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Dust acoustic shock wave in electronegative dusty plasma: Roles of weak magnetic field

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The effects of nonsteady dust charge variations and weak magnetic field on small but finite amplitude nonlinear dust acoustic wave in electronegative dusty plasma are investigated. The dynamics of the nonlinear wave are governed by a Korteweg–de Vries Burger equation that possesses dispersive shock wave. The weak magnetic field is responsible for the dispersive term, whereas nonsteady dust charge variation is responsible for dissipative term, i.e., the Burger term. The coefficient of dissipative term depends only on the obliqueness of the magnetic field. It is found that for parallel propagation the dynamics of the nonlinear wave are governed by the Burger equation that possesses monotonic shock wave. The relevances of the findings to cometary dusty plasma, e.g., Comet Halley are briefly discussed. © *2008 American Institute of Physics*.
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I. INTRODUCTION

Dusty plasmas with negative ions, i.e., electronegative dusty plasmas are quite obvious in astrophysical cometary plasma[1–5] as well as in modern technological applications.[6–8] The linear collective phenomena,[9] the spatial profiles of number densities of electrons, positive ions, electrons temperature, and dust charge are investigated in electronegative dusty plasma.[10]

The presence of the magnetic field makes a dusty plasma anisotropic and depending on the field strength as well as on dust grain radius, it changes the charging characteristics of dust grains. In the case of sufficiently large electron (ion) gyroradius compared to dust grain radius, the curvature effect of the trajectory of an electron (ion) impinging on the dust grains can be neglected so that the effect of magnetic field on charging processes is insignificant.[11] In the opposite case, the curvature effect cannot be neglected and in this case the dust charge decreases (when the electrons in the charging process start to be magnetized) and then, with a further increase of magnetic field, the dust charges increase (when the ions in the charging process start to be magnetized).[12]

On the other hand, the presence of negative ions due to their lower thermal velocity reduces the floating potential acquired by a dust grains and thereby reduces the magnitude of the charge of a dust grains.[13]

However, dust grains immersed in a plasma can exhibit self-consistent charge variations in response to the surrounding plasma oscillations and thus become a time dependent dynamical variable. The nonsteady dust charge variation due to collective perturbations modifies the dust floating potential which in turn self-consistently opposes the buildup of the plasma currents at the surface of the dust grains and consequently causes dissipative effects in both unmagnetized and magnetized dusty plasma. In the case of linear propagation, these nonsteady charge variations cause a collisionless non-Landau damping of wave modes,[8,14] whereas, in the nonlinear regime these nonsteady charge variations cause anomalous dissipation in the plasma, which leads to the formation of collisionless shock wave in both unmagnetized and magnetized dusty plasma.[15–17]

The effects of both high and low charging rates of dust grains compared to dust oscillation frequency on dust acoustic solitary waves are investigated in electronegative dusty plasma without magnetic field.[18] Also recent theoretical investigation reveals that the nonsteady dust charge variations produce a dissipative effect that generates a shock wave in electronegative unmagnetized dusty plasma.[19] However, the effects of both nonsteady dust charge variation and external magnetic field on the dust acoustic wave (DAW) (Ref. 20) in electronegative dusty plasma are not investigated.

Thus, in this paper the effects of both nonsteady dust charge variations and weak magnetic field on small but finite amplitude DAWs in electronegative dusty plasma are investigated. It is seen that the Korteweg–de Vries (KdV) Burger equation governs the dynamics of nonlinear DAW. The dispersive term arises only due to the presence of magnetic field and in absence of magnetic field or for parallel propagation the nonlinear wave is governed by the Burger equation.

The paper is organized in the following manner: Physical assumptions and formulation of the problem are given in Sec. II. The basic equations are given in Sec. III. The nonlinear evolution equations are derived in Sec. IV. The nature of the steady state solution of the KdV Burger equation is discussed in Sec. V. Numerical results related to Halley's comet are discussed in Sec. VI. Finally, the findings of the present investigation are summarized in Sec. VII.

II. PHYSICAL ASSUMPTIONS

The following basic assumptions are made that will help to formulate the physical problem and to find the explicit final results:

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(i) The electronegative magnetized dusty plasma is homogeneous, collisionless, and unbounded. The plasma constituents are electrons (e) with number density n_e, positive ions $(+)$ with number density n_+, negative ions $(-)$ with number density n_- and mobile negatively charged dust grains (d) with number density n_d. The dust grains are negatively charged and the charge varies continuously with time. The constant external weak magnetic field $\rightarrow B$ lies in the $x-z$ plane making an angle θ with the x axis and the wave propagation vector lies along the x axis.

(ii) At far upstream, there is a plasma flow V_0 in the direction of the wave propagation, i.e., along the x direction and the plasma is assumed to be in its equilibrium state defined by $\phi=0$, $n_e=n_{e0}$, $n_+=n_{+0}$, $n_-=n_{-0}$, $n_d=n_{d0}$, and $q_d=-z_d e$ so that the plasma is quasineutral

$$n_{e0} + z_d n_{d0} + n_{-0} = n_{+0}. \tag{1}$$

(iii) The wavelength $\lambda(=2\pi/k)\ll$, the gyroradius of the $s(=e,+,-)$th species particle $\rho_s=V_{ts}/\Omega_s$ (Ω_s, and V_{ts} are, respectively, the cyclotron frequency and thermal velocity of the sth species particle) so that only dust grains are magnetized but electrons, positive ions, and negative ions are unmagnetized. Thus for low frequency motion in magneto plasma satisfying $(\omega/k)\ll V_{ts}$ and $\lambda \ll \rho_s$, yields the Boltzmann distribution for electrons, positive ions, and negative ions. Hence the number densities of electrons and ions are given by[13,18,19]

$$n_e = n_{e0} z \exp(\Phi); \quad n_+ = n_{+0} \exp\left(-\frac{\Phi}{\sigma_+}\right);$$
$$n_- = n_{-0} \exp\left(\frac{\Phi}{\sigma_-}\right), \tag{2}$$

where $\sigma_+=T_+/T_e$, $\sigma_-=T_-/T_e$, T_e, T_+, and T_- are the temperatures of electrons, positive ions, and negative ions, respectively, and $\Phi(=e\phi/T_e)$ is the normalized plasma potential.

(iv) The dust grain radius $(r_0)\ll$ and the electron gyroradius (ρ_e) of the electron and ion (positive and negative) current can be approximated by the expressions of the unmagnetic case.[11,12] Also, dust flow velocity $V_0 \ll V_{ts}(s=e,+,-)$. Thus the normalized expressions for the electron current (I_e), negative ion current (I_-), and positive ion current (I_+) for spherical dust grain of radius r_0, are as follows:

$$I_e = -J_e \exp[\Phi + z(Q-1)];$$
$$I_- = -J_- \exp\left[\frac{\Phi + z(Q-1)}{\sigma_-}\right], \tag{3}$$
$$I_+ = J_+\left[1 - \frac{z(Q-1)}{\sigma_+}\right]\exp\left(-\frac{\Phi}{\sigma_+}\right),$$

where $J_s=\pi r_0^2 e n_{s0}\sqrt{8T_s/m_s}$, m_s, and T_s are the mass and temperature of the sth species and $z=z_d e^2/4\pi\varepsilon_0 r_0 T_e$, $4\pi\varepsilon_0 r_0$ is the capacitance of the spherical dust grain. Note that dust charge $q_d=q_d^{\mathrm{eq}}+\delta q_d=-z_d e+\delta q_d$, δq_d is the fluctuating dust charge so that $q_d/z_d e=-1+Q$; $Q=\delta q_d/z_d e$ (normalized in units of equilibrium dust charge).

(v) The ratio of dust oscillation frequency $\omega_{pd}(=\sqrt{z_d^2 e^2 n_{d0}/\varepsilon_0 m_d})$ to dust charging frequency (ν_{ch}) is finite, i.e., $\omega_{\mathrm{ch}}=\omega_{pd}/\nu_{\mathrm{ch}}$ is finite, i.e., $\omega_{\mathrm{ch}}\sim O(1)$, which is in contrast to the assumption ω_{ch} is small, but $\neq 0$, i.e., $\omega_{\mathrm{ch}}\sim O(\sqrt{\varepsilon})$, where ε is the usual expansion parameter.[17,19] On the other hand, because of the weak magnetic field, the ratio of dust cyclotron frequency $\Omega_d(=z_d e B_0/m_d; B_0=|B|)$ to ω_{pd}, i.e., $\omega_{cd}=\Omega_d/\omega_{pd}$ is small but $\neq 0$, i.e., $\omega_{cd}\sim O(\sqrt{\varepsilon})$, which is also in contrast to the assumption $\omega_{cd}\sim O(1)$.[17] Later, in Sec. VI, it has been shown that these assumptions are justified in cometary dusty plasma, e.g., Comet Halley.

III. BASIC EQUATIONS

On the basis of the assumption, stated in the previous section, the external magnetic field $B=B_0\cos\theta\hat{x}+B_0\sin\theta\hat{z}$ and the relative dust fluid velocity $V_d(x)=V_{dx}\hat{x}+V_{dy}\hat{y}+V_{dz}\hat{z}$. Thus the nonlinear dynamics of low phase velocity DAW are governed by the normalized equations,

$$\frac{\partial N_d}{\partial T} + \frac{\partial(N_d V_{dx})}{\partial X} = 0, \tag{4}$$

$$\frac{\partial V_{dx}}{\partial T} + V_{dx}\frac{\partial V_{dx}}{\partial X} = -(Q-1)\left(\frac{1}{\alpha_d}\frac{\partial\Phi}{\partial X} - \omega_{cd}V_{dy}\sin\theta\right) - \frac{\gamma_d\sigma_d}{\alpha_d}N_d^{\gamma_d-2}\frac{\partial N_d}{\partial X}, \tag{5}$$

$$\frac{\partial V_{dy}}{\partial T} + V_{dx}\frac{\partial V_{dy}}{\partial X} = \omega_{cd}(Q-1)(V_{dz}\cos\theta - V_{dx}\sin\theta), \tag{6}$$

$$\frac{\partial V_{dz}}{\partial T} + V_{dx}\frac{\partial V_{dz}}{\partial X} = -\omega_{cd}(Q-1)V_{dy}\cos\theta, \tag{7}$$

$$\gamma\frac{\partial^2\Phi}{\partial X^2} = \delta_+\exp(\Phi) + \delta_-\exp\left(\frac{\Phi}{\sigma_-}\right) - \exp\left(-\frac{\Phi}{\sigma_+}\right) - \Delta N_d(Q-1), \tag{8}$$

where $N_d=n_d/n_{d0}$, $\delta_d=T_d/z_d T_e$, $\Delta=1-\delta_+-\delta_-$, $\delta_+=n_{e0}/n_{+0}$, $\delta_-=n_{-0}/n_{+0}$, $\alpha_d=z_d n_{d0}/\gamma n_{+0}$, $\gamma=(\delta_++1/\sigma_++\delta_-/\sigma_-)$, T_d is the dust temperature and γ_d is the adiabatic index.

The normalized charge variable Q is determined by the charging equation $(dq_d/dt=\Sigma_{s=e,+,-}I_s)$, which in normalized form reads as

$$\omega_{\mathrm{ch}}\frac{dQ}{dT} = \frac{\sigma_+\beta_{\mathrm{ch}}}{(1+\sigma_++\gamma_2^-)}\left[\left(1-\frac{zQ}{z+\sigma_+}\right)\exp\left(-\frac{\Phi}{\sigma_+}\right) - A_+\exp(\Phi+zQ) - A_-\exp\left(\frac{\Phi+zQ}{\sigma_-}\right)\right], \tag{9}$$

where ν_{ch}, β_{ch}, A_+, and A_- are, respectively,[18,19]

$$\nu_{ch} = \frac{r_0}{\sqrt{2\pi}}\frac{\omega_{p+}^2}{V_{t+}}\frac{(z+\sigma_+)(1+\sigma_++\gamma_2^-)}{z\sigma_+\beta_{\mathrm{ch}}};$$

$$\beta_{\rm ch} = \frac{(z+\sigma_+)(1+\sigma_+ + \gamma_2^-)}{z\sigma_+[1+z+\sigma_+ + \gamma_1^-]};$$

$$\gamma_1^- = \frac{(z+\sigma_+)(1-\sigma_-)}{\sigma_-}A_-; \quad \gamma_2^- = \frac{\sigma_+(1-\sigma_-)}{\sigma_-}A_-, \tag{10}$$

$$A_+ = \frac{\delta_+ \exp(-z)}{(z+\sigma_+)}\sqrt{\frac{\sigma_+ m_+}{m_e}};$$

$$A_- = \frac{\delta_- \exp(-z/\sigma_-)}{(z+\sigma_+)}\sqrt{\frac{\sigma_+\sigma_- m_+}{m_-}}.$$

Note that equilibrium current balance equation $I_{e0}+I_{+0}+I_{-0}=0$ yields $A_+ + A_- = 1$.

In the above, the time (T) and space (X) scales are, respectively, normalized in the units of ω_{pd} and the plasma Debye length $\lambda_D(=\sqrt{\lambda_{De}^{-2}+\lambda_{D+}^{-2}+\lambda_{D-}^{-2}})$, $\lambda_{Ds}(=\sqrt{\varepsilon_0 T_s/n_{s0}e^2})$ is the sth species particle Debye length. The dust fluid velocity V_d is normalized in units of dust acoustic speed $c_d(=\sqrt{z_d T_e \alpha_d/m_d})$.

IV. NONLINEAR ANALYSIS: KdV BURGER EQUATION

To study the small but finite amplitude nonlinear DAW using reductive perturbation technique, the independent variables are stretched as

$$\xi = \varepsilon(X - V_{\rm ph}T); \quad \tau = \varepsilon^2 T, \tag{11}$$

where $V_{\rm ph}$ is the normalized phase velocity of the linear DAW and ε is a small parameter characterizing the strength of the nonlinearity. The dynamical variables are expanded in powers of ε as follows:

$$\left.\begin{array}{l} f = f_0 + \varepsilon f^{(1)} + \varepsilon^2 f^{(2)} + \cdots \\ V_{dy} = \varepsilon^{3/2}V_{dy}^{(1)} + \varepsilon^{5/2}V_{dy}^{(2)} + \cdots \end{array}\right]; \quad f = N_d, V_{dx}, V_{dz}, \Phi, Q, \tag{12}$$

where $f^{(0)}=1$, V_0 for N_d, V_{dx} and $=0$ for V_{dz}, Φ and Q, respectively.

Also because of the assumption (v) in Sec. II, for consistent perturbation expansion, it is assumed that

$$\omega_{cd} = \frac{\Omega_d}{\omega_{pd}} = O(\sqrt{\varepsilon}). \tag{13}$$

Introducing Eqs. (11)–(13) into Eqs. (4)–(9) and equating to lowest powers of ε the following relations are obtained:

$$\begin{aligned} V_{dz}^{(1)} &= \tan\theta V_{dx}^{(1)}, \\ Q^{(1)} &= \left(1 - \Lambda^2\sec^2\theta + \frac{\gamma_d\sigma_d}{\alpha_d}\right)N_d^{(1)}, \\ V_{dy}^{(1)} &= -\frac{\Lambda^2 \sin\theta \sec^2\theta}{\omega_{cd}}\frac{\partial N_d^{(1)}}{\partial\xi}, \\ Q^{(1)} &= \alpha_d\beta_{\rm ch}\left(\Lambda^2\sec^2\theta - \frac{\gamma_d\sigma_d}{\alpha_d}\right)N_d^{(1)}, \end{aligned} \tag{14}$$

where $\Lambda = V_{\rm ph} - V_0$. This set of Eqs. (14) self-consistently yields the following linear DAW phase velocity:

$$V_{\rm ph} = V_0 - \cos\theta\left(\frac{\gamma_d\sigma_d}{\alpha_d} + \frac{1}{1+\alpha_d\beta_{\rm ch}}\right)^{1/2} \Rightarrow \Lambda = \cos\theta\left(\frac{\gamma_d\sigma_d}{\alpha_d} + \frac{1}{1+\alpha_d\beta_{\rm ch}}\right)^{1/2}. \tag{15}$$

Then equating the terms of next higher powers of ε the following relations are obtained:

$$\frac{\partial N_d^{(1)}}{\partial\tau} + \frac{\partial(N_d^{(1)}V_{dx}^{(1)})}{\partial\xi} = -\frac{\partial(\Lambda N_d^{(2)} + V_{dx}^{(2)})}{\partial\xi}, \tag{16}$$

$$\frac{\partial V_{dx}^{(1)}}{\partial\tau} + V_{dx}^{(1)}\frac{\partial V_{dx}^{(1)}}{\partial\xi} + \frac{1}{\alpha_d}\left(Q^{(1)}\frac{\partial\Phi^{(1)}}{\partial\xi} + \gamma_d(\gamma_d-2)\sigma_d N_d^{(1)}\frac{\partial N_d^{(1)}}{\partial\xi}\right) = \frac{1}{\alpha_d}\frac{\partial(\Phi^{(2)} - \gamma_d\sigma_d N_d^{(2)})}{\partial\xi} - \Lambda\frac{\partial V_{dx}^{(2)}}{\partial\xi} + \omega_{cd}(Q^{(1)}V_{dx}^{(1)} - V_{dy}^{(2)})\sin\theta, \tag{17}$$

$$\Lambda\frac{\partial V_{dy}^{(1)}}{\partial\xi} + \omega_{cd}Q^{(1)}(V_{dx}^{(1)}\sin\theta - V_{dz}^{(1)}\cos\theta) = \omega_{cd}(V_{dx}^{(2)}\sin\theta - V_{dz}^{(2)}\cos\theta), \tag{18}$$

$$\frac{\partial V_{dz}^{(1)}}{\partial\tau} + V_{dx}^{(1)}\frac{\partial V_{dz}^{(1)}}{\partial\xi} = -\Lambda\frac{\partial V_{dz}^{(2)}}{\partial\xi} - \omega_{cd}(Q^{(1)}V_{dy}^{(1)} - V_{dy}^{(2)})\cos\theta, \tag{19}$$

$$\Phi^{(2)} + \frac{1}{2\gamma}\left(\delta_+ + \frac{\delta_-}{\sigma_-^2} - \frac{1}{\sigma_+^2}\right){\Phi^{(1)}}^2 - \alpha_d(Q^{(2)} - N_d^{(2)} + N_d^{(1)}Q^{(1)}) = 0, \tag{20}$$

$$Q^{(2)} + \beta_{\rm ch}\Phi^{(2)} + \Lambda\omega_{\rm ch}\frac{\partial Q^{(1)}}{\partial\xi} + \frac{\sigma_+\beta_{\rm ch}}{2(1+\sigma_+ + \gamma_2^-)} \times \left[\begin{array}{l} z^2(1+B_-){Q^{(1)}}^2 - \dfrac{(1-\sigma_+^2(1+B_-))}{\sigma_+^2}{\Phi^{(1)}}^2 \\ -\dfrac{z(1-\sigma_+(z+\sigma_+)(1+B_-))}{\sigma_+(z+\sigma_+)}Q^{(1)}\Phi^{(1)} \end{array}\right] = 0, \tag{21}$$

where $B_- = (1-\sigma_-^2)A_-/\sigma_-^2$.

Finally, eliminating all the second order quantities from Eqs. (16)–(21) and using relations of Eq. (14), the following KdV Burger equation is obtained:

$$\frac{\partial N_d^{(1)}}{\partial \tau} - \alpha N_d^{(1)} \frac{\partial N_d^{(1)}}{\partial \xi} - \beta \frac{\partial^3 N_d^{(1)}}{\partial \xi^3} = \mu_{\rm ch} \frac{\partial^2 N_d^{(1)}}{\partial \xi^2}. \tag{22}$$

The coefficients of nonlinearity α, dispersion β, and Burger term $\mu_{\rm ch}$ are as follows:

$$\alpha = \frac{\cos\theta}{(1+\alpha_d\beta_{\rm ch})^{1/2}}\left[\frac{\gamma_d(\gamma_d+1)\sigma_d}{\alpha_d} + \frac{1}{(1+\alpha_d\beta_{\rm ch})^2}\left(\frac{3\gamma+\delta_+ + \dfrac{\delta_-}{\sigma_-^2} - \dfrac{1}{\sigma_+^2}}{\gamma} + \frac{\sigma_+\alpha_d^2\beta_{\rm ch}C}{(1+\alpha_d\beta_{\rm ch})(1+\sigma_+ + \gamma_2^-)}\right)\right], \tag{23}$$

where

$$C = (1+B_-)(z\beta_{\rm ch}-1)^2 + \frac{2z\beta_{\rm ch}}{\sigma_+(z+\sigma_+)} - \frac{1}{\sigma_+^2}, \tag{24}$$

$$\beta = \frac{\sin^2\theta\cos\theta}{2\omega_{cd}^2}\left(\frac{\gamma_d\sigma_d}{\alpha_d} + \frac{1}{1+\alpha_d\beta_{\rm ch}}\right)^{3/2}, \tag{25}$$

$$\mu_{\rm ch} = \frac{\alpha_d\beta_{\rm ch}\omega_{\rm ch}\cos^2\theta}{2(1+\alpha_d\beta_{\rm ch})^2} = \frac{\alpha_d\beta_{\rm ch}\cos^2\theta}{2(1+\alpha_d\beta_{\rm ch})^2}\left(\frac{\omega_{pd}}{\nu_{\rm ch}}\right). \tag{26}$$

The expression for α [Eq. (23)] shows that the coefficient of nonlinear term is proportional to the obliqueness of the magnetic field.

The expression for β [Eq. (25)] shows that the dispersive term is directly proportional to the obliqueness of the magnetic field and inversely proportional to the magnitude of the magnetic field. Thus the dissipative term arises only due to the presence of external magnetic field. The dispersive term vanishes (β=0) for parallel propagation (θ=0) and the nonlinear wave is governed by the well-known Burger equation

$$\frac{\partial N_d^{(1)}}{\partial \tau} - \alpha N_d^{(1)} \frac{\partial N_d^{(1)}}{\partial \xi} = \mu_{\rm ch} \frac{\partial^2 N_d^{(1)}}{\partial \xi^2}. \tag{27}$$

The expression for $\mu_{\rm ch}$ [Eq. (26)] shows that the Burger term is proportional to $\omega_{\rm ch}(=\omega_{pd}/\nu_{\rm ch})$ and arises due to nonsteady dust charge variations. Note that $\Delta(=1-\delta_+-\delta_- = z_d n_{d0}/n_{+0})=0 \Rightarrow \mu_{\rm ch}=0$, this is expected as $\Delta=0$ implies the absence of charged dust grains and consequently no charge fluctuation. Hence the Burger term, which is responsible for the generation of shock wave, originates from the nonsteady dust charge variation under the assumption that $\omega_{\rm ch}$ is finite. Also note that the coefficient of the Burger term is proportional to $\cos^2\theta$ (other plasma parameter remains constant) thus the dissipative term depends only on the obliqueness of the magnetic field, not on the magnitude of the magnetic field.

It is interesting to note that for *adiabatic* dust charge variation[17(a),18,21] $\omega_{\rm ch}\approx 0$ [e.g., for typical laboratory dusty plasma, dust oscillation frequency $\omega_{pd}\approx 10^2$ s^{-1} and dust charging frequency $\nu_{\rm ch}\approx 10^8$ s^{-1},[20(b)] implies $\omega_{\rm ch}(\approx 10^{-6})\approx 0$] and the charging equation (9) is approximated by $I_e+I_++I_-\approx 0$ so that the charge on the dust grains instantaneously reaches its equilibrium value and hence does not play any dissipative role. Thus in this case $\mu_{\rm ch}\approx 0$, i.e., the Burger term vanishes in Eq. (22) and thereby the nonlinear dynamics of DAW is governed by the well-known Korteweg–de Vries equation that exhibits only the soliton solution instead of the shock wave.

V. STEADY STATE SOLUTION: GENERATION OF SHOCK WAVE

It is well known that the KdV Burger equation describes the shock wave profile. The criteria for the formation of the shock wave is that the coefficient of the Burger term $\mu_{\rm ch}$ which arises due to nonsteady dust charge variations should be positive (here $\mu_{\rm ch}>0$), otherwise it would not be possible to get a stable solution of the Burger equation. A particular solution of the above KdV Burger equation [Eq. (22)] is of the following form:[22]

$$N_d^{(1)}(\xi,\tau) = -\frac{3\mu_{\rm ch}^2}{25\alpha\beta}\left[1+\tanh\frac{\mu_{\rm ch}}{10\beta}\left(\frac{6\mu_{\rm ch}^2}{25\beta}\tau - \xi\right)\right]^2. \tag{28}$$

On the other hand, for parallel propagation (θ=0), one can easily find the following analytic solution of the Burger equation (27):

$$N_d^{(1)}(\xi,\tau) = N\left[1+\tanh\left(\frac{\eta}{L_w}\right)\right] \tag{29}$$

subject to the boundary conditions $N_d^{(1)}(\xi,\tau), d_\eta N_d^{(1)}(\xi,\tau) \to 0$ as $\eta\to-\infty$, which exhibits monotonic shock solution. In the above N is the initial shock amplitude, L_w is the shock width, V_f is the shock velocity, and

$$\eta = V_f\tau + \xi = \frac{\varepsilon[c_d(\varepsilon V_f - V_{\rm ph})t + x]}{\lambda_D} \tag{30}$$

is the wave frame.

However, for steady state numerical solution transforming to the wave frame η the KdV Burger equation [Eq. (22)] with $\psi = N_d^{(1)}$ reduces to

$$\frac{d^2\psi}{d\eta^2} = \left(\frac{V_f}{\beta}\right)\psi - \left(\frac{\alpha}{2\beta}\right)\psi^2 - \left(\frac{\mu_{\rm ch}}{\beta}\right)\frac{d\psi}{d\eta}. \tag{31}$$

This Eq. (31) has a well-known mechanical analogy; it describes a damped anharmonic oscillator where ψ plays a role of the generalized coordinate and η plays the role of time. In the $(\psi, d\psi/d\eta)$ plane, Eq. (31) has two singular points (0,0) and $(2V_f/\alpha, 0)$ the former (with $\psi=\psi_1=0$) corresponds to the upstream state and latter (with $\psi=\psi_2=2V_f/\alpha$) corresponds to the equilibrium downstream state. The singular point (0,0) is always a saddle point, whereas the singular point $(2V_f/\alpha,0)$ is a nodal point if $(\mu_{\rm ch}^2>4V_f\beta)$ and a focal point if $(\mu_{\rm ch}^2<4V_f\beta)$. In the absence of dissipations (i.e., $\mu_{\rm ch}=0$), the downstream singular point becomes a center and no integral curve can enter this point so that there is no way of passing from $\psi=\psi_1$ to $\psi=\psi_2$ state and hence there is no shock wave. In this case the solution that leads from the upstream singular point is the one that returns to it and with the boundary conditions $\psi(\eta=-\infty)=\psi_\eta(\eta=-\infty)=0$ is a soliton solution for $V_0=0$. In the presence of a flow velocity $V_0\neq 0$, the boundary conditions give rise to periodic waves other than solitary wave solutions. We define

$$V_a = V_0 - (V_{\rm ph} - \varepsilon V_f) = \Lambda + \varepsilon V_f, \qquad (32)$$

where V_a is the DAW velocity ahead of the shock front relative to a moving wavefront. The Mach number M is defined as

$$M = \frac{V_a}{\Lambda} = 1 + \varepsilon\frac{V_f}{\Lambda} \Rightarrow V_f = \frac{\Lambda(M-1)}{\varepsilon}. \qquad (33)$$

A stable focus corresponds to oscillatory shock, whereas the stable node corresponds to monotonic shock and hence the condition for oscillatory shock (monotonic shock) is

$$\frac{\omega_{pd}}{\nu_{\rm ch}} < (>) \frac{8\tan\theta(1+\alpha_d\beta_{\rm ch})^2[(\gamma_d\sigma_d/\alpha_d) + 1/(1+\alpha_d\beta_{\rm ch})]\sqrt{M-1}}{\alpha_d\beta_{\rm ch}\omega_{cd}}. \qquad (34)$$

VI. NUMERICAL ANALYSIS AND DISCUSSIONS

Horanyi and Mendis[23,24] calculated the trajectories of micron and submicron sized dust grains that are expected to be released from the cometary nucleus. It was shown that the electromagnetic forces associated with the motion of the grains (which are electrically charged by virtue of plasma environment) through the magnetized plasma played a crucial role in their dynamics. Different spacecrafts (*Vega 1*, *Vega 2*, and *Giotto*) observations on comets reveal that Comet Halley contains electrons, ice dust grains, and different (positive and negative) ions such as (H^+,H^-), (O^+,O^-), (Si^+,Si^-), (OH^+,OH^-), and etc.[1] For the numerical analysis of the present findings, (H^+,H^-) and pure ice dust grains are to be considered. The approximated physical parameters of Comet Halley ($\sim 10^4$ km from the nucleus) are[1–5] $n_{+0}\sim 2\times 10^8$ m^{-3}, $n_{d0}\sim 1$ m^{-3}, $T_e=T_+\sim 100$ eV, $m_+=m_-\sim 1.6726\times 10^{-27}$ kg, $B_0\sim 7.5\times 10^{-3}$ Tesla, the dust mass density (for ice dust grains) $\rho_d\sim 9\times 10^2$ kg m^{-3}, $r_0\sim 5$ μm, and $\gamma_d\sim 5/3$.

For numerical computations the following relation that is derived from equilibrium current balance equation $I_{e0}+I_{+0}+I_{-0}=0$ and charge neutrality condition (1) is used to find the negative ion-positive ion density ratio $\delta_-(=n_{-0}/n_{+0})$ as a function of nondimensional dusty plasma parameter z,

$$\delta_- = \frac{(1-\Delta)e^{-z} - (z+\sigma_+)\sqrt{\dfrac{m_e}{\sigma_+ m_+}}}{e^{-z} - e^{-z/\sigma_-}\sqrt{\dfrac{\sigma_- m_e}{m_-}}}, \quad \Delta = zP, \qquad (35)$$

where $P=4\pi r_0 n_{d0}L^2$, $L(=\sqrt{\varepsilon_0 T_e/n_{+0}e^2})$ is a characteristic length[19] and for the present model, $P\approx 1.74\times 10^{-3}$. The above physical parameters with $\sigma_-=1$ also estimate $V_{t+}(V_{t-})\sim 9.8\times 10^4$ ms^{-1}, $\Omega_+(\Omega_-)\sim 7.2\times 10^5$ s^{-1}, $V_{te}\sim 4.2\times 10^6$ ms^{-1}, $\Omega_e\sim 1.3\times 10^9$ s^{-1}, $\rho_+(\rho_-)\sim 1.4\times 10^{-1}$ m, and $\rho_e\sim 3.2\times 10^{-3}$ m. The values of the ratios $r_0/\rho_e\sim 1.6\times 10^{-3}\ll 1$and $r_0/\rho_+\sim 3.6\times 10^{-5}\ll 1$ justifies the assumption (iv). Also note that a spacecraft flying through the comet and dust cloud will impact the dust at the spacecraft-comet relative speeds. The relative speeds of the 1986 flyby of *Vega 1*, *Vega 2*, and *Giotto* spacecraft at Comet Halley are, respectively, 7.9×10^4 ms^{-1}, 7.7×10^4 ms^{-1}, and 6.8×10^4 ms^{-1},[25] which are less than the thermal velocities of electron and ions (positive and negative) and thereby the assumption $V_0\ll V_{ts}$ on the basis of which the expressions for electron and ion currents (3) are approximated is also justified. It is also observed from Eq. (35), $(\sigma_-=1)$ that $z\sim 2.49$, $1.37\Rightarrow\delta_-\sim 0.011, 0.8$ implying $\omega_{\rm ch}(=\omega_{pd}/\nu_{\rm ch})\sim 2.14$, 1.56 which is finite and $\omega_{cd}(=\Omega_d/\omega_{pd})\sim 3.3\times 10^{-2}\ll 1$ which justifies the assumption (v) and the scaling Eq. (13) on the basis of which the nonlinear equations are derived.

On the basis of the plasma parameters relevant to Comet Halley,[1–5] the variations of dissipation, i.e., the coefficient of the Burger term $\mu_{\rm ch}$ with negative ion-positive ion number density ratio $\delta_-(=n_{-0}/n_{+0})$ are drawn in Fig. 1 for different negative ion-electron temperature ratio $\sigma_-(=T_-/T_e)$ and incident angle θ of the magnetic field. This figure shows that $\mu_{\rm ch}$ decreases with the increase of δ_-. Different curves of this figure also show that $\mu_{\rm ch}$ increases with the increase of σ_- [solid curves: (a), (b)], but decreases with the increase of θ [dotted curves: (a), (b)].

Applying the same plasma parameters of Comet Halley,[1–5] the numerical integration of Eq. (31) using a fifth order Runge-Kutta scheme with boundary conditions $\psi=\psi_\eta=0$ at $\eta=-\infty$ is carried out starting from the saddle point at the upstream and is found to be stable. The solutions are depicted in Fig. 2. This figure shows that the shock wave is more dispersive in nature for a larger value of magnetic field than that of for lower value. On the other hand, for parallel propagation the shock is monotonic in nature, which is shown in Fig. 3.

Note that as the shock moves forward i.e. from upstream to downstream side the pressure as well as temperature increase. For the present investigation, the values of the

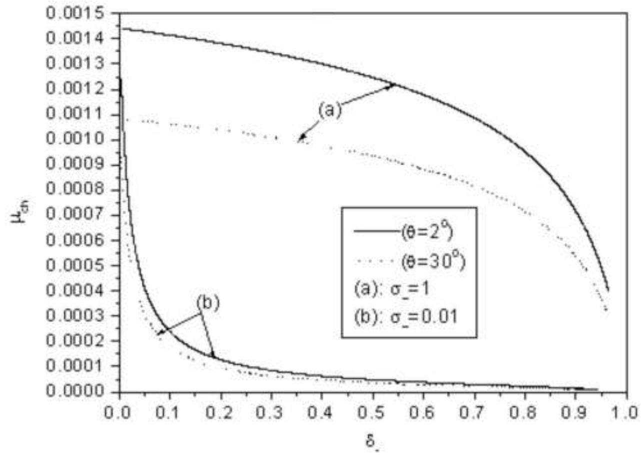

FIG. 1. Coefficient of the Burger term μ_{ch} as given by Eq. (26) plotted against δ_{-}.

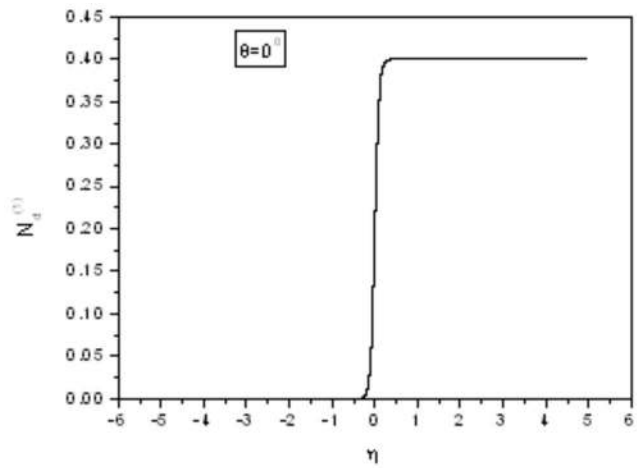

FIG. 3. Monotonic shock amplitude obtained as the solution of Eq. (29) plotted against η for $\theta=0^{\circ}$, $\delta_{-}=0.011$, $\sigma_{-}=1$, and $N=0.2$.

normalized pressure (P_d) and temperature (T_d) become $P_d \sim [1+N_d^{(1)}]^{\gamma_d} \sim 2.1$ and $T_d \sim [1+N_d^{(1)}]^{\gamma_d-1} \sim 1.33$. The shock is compressive $[N_d^{(1)}>0]$ in nature and as the potential is negative $\{\Phi^{(1)}=-[\alpha_d/(1+\alpha_d\beta_{ch})]N_d^{(1)};N_d^{(1)}>0\}$, the negatively charged dust grain is energized by passing shock wave to

$$E=-z_dT_e\Phi=-z_dT_e[\varepsilon\Phi^{(1)}]_{max}=\frac{z_dT_e\alpha_d}{(1+\alpha_d\beta_{ch})}[\varepsilon N_d^{(1)}]_{max}=\frac{2z_dT_e\alpha_d\Lambda(M-1)}{\alpha(1+\alpha_d\beta_{ch})}. \tag{36}$$

VII. SUMMARY

The results of the present investigation can be summarized as follows:

(a) The nonsteady dust charge variation produces anomalous dissipation that leads to the shock wave generation in electronegative magnetized dusty plasma. The magnetic field modifies the nonlinearity, dispersive as well as dissipative properties of the nonlinear wave.

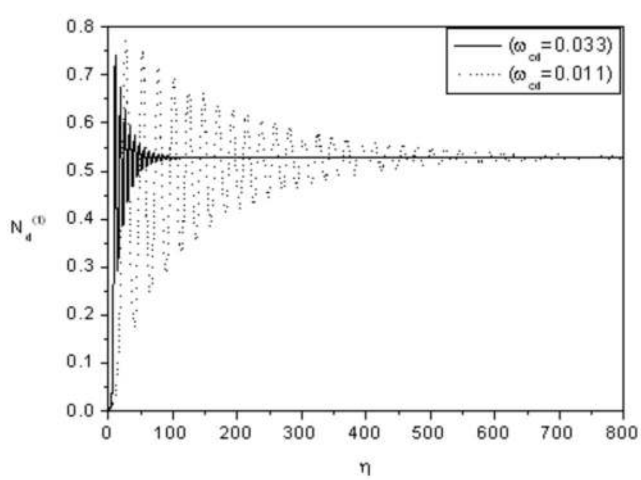

FIG. 2. Oscillatory shock amplitudes obtained as the solution of Eq. (31) plotted against η for $\theta=2^{\circ}$, $\delta_{-}=0.011$, $\sigma_{-}=1$, and $M=2$.

(b) Nonsteady dust charge variation induced dissipation increases with the increase of the negative ion-electron temperature ratio, but decreases with the increase of negative ion-positive ion density ratio as well as with the obliqueness of the external magnetic field [Fig. 1, curves (a) and (b)].

(c) The magnitude of the magnetic field modifies the dispersion properties of the shock wave (Fig. 2, solid and dotted curves). In the case of parallel propagation DAW possesses only monotonic shock (Fig. 3).

(d) The observed shock wave is a compressional one providing sufficient dust density enhancement, which has a significant importance in astrophysical context. The generation of the shock wave is quite common feature in cometary plasma. When a dust grains impacts a solid surface, such as a spacecraft, compressional shock waves are created in both the dust grain and the spacecraft surface. The foreshock and bow shock wave is observed at Halley's comet.[5] Also the dust grains emitted from a comet get electrically charged in its plasma environment. Thus the results of the present investigation are useful for understanding the physics of formation of shock wave at Halley's comet.

(e) It is clear from Eqs. (23), (25), and (26), that for a large angle ($\theta \to 90°$), coefficient of nonlinearity $\alpha \to 0$, the coefficients of dispersion $\beta \to 0$, and Burger term $\mu_{ch} \to 0$. Thus for large angle this model is no longer valid and in this case, the stretching coordinates will be different. Thus the present model is valid for $0 \le \theta < 90°$, $r_0 \ll \rho_e$ and also for weak magnetic field so that $\omega_{cd}=O(\sqrt{\varepsilon})$.

(f) Finally, at first step only 1D nonlinear DAW in electronegative magnetized dusty plasma in the presence of weak magnetic field and nonsteady dust charge variation is investigated. The 2D and 3D nonlinear structures (e.g., soliton, shock, etc.) of the DAW in such plasma in the presence of strong magnetic field is our future area of investigation.

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