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Dissipative solitons in pair-ion plasmas

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The effects of ion-neutral collisions on the dynamics of the nonlinear ion acoustic wave in pair-ion plasma are investigated. The standard perturbative approach leads to a Korteweg-de Vries equation with a linear damping term for the dynamics of the finite amplitude wave. The ion-neutral collision induced dissipation is responsible for the linear damping. The analytical solution and numerical simulation reveal that the nonlinear wave propagates in the form of a weakly dissipative compressive solitons. Furthermore, the width of the soliton is proportional to the amplitude of the wave for fixed soliton velocity. Results are discussed in the context of the fullerene pair-ion plasma experiment. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4862033]

I. INTRODUCTION

Ordinary plasmas consisting of electrons and ions are space-time asymmetric due to the wide mass differences between electrons and ions. These mass differences give rise to different time scales, which can be used to study the long and short wavelength collective phenomena in electron-ion plasma. On the contrary, such scales collapse in pair plasmas consisting of only positive and negative charged particles of equal masses. This makes the pair plasma space-time symmetric. The symmetric pair plasma consisting of electrons and positrons has been produced experimentally.[1–5] Different collective phenomena (linear and nonlinear) in such electron-positron pair plasmas have also been investigated theoretically.[6–13] Pair plasmas consisting of electrons and positrons are found to exist in inertial confinement fusion reactor using ultraintense lasers[4] and astrophysical environment (pulsar magnetospheres, early universe, active galaxy nuclei, etc.).[14,15]

Recently, Oohara and Hatakeyama[16] have developed a novel method for generating a symmetric pair plasma consisting of only positive and negative ions with an equal mass by using positive and negative fullerene ions (C_{60}^{+}, C_{60}^{-}) as the ion source. Three types of electrostatic collective modes parallel to external magnetic field are observed in pair-ion plasma experiment.[17–19] These three collective modes are the low-frequency ion acoustic wave (IAW), high-frequency ion plasma wave (IPW), and the intermediate-frequency wave (IFW). Later, the IFW is identified as the incompressible (surface) ion wave in pair-ion plasma.[20] Such pair-ion plasma is expected to be useful for nanotechnology as well as for the synthesis of dimers directly from carbon allotropes.

Some theoretical works have already been done, which concern the elementary properties, linear and nonlinear collective modes in pair-ion plasma.[21–28] All these investigations are in collisionless limit. However, in fullerene pair-ion plasma, the negative C_{60}^{-} ions collide with the positive C_{60}^{+} ions and/or neutral fullerenes yielding the collisional product as dimer C_{121}, which are useful in nanotechnology and fusion plasmas.[29,30] Thus, the collisions also play an important role in the collective processes in such pair-ion plasma.

Our aim, here, is to investigate the weakly nonlinear localized structures of IAWs in one-spatial dimension in pair-ion plasma in presence of ion-neutral collisions. We employ the well-known reductive perturbation theory (RPT) to study the nonlinear dynamics of the wave. The plasma is consisting of only two species of ions (positive and negative) and, therefore, we consider the two-fluid collisional plasma model. Using the RPT, we derive a Korteweg de Vries (KdV) equation with a linear damping term that arises due to the ion-neutral collision. This damped KdV equation is analyzed by both analytically and numerically. Both positive and negative potential ion acoustic solitons, whose width is proportional to the amplitude for fixed soliton velocity, are found to exist. These properties distinguish the ion acoustic soliton in pair-ion plasma from those found in a conventional electron-ion plasma. However, in both the cases, the solitons are compressive in nature. The weak ion-neutral collisional effect makes these solitons weakly dissipative.

This paper is organized as follows: In Sec. II, we demonstrate the complete set of two-fluid equations to describe the model. We derive the damped KdV equation using RPT in Sec. III. The effect of ion-neutral collision on ion acoustic solitary wave and an approximate analytical solution of the damped KdV equation are derived in Sec. IV. The results of numerical simulation are provided in Sec. V. Finally, we conclude our results in Sec. VI.

II. PHYSICAL ASSUMPTIONS AND BASIC EQUATIONS

We consider a homogeneous unbounded pair-ion plasma (without electrons) consisting of fullerene positive and negative ions (C_{60}^{+} and C_{60}^{-}) in absence of magnetic field. The masses of these ions (positive and negative) are equal because they are generated by the same source (fullerene ion

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source), but their temperatures are slightly different [range of $(0.3-0.5)$eV] due to the different charging processes of both the positive and negative fullerene ions.[16–19] Therefore, according to the experimental observation, we consider the pair-ion plasma with equal mass $m_+ = m_- = m$(say) (where $m_\pm$ is the positive (negative) ion mass), but slightly different in temperature $T_+ \neq T_-$ (where $T_\pm$ is the positive (negative) ion temperature). Moreover, these ions collide with neutral fullerenes that produce the dimer C_{121}.[29,30] Therefore, we also consider the effects of the ion-neutral collisions on nonlinear collective phenomena in pair-ion plasma.

The plasma is overall quasi-neutral and we assume that the ions are singly charged so that $n_{+0} = n_{-0} = n_0$(say), where $n_{\pm 0}$ is the equilibrium number density of positive (negative) ions. Before going to the details of the basic two-fluid equations, we introduce a new temperature variable $T = (T_+ + T_-)/2$. The pressure of both the ions is assumed to be adiabatic, i.e., $p_\pm = Cn_\pm^\gamma$, where C is a constant, γ is the adiabatic index, and $\gamma = 3$ for one-dimension. Then, the characteristic length scale and the frequency of this pair-ion plasma are, respectively, the plasma Debye length $\lambda_D = \left(\epsilon_0 \gamma T/n_0 e^2\right)^{1/2}$ and plasma frequency $\omega_p = \left(n_0 e^2/\epsilon_0 m\right)^{1/2}$, where ϵ_0 is the permittivity of the free space. This defines the acoustic speed $C_s (= \omega_p \lambda_D) = (\gamma T/m)^{1/2}$. The pair-ion moves with the fluid velocity $u_\pm$ and they collide with the neutral atom with frequency $\nu_n^\pm$. Here, we focus only on the nonlinear collective behavior of IAWs in pair-ion plasma in presence of ion-neutral collisions. To incorporate the effects of collisions, we assume that the ion-neutral collision frequency is much smaller than the plasma frequency, i.e., $\nu_n^\pm \ll \omega_p$.

The one-dimensional normalized collisional two-fluid equations for a pair-ion plasma in absence of magnetic field are as follows:

$$\frac{\partial n_\pm}{\partial t} + \frac{\partial}{\partial x}(n_\pm u_\pm) = 0, \tag{1}$$

$$\frac{\partial u_\pm}{\partial t} + u_\pm \frac{\partial u_\pm}{\partial x} = \mp \frac{\partial \phi}{\partial x} - \sigma_\pm n_\pm \frac{\partial n_\pm}{\partial x} - \tilde{\nu}_n^\pm u_\pm, \tag{2}$$

and

$$\frac{\partial^2 \phi}{\partial x^2} = n_- - n_+. \tag{3}$$

In writing the above equations, the time and space scales are normalized in units of ω_p^{-1} and λ_D. The fluid velocity and electrostatic potential are normalized in units of C_l and e/T. The number density and collisional frequency are normalized, respectively, in units of n_0 and plasma frequency ω_p. The other physical parameter $\sigma_\pm = T_\pm/T$.

III. NONLINEAR WAVE EQUATIONS

To study the nonlinear propagation characteristic of IAWs in pair-ion plasmas, we employ the reductive perturbation technique and introduce the following stretched coordinates:

$$\xi = \epsilon^{\frac{1}{2}}(x - Mt), \quad \tau = \epsilon^{\frac{3}{2}} t, \tag{4}$$

where M determines the (normalized) phase velocity of the linear wave and ϵ characterizes the strength of the nonlinearity. The physical variables $n_\pm, u_\pm$ and ϕ are expanded in the power series of ϵ as

$$\begin{pmatrix} n_\pm \\ u_\pm \\ \phi \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \epsilon \begin{pmatrix} n_\pm^{(1)} \\ u_\pm^{(1)} \\ \phi^{(1)} \end{pmatrix} + \epsilon^2 \begin{pmatrix} n_\pm^{(2)} \\ u_\pm^{(2)} \\ \phi^{(2)} \end{pmatrix} + \cdots \tag{5}$$

To include the collisional effect and also for the consistent perturbation, we consider the following scaling:

$$\tilde{\nu}_n^\pm \equiv \frac{\nu_n^\pm}{\omega_p} = \nu_\pm \epsilon^{\frac{3}{2}}, \tag{6}$$

which is compatible with the assumption that the ion-neutral collision frequency is low compared to the ion plasma frequency.

Now, substituting the stretching coordinate (4), perturbation expansion (5), and the scaling (6) into the two-fluid system [Eqs. (1)–(3)], we obtain the following relations in the lowest powers of ϵ:

$$\begin{aligned} u_\pm^{(1)} - Mn_\pm^{(1)} &= 0, \\ -Mu_\pm^{(1)} \pm \phi^{(1)} + \sigma_\pm n_\pm^{(1)} &= 0, \\ n_+^{(1)} - n_-^{(1)} &= 0, \end{aligned} \tag{7}$$

This system of equations self-consistently determines the phase velocity of the linear wave

$$M^2 = \left(\frac{\sigma_- + \sigma_+}{2}\right) = 1. \tag{8}$$

According to the definition of $\sigma_\pm$ and T. It is to be noted that from Eq. (8), we readily recover the dispersion relation (in dimensional form) of IAW in pair-ion plasma:[17–19] $\omega^2 = C_s^2 k^2$, where $C_s = (\gamma T/m)^{1/2}$ (for one-dimension $\gamma = 3$).

The next highest power of ϵ yields the following second order equations (with $M = 1$):

$$\begin{aligned} -\frac{\partial n_\pm^{(2)}}{\partial \xi} + \frac{\partial u_\pm^{(2)}}{\partial \xi} = &\mp \frac{1}{(1-\sigma_\pm)} \frac{\partial \phi^{(1)}}{\partial \tau} \\ &- \frac{1}{(1-\sigma_\pm)^2} \frac{\partial \phi^{(1)2}}{\partial \xi}, \end{aligned} \tag{9}$$

$$\begin{aligned} -\frac{\partial u_\pm^{(2)}}{\partial \xi} \pm \frac{\partial \phi^{(2)}}{\partial \xi} + \sigma_\pm \frac{\partial n_\pm^{(2)}}{\partial \xi} = \mp \Bigg[&\frac{1}{(1-\sigma_\pm)} \frac{\partial \phi^{(1)}}{\partial \tau} \\ &\pm \frac{(1+\sigma_\pm)}{2(1-\sigma_\pm)^2} \frac{\partial \phi^{(1)2}}{\partial \xi} \\ &+ \frac{\nu_\pm \phi^{(1)}}{(1-\sigma_\pm)} \Bigg], \end{aligned} \tag{10}$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = n_-^{(2)} - n_+^{(2)}. \tag{11}$$

Finally, eliminating $n_\pm^{(2)}$, $u_\pm^{(2)}$, and $\phi^{(2)}$ from Eqs. (9)–(11), we obtain the following Korteweg -de Vries (KdV) equation with a linear damping term with $[\phi^{(1)} = \psi]$:

$$\frac{\partial\psi}{\partial\tau} + \alpha\psi\frac{\partial\psi}{\partial\xi} + \beta\frac{\partial^3\psi}{\partial\xi^3} + \nu\psi = 0, \tag{12}$$

where α, β, and ν are as follows:

$$\alpha = \frac{4}{(1-\sigma_+)} = 4\left(\frac{T_- + T_+}{T_- - T_+}\right), \tag{13}$$

$$\beta = \frac{(1-\sigma_+)^2}{2} = \frac{1}{2}\left(\frac{T_- - T_+}{T_- + T_+}\right)^2, \tag{14}$$

and

$$\nu = \frac{(\nu_+ + \nu_-)}{2}. \tag{15}$$

It is clear from the expression (13) [coefficient of nonlinearity] that α changes its sign according as

$$\begin{aligned} \alpha > 0 \quad &\text{if} \quad T_- > T_+ \\ \alpha < 0 \quad &\text{if} \quad T_- < T_+, \end{aligned} \tag{16}$$

i.e., depending on the temperature ratio of pair-ions. However, the coefficient of dispersion $\beta \propto \alpha^{-2}$ is always positive. It is clear from the expression (15) that the linear damping term ν is arising due to the (both positive and negative) ion-neutral collisions.

IV. ANALYTICAL SOLUTION: DISSIPATIVE SOLITONS

In the absence of the ion-neutral collision, i.e., for $\nu = 0$, we recover the usual KdV equation for IAW in pair-ion plasma. This KdV equation (Eq. (12) with $\nu = 0$) represents a completely integrable Hamiltonian system, which has an infinite set of conservation laws.[31,32] Let us consider the following energy conservation equation of the KdV equation (Eq. (12) with $\nu = 0$) by $\psi(\xi,\tau)$ subject to the solitary wave boundary conditions $\psi(\xi,\tau), \partial_\xi\psi(\xi,\tau)$ and $\partial_\xi^2\psi(\xi,\tau)$ (∂_ξ partial derivative with respect to ξ) all $\to 0$ as $|\xi| \to \infty$

$$\frac{\partial\mathcal{E}}{\partial\tau} = 0, \quad \text{where } \mathcal{E} = \int_{-\infty}^{\infty}\psi^2(\xi,\tau)d\xi \tag{17}$$

is the soliton energy. Thus, in absence of damping ($\nu = 0$), the energy $\mathcal{E}$ is conserved and the KdV equation possesses the single soliton solution

$$\psi(\xi,\tau) = \Psi\ \mathrm{sech}^2\sqrt{\frac{\alpha\Psi}{12\beta}}\left(\xi - \frac{\alpha\Psi\tau}{3}\right), \tag{18}$$

where $\Psi = 3U/\alpha$ is the soliton amplitude, U is the soliton velocity, and $L = (\alpha\Psi/(12\beta))^{-1/2}$ is the spatial width of the soliton. Thus, with the soliton amplitude, the soliton velocity increases, but the spatial width decreases so that amplitude $\times\text{width}^2 = 12\beta/|\alpha| = \text{constant}$(for fixed plasma parameters), the well known property of the KdV soliton. This well known KdV soliton property provides the following relation for spatial width (of KdV soliton in pair-ion plasma):

$$L^2 = \frac{12\beta}{\alpha\Psi} = \frac{32}{9U^3}\Psi^2,$$

where $\beta = 8\alpha^{-2}$ and $\alpha = 3U/\Psi$. Thus, for fixed soliton velocity U, the width of the soliton L is proportional to the amplitude Ψ, which is an interesting difference from the ion acoustic solitons in conventional electron-ion plasma.

However, in presence of damping ($\nu \neq 0$), Eq. (12) is not a completely integrable Hamiltonian system and, in this case, the above energy Eq. (17) becomes

$$\frac{\partial\varepsilon}{\partial\tau} = -2\nu\varepsilon. \tag{19}$$

This indicates that the damped KdV Eq. (12) is not exactly analytically solvable. However, for weak dissipation (damping due to ion-neutral collisions), we can obtain an approximate analytical solution of Eq. (12) by the soliton perturbation analysis[33,34] with $\nu(\ll 1)$ as the perturbed parameter. For this purpose, we assume the slow time dependent form of the soliton parameter $\Psi = \Psi(\tau)$. Then, the leading order one-soliton solution of Eq. (12) in this perturbation analysis[33,34] becomes

$$\psi(\xi,\tau) = \Psi(\tau)\mathrm{sech}^2\sqrt{\frac{\alpha\Psi(\tau)}{12\beta}}\left(\xi - \frac{\alpha}{3}\int_0^\tau\Psi(\bar\tau)d\bar\tau\right), \tag{20}$$

where $(\alpha/3)\frac{d}{d\tau}\int_0^\tau\Psi(\tau)$ is the soliton velocity. To explain the effects of disturbance (here damping) on the initial soliton (leading order), the judicial choice is the use of conservation laws.[35] Accordingly,[33,34] we apply the energy conservation Eq. (22). Finally, substitution of (20) in (19), yields the following expressions for soliton energy and amplitude

$$\begin{aligned} \mathcal{E} &= \mathcal{E}(0)\exp(-2\nu\tau), \quad \text{and} \\ \Psi(\tau) &= \Psi(0)\exp\left(-\frac{4\nu}{3}\tau\right), \end{aligned} \tag{21}$$

where $\mathcal{E}(0)$ and $\Psi(0)$ are the initial soliton energy and amplitude. The soliton velocity and width are given, respectively

$$\begin{aligned} U &= \frac{\alpha\Psi(0)}{3}\exp\left(-\frac{4\nu}{3}\tau\right), \quad \text{and} \\ L &= \sqrt{\frac{12\beta}{\alpha\Psi(0)}}\exp\left(\frac{2\nu}{3}\tau\right). \end{aligned} \tag{22}$$

Note that the above solution is the approximated (leading order) soliton solution, but holds fairly well for dissipative perturbations.[33–35] This can also be seen from the numerical simulation results. The higher order terms in the perturbation analysis introduce only corrections (the change in amplitude, velocity, and width remain same as obtained in the leading order approximation).[33,34]

The above solutions reveal that the ion-neutral collisional effect causes the soliton amplitude Ψ, soliton energy $\mathcal{E}$, and soliton velocity U to decay exponentially with time

(τ) according as Eq. (21) and (22), whereas the soliton width L increases with time according as (22). But, the product of the amplitude and the square of the width remains constant $\left[\text{amplitude} \times \text{width}^2 = 12\beta/\alpha\right]$. Also, the solutions show that the damped soliton moves with a gradually diminishing velocity. As a result, it will propagate only a finite distance $L_{damp} = \Psi(0)/4\nu$ before it dies out ($\tau \to \infty$). Thus, the solutions are weakly dissipative in nature.[36,37]

Several features of physical interest arise depending on the values of the coefficients α and β of the KdV Eq. (12). It is evident from Eq. (18) (or (20)) that for a solitary wave, we must have

$$\frac{\alpha}{\beta}\Psi > 0.$$

This condition has the following consequences:

$$\alpha \gtrless 0 \Rightarrow \Psi \gtrless 0 \Rightarrow n_{\pm}^{(1)} > 0, \quad (23)$$

where $\beta(\propto \alpha^{-2})$ is always positive [Eq. (14)] and $n_{\pm}^{(1)} = \alpha\phi^{(1)}/4$ [Eq. (7)]. Then, according to the condition (16), for $T_- > T_+$, the positive potential ion acoustic solitons exist, whereas in the opposite case $T_- < T_+$, the negative potential ion acoustic solitons exist. However, in both the cases, the solitons are compressive ($n_{\pm}^{(1)} > 0$) in nature. All these features can also be seen from the numerical simulation. Thus, IAWs in pair-ion plasma possess both positive and negative potential solitons, unlike in electron-ion plasma. This indicates the possibility of double-layer structures in pair-ion plasma. Also note that when $T_+ = T_-$, $\beta = 0$, and $\alpha = \infty$ so that no soliton solution exists and, therefore, in this situation RPT is not applicable.

V. NUMERICAL SIMULATION

We are interested to find the solution of Eq. (12) with its full generality. We have already seen in Sec. IV that in presence of dissipation ($\nu \neq 0$), Eq. (12) is not an exactly integrable Hamiltonian system. Therefore, to investigate the effect of ion-neutral collision on weakly nonlinear IAWs in pair-ion plasmas, we solve the nonlinear damped KdV Eq. (12) numerically with the help of MATHEMATICA based finite difference scheme. The numerical computations are carried out for the representative parameters of the pair-ion (fullerene) plasma experiment.[16–19] The plasma parameters used in the computations are: $n_0 = (1-2)\times 10^7\,\text{cm}^{-3}$ and $T_\pm = (0.3-0.5)\text{eV}$.

In absence of collision ($\nu = 0$), the KdV Eq. (12) possesses a single soliton solution. Therefore, for the time-dependent numerical simulation, we use the single soliton solution as the initial waveform: $\psi(\xi,0) = (3U/\alpha)\text{sech}^2(\sqrt{U/4\beta}\xi), \xi \in [-L, L]$, where $U = \alpha\Psi(0)/3$ is the soliton velocity and L is the spatial length. The boundary conditions are: $\psi(\pm L,\tau) = (3U/\alpha)\text{sech}^2(\pm\sqrt{U/4\beta}L)$ and $\psi_\xi(-L,\tau) = 0 = \psi_\xi(L,\tau)$. To obtain adequate results for the computation, we take $L = 15$ and $U = 1$. At first, we solve the KdV equation [(12) with $\nu = 0$] for different ion temperature ratios which are plotted in Fig. 1. The left figure shows the solitary wave with positive potential, whereas the right figure shows the solitary wave with negative potential. This indicates the possibility of the existence of potential double-layer structures in pair-ion plasma.

Also the comparative study between the curves in Fig. 1 shows that for fixed soliton velocity $U = 1$, the space scale becomes narrower with the decrease of the amplitude. This confirms the analytical result that for fixed soliton velocity, width is proportional to the amplitude.

Then, we introduce the effect of ion-neutral collision in our numerical simulation. For adequate result, we take $\nu = 0.1$. The results of the time-dependent numerical simulation are plotted in Fig. 2, which confirms the existence of weakly dissipative soliton structures in pair-ion plasmas. The potential profiles at different times in these figures show that with the increase of the time, the soliton amplitude and velocity decrease, but the width of the soliton increases, which preserves the well-known soliton property $\left(\text{amplitude} \times \text{width}^2 = \text{constant}\right)$. The contour of the potential picks in these figures clearly demonstrates the exponential decay nature of the amplitudes. Thus, the time-dependent numerical simulation exhibits the same nature of the weakly dissipative solitons as predicted by the analytical time-dependent analysis.

The time-evolutions of the ion density are plotted in Fig. 3. These figures clearly show the compressive nature of the solitons. Also the solitons are weakly dissipative in nature due to the presence of ion-neutral collisions.

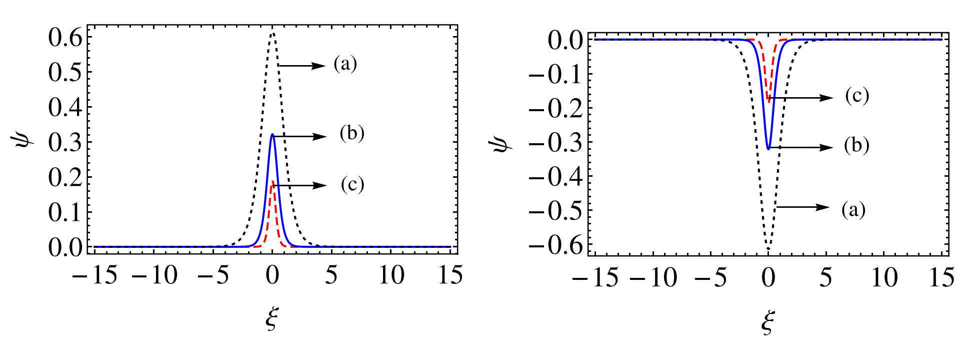

FIG. 1. Numerical solution of Eq. (12) in absence of ion-neutral collision ($\nu = 0$). The initial condition is $\psi(\xi,0) = (3U/\alpha)\text{sech}^2[\sqrt{U/4\beta}\xi]$. The nonlinear wave velocity $U = 1$. The left figure is for $T_- > T_+$. The curves in left figure are: (a) $T_+/T_- = 0.1$, (b) $T_+/T_- = 0.4$, and (c) $T_+/T_- = 0.6$. The right figure is for $T_- < T_+$. The curves in right figure are: (a) $T_+/T_- = 10$, (b) $T_+/T_- = 2.5$, and (c) $T_+/T_- = 1.7$. These figures show the existence of both positive and negative potential ion acoustic solitons.

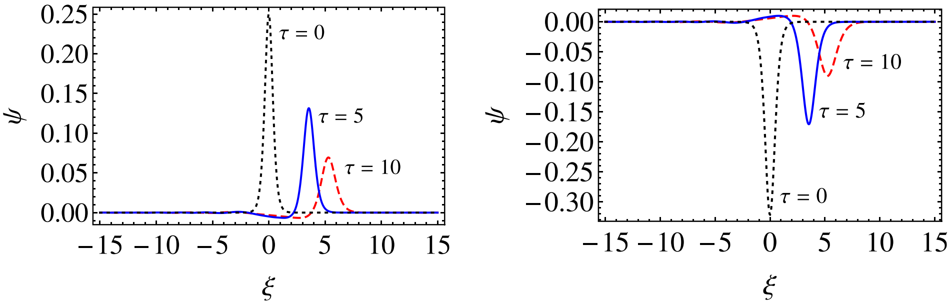

FIG. 2. Time-evaluation of ion acoustic solitons in presence ion-neutral collision. The numerical solution of Eq. (12) with $U = 1$, $\nu = 0.1$, and the initial condition is same as in Fig. 1. As in Fig. 1, the left figure is for $T_- > T_+$ ($T_+/T_- = 0.5$) and the right figure is for $T_- < T_+ (T_+/T_- = 2.5)$.

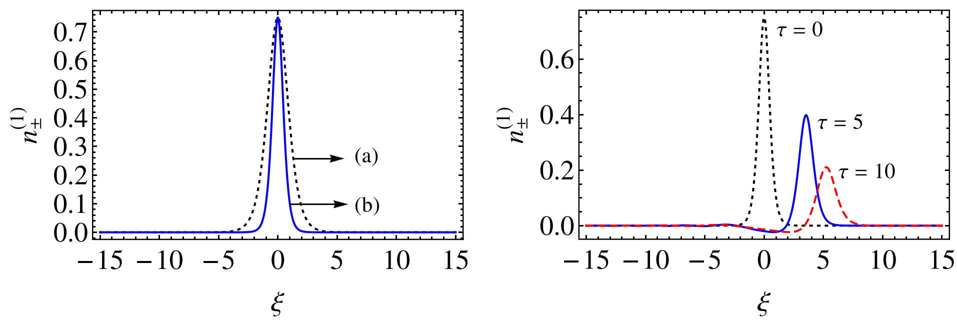

FIG. 3. Time-evaluation of ion acoustic solitons. The numerical solution of Eq. (12) with initial condition as in Fig. 1. The left figure is drawn without collision. The curves in left figure are: (a) $T_+/T_- = 0.1$ and (b) $T_+/T_- = 2.5$. The right figure is drawn with collision for parameters $T_+/T_- = 0.4$ and $\nu = 0.1$. In both the figures, soliton velocity $U = 1$. The left figure shows the compressive nature of the solitons, whereas, the figure shows the weakly dissipative natures of the compressive solitons due to the collision.

VI. CONCLUSIONS

In this paper, we investigate the nonlinear localized structures of IAWs in fullerene pair-ion plasma with equal mass but different temperatures incorporating the ion-neutral collision under the assumption that the collision frequency is much smaller than the plasma frequency. The nonlinear wave is governed by a damped KdV equation [Eq. (12)]. The analytical solution and numerical simulation predict the formation of weakly dissipative solitons. For ion acoustic solitons in pair-ion plasma, present investigations yield the following interesting differences from the conventional electron-ion plasma:

(i) Both positive and negative potential ion acoustic solitons are found to exist.
(ii) The width of the soliton is proportional to the amplitude of the wave for fixed soliton velocity.

In case of equal ion temperature $T = T_+ \approx T_-$, the phase speed of IAW $[V_{Ph} = (\gamma T/m)^{1/2}]$ is very close to the ion thermal speed $[V_{Th} = (T/m)^{1/2}]$ and, therefore, the collective processes in such pair-ion plasma are expected to suffer very strong damping.[38] Moreover, in this case $(T_+ \approx T_-)$, the coefficient of nonlinearity α becomes infinitely large and coefficient of dispersion β becomes vanishingly small [Eqs. (13) and (14)]. Therefore, the RPT in not applicable to study the small but finite amplitude nonlinear electrostatic modes in pair-ion plasma, instead one has to apply the technique of Ref. 8. The investigation of collective phenomena in pair-ion plasma is extremely important from a diagnostic point of view[16,18] and thus the findings of the present investigation can be used in diagnosing the fullerene pair-ion plasma. Also the existence of the positive and negative potential solitons predicts the potential double-layer structures in pair-ion plasma. Therefore, we suggest the future experiments should consider the possibility of observing ion acoustic solitons in pair-ion plasmas, similar to what has been done in the experiment[39] for the investigation of the dynamical evolution of plasma structures (double-layer induced solitary pulses) due to local production of massive ions.

Finally, we remark that for simplicity, we have considered only about the existence of the single soliton solutions. However, depending on the strength of the initial perturbation, multi-solitons can also be observed. According to the qualitative approach of Kadomtsev,[40] for the present physical problem in absence of damping, one can estimate the number of solitons by introducing the parameter P defined by

$$P = \frac{\alpha L^2}{12\beta}\Psi,$$

where Ψ and L are the amplitude of the perturbation and spatial scale of the perturbation (spatial width of the soliton). Note that $P = 1$ for a single soliton (as for single soliton $L^2 = 12\beta/(\alpha\Psi)$), but, if $P < 1$, no more than single soliton can be generated, whereas if $P \gg 1$, one can generate multi-solitons and the estimated number of soliton is $N_{sol} \sim \sqrt{P}$.

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