepsilon^{(3)}. \tag{52}$$

Following the same procedure as getting (41) from (35), we get the following equations from (51):

$$ik[\hat{n}_e^{(3)}-(1-\beta)\hat{\varphi}^{(3)}]=-\frac{1}{4}iV\alpha_1(4-3\beta)\sqrt{\frac{\pi}{2}}\,\mathrm{sgn}(k)\hat{x}_2+\hat{y}_2+\frac{1}{2}(1+3\beta)\hat{z}_2. \tag{53}$$

Taking a Fourier inversion of the above equation, we get

$$\frac{\partial n_e^{(3)}}{\partial \zeta}=(1-\beta)\frac{\partial \varphi^{(3)}}{\partial \zeta}+\frac{\partial}{\partial \zeta}(\varphi^{(1)}\varphi^{(2)})+\frac{1}{2}(1+3\beta)\times[\varphi^{(1)}]^2\frac{\partial \varphi^{(1)}}{\partial \zeta}-\frac{1}{4\sqrt{2\pi}}V\alpha_1(4-3\beta)P\times\int_{-\infty}^{\infty}\frac{\partial \varphi^{(1)}}{\partial \xi'}\frac{d\zeta'}{\zeta-\zeta'}. \tag{54}$$

Eliminating $\partial n^{(3)}/\partial \zeta$ and $\partial n_e^{(3)}/\partial \zeta$ from Eqs. (48) and (54) with the help of Eq. (49), we get the following evolution equation for $\varphi^{(1)}$, in which the term containing $\partial \varphi^{(3)}/\partial \zeta$ disappears due to the linear dispersion relation. Also the term containing $\partial/\partial \zeta(\varphi^{(1)}\varphi^{(2)})$ disappears due to the critical condition $B=0$.

$$\frac{\partial \varphi^{(1)}}{\partial \tau}+AB'[\varphi^{(1)}]^2\frac{\partial \varphi^{(1)}}{\partial \zeta}+\frac{1}{2}A\frac{\partial^2 \varphi^{(1)}}{\partial \zeta^3}+\frac{1}{2}AD\frac{\partial}{\partial \zeta}\left(\frac{\partial^2 \varphi^{(1)}}{\partial \xi^2}+\frac{\partial^2 \varphi^{(1)}}{\partial \eta^2}\right)+\frac{1}{2}AE\alpha_1 P\int_{-\infty}^{\infty}\frac{\partial \varphi^{(1)}}{\partial \zeta'}\frac{d\zeta'}{\zeta-\zeta'}=0, \tag{55}$$

where A, D, and E are same as in (44) and B' is given by

$$B'=\frac{1}{2}\left\{(1-\beta)^4\left[\frac{10\sigma}{27}-6V^2+\frac{3}{2}(1-\beta)\left(3V^2-\frac{5\sigma}{9}\right)^2\right]-\frac{1}{2}(1+3\beta)\right\}. \tag{56}$$

Now if we neglect the electron to ion mass ratio, i.e., if we set $\alpha_1=0$ in Eq. (55) then it reduces to the MKdV–ZK equation (14) of Bandyopadhyay and Das.[6] Now the linear dispersion relation (27) and the critical condition $B=0$ give V and σ as a function of β only and these expressions of V and σ are given by

$$V=\{\tfrac{1}{8}(1-\beta)^{-3}[3-(1-\beta)^2]\}^{1/2},\quad \sigma=\tfrac{9}{40}(1-\beta)^{-3}[1-3(1-\beta)^2]. \tag{57}$$

Now $\sigma \geqslant 0$ imposes a restriction on β and this gives the following inequality:

$$0.423\leqslant\beta<1, \tag{58}$$

and under this restriction on β, one can easily verify that $B'>0$. Consequently the coefficients A, B', D, E of Eq. (55) are all positives.

V. SOLITARY WAVE SOLUTION

If we neglect the electron to ion mass ratio, i.e., if we set $\alpha_1=0$ in Eqs. (37) and (55), then the above two equations reduce, respectively, to the usual KdV–ZK and MKdV–ZK equations, the solitary wave solutions of which have extensively been investigated by Bandyopadhyay and Das.[6] In fact, the solitary wave solution of Eqs. (37) and (55) for $\alpha_1=0$, propagating at an arbitrary angle δ to the external magnetic field lying in the $\xi\zeta$-plane are given, respectively, by the following equations:[6]

$$\varphi^{(1)}\equiv\varphi_0(z)=a\,\mathrm{sech}^2\,pz, \tag{59}$$

$$\varphi^{(1)}\equiv\varphi_0(z)=\pm\bar{a}\,\mathrm{sech}\,\bar{p}z, \tag{60}$$

where

$$z=\xi\sin\delta+\zeta\cos\delta-U\tau,\quad p=\sqrt{\frac{U}{4a_5}},\quad \bar{p}=\sqrt{\frac{U}{a_5}},\quad a=\frac{3U}{a_1},\quad \bar{a}=\sqrt{\frac{6U}{a_2}},\quad a_1=AB\cos\delta,\quad a_2=AB'\cos\delta,\quad a_5=\tfrac{1}{2}A\cos^3\delta(1+D\tan^2\delta). \tag{61}$$

Now using Eq. (61), Eqs. (59) and (60) can be written, respectively, as follows:

$$\varphi^{(1)}\equiv\varphi_0(z)=a\,\mathrm{sech}^2\left[\sqrt{\frac{aa_1}{12a_5}}\left(\xi\sin\delta+\zeta\cos\delta-\frac{1}{3}a_1 a\tau\right)\right], \tag{62}$$

$$\varphi^{(1)}\equiv\varphi_0(z)=\pm\bar{a}\,\mathrm{sech}\left[\sqrt{\frac{a_2}{62a_5}}\bar{a}\left(\xi\sin\delta+\zeta\cos\delta-\frac{1}{6}a_2\bar{a}^2\tau\right)\right]. \tag{63}$$

For $\alpha_1\neq 0$, we shall discuss the solitary wave solutions of Eqs. (37) and (55) separately in the following two subsections in which Sec. V A deals with the solitary wave solution of the evolution equation (37) and Sec. V B deals with the solitary wave solution of the modified evolution equation (55).

A. Solitary wave solution of the evolution equation

In this case, following Ott and Sudan,[9] one can show that

$$\frac{\partial}{\partial \tau}\int\int\int_{-\infty}^{\infty}[\varphi^{(1)}]^2 d\xi d\eta d\zeta\leqslant 0, \tag{64}$$

which states that an initial perturbation in the form (59) will decay to zero. Therefore, the wave amplitude a is not a constant but decreases slowly with time. So following Ott and Sudan,[9] we have introduced a new space coordinate in a frame moving with soliraty wave and normalized to its width defined by

$$z=\sqrt{\frac{a_1}{12a_5}}\sqrt{a}\left(\xi\sin\delta+\zeta\cos\delta-\frac{1}{3}a_1\int_0^{\tau}a\,d\tau\right), \tag{65}$$

in which a is a slowly varying function of time. Obviously this co-ordinate varies only along the direction of propagation of the solitary wave.

Under this change of variable equation (37) assumes the following form, where we assume that $\varphi^{(1)}$ is a function of z, τ only.

$$\frac{\partial\varphi^{(1)}}{\partial\tau}+\left[-\frac{1}{3}a_1pa+\frac{z}{2a}\frac{da}{d\tau}\right]\frac{\partial\varphi^{(1)}}{\partial z}+a_1p\varphi^{(1)}\frac{\partial\varphi^{(1)}}{\partial z}+p^3a_5\frac{\partial^3\varphi^{(1)}}{\partial z^3}+\frac{1}{2}AE\alpha_1p\cos\delta P\int_{-\infty}^{\infty}\frac{\partial\varphi^{(1)}}{\partial z'}\frac{dz'}{z-z'}=0, \tag{66}$$

where we have used the following notation:

$$\frac{\partial\varphi^{(1)}}{\partial z'}=\frac{\partial\varphi^{(1)}}{\partial z}\quad\text{at } z=z'.$$

Now to investigate the solution of Eq. (66), we follow Ott and Sudan[9] and generalizing the multiple-time scale analysis with respect to α_1, by setting

$$\varphi^{(1)}(z,\tau)=q^{(0)}+\alpha_1q^{(1)}+\alpha_1^2q^{(2)}+\alpha_1^3q^{(3)}+\cdots, \tag{67}$$

where each $q^{(j)}$ $(j=0,1,2,3,...)$ are the function of $\tau=\tau_0,\tau_1,\tau_2,\ldots$. Here τ_j is given by

$$\tau_j=\alpha_1^j\tau,\quad j=0,1,2,3,... . \tag{68}$$

Substituting (67) into Eq. (66) and then equating the coefficient of different power of α_1 on each side of Eq. (66), we get a sequence of equations of which zeroth- and first-order equations are, respectively, given by

$$\rho\left[\frac{\partial}{\partial\tau}+\frac{z}{2a}\frac{\partial a}{\partial\tau}\frac{\partial}{\partial z}\right]q^{(0)}+L\partial_zq^{(0)}=0, \tag{69}$$

$$\rho\left[\frac{\partial}{\partial\tau}+\frac{z}{2a}\frac{\partial a}{\partial\tau}\frac{\partial}{\partial z}\right]q^{(1)}+\partial_zLq^{(1)}=\rho Mq^{(0)}, \tag{70}$$

where

$$L=\frac{\partial^2}{\partial z^2}+4\left(3\frac{q^{(0)}}{a}-1\right),\quad \partial_z=\frac{\partial}{\partial z},$$

$$\rho=24\sqrt{\frac{3a_5}{a_1^3}}a^{-3/2},$$

$$Mq^{(0)}=-\left[\frac{\partial q^{(0)}}{\partial\tau_1}+\frac{z}{2a}\frac{\partial a}{\partial\tau_1}\frac{\partial q^{(0)}}{\partial z}+\frac{1}{2}AEp\cos\delta P\int_{-\infty}^{\infty}\frac{\partial q^{(0)}}{\partial z'}\frac{dz'}{z-z'}\right]. \tag{71}$$

Now it can be easily be verified that $q^{(0)}=a\,\mathrm{sech}^2 z$ is the soliton solution of the equation $L\partial_zq^{(0)}=0$ and consequently $q^{(0)}=a\,\mathrm{sech}^2 z$ will be the soliton solution of (69) if and only if

$$\frac{\partial a}{\partial\tau}=0. \tag{72}$$

Using (72), we can write Eq. (70) as follows:

$$\rho\frac{\partial q^{(1)}}{\partial\tau}+\partial_zLq^{(1)}=\rho Mq^{(0)}. \tag{73}$$

Now for the solution of Eq. (73) to exist, its right-hand must be perpendicular to the kernal of the operator adjoint to the operator ∂_zL; this kernel which must tend to zero as $|z|\to\infty$ is $\mathrm{sech}^2 z$. Thus we get the following equation:

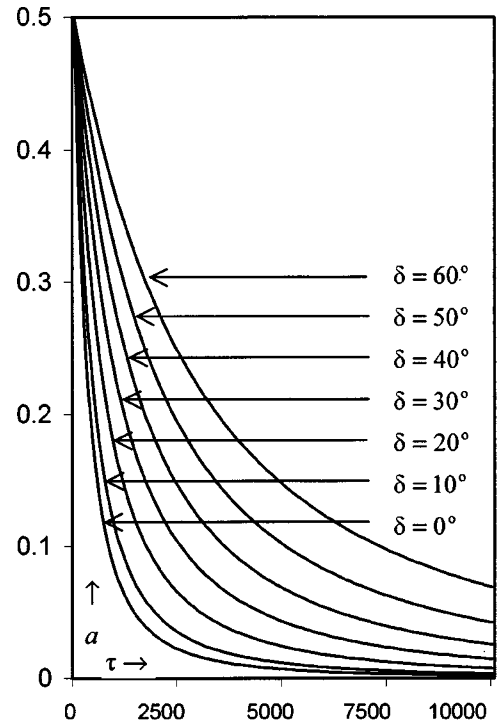

FIG. 2. The amplitude of the KdV-solitons is plotted against τ for different values of δ.

$$\int_{-\infty}^{\infty}(\mathrm{sech}^2 z)Mq^{(0)}dz=0. \tag{74}$$

The above equation gives the following differential equation for the solitary wave amplitude a:

$$\frac{\partial a}{\partial\tau_1}+\frac{1}{2}AE\cos\delta\sqrt{\frac{B}{6(\cos^2\delta+D\sin^2\delta)}}a^{3/2}P\times\int\int_{-\infty}^{\infty}\mathrm{sech}^2 z\frac{\partial}{\partial z'}(\mathrm{sech}^2 z')\frac{dz'dz}{z-z'}=0. \tag{75}$$

Solving Eq. (75), we get

$$a=a_0\left(1+\frac{\tau}{\tau'}\right)^{-2}, \tag{76}$$

where a_0 is the initial value of a, i.e., the value of a when $\tau=0$ and τ' is given by the following equation:

$$\tau'=\left[\frac{1}{4}AE\cos\delta\sqrt{\frac{Ba_0}{6(\cos^2\delta+D\sin^2\delta)}}\alpha_1P\times\int\int_{-\infty}^{\infty}\mathrm{sech}^2 z\frac{\partial}{\partial z'}(\mathrm{sech}^2 z')\frac{dz'dz}{z-z'}\right]^{-1}. \tag{77}$$

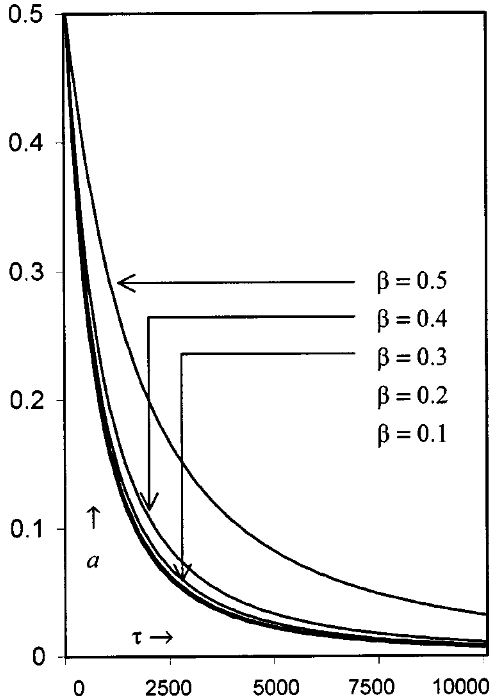

FIG. 3. The amplitude of the kdV-solitons is plotted against τ for different values of β.

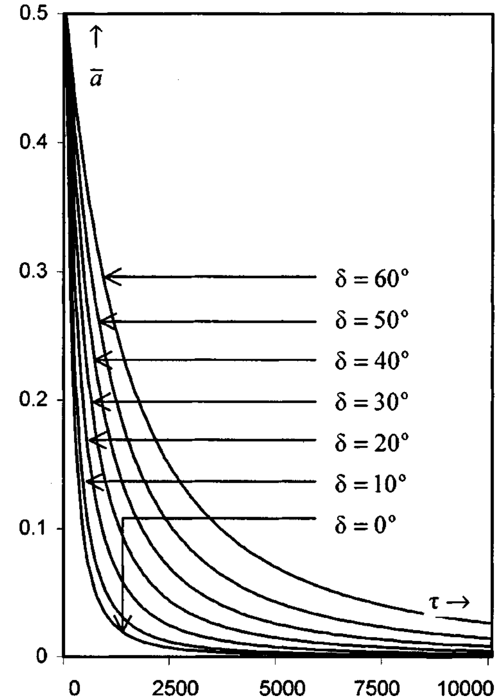

FIG. 4. The amplitude of the MKdV-solitons is plotted against τ for different values of δ.

In Fig. 2, the amplitude a of the solitary wave is plotted against τ for different values of δ with $\sigma=0.0001$, $\beta=0.3$, $a_0=0.5$, and $\omega_c=0.2$. This graph shows that for any fixed value of δ, the amplitude of the solitary wave decreases. Again from this graph we find for any value of τ, the amplitude of the solitary wave increases with δ. In Fig. 3, the amplitude of the solitary wave is plotted against τ for different values of β with $\sigma=0.0001$, $a_0=0.5$, $\omega_c=0.2$, and $\delta=25°$. This graph shows that the amplitude of the solitary wave decreases with τ for any fixed value β and for any fixed value of τ, the amplitude increases with β.

B. Solitary wave solution of the modified evolution equation

In this case also it can easily be verified that the inequality (64) holds and, consequently, the wave amplitude $\bar{a}$ is no longer a constant but decreases slowly with time. Therefore, according to the solitary structure as given in Eq. (63), we have introduced a new space coordinate in a frame moving with solitary wave and normalized to its width defined by

$$z=\bar{a}\sqrt{\frac{a_2}{6a_5}}\left(\xi\sin\delta+\zeta\cos\delta-\frac{1}{6}a_2\int_0^\tau \bar{a}^2 d\tau\right), \tag{78}$$

in which $\bar{a}$ is a slowly varying function of time. Obviously this coordinate z varies only along the direction of propagating of the solitary wave.

Under the above change of variables and assuming that $\varphi^{(1)}$ is a function of z, τ only, Eq. (55) assumes the following form:

$$\frac{\partial\varphi^{(1)}}{\partial\tau}+\left[-\frac{1}{6}\bar{p}a_2\bar{a}^2+\frac{z}{\bar{a}}\frac{d\bar{a}}{d\tau}\right]\frac{\partial\varphi^{(1)}}{\partial z}+a_2\bar{p}[\varphi^{(1)}]^2\frac{\partial\varphi^{(1)}}{\partial z}$$
$$+\bar{p}^3a_5\frac{\partial^3\varphi^{(1)}}{\partial z^3}+\frac{1}{2}AE\alpha_1\bar{p}\cos\delta P\int_{-\infty}^{\infty}\frac{\partial\varphi^{(1)}}{\partial z'}\frac{dz'}{z-z'}=0, \tag{79}$$

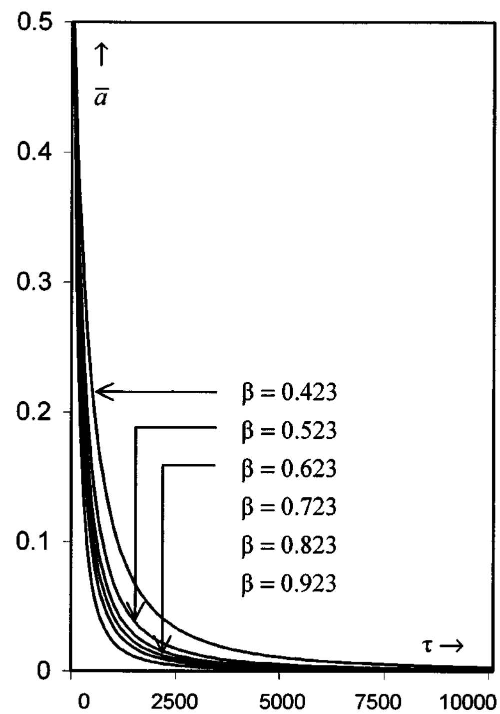

FIG. 5. The amplitude of the MKdV-solitons is plotted against τ for different values of β.

where $\varphi^{(1)}=\varphi^{(1)}(z,\tau)$ and we have used the following notation:

$$\frac{\partial\varphi^{(1)}}{\partial z'}=\frac{\partial\varphi^{(1)}}{\partial z}\quad\text{at } z=z'.$$

Now according to multiple-time scale analysis with respect to the small parameter α_1, we take the same perturbation equation of $\varphi^{(1)}(z,\tau)$ as given by Eq. (67). Substituting (67) into Eq. (79) and then equating the coefficients of different powers of α_1 on both sides of Eq. (79), we get a sequence of equations. The zeroth- and first-order equations of the above sequence are, respectively, given by

$$\bar{\rho}\left[\frac{\partial}{\partial\tau}+\frac{z}{\bar{a}}\frac{\partial\bar{a}}{\partial\tau}\frac{\partial}{\partial z}\right]q^{(0)}+\bar{L}\partial_z q^{(0)}=0, \tag{80}$$

$$\bar{\rho}\left[\frac{\partial}{\partial\tau}+\frac{z}{\bar{a}}\frac{\partial\bar{a}}{\partial\tau}\frac{\partial}{\partial z}\right]q^{(1)}+\partial_z\bar{L}q^{(1)}=\bar{\rho}\bar{M}q^{0}, \tag{81}$$

where

$$\bar{L}=\frac{\partial^2}{\partial z^2}+6\frac{[q^{(0)}]^2}{\bar{a}^2}-1,$$

$$\bar{M}q^{(0)}=-\left[\frac{\partial q^{(0)}}{\partial\tau_1}+\frac{z}{\bar{a}}\frac{\partial\bar{a}}{\partial\tau_1}\frac{\partial q^{(0)}}{\partial z}+\frac{1}{2}AE\bar{\rho}\cos\delta P\int_{-\infty}^{\infty}\frac{\partial q^{(0)}}{\partial z'}\frac{dz'}{z-z'}\right], \tag{82}$$

$$\bar{\rho}=6\sqrt{\frac{6a_5}{a_2^3}}\,\bar{a}^{-3}.$$

Now one can easily verify that $q^{(0)}=\pm\bar{a}\,\mathrm{sech}\,z$ is the solution of the equation $\bar{L}\partial_z q^{(0)}=0$ and consequently $q^{(0)}=\pm\bar{a}\,\mathrm{sech}\,z$ will be the soliton solution of (80) if and only if

$$\frac{\partial\bar{a}}{\partial\tau}=0. \tag{83}$$

Using (83), Eq. (82) can be written as

$$\bar{\rho}\frac{\partial q^{(1)}}{\partial\tau}+\partial_z\bar{L}q^{(1)}=\bar{\rho}\bar{M}q^{(0)}. \tag{84}$$

Now for the existence of the solution of Eq. (84), its right hand must be perpendicular to the kernel of the operator adjoint to the operator $\partial_z\bar{L}$; this kernel, which must tend to zero as $|z|\to\infty$ is sech z. Thus we get the following equation:

$$\int_{-\infty}^{\infty}\mathrm{sech}\,z(\bar{M}q^{(0)})dz=0. \tag{85}$$

Equation (85) gives the following differential equation for $\bar{a}$:

$$\frac{\partial\bar{a}}{\partial\tau_1}+\frac{1}{2}\bar{a}^2AE\cos\delta\sqrt{\frac{B'}{3(\cos^2\delta+D\sin^2\delta)}}P\times\int\int_{-\infty}^{\infty}\mathrm{sech}\,z\frac{\partial}{\partial z'}(\mathrm{sech}\,z')\frac{dzdz'}{z-z'}=0. \tag{86}$$

Solving the above equation we get

$$\bar{a}=\bar{a}_0(1+\tau/\tau')^{-1}, \tag{87}$$

where $\bar{a}_0$ is the initial value of $\bar{a}$, i.e., the value of $\bar{a}$ when $\tau=0$ and τ' is given by the following equation:

$$\tau'=\left[\frac{1}{2}AE\alpha_1\bar{a}_0\cos\delta\sqrt{\frac{B'}{3(\cos^2\delta+D\sin^2\delta)}}P\times\int\int_{-\infty}^{\infty}\mathrm{sech}\,z\frac{\partial}{\partial z'}(\mathrm{sech}\,z')\frac{dzdz'}{z-z'}\right]^{-1}. \tag{88}$$

In Fig. 4, the amplitude $\bar{a}$ of the solitary wave having sech-profile is plotted against τ for different values of δ with $\beta=0.423$, $a_0=0.5$, and $\omega_c=0.2$. From this graph we find that the amplitude decreases with τ for any fixed value of δ whereas for any fixed value of τ, the amplitude increases with δ. The amplitude ($\bar{a}$) is plotted in Fig. 5 versus τ for different value of β with $a_0=0.5$, $\omega_c=0.2$, and $\delta=25°$. This graph shows that the amplitude of the solitary wave decreases with β for any fixed value of τ.

VI. CONCLUSIONS

The evolution equation describing weakly nonlinear and weakly dispersive ion-acoustic waves in a nonthermal magnetized plasma with warm ions including the effect of Landau damping has been derived. This equation reduces to the KdV–ZK equation of Bandyopadhyay and Das[6] if electron to ion mass ratio is set equal to zero. If the coefficient of nonlinear term of this evolution equation vanishes, then the nonlinear behavior of ion-acoustic wave including the effect of Landau damping is described by a modified evolution equation and this equation reduces to the MKdV–ZK equation of Bandyopadhyay and Das[6] if electron to ion mass ratio is set equal to zero. The multiple time scale method of Ott and Sudan[9] has been generalized here to solve those evolution equation. Both the evolution equations admit solitary wave solutions, the former having sech^2-profile and the later having sech-profile. In either case, the amplitude of the solitary wave slowly decreases with time. The amplitude of the solitary waves having sech^2-profile increases with increasing value of the nonthermal parameter β whereas the amplitude of the solitary waves having a sech-profile decreases with increasing β.

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