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Dust acoustic shock waves in two-component dusty plasma

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Abstract. The effect of electron–dust collision on small-amplitude nonlinear dust acoustic (DA) waves in two-component thermal dusty plasma consisting of positively charged (due to thermionic emission) dust grains and electrons has been investigated incorporating the nonadiabaticity of dust-charge variation arising due to delays in the dust charging, i.e. due to small nonzero values of $\omega_{\mathrm{pd}}/\nu_{\mathrm{ch}}$, where ω_{pd} is the dust-plasma frequency and ν_{ch} is the dust-charging frequency. The propagation of small-amplitude DA waves is governed by a modified Korteweg–de Vries–Burger equation in which the Burger term arising due to the charge delay induced dissipation. Numerical investigations reveal that this equation has a shock-wave solution. Numerical investigations also reveal that in the absence of collision-induced dissipation the charge-delay-induced dissipation also causes the generation of a DA shock wave in two-component dusty plasma.

Contents

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1. Introduction

It is well known that low-frequency oscillations in so-called three-component 'complex' plasmas consisting of electrons, ions and negatively charged dust grains give rise to some new low-frequency eigenmodes, e.g. 'dust ion acoustic' (DIA) [1] and 'dust acoustic' (DA) [2]. The dust grains immersed in a plasma are electrically charged. In a laboratory plasma system, the dust grains are usually negatively charged. However, in space as well as in the laboratory, dust grains can be charged positively by UV irradiation [3, 4] and by thermionic emission [5]–[7]. The charge on the dust grains is not fixed but fluctuates due to the fluctuating electric field. In a three-component dusty plasma with fluctuating dust charges and high dust densities, there also exists another wave mode called the 'dust Coulomb (DC) wave' or the 'dust electro-acoustic (DEA) wave' [8]. The damping or instabilities of the DA wave due to the effects of dust-charge variations and collisions have been studied both theoretically and experimentally [9]–[15].

The charge q_d on the dust grain is an extra dynamical variable which controls grain motion but itself must be determined from the orbit motion limited (OML) grain charging equation

$$\frac{\omega_{pd}}{\nu_{ch}}\frac{dq_d}{d(\omega_{pd}t)} = \frac{I}{\nu_{ch}},$$

where I is the total current flowing to the dust grain surface, ω_{pd} $(=\sqrt{(q_d^2 n_{d0})/(\epsilon_0 m_d)})$ is the dust-plasma frequency and ν_{ch} $[\sim\partial I/\partial q_d]$ is the dust-charging frequency. For nonzero values of ω_{pd}/ν_{ch} the linear DA wave mode in weakly coupled dusty plasma becomes damped [9]–[11]. Experimental observations [15] reveals that in inhomogeneous strongly coupled dusty plasma the DA wave becomes unstable due to the delay ($\omega_{pd}/\nu_{ch} \ll 1$ but is nonzero; in the experiment in [15], it is $O(10^{-4})$) in the charging of dust grains. The nonlinear analysis of DA, DIA and DC in a weakly coupled three-component dusty plasma shows that this delay in the charging (which is usually called nonadiabatic dust-charge variations) causes a dissipation in dusty plasma. This dissipation leads to the formation of both DA shock, DC(DAE) shock waves [16] and DIA shock waves [17]–[19]. Another experimental observation [20] reveals that the DIA shock is generated due to ion–dust collision through ion viscosity [21, 22] described by the well known Korteweg–de Vries–Burger (KdVB) equation. Various researchers have investigated the basic properties of nonlinear coherent structures, namely solitons, shocks, vortices etc of DA, DC(DAE), DL and DIA waves in dusty plasma [23]–[26].

Recently Shukla [27] investigated the linear DA wave in two-component dusty plasma consisting of electrons and positively charged dust grains incorporating the dust neutral collision and dust-charge variations. Later, Khrapak and Morfill [28] reinvestigated the problem, taking into account the ionization and recombination processes on the dust surface, the electron–neutral, dust–neutral and electron–dust elastic collisions, the electron drag on dust grains, in addition to the charge variations, to generalize the result obtained by Shukla. Motivated by these two investigations, in this paper we have studied the small-amplitude nonlinear DA wave in a two-component thermal dusty plasma consisting of electrons and positively charge (by thermionic emission) dust grains by the reductive perturbation technique. The effects of electron–dust elastic collisions under the assumption that the collision frequency is much greater than the dust-plasma frequency (later we shall show that this assumption is justified) and nonadiabatic (ω_{pd}/ν_{ch} is small but nonzero) dust-charge variations are also incorporated. It is seen that due to the presence of electron–dust collisions and nonadiabatic dust-charge variations, the nonlinear DA wave in governed by a modified KdV–Burger (mKdVB) equation. In the absence of nonadiabatic dust-charge variations, this mKdVB reduces to a mKdV equation which is analogous to a nonlinear equation derived by Kakutani and Kawahara [29] for electron–ion plasma. On the other hand, in the absence of electron–dust collision, this mKdVB equation reduces to the well known KdV–Burger equation [16]. Numerical investigations show that this equation possesses a compressional shock-wave solution and depending on the magnitude of the electron–dust collision and ω_{pd}/ν_{ch}, it exhibits both monotonic and oscillatory shock.

The paper is organized as follows. Formulation of the problem is given in section 2. Section 3 deals with the nonlinear evolution equations describing nonlinear DA waves in two-component electron–dust plasma. The shock-wave solution of the mKdVB equation is given in section 4. Section 5 contains the numerical results and discussions. A summary of the result is given in section 6.

2. Formulation of the problem

We consider two-component, unmagnetized, nonrelativistic thermal dusty plasma consisting of positively charged dust grains and electrons. The positively charged dust grains undergo elastic collisions with the electrons. The charges on the dust grains fluctuate according to the electric field E. In this situation at equilibrium, $E = 0$, the overall charge neutrality condition becomes

$$n_{e0} = z_{d0} n_{d0}, \tag{1}$$

where n_{e0} (n_{d0}) is the electron (dust) equilibrium number density and $z_{d0}e$ is the (positive) charge residing on the dust grains.

To incorporate the effect of the friction-dominated dust fluid momentum loss through electron–dust elastic collisions on the DA wave, we assume that the electron–dust collision frequency is much larger than the dust-plasma frequency. If $R_d = -m_d n_d \nu'_{de}(v_e - v_d)$ and $R_e = m_e n_e \nu'_{ed}(v_e - v_d)$, are the momentum losses of dust grains and electrons per unit volume due to electron–dust elastic collisions, where m_d (m_e) is the dust (electron) mass, n_d (n_e) is the dust (electron) number density, v_d (v_e) is the dust (electron) velocity, ν'_{ed} and ν'_{de} are the electron–dust and dust–electron collision frequency, then Krook's model suggests that

$$R_d + R_e = 0 \Rightarrow \nu'_{de} = \left(\frac{m_e}{m_d}\right)\left(\frac{n_e}{n_d}\right)\nu'_{ed} \tag{2}$$

where the electron–dust collision frequency [30] is given by

$$\nu'_{ed} = \frac{\Lambda z_{d0}^2 e^4 n_{d0}}{4\pi\epsilon_0^2\sqrt{m_e}T_e^{\frac{3}{2}}} = 4\pi a^2 n_{d0} V_{te} z^2. \tag{3}$$

In the above expression $\Lambda = \ln[\sqrt{5}\lambda_{De}/za]$ is the Coulomb logarithm, where $\lambda_{De} = \sqrt{(\epsilon_0 T_e)/(n_{e0}e^2)}$ is the electron plasma Debye length, a is the dust grain radius, $z = (z_{d0}e^2)/(4\pi\epsilon_0 a T_e)$ is the nondimensional dusty plasma parameter and $V_{te} = \sqrt{T_e/m_e}$ is the electron thermal velocity.

The one-dimensional behaviour of the thermal plasma under consideration may be described by the following normalized equations for the two-fluid model:

$$\frac{\partial N_d}{\partial T} + \frac{\partial (N_d V_d)}{\partial X} = 0 \tag{4}$$

$$\frac{\partial V_d}{\partial T} + V_d\frac{\partial V_d}{\partial X} = Q_d E + \delta\nu_{ed}\frac{N_e}{N_d}(V_e - V_d) - \frac{\sigma}{N_d}\frac{\partial N_d}{\partial X} \tag{5}$$

$$\delta\left[\frac{\partial V_{\rm e}}{\partial T}+V_{\rm e}\frac{\partial V_{\rm e}}{\partial X}\right]=-E-\frac{1}{N_{\rm e}}\frac{\partial N_{\rm e}}{\partial X}-\delta\nu_{\rm ed}(V_{\rm e}-V_{\rm d}) \tag{6}$$

$$\frac{\partial E}{\partial X}=Q_{\rm d}N_{\rm d}-N_{\rm e}. \tag{7}$$

To close this system, instead of using an electron continuity equation we consider the following current displacement equation for electrons and dust fluids:

$$\frac{\partial E}{\partial T}=N_{\rm e}V_{\rm e}-Q_{\rm d}N_{\rm d}V_{\rm d}. \tag{8}$$

In the above set of equations $\delta=(z_{\rm d0}m_{\rm e})/m_{\rm d}$, $\sigma=T_{\rm d}/(z_{\rm d0}T_{\rm e})$, $N_{\rm e}=n_{\rm e}/n_{\rm e0}$, $N_{\rm d}=n_{\rm d}/n_{\rm d0}$, $Q_{\rm d}=q_{\rm d}/(z_{\rm d0}e)$ and $q_{\rm d}$ is the total charge on the dust grains. The timescale T is normalized in units of the dust-plasma frequency, $\omega_{\rm pd}$ $(=\sqrt{(z_{\rm d0}^2n_{\rm d0}e^2)/(\epsilon_0m_{\rm d})})$. The normalized electron–dust collision frequency is $\nu_{\rm ed}=\nu'_{\rm ed}/\omega_{\rm pd}$. The space scale X and dust velocity $V_{\rm d}$, electron velocity $V_{\rm e}$ are normalized in units of electron Debye length $\lambda_{\rm De}$ $(=\sqrt{(\epsilon_0T_{\rm e})/(n_{\rm e0}e^2)})$ and DA speed $C_{\rm d}$ $(=\sqrt{(z_{\rm d0}T_{\rm e})/(m_{\rm d})})$. The electric field E is normalized by $e/(\lambda_{\rm De}T_{\rm e})$.

In the dust momentum conservation equation (5), the momentum loss due to viscous drag arising from dust–dust and dust–electron collisions through the dust viscous stress $\eta_{\rm de}\partial^2v_{\rm d}/\partial x^2$ is ignored compared to that from the frictional force $\nu'_{\rm de}v_{\rm d}$. This is because the ratio of the friction force to viscous stress is of the order

$$\mathrm{O}\left(\frac{|\nu'_{\rm de}v_{\rm d}|}{|\eta_{\rm de}\frac{\partial^2v_{\rm d}}{\partial x^2}|}\right)=\frac{\nu'_{\rm de}\lambda_{\rm De}^2}{\eta_{\rm de}}=\frac{z_{\rm d0}T_{\rm e}}{T_{\rm d}}\frac{\nu'^2_{\rm de}}{\omega_{\rm pd}^2}\gg1 \qquad \text{as } \eta_{\rm de}=\frac{T_{\rm d}}{m_{\rm d}}\nu'^{-1}_{\rm de}.$$

The inequality holds good for $(z_{\rm d0}T_{\rm e})/T_{\rm d}\gg1$ (even though for thermal dusty plasma $T_{\rm d}=T_{\rm e}$ as $z_{\rm d0}\gg1$) and $\nu'_{\rm de}/\omega_{\rm pd}\gg1$ (our assumption). Relation (2) has been used to derive the dust momentum conservation equation (5). The momentum loss due to electron–dust collisions has been considered in both the dust and electron momentum conservation equations (5) and (6). It is assumed that the electron–dust collision frequency $\nu'_{\rm ed}$ is much larger than the dust-plasma frequency $\omega_{\rm pd}$, i.e. $\nu'_{\rm ed}/\omega_{\rm pd}\gg1$ and $\delta=(z_{\rm d0}m_{\rm e})/m_{\rm d}=\omega_{\rm pd}^2/\omega_{\rm pe}^2\ll1$ so that $\delta\nu_{\rm ed}=(z_{\rm d0}m_{\rm e}/m_{\rm d})(\nu'_{\rm ed}/\omega_{\rm pd})\gg\mathrm{O}(\omega_{\rm pd}^2/\omega_{\rm pe}^2)$. Hence by neglecting the term $\mathrm{O}(\omega_{\rm pd}^2/\omega_{\rm pe}^2)$, equation (6) can be rewritten as

$$\frac{\partial N_{\rm e}}{\partial X}=-N_{\rm e}E-\delta\nu_{\rm ed}N_{\rm e}(V_{\rm e}-V_{\rm d}). \tag{9}$$

The normalized charge, $Q_{\rm d}$, on the dust grain is determined by the following OML dust-charging equation:

$$\frac{\omega_{\rm pd}}{\nu_{\rm ch}}\left[\frac{\partial Q_{\rm d}}{\partial T}+V_{\rm d}\frac{\partial Q_{\rm d}}{\partial X}\right]=\frac{I_{\rm e}^{\rm Th}+I_{\rm e}}{z_{\rm d0}e\nu_{\rm ch}}, \tag{10}$$

where $I_{\rm e}^{\rm Th}$ is the thermionic emission current [5]–[7] and $I_{\rm e}$ is the electron plasma current. The normalized expressions of the $I_{\rm e}^{\rm Th}$ and $I_{\rm e}$ currents for spherical dust grains of radius a are as follows:

$$I_{\rm e}^{\rm Th}=2\pi a^2e\left(\frac{m_{\rm e}T_{\rm e}}{2\pi\hbar^2}\right)^{\frac{3}{2}}\sqrt{\frac{8T_{\rm e}}{\pi m_{\rm e}}}(1+zQ_{\rm d})\exp\left(-\frac{W_{\rm e}}{T_{\rm e}}-zQ_{\rm d}\right) \tag{11}$$

and

$$I_e = -\pi a^2 e \sqrt{\frac{8T_e}{\pi m_e}} n_{e0} N_e (1 + zQ_d) \tag{12}$$

where W_e is the work function, $z = z_{d0} e^2 / 4\pi\epsilon_0 a T_e$ is the nondimensional dusty plasma parameter and $4\pi\epsilon_0 a$ is the capacitance of the spherical dust grains of radius a.

The dust-charging frequency ν_{ch} is given by

$$\nu_{ch} = -\frac{\partial(I_e^{Th} + I_e)}{\partial Q_d} = \pi a^2 n_{d0} \sqrt{\frac{8T_e}{\pi m_e}} z(1+z) = \frac{a}{\sqrt{2\pi}} \frac{\omega_{pe}^2}{V_{te}}(1+z), \tag{13}$$

where ω_{pe} is the electron plasma frequency and V_{te} is the electron thermal velocity.

3. Nonlinear evolution equations

Our aim is to find the equations for a single fluid (dust), which describe the behaviour of nonlinear DA waves. By eliminating N_e and V_e from equations (5), (7), (8) and (9)–(12), we obtain

$$\frac{\partial N_d}{\partial T} + \frac{\partial (N_d V_d)}{\partial X} = 0 \tag{14}$$

$$N_d \left[\frac{\partial V_d}{\partial T} + V_d \frac{\partial V_d}{\partial X} \right] = E \frac{\partial E}{\partial X} + \frac{\partial^2 E}{\partial X^2} - \sigma \frac{\partial N_d}{\partial X} - \frac{\partial((1+\Delta Q)N_d)}{\partial X} \tag{15}$$

$$\delta \nu_{ed} \left[\frac{\partial E}{\partial T} + V_d \frac{\partial E}{\partial X} \right] = E \frac{\partial E}{\partial X} + \frac{\partial^2 E}{\partial X^2} - \frac{\partial((1+\Delta Q)N_d)}{\partial X} - (1+\Delta Q) N_d E. \tag{16}$$

The grain-charging equation (10) together with (11) and (12), becomes

$$\frac{\omega_{pd}}{\nu_{ch}} \left[\frac{\partial \Delta Q}{\partial T} + V_d \frac{\partial \Delta Q}{\partial X} \right] = \beta_d \left[((1+z) + z\Delta Q) \left(\exp(-z\Delta Q) - (1+\Delta Q) N_d + \frac{\partial E}{\partial X} \right) \right], \tag{17}$$

where $\beta_d = 1/(z(z+1))$ and ΔQ is the perturbed part of the normalized dust charge Q_d, i.e. $Q_d = 1 + \Delta Q$.

In order to study the small-amplitude nonlinear DA wave in electron–dust two-component plasma, the reductive perturbation technique has been employed and the following stretched coordinate introduced:

$$\xi = \epsilon^{\frac{1}{2}}(X - \lambda T); \qquad \tau = \epsilon^{\frac{3}{2}} T, \tag{18}$$

where λ is the phase velocity of the linear DA wave normalized by the DA speed and ϵ measures the order of the smallness of the perturbations.

The dynamical variables N_d, V_d, ΔQ and the electric field E are expanded as

$$f = f^{(0)} + \epsilon f^{(1)} + \epsilon^2 f^{(2)} + \cdots; \qquad E = \epsilon^{\frac{3}{2}} E^{(1)} + \epsilon^{\frac{5}{2}} E^{(2)} + \cdots, \tag{19}$$

where $f^{(0)} = 1$ for $f = N_d$ and $f^{(0)} = 0$ for $f = V_d, \Delta Q$.

To employ the reductive perturbation technique, we first consider the dust grain charging equation (17). Using stretching (18), perturbation expansion (19) for all the variables and expressions (11) and (12) we arrive at

$$\frac{\omega_{\rm pd}}{\nu_{\rm ch}}\left[(-\lambda\epsilon^{\frac{1}{2}}+\epsilon^{\frac{3}{2}}V_{\rm d}^{(1)})\frac{\partial}{\partial\xi}+\epsilon^{\frac{3}{2}}\frac{\partial}{\partial\tau}\right](\epsilon\Delta Q^{(1)}+\epsilon^2\Delta Q^{(2)})$$
$$= -\beta_{\rm d}(1+z)\left[\epsilon((1+z)\Delta Q^{(1)}+N_{\rm d}^{(1)})+\epsilon^2\left(N_{\rm d}^{(2)}+(1+z)\Delta Q^{(2)}\right.\right.$$
$$\left.\left.+\frac{1+2z}{z+1}N_{\rm d}^{(1)}\Delta Q^{(1)}+\frac{z(2-z)}{2}\Delta Q^{(1)2}-\frac{\partial E^{(1)}}{\partial\xi}\right)\right]. \tag{20}$$

To include the effect of charge delay in nonlinear evolution equations and to make the nonlinear perturbation consistent with that of (18) and (19), we chose the following scaling [16]:

$$\frac{\omega_{\rm pd}}{\nu_{\rm ch}}=\nu_{\rm d}\sqrt{\epsilon}, \tag{21}$$

where $\nu_{\rm d}\approx$ O(1). Using the above scaling in equation (20) and equating the terms in lowest powers of ϵ, i.e. the term O(ϵ) and the terms O(ϵ^2), we get the following equations:

$$\Delta Q^{(1)}=-z\beta_{\rm d}N_{\rm d}^{(1)} \tag{22}$$

$$\Delta Q^{(2)}=-z\beta_{\rm d}\left[N_{\rm d}^{(2)}+(1+2z)z\beta_{\rm d}N_{\rm d}^{(1)}\Delta Q^{(1)}+\frac{z(2-z)}{2}\Delta Q^{(1)2}-\frac{\partial E^{(1)}}{\partial\xi}\right]$$
$$+\frac{\nu_{\rm d}\lambda}{\beta_{\rm d}(z+1)^2}\frac{\partial\Delta Q^{(1)}}{\partial\xi}. \tag{23}$$

The expression for the normalized electron–dust-collision frequency in terms of the dust-charging frequency can be rewritten as

$$\nu_{\rm ed}=\frac{\nu'_{\rm ed}}{\omega_{\rm pd}}\Rightarrow\left(\frac{\sqrt{2\pi}\Lambda z}{z+1}\right)\left(\frac{\nu_{\rm ch}}{\omega_{\rm pd}}\right). \tag{24}$$

To make the nonlinear perturbation consistent with that of (19) and (21), we assume the following scaling:

$$\nu_{\rm ed}=\nu_{\rm c}\epsilon^{-\frac{1}{2}} \tag{25}$$

where $\nu_{\rm c}\approx$ O(1). The justification of scalings (21) and (25) is discussed in section 5.

Applying equations (19) and (25) to dynamical equations (14)–(16) and equating the terms at the lowest powers of ϵ i.e. O($\epsilon^{\frac{3}{2}}$) and eliminating $V_{\rm d}^{(1)}=\lambda N_{\rm d}^{(1)}$, we get

$$N_{\rm d}^{(1)}=\frac{\Delta Q^{(1)}}{\lambda^2-1-\sigma} \tag{26}$$

$$\frac{\partial N_{\rm d}^{(1)}}{\partial\xi}=\lambda\nu_{\rm c}\delta\frac{\partial E^{(1)}}{\partial\xi}-E^{(1)}-\frac{\partial\Delta Q^{(1)}}{\partial\xi}. \tag{27}$$

Equating the O($\epsilon^{\frac{5}{2}}$) terms, we get from (14)–(16)

$$\frac{\partial N_{\rm d}^{(1)}}{\partial\tau}+2\lambda N_{\rm d}^{(1)}\frac{\partial N_{\rm d}^{(1)}}{\partial\xi}=\lambda\frac{\partial N_{\rm d}^{(2)}}{\partial\xi}-\frac{\partial V_{\rm d}^{(2)}}{\partial\xi} \tag{28}$$

$$\lambda\frac{\partial N_{\rm d}^{(1)}}{\partial\tau}-\frac{\partial^2E^{(1)}}{\partial\xi^2}+\frac{\partial(\Delta Q^{(1)}N_{\rm d}^{(1)})}{\partial\xi}=\lambda\frac{\partial V_{\rm d}^{(2)}}{\partial\xi}-(1+\sigma)\frac{\partial N_{\rm d}^{(2)}}{\partial\xi}-\frac{\partial\Delta Q^{(2)}}{\partial\xi} \tag{29}$$

$$\delta\nu_{\rm c}\frac{\partial E^{(1)}}{\partial\tau}+(N_{\rm d}^{(1)}+\Delta Q^{(1)})E^{(1)}+\delta\lambda\nu_{\rm c}N_{\rm d}^{(1)}\frac{\partial E^{(1)}}{\partial\xi}-\frac{\partial^2E^{(1)}}{\partial\xi^2}+\frac{\partial(\Delta Q^{(1)}N_{\rm d}^{(1)})}{\partial\xi}$$
$$=\delta\lambda\nu_{\rm c}\frac{\partial E^{(2)}}{\partial\xi}-E^{(2)}-\frac{\partial\Delta Q^{(2)}}{\partial\xi}-\frac{\partial N_{\rm d}^{(2)}}{\partial\xi}. \tag{30}$$

For DA waves the value of λ follows from equations (22) and (26)

$$\lambda=\sqrt{1+\sigma-z\beta_{\rm d}}. \tag{31}$$

Here $\beta_{\rm d}$ arises due to the dust-charge variation. The above expression thus shows that the linear DA wave phase velocity is modified due to the dust-charge variation.

Eliminating $\partial N_{\rm d}^{(2)}/\partial\xi$ from equations (29) and (30) and then using (22), the following relationship can be obtained:

$$\frac{2\lambda}{1+\sigma}\frac{\partial N_{\rm d}^{(1)}}{\partial\tau}-\delta\nu_{\rm c}\frac{\partial E^{(1)}}{\partial\tau}-(1-z\beta_{\rm d})N_{\rm d}^{(1)}E^{(1)}+\frac{\sigma}{1+\sigma}\frac{\partial^2E^{(1)}}{\partial\xi^2}+\frac{2\beta_{\rm d}\sigma}{(1+\sigma)}N_{\rm d}^{(1)}\frac{\partial N_{\rm d}^{(1)}}{\partial\xi}$$
$$-\delta\lambda\nu_{\rm c}N_{\rm d}^{(1)}\frac{\partial E^{(1)}}{\partial\xi}=\frac{\lambda}{1+\sigma}\frac{\partial V_{\rm d}^{(2)}}{\partial\xi}+E^{(2)}-\delta\lambda\nu_{\rm c}\frac{\partial E^{(2)}}{\partial\xi}+\frac{\sigma}{1+\sigma}\frac{\partial\Delta Q^{(2)}}{\partial\xi}. \tag{32}$$

Adding equation (30) with the resulting equation obtained by the elimination of $\partial V_{\rm d}^{(2)}/\partial\xi$ from equations (28) and (32), we arrive at the following equation:

$$2\lambda\frac{\partial N_{\rm d}^{(1)}}{\partial\tau}+2(\lambda^2-2z\beta_{\rm d})N_{\rm d}^{(1)}\frac{\partial N_{\rm d}^{(1)}}{\partial\xi}-\frac{\partial^2E^{(1)}}{\partial\xi^2}=(\lambda^2-1-\sigma)\frac{\partial N_{\rm d}^{(2)}}{\partial\xi}-\frac{\partial\Delta Q^{(2)}}{\partial\xi}. \tag{33}$$

Finally using equations (22), (23) in (33) and then eliminating $E^{(1)}$ with the help of equation (27), we get the following mKdVB equation:

$$\frac{\partial N_{\rm d}^{(1)}}{\partial\tau}+\alpha N_{\rm d}^{(1)}\frac{\partial N_{\rm d}^{(1)}}{\partial\xi}+\beta\frac{\partial^3N_{\rm d}^{(1)}}{\partial\xi^3}=\nu\frac{\partial}{\partial\xi}\left[\frac{\partial N_{\rm d}^{(1)}}{\partial\tau}+\alpha N_{\rm d}^{(1)}\frac{\partial N_{\rm d}^{(1)}}{\partial\xi}\right]+\mu\frac{\partial^2N_{\rm d}^{(1)}}{\partial\xi^2}, \tag{34}$$

where

$$\alpha=\frac{1}{\sqrt{1+\sigma-z\beta_{\rm d}}}\left[1+\sigma-2z\beta_{\rm d}+\frac{z^3\beta_{\rm d}^3}{2}((1+z)^2+1)\right] \tag{35}$$

$$\beta=\frac{(1-z\beta_{\rm d})^2}{2\sqrt{1+\sigma-z\beta_{\rm d}}}+\nu_{\rm c}\nu_{\rm d}\frac{\lambda(z+1)\delta\beta_{\rm d}}{2} \tag{36}$$

$$\nu=\nu_{\rm c}\delta\sqrt{1+\sigma-z\beta_{\rm d}} \tag{37}$$

$$\mu=\nu_{\rm d}\frac{(z+1)\beta_{\rm d}}{2}. \tag{38}$$

The above expressions show that the dust-charge variation modifies both the coefficient of nonlinearity α and the coefficient of dispersion β, as the term $\beta_{\rm d}$ arising due to dust-charge

Table 1. Values of ν_{ed} for different plasma parameters.

$T_e = T_d$ (eV)	W_e (eV)	z_{d0}	$z = \frac{z_{d0}e^2}{4\pi\epsilon_0 a T_e}$	$\frac{\nu_{ch}}{\omega_{pd}}$	ν_{ed}
5	0.5	1.126×10^5	32.436	3.625×10^3	3.47×10^4
	2	1.115×10^5	32.136	3.609×10^3	3.462×10^4
	5	1.096×10^5	31.556	3.579×10^3	3.44×10^4
0.5	0.5	9.75×10^3	28.082	3.3892×10^4	2.403×10^5
	2	8.709×10^3	25.082	3.216×10^4	2.358×10^5
	5	6.695×10^3	19.282	2.852×10^4	2.246×10^5

variation is present in both the expressions, whereas in the case of two-component electron–ion plasma $\alpha \approx 1$ and $\beta \approx 0.5$. The right-hand side (RHS) of equation (34) implies the generation of shock waves due to electron–dust collisions and also due to nonadiabatic dust-charge variations in a two-component electron–dust plasma (figures 3, 4). The RHS of (34) also shows that the in absence of nonadiabatic dust-charge variations i.e. if we neglect the charge delay ($\nu_d \approx 0 \Rightarrow \mu \approx 0$), this equation reduces to a mKdV equation which is similar to a nonlinear equation derived by Kakutani and Kawahara [29] for two-component electron–ion plasma, and the dissipation is proportional to the electron–dust collision frequency ν_{ed}. On the other hand, in the absence of electron–dust collisions ($\nu_c \approx 0 \Rightarrow \nu \approx 0$), we recover the well known KdV–Burger equation and the dissipation is proportional to μ arising due to the delay ($\nu_d \neq 0$) in the charging. Hence the charge delay plays a dissipative role in dusty plasma and this dissipation leads to the formation of a shock wave (figure 3 with $\nu = 0$, dotted curve) in two-component dusty plasma. Again, with the help of relation (25) and by virtue of equation (24) the expression for ν can be rewritten as

$$\nu = \sqrt{\epsilon}\left(\frac{\sqrt{2\pi}\,\Lambda z}{z+1}\right)\left(\frac{\omega_{pd}}{\omega_{pe}}\right)^2\left(\frac{\nu_{ch}}{\omega_{pd}}\right)\sqrt{1+\sigma - z\beta_d}. \tag{39}$$

Similarly with the help of (21) the expression for μ can be rewritten as

$$\mu = \sqrt{\epsilon}\frac{(\nu_{ch}/\omega_{pd})(z+1)\beta_d}{2} = \sqrt{\epsilon}\frac{1}{2z}\left(\frac{\nu_{ch}}{\omega_{pd}}\right) \tag{40}$$

with $\beta_d = 1/(z(z+1))$. From (39) and (40), we have

$$\frac{\nu}{\mu} = \frac{\sqrt{8\pi}\,\Lambda z^2\sqrt{1+\sigma - z\beta_d}}{(z+1)}\left(\frac{\omega_{pd}}{\omega_{pe}}\right)^2. \tag{41}$$

Using the numerical values given in table 1, we see that

$$\nu \ll \mu. \tag{42}$$

Thus the dissipation arising due to the delay in the charging is much greater than the dissipation arising due to electron–dust collisions under the assumption that the electron–dust collision frequency is much greater than the dust-plasma frequency in two-component dusty plasma.

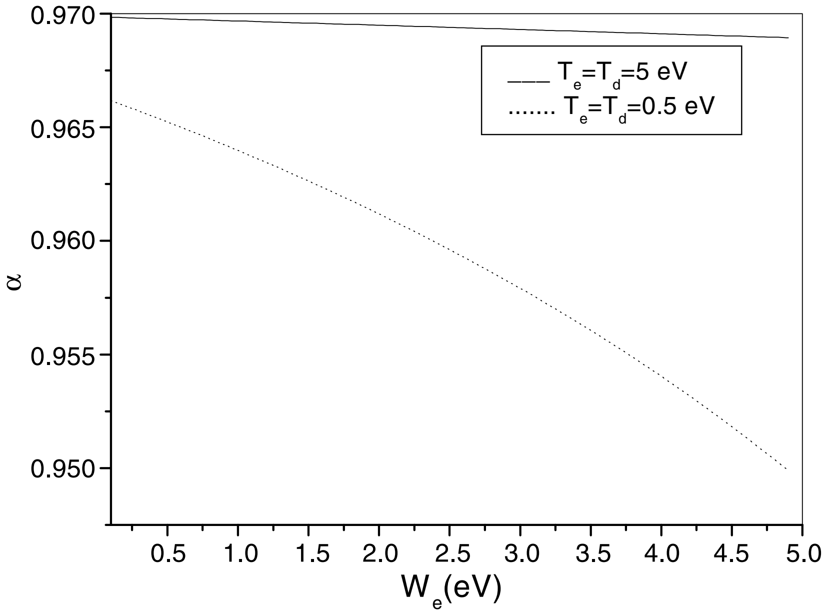

Figure 1. Variation of coefficient of nonlinear α (equation (35)) with the work function, W_e, for $T_d = T_e$. The solid curve represents $T_e = 5$ eV and the dotted curve represents $T_e = 0.5$ eV.

4. Shock-wave solution

On transforming to the wave frame

$$\eta = V\tau - \xi = \sqrt{\epsilon}\frac{[C_d(\lambda + \epsilon V)t - x]}{\lambda_{De}} \tag{43}$$

the modified KdVB equation (34) reduces to

$$\frac{d^2 N_d^{(1)}}{d\eta^2} = \frac{V}{\beta}N_d^{(1)} - \frac{\alpha}{2\beta}N_d^{(1)2} - \frac{\mu - \nu V}{\beta}\frac{dN_d^{(1)}}{d\eta} - \frac{\alpha\nu}{\beta}N_d^{(1)}\frac{dN_d^{(1)}}{d\eta}. \tag{44}$$

On introducing the transformations

$$N_d^{(1)} = u; \qquad \frac{dN_d^{(1)}}{d\eta} = v \tag{45}$$

equation (44) is rewritten as a set of simultaneous equations for u and v

$$\frac{du}{d\eta} = v; \qquad \frac{dv}{d\eta} = \frac{V}{\beta}u - \frac{\alpha}{2\beta}u^2 - \frac{\mu - \nu V}{\beta}v - \frac{\alpha\nu}{\beta}vu. \tag{46}$$

This system of equations has two singular points at $(u, v) = (0, 0)$ and $(2V/\alpha, 0)$. The first one, i.e. $(u, v) = (0, 0)$, is a saddle point, whereas $(u, v) = (2V/\alpha, 0)$ is a stable node or a stable focus according to

$$(\mu + \nu V)^2 > \text{ or } < 4\beta V. \tag{47}$$

A stable node corresponds to a monotonic shock (dissipation dominant) front (figure 4) while a stable focus implies that the shock structure is oscillatory (dispersion dominant) (figure 3).

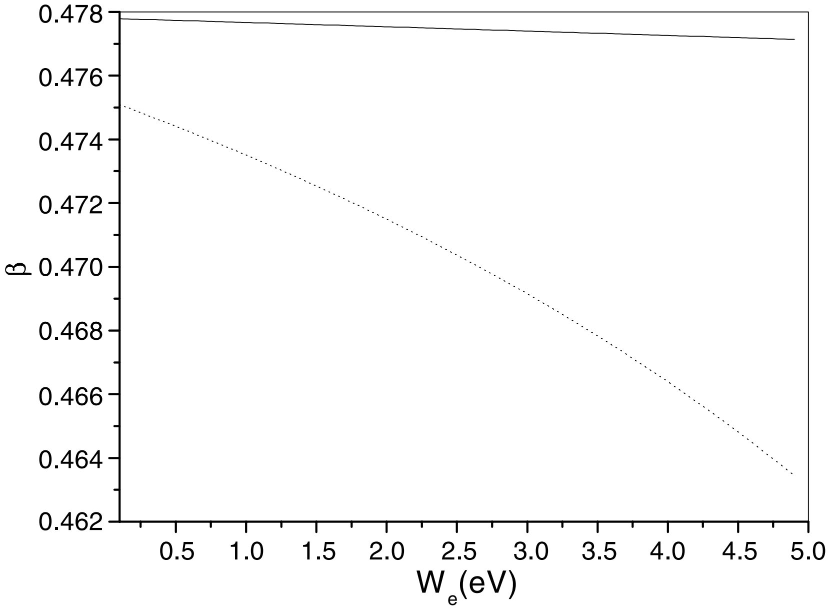

Figure 2. Variation in coefficient of dispersion, β (equation (36)), with the work function, W_e. The other plasma parameters are the same as in figure 1.

Introducing the shock Mach number, M, defined as the ratio of the velocity $C_d(\lambda + \epsilon V)$ of the nonlinear wave to the DA(linear) wave velocity $C_d\lambda$

$$M = 1 + \epsilon \frac{V}{\lambda} \tag{48}$$

criterion (47) can be re-expressed as

$$[\delta V \lambda \nu'_{ed} + 0.5(z+1)\beta_d \nu_{ch}]^2 > \text{ or } < \frac{4V^2 \omega_{pd}^2 \beta}{\lambda(M-1)} \tag{49}$$

using equations (25) and (48).

5. Numerical results and discussions

In our numerical analysis, we considered the following dusty plasma parameters: plasma temperature $T_e = T_d = 5$ eV, 0.5 eV; dust number density $n_{d0} \approx 5 \times 10^{13} m^{-3}$; average dust grain radius $a \approx 1\ \mu$m; dust mass density $\rho_d \approx 10^3$ kg m^{-3} so that the dust mass $m_d \approx \frac{4}{3}\pi\rho_d a^3 \approx 4.19 \times 10^{-15}$ kg; electron mass $m_e \approx 9.1094 \times 10^{-31}$ kg implies $m_e/m_d \approx 2.17 \times 10^{-16}$. The values of the ratio of the dust-charging frequency (ν_{ch}) to the dust-plasma frequency (ω_{pd}) and the values of the normalized electron–dust collision frequency (ν_{ed}) (equation (24)) are shown in table 1 for different values of the plasma temperature, T_e, and the work function, W_e. It is found that for $T_e = T_d = 5$ eV, $\omega_{pd}/\nu_{ch} \approx O(10^{-3})$ and for $T_e = T_d = 0.5$ eV, it is $O(10^{-4})$. Hence, to include the effects of charge delay, these values justify the scaling $\omega_{pd}/\nu_{ch} \approx \nu_d \epsilon^{1/2}$ (equation (21)) on the basis of which charging equation (equation (18)) is approximated. This table also shows that for $T_e = T_d = 5$ eV, the normalized (normalized by the dust-plasma frequency, ω_{pd}) electron–dust collision frequency is $1/\nu_{ed} \approx O(10^{-4})$) and for

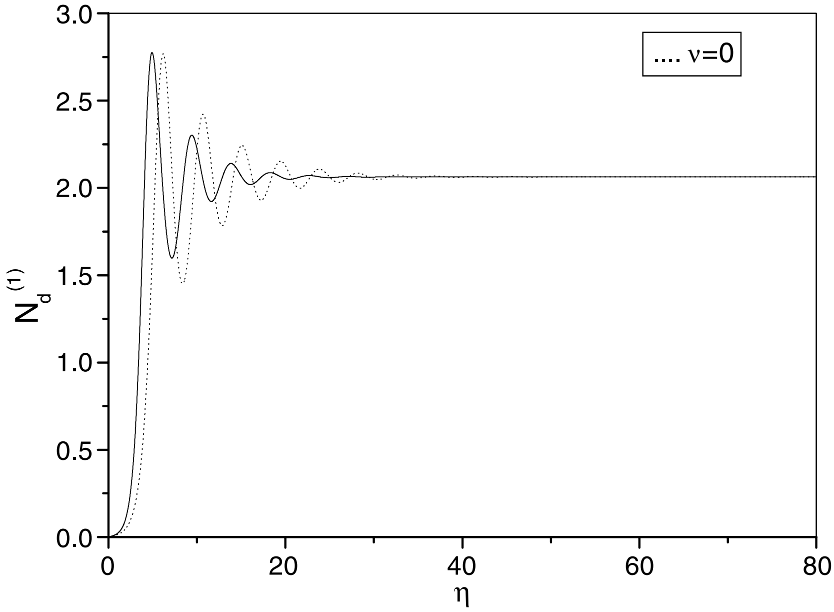

Figure 3. Oscillatory shock for weak dissipation with $M = 2$, $T_e = 5$ eV and the work function $W_e = 2$ eV. The solid curve represents the oscillatory shock for both dissipations (collisional and nonadiabatic), whereas the dotted curve represents the dissipation due to just nonadiabatic dust-charge variation.

$T_e = T_d = 0.5$ eV, it is $O(10^{-5})$ so that $\nu_{ed} \gg 1$ and hence it can be assumed that $1/\nu_{ed} \approx O(\sqrt{\epsilon})$. Thus the numerical values justify our assumption that the electron–dust collision frequency is greater than the dust-plasma frequency and also justify the scaling $\nu_{ed} \approx \nu_c \epsilon^{-1/2}$ (equation (25)). This table also suggests that with the increased work function W_e, the dust-charging frequency decreases, whereas with the increase in plasma temperature, the charging frequency increases.

Figures 1 and 2 show the variations in the coefficient of the nonlinear term, α, and the coefficient of the dispersive term, β, with the work function, W_e, for different plasma temperatures. Figure 2 shows that the nonlinearity of the DA wave decreases with W_e and increases with the plasma temperature. Figure 3 shows that β behaves in a qualitatively similar way to α.

The system of equations (46) have been solved numerically by the usual Runge–Kutta–Felhberg method with the help of the values of the above-mentioned parameters. Figures 3 and 4 are drawn for $W_e = 2$, $T_d = T_e = 5$ eV and Mach number $M = 2$. In figure 3 the solid curve represents the oscillatory shock structure for the dissipation arising due to both the electron–dust collision and nonadiabatic dust-charge variations. The dotted curve represents the same for the dissipation arising only due to the nonadiabatic dust-charge variation, i.e. dissipation arising due to a delay in the charging. This figure shows that the DA wave in a two-component electron–dust plasma exhibits an oscillatory shock wave. Figure 4 is drawn for $T_e = T_d = 5$ eV and $W_e = 2$ eV and shows the monotonic nature of the DA shock wave. These two figures also show the transition of an oscillatory shock to a monotonic shock when the dissipation increases.

Figure 5 shows that the shock strength given by

$$[\epsilon N_d^{(1)}]_{max} = \epsilon \frac{2V}{\alpha} = \frac{4(M-1)(1+\sigma-z\beta_d)}{2(1+\sigma-z\beta_d)+z^3\beta_d^3((1+z)^2+1)}$$

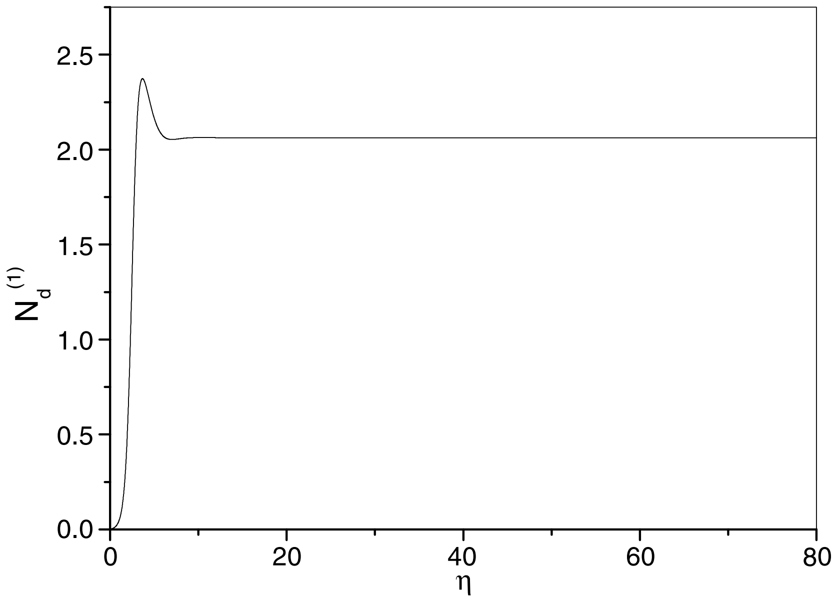

Figure 4. Monotonic shock structure for moderate dissipation.

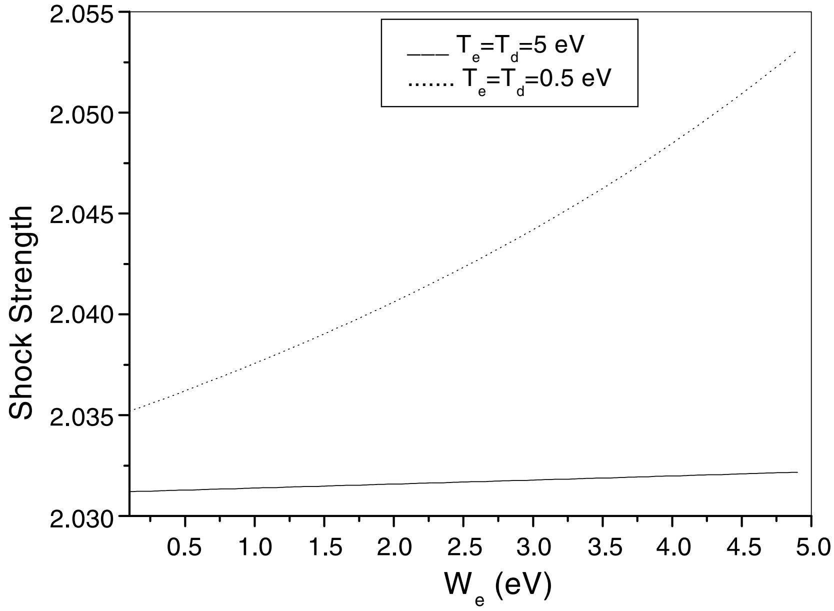

Figure 5. Variation in shock strength with the work function, W_e, for $T_d = T_e = 5$ eV (solid curve) and $T_d = T_e = 0.5$ eV (dotted curve).

increases as the work function, W_e, increases. This figure also shows that as the plasma temperature increases, the shock strength decreases. Thus the DA shock speed increases as the work function increases and the plasma temperature decreases.

6. Summary

We summarize the results as follows.

(1) In this paper the nonlinear characteristics of a DA wave in two-component electron–dust (positively charged due to thermionic emission) dusty plasma has been studied incorporating nonadiabatic dust-charge variations (i.e. the charge delay has been included) under the assumption that the electron–dust collision frequency is greater than the dust-plasma frequency (by equation (25) and table 1). It is seen that the nonlinear DA wave is governed by a mKdVB equation (34). The dissipation arising due to the delay in the charging leads to the formation of a shock wave in two-component dusty plasma.

(2) Numerical investigation shows that the nonlinear DA wave described by the modified mKdVB equation possesses both compressional oscillatory (dispersion dominant) (figure 3) and monotonic (dissipation dominant) (figure 4) shock-wave solutions. The dissipation causes the shock wave to arise due to the electron–dust collision and the delay in the charging of the dust grains. The shock strength increases as the work function, W_e, increases and decreases as the plasma temperature decreases(figure 5).

(3) The thermionic emission modifies the nonlinear propagation characteristics of a DA wave in a two-component electron–dust plasma (figures 1 and 2).

(4) The dissipation arising due to a delay in the charging has larger effect on the DA wave than the electron–dust collision-induced dissipation under the assumption $\nu_{ed} = \nu'_{ed}/\omega_{pd} \gg 1$ in two-component electron–dust dusty plasma (equation (42)).

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