Dust acoustic solitary wave with variable dust charge: Role of negative ions

Samiran Ghosh

Citation: Physics of Plasmas (1994-present) **12**, 094504 (2005); doi: 10.1063/1.2041652
View online: http://dx.doi.org/10.1063/1.2041652
View Table of Contents: http://scitation.aip.org/content/aip/journal/pop/12/9?ver=pdfcov
Published by the AIP Publishing

Dust acoustic solitary wave with variable dust charge: Role of negative ions

Samiran Ghosh[a)]
Government College of Engineering and Textile Technology, 4, Cantonment Road, Berhampore, Murshidabad 742101 West Bengal, India

(Received 9 June 2005; accepted 26 July 2005; published online 9 September 2005)

The role of negative ions on small but finite amplitude dust acoustic solitary wave including the effects of high and low charging rates of dust grains compared to the dust oscillation frequency in electronegative dusty plasma is investigated. In the case of high charging rate, the solitary wave is governed by Korteweg–de Vries (KdV) equation, but in the case of low charging rate, it is governed by KdV equation with a linear damping term. Numerical investigations reveal that in both cases dust acoustic soliton sharpens (flatens) and soliton width decreases (increases) with the increase of negative-ion number density (temperature). Also, the negative ions reduce the damping rate. © *2005 American Institute of Physics*. [DOI: 10.1063/1.2041652]

During the last several years, the collective processes of both linear[1,2] and nonlinear (such as soliton,[1,3–6] shock,[7,8] etc.) dust acoustic waves in space as well as laboratory environment have been well studied for (electropositive) dusty plasmas without negative ions.

However, nowadays the physics of electronegative dusty plasmas which are highly chemically reactive becomes very important because of its wide technological applications such as in plasma processing of materials useful for etching and deposition, synthesis of novel nanomaterials, plasma coating, environmental remediation, etc.[9–12] The electronegative dusty plasmas are also ubiquitous in astrophysical plasmas specially in the Earth's lower ionospheric plasmas, etc.[13,14] In most of the astrophysical and laboratory plasmas, the electron number density decreases because of their attachment on the dust grain surface and hence in this situation the negative ions play a crucial role. Furthermore, in such dusty plasmas the additional forces due to the interaction between negative ions and dust grains can appear.[15] The interesting phenomenon of such dusty plasmas is the surface potential acquired by a dust grain in the presence of negative ions. Due to the lower thermal velocity of negative ions compared to electrons, there is a reduction of dust surface potential and hence the dust charge magnitude in electronegative dusty plasmas.[16,17] Thus it is important to study the collective behaviors of both linear and nonlinear dust acoustic waves in charge varying electronegative dusty plasmas. Recently, in the linear regime, the effects of negative ions on the collective behaviors of dust acoustic wave[18] and ion acoustic wave[19] in electronegative dusty plasma have been investigated.

However, in this Brief Communication (BC), it is shown how the negative-ion concentration, negative-ion temperature, and also the dust charging rates (high and low) affect the dust acoustic solitary wave in unmagnetized electronegative laboratory and cosmic dusty plasmas.

We consider electronegative dusty plasmas whose constituents are Boltzmann-distributed electrons, singly charged positive and negative ions, and charge fluctuating negatively charged cold dust grains. Hence the nonlinear dynamics of low phase velocity dust acoustic waves are governed by the normalized dust dynamical equations:

$$\begin{aligned} &\partial_T N_d + \partial_X (N_d V_d) = 0, \\ &\partial_T V_d + V_d \partial_X V_d = -(-1+Q)\partial_X \Phi, \end{aligned} \tag{1}$$

and the Poisson's equation

$$\begin{aligned} \Delta \partial_X^2 \Phi = {}& \delta \exp(\Phi) + \delta_- \exp(\Phi/\sigma_-) - \exp(-\Phi/\sigma_+) \\ & - \Delta N_d(-1+Q). \end{aligned} \tag{2}$$

In deriving this equation, we use the charge neutrality condition $n_{e0}+z_d n_{d0}+n_{-0}=n_{+0}$, where $n_{e0}(n_{d0},n_{+0},n_{-0})$ is the electron (dust, positive-ion, and negative-ion) equilibrium number density and $-z_d e$ is the charge residing on the dust grains.

The normalized dust charging equation is

$$(\omega_{\rm pd}/\nu_{\rm ch})(\partial_T Q + V_d \partial_X Q) = (I_e + I_+ + I_-)/z_d e \nu_{\rm ch}, \tag{3}$$

where $\nu_{\rm ch}=(r_0/\sqrt{2\pi})(\omega_{p+}^2/V_{t+})(1+\sigma_+ + z + \mu_-)$ is the dust charging frequency and $\mu_- = \delta_- \sqrt{m_+ T_+ / m_- T_-}(1-\sigma_-)e^{-z/\sigma_-}$. The time ($T$) and space ($X$) scales are normalized in units of the dust plasma frequency $\omega_{\rm pd}(=\sqrt{z_d^2 e^2 n_{d0}/\epsilon_0 m_d})$ and the dust Debye length $\lambda_d(=\sqrt{\epsilon_0 T_e / n_{d0} z_d e^2})$, respectively. The dust velocity V_d and electrostatic potential Φ are normalized by dust acoustic speed $c_d(=\sqrt{z_d T_e/m_d})$ and e/T_e, respectively. The dust number density N_d and dust charge q_d are normalized by their equilibrium values n_{d0} and $z_d e$. Also we denote $\delta = n_{e0}/n_{+0}$, $\delta_- = n_{-0}/n_{+0}$, $\Delta = 1-\delta-\delta_-$, $\sigma_+ = T_+/T_e$, $\sigma_- = T_-/T_e$, and $z = z_d e^2 / 4\pi\epsilon_0 r_0 T_e$. The normalized expressions for the electron current (I_e), positive-ion current (I_+), and negative-ion current (I_-) for spherical dust grain of radius r_0 are as follows:[16]

[a)]Electronic mail: sran_g@yahoo.com

$$I_e = -J_e \exp[\Phi + z(-1+Q)],$$

$$I_+ = J_+ \exp\left(-\frac{\Phi}{\sigma_+}\right)\left[1 - \frac{z(-1+Q)}{\sigma_+}\right], \quad (4)$$

$$I_- = -J_- \exp\left[\frac{\Phi + z(-1+Q)}{\sigma_-}\right],$$

where $J_j = \pi r_0^2 e\sqrt{8T_j/\pi m_j} n_{j0}$, $j = e, +, -$.

To study the small but finite amplitude dust acoustic solitary wave, we introduce the stretched coordinates $\xi = \epsilon^{1/2}(X - \lambda T)$ and $\tau = \epsilon^{3/2} T$ and expand the dynamical variables N_d, V_d, Q, and Φ in power series of ϵ as follows:

$$f = f^{(0)} + \sum_{i=1}^{\infty} \epsilon^i f^{(i)}, \quad f = N_d, Q, V_d, \Phi, \quad (5)$$

where $f^{(0)}=1$ for N_d and $=0$ for V_d, Q, and Φ, respectively.

CASE I: HIGH CHARGING RATE

Here, we assume that the dust charging frequency $(\sim \nu_{ch}) \gg$ the dust oscillation frequency $(\sim \omega_{pd})$. To justify this assumption, we consider the representative parameters of $0.1C_4F_8 - 0.9Ar$ fluorocarbon laboratory-type electronegative dusty plasmas,[19] which are often used for ultrafine and highly selective etching of polysilicons[20] ($n_{+0} \sim 4 \times 10^{17}$ m^{-3}, $n_{d0} \sim 4 \times 10^{13}$ m^{-3}, $r_0 = 1$ μm, $T_e = 2$ eV, $T_- = 0.1$ eV, $T_+ = 0.2$ eV, $m_- \sim 3.2 \times 10^{-26}$ kg, $m_+ \sim 6.69 \times 10^{-26}$ kg, and dust mass density $\rho_d \sim 1.5 \times 10^3$ kg m^{-3}) and observe that for $\delta_- = 0.4$, $\omega_{pd} \sim 1.46 \times 10^3$ s^{-1} and $\nu_{ch} \sim 3.5 \times 10^7$ s^{-1}. Thus $\omega_{pd}/\nu_{ch} \approx 4.2 \times 10^{-4} \Rightarrow \omega_{pd} \ll \nu_{ch}$, which justifies the assumption. Hence this low value of ω_{pd}/ν_{ch}, can be assumed to be $\omega_{pd}/\nu_{ch} \approx 0$ and this reduces Eq. (3) to

$$I_e + I_+ + I_- \approx 0. \quad (6)$$

In this case the dust charge q_d instantaneously reaches its equilibrium value at each space-time point determined by the local electrostatic potential $\phi(x,t)$ and hence does not give rise to any dissipative effects (which is known as *adiabatic* dust charge variation[6]). Now using perturbation expansion (5) in (1), (2), and (6) together with (4) and then equating the terms in lowest order of ϵ, we obtain $\lambda = \sqrt{\Delta(\beta_{ch}\Delta + 1/\sigma_+ + \delta_-/\sigma_- + \delta)^{-1}}$, where $\beta_{ch} = (1+\sigma_+)(z+\sigma_+)/[z\sigma_+(1+z+\sigma_+ + \mu_-)]$. Finally, equating the terms of next higher order of ϵ from (1), (2), and (6) and then eleminating the second-order terms, we derive the following Korteweg–de Vries (KdV) equation in electronegative dusty plasmas:

$$\partial_\tau N_d^{(1)} + \alpha N_d^{(1)} \partial_\xi N_d^{(1)} + \beta \partial_\xi^3 N_d^{(1)} = 0. \quad (7)$$

The coefficient of nonlinearity α and coefficient of dispersion β are given by

$$\alpha = \frac{\lambda}{2}\left(3\left\{1 - \lambda^2\beta_{ch}) + \lambda^4\left[\Delta^{-1}\left(\frac{\delta_-}{\sigma_-^2} - \frac{1}{\sigma_+^2} + \delta\right) + \frac{\beta_{ch}\sigma_+}{(1+\sigma_+)} A\right]\right\}\right), \quad \beta = \frac{\lambda^3}{2}, \quad (8)$$

where $A = \{1 + [\mu_-(1+\sigma_-)/\sigma_-(z+\sigma_+)]\}(z\beta_{ch} - 1)^2 + 2z\beta_{ch}/[\sigma_+(z+\sigma_+)] - (1/\sigma_+^2)$. The expressions for α and β [Eq. (8)] show the dust charge variation ($\propto \beta_{ch}$) and also the negative ions ($\propto \delta_-$) significantly modify the characteristics of dust acoustic solitary waves in electronegative dusty plasma. The variations of α and β with δ_- for different σ_- are drawn in Fig. 1. The single soliton solution of KdV equation (7) is $N_d^{(1)}(\xi,\tau) = (3V/\alpha)\text{sech}^2[\sqrt{V/4\beta}(\xi - V\tau)]$, where V, $3V/\alpha$, and $\sqrt{4\beta/V}$ are the velocity, amplitude, and spatial width of the solitary wave, respectively.

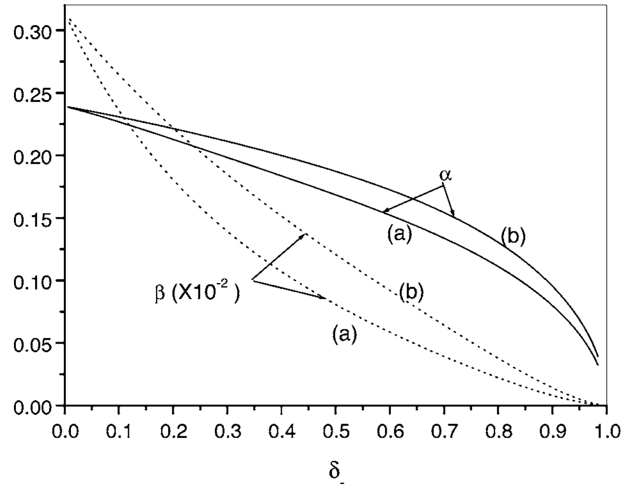

FIG. 1. Variation of coefficients of nonlinearity α (solid curve) and dispersion β (dotted curve) [Eq. (8)] with $\delta_-(=n_{-0}/n_{+0})$ for $\sigma_+(=T_+/T_e)=0.1$ and for different $\sigma_-(=T_-/T_e)$. The different curves are (a) $\sigma_-=0.05$ and (b) $\sigma_-=0.1$.

CASE II: LOW CHARGING RATE

Here, we assume $\nu_{ch} \ll \omega_{pd}$. To justify this, we consider the cosmic (e.g., Noctilucent Cloud[14,21]) electronegative dusty plasma parameters [$n_{+0} \sim 10^9$ m^{-3}, $n_{d0} \sim 10^7$ m^{-3}, $r_0 = 0.1$ μm, $T_e = T_+ = 0.013$ eV, $m_+ = m_- \sim 2.7 \times 10^{-26}$ kg, and dust mass density $\rho_d \sim 292$ kg m^{-3} (for ice dust grains)] and observe that for $\delta_- = 0.4$, $\omega_{pd} \sim 0.25$ s^{-1} and $\nu_{ch} \sim 0.07$ s^{-1}. Thus $\nu_{ch}/\omega_{pd} \approx 0.28$, which indicates that $\nu_{ch}/\omega_{pd} \ll 1, \neq 0$. Hence for consistent perturbation expansion, we can consider the following scaling:

$$\nu_{ch}/\omega_{pd} = \nu_d \epsilon^{3/2}, \quad (9)$$

where $\nu_d \approx O(1)$. In this case due to the nonzero value of ν_{ch}/ω_{pd}, the dust charge q_d does not instantaneously reach its equilibrium value but instead does play a dissipative role. Using the above scaling in Eq. (3) together with (4) and then equating the terms in lowest order and next to the lowest order of ϵ, we obtain

$$\partial_\xi Q^{(1)} = 0 \Rightarrow Q^{(1)} = Q^{(1)}(\tau) = 0, \quad \lambda \partial_\xi Q^{(2)} = \nu_d \beta_{ch} \Phi^{(1)}, \quad (10)$$

by virtue of the boundary condition that all perturbations vanish at $X = -\infty$ ($\xi = -\infty$) for all time scales slow or fast. As before, equating all the terms of lowest order of ϵ and using (10), we obtain $\lambda = \sqrt{\Delta(1/\sigma_+ + \delta_-/\sigma_- + \delta)^{-1}}$. Applying the same procedure as in case I, we derive the following KdV equation with a linear damping term:

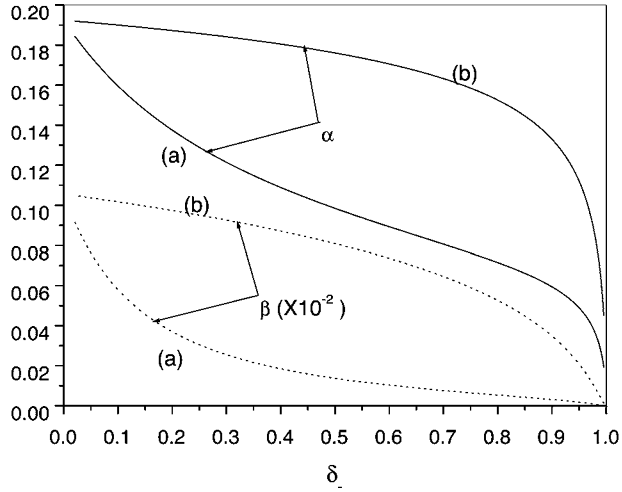

FIG. 2. Variation of coefficients of nonlinearity α (solid curve) and dispersion β (dotted curve) [Eq. (12)] with δ_- for $\sigma_+=1$ and for different σ_-. The different curves are (a) $\sigma_-=0.1$ and (b) $\sigma_-=1$.

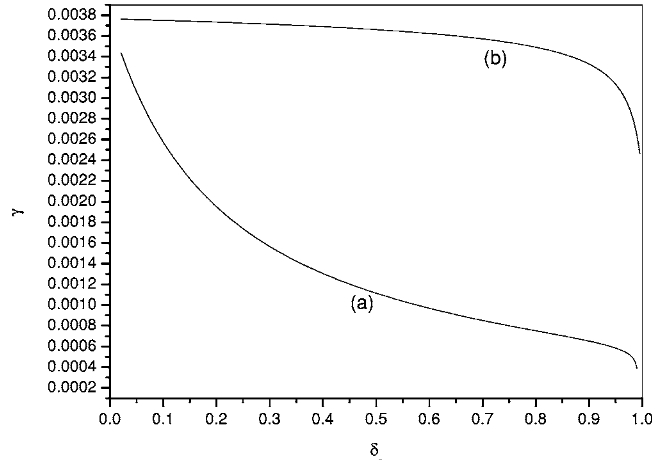

FIG. 3. Variation of γ [Eq. (12)] with δ_- for $\nu_d=1$. The plasma parameters are the same as in Fig. 2.

$$\partial_\tau N_d^{(1)} + \alpha N_d^{(1)}\partial_\xi N_d^{(1)} + \beta\partial_\xi^3 N_d^{(1)} + \gamma N_d^{(1)} = 0. \tag{11}$$

The coefficient of nonlinearity α and coefficient of dispersion β are given by

$$\alpha = \frac{\lambda}{2}\left[3 + \lambda^4\Delta^{-1}\left(\frac{\delta_-}{\sigma_-^2} - \frac{1}{\sigma_+^2} + \delta\right)\right],$$
$$\beta = \frac{\lambda^3}{2}, \quad \gamma = \frac{\nu_d}{2}\beta_{\mathrm{ch}}\Delta\left(\frac{1}{\sigma_+} + \frac{\delta_-}{\sigma_-} + \delta\right)^{-1}. \tag{12}$$

In this case, the expressions for α and β show that only negative ions ($\delta_- \neq 0$) [by Eq. (12)] modify the characteristics of dust acoustic solitary waves in electronegative dusty plasma. The variations of α and β with δ_- for different σ_- are depicted in Fig. 2. The expression (12) shows that the damping γ disappears if we put $\nu_d=0$, this should be so because $\nu_d=0$ implies the right-hand side of Eq. (3) $=0$ ($dQ/dT=0$), i.e., there is no dust charge variation [by Eq. (10); $Q^{(1)}=Q^{(2)}=0 \Rightarrow$ fixed charge on the dust grain surface]. Also note that $\gamma=0$ for $\Delta\,(=1-\delta-\delta_-=z_d n_{d0}/n_{+0})=0$ (by charge neutrality condition); this is expected as $\Delta=0$ implies the absence of charged dust grains and consequently no charge variation. Hence the damping of the dust acoustic solitary wave arises due to the low charging rate of dust grains compared to the dust oscillation frequency. The variation of γ with δ_- for different σ_- is depicted in Fig. 3. Employing Karpman and Maslov's method and using the boundary conditions that all perturbed variables vanish at $|\xi|=\infty$, we obtain the approximate analytical time evolution sech^2 solution of (11) as $N_d^{(1)}(\xi,\tau)=N(0)e^{-(4/3)\gamma\tau}\,\mathrm{sech}^2\{[\xi - V(\tau)\tau]/W(\tau)\}$, where $V(\tau)=[\alpha N(0)/3]e^{-(4/3)\tau}$ and $W(\tau) = \sqrt{12\beta e^{(4/3)\tau}/\alpha N(0)}$ are the velocity and width of the solitary wave, respectively, with $N(0)=N(\tau=0)=3V/\alpha$. These show that due to the low charging rate compared to ω_{pd} ($\nu_{\mathrm{ch}}/\omega_{\mathrm{pd}} \ll 1, \neq 0$), $\gamma \neq 0$, the soliton amplitude and velocity decay exponentially with time τ at decay rate γ, but the spatial width of the soliton increases exponentially with τ.

The results of this BC can be summarized as follows:

(1) In laboratory-type electronegative dusty plasmas,[19] the dust charging rate is very high compared to the dust oscillation frequency ($\omega_{\mathrm{pd}} \ll \nu_{\mathrm{ch}}$) and the dust acoustic solitary wave is governed by KdV equation [Eq. (7)]. On the other hand, in the case of cosmic electronegative dusty plasma (e.g., Noctilucent Cloud),[14,21] the dust charging rate is low but comparable to dust oscillation frequency [$\nu_{\mathrm{ch}}/\omega_{\mathrm{pd}} < 1, \neq 0$, Eq. (9)] and this produces a dissipative effect for which the dust acoustic solitary wave is governed by KdV equation with a linear damping term [Eq. (12)].

(2) In both cases, the coefficient of nonlinearity α decreases (increases) with the increase of $\delta_-(\sigma_-)$ [Figs. 1 and 2, solid curves]. Thus in both cases the soliton amplitude which is $\propto 1/\alpha$ increases (decreases), i.e., the dust acoustic soliton sharpens (flatens) as the negative-ion density (temperature) increases. Also in both cases the dust acoustic solitary wave admits compressive soliton $N_d^{(1)}(\xi,\tau) > 0$ with dust density enhancement as $\alpha > 0$.

(3) Also in both cases, coefficient of dispersion β behaves in a qualitatively similar fashion as done by α [Figs. 1 and 2, dotted curves]. Thus the soliton width which is $\propto\sqrt{\beta}$ decreases (increases) with the increase of negative-ion density (temperature).

(4) The damping rate γ [Eq. (12)] decreases (increases) with the increase of $\delta_-(\sigma_-)$ [Fig. 3]. This is happening due to the fact that the increase of negative-ion number density implies the reduction of electrons from the system by charge neutrality condition which decreases the magnitude of the dust grain surface potential z and hence decreases the damping rate γ as it arises due to dust charge variation. But, the magnitude of dust surface potential z increases with the increase of negative-ion temperature and hence γ increases with negative-ion temperature.

The author would like to thank Professor M. R. Gupta of Centre for Plasma Studies, Faculty of Science, Jadavpur University for his stimulating influence and helpful suggestions.

[1]N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. **38**, 543 (1990); A. Barkan, R. L. Merlino, and N. D'Angelo, Phys. Plasmas **2**, 3563 (1995).
[2]R. K. Varma, P. K. Shukla, and V. Krishan, Phys. Rev. E **47**, 3612 (1993); F. Melandso, T. Aslaksen, and O. Havnes, Planet. Space Sci. **41**, 321 (1993); V. E. Fortov, A. G. Khrapak, V. I. Molotkov, A. P. Nefedov, O. F. Petrov, and V. M. Torchinsky, Phys. Plasmas **7**, 1374 (2000).
[3]A. Mamun, R. A. Cairns, and P. K. Shukla, Phys. Plasmas **3**, 702 (1996); F. Verheest, Planet. Space Sci. **40**, 1 (1992).
[4]S. Ghosh, T. K. Chaudhuri, S. Sarkar, M. Khan, and M. R. Gupta, Phys. Rev. E **65**, 037401 (2002); Astrophys. Space Sci. **278**, 463 (2001).
[5]N. N. Rao and P. K. Shukla, Planet. Space Sci. **42**, 221 (1994).
[6]J. X. Ma and J. Liu, Phys. Plasmas **4**, 253 (1997); B. S. Xie, K. He, and Z. Huang, Phys. Lett. A **247**, 403 (1998); S. Ghosh, S. Sarkar, M. Khan, M. R. Gupta, and K. Avinash, Phys. Lett. A **298**, 49 (2002).
[7]S. I. Popel, A. A. Gisko, A. P. Golub, T. V. Losseva, R. Bingham, and P. K. Shukla, Phys. Plasmas **7**, 2410 (2000).
[8]M. R. Gupta, S. Sarkar, S. Ghosh, M. Debnath, and M. Khan, Phys. Rev. E **63**, 046406 (2001); S. Ghosh, S. Sarkar, M. Khan, K. Avinash, and M. R. Gupta, Phys. Plasmas **10**, 977 (2003).
[9]M. A. Lieberman and A. J. Lichtenberg, *Principles of Plasma Discharges and Materials Processing*, (Wiley, New York, 1994), Chap. 6; J. Goree, Plasma Sources Sci. Technol. **3**, 400 (1994); G. S. Selwyn, Jpn. J. Appl. Phys., Part 1 **32**, 3068 (1993).
[10]A. Bouchoule and L. Boufendi, Plasma Sources Sci. Technol. **3**, 262 (1994).
[11]S. V. Vladimirov and K. Ostrikov, Phys. Rep. **393**, 175 (2004).
[12]K. Ostrikov, Rev. Mod. Phys. **77**, 489 (2005).
[13]L. Spitzer, Astrophys. J. **93**, 369 (1941); *Diffuse Matter in Space* (Interscience, New York, 1968), Chap. 4; B. A. Klumov, S. I. Popel, and R. Bingham, JETP Lett. **72**, 364 (2000).
[14]M. Dadsden and W. Schroder, *Noctilucent Clouds* (Spinger, New York, 1989), Chap. 9.
[15]I. Denysenko, M. Y. Yu, L. Stenflo, and N. A. Azarenkov, Phys. Plasmas **12**, 042102 (2005).
[16]A. Mamun and P. K. Shukla, Phys. Plasmas **10**, 1518 (2003); M. Djebli, Phys. Plasmas **10**, 4910 (2003); B. M. Annaratone and J. E. Allen, J. Phys. D **38**, 26 (2005).
[17]B. M. Annaratone, T. Antonoua, H. M. Thomas, and G. E. Morfill, Phys. Rev. Lett. **93**, 185001 (2004).
[18]N. D'Angelo, J. Phys. D **37**, 860 (2004).
[19]S. V. Vladimirov, K. Ostrikov, M. Y. Yu, and G. E. Morfill, Phys. Rev. E **67**, 036406 (2003).
[20]H. Kokura, S. Yoneda, K. Nakamura, N. Mitsuhira, M. Nakamura, and H. Sugai, Jpn. J. Appl. Phys., Part 1 **38**, 5256 (1999).
[21]D. A. Mendis and M. Rosenberg, IEEE Trans. Plasma Sci. **20**, 929 (1992).