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Determination of Propagation Characteristics of Crucial Planar Wave Guide Structures by Finite Difference Method

Abstract: Based on finite difference (FD) method, recently applied to investigate propagation characteristics of optical waveguides, we verify its applicability to predict the electric field and propagation parameter of planar waveguide for the entire range of normalized frequencies. As a test case, we crosscheck our FD result for symmetric step index planar waveguide and then predict the propagation characteristics of crucial planar waveguide structures like exponential index and Epstein layer model profile. Our findings show high degree of accuracy when compared with published results.

Keywords: optical planar waveguide, step index profile, exponential index profile, Epstein model profile

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1 Introduction

Novel methods of solution of the electromagnetic field equations relevant to various refractive index structures are still being proposed in literature of integrated photonics. In majority of cases, analytical solutions do not exist for electromagnetics of such profiles and one has to take resort to various approximate and numerical methods. The approximate methods include perturbation method [1], variational method [2], etc. whereas the deeply involved numerical methods include spectral index method [3], Galerkin method [4] and beam propagation method [5].

All the techniques take care not only of the field equations but also of the matching of the fields and derivatives at various boundaries. The problem becomes intricate when the structure consists of deployed grids of alternative dielectric materials in another dielectric matrix like photonic crystal and holy fibers [6] and geometry changes from one dimension to two or three dimensions. However any method ready for application to two dimensional case should be full proof in one dimensional case.

Out of the available numerical methods, very recently the finite difference (FD) method [7] is receiving tremendous attention for prediction of propagation characteristics of one and two dimensional refractive index structural arrangements. Apart from simplicity, the elegance and power of these methods are the independence of the initial choice of the electromagnetic fields. Thus choosing any arbitrary field as an initial choice, the entire simulation process takes care of its automatic transformation into the proper mode of the system describing the propagation characteristics of the engineered refractive index structure accurately.

It may be relevant to point out in this connection that in the earlier investigation, involving FD method [7], normalized propagation parameter has been analyzed for a particular normalized frequency for core-cladding symmetric and asymmetric step-index and other interesting profiles of practical interest and the applicability of the formalism for the entire range of normalized frequencies is not seen to be reported to the best of our knowledge. In this paper, we investigate and report the merit of the method for prediction of propagation parameter for entire range of normalized frequencies in case of known one dimensional step core-cladding symmetric profile to cross check and compare our analysis with available results [8, 9, 10].

Then we predict the entire range of propagation parameters for entire range of normalized frequencies in case of step index profile, exponential index profile and Epstein-layer model profiles first time by the FD method and compare our results with those obtained by the other methods. In the next section, we present our analysis and model of refractive index distribution with results and discussion in following section.

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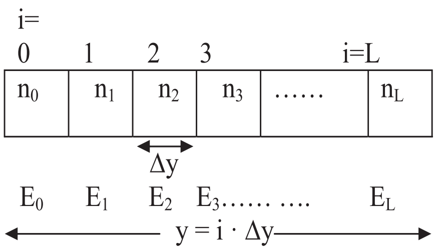

Fig. 1: Discretization of refractive index profile in transverse direction of the waveguide

2 Analysis

2.1 Formalism

For harmonic wave propagation in isotropic, linear, lossless, non conducting and non magnetic one dimensional structures such as optical planar waveguides with refractive index n(y), the modal field under weakly guiding condition satisfies Helmholtz's equation [11]:

$$\frac{\partial^2\phi}{\partial y^2}+k_0^2(n(y)^2-n_{eff}^2)\phi(y)=0 \quad (1)$$

where n_{eff} represents the effective refractive index of the propagating mode.

In FD method, with usual discretization in transverse y-direction as $y=i\times\Delta y$, i being an integer as shown in Fig. 1, the one dimensional refractive index profile is first placed in the array of $(N+1)$ points in the computational domain. Here, $(N+1)\times\Delta y$ is the total length of the waveguide in transverse dimension. By applying FD method approximation on Eq. (1) [7], to investigate TE Mode, we can write

$$e_{i+1}=\{2-(\Delta y)^2k_0^2(n_i^2-n_{eff}^2)\}e_i-e_{i-1} \quad (2)$$

The field convergence algorithm [7] has been used here. The method is simple and based on FD discretization applied to the scalar or semivectorial form of Helmholtz's equation, which transforms an arbitrary field to a mode.

Now for any arbitrary value of n_{eff} and appropriate choice of boundary value of electric field we evaluate the electric field by using Eq. (2). Then we proceed to have a more relevant expression of n_{eff} [11] as

$$n_{eff}^2=\frac{\int_{-\infty}^{\infty}\{(\Delta y)^2k_0^2n_i^2e_i^2+(e_{i+1}-2e_i+e_{i-1})e_i\}dy}{(\Delta y)^2k_0^2\int_{-\infty}^{\infty}e_i^2dy} \quad (3)$$

Using the calculated electric field value from Eq. (2), we calculate the value of $n_{eff.}$ from Eq. (3). This new n_{eff} is now inserted in Eq. (2) and the value of electric field is calculated. Again these field values are inserted in Eq. (3), which gives a new n_{eff}. This iteration process is continued until a desired precision is achieved yielding both the mode field as well as the effective mode index.

For TM modes Eq. (2) [7] can be re-written as

$$\frac{2n_{i-1}^2}{n_{i-1}^2+n_i^2}e_{i-1}+\frac{2n_{i+1}^2}{n_{i-1}^2+n_i^2}e_{i+1}+[(\Delta y)^2k_0^2(n_i^2-n_{eff}^2)e_i-2-\frac{n_i^2-n_{i-1}^2}{n_{i-1}^2+n_i^2}-\frac{n_i^2-n_{i+1}^2}{n_{i+1}^2+n_i^2}]e_i=0 \quad (4)$$

To calculate the effective refractive index, n_{eff} for TM mode, we use the following equation [7]:

$$n_{eff}^2=\frac{\int_{-\infty}^{\infty}[\frac{2n_{i-1}^2}{n_{i-1}^2+n_i^2}e_{i-1}+\frac{2n_{i+1}^2}{n_{i+1}^2+n_i^2}e_{i+1}+\{(\Delta y)k_0^2n_i^2-2-\frac{n_i^2-n_{i-1}^2}{n_{i-1}^2+n_i^2}-\frac{n_i^2-n_{i+1}^2}{n_{i+1}^2+n_i^2}\}]dy}{(\Delta y)^2k_0^2n_i^2\int_{-\infty}^{\infty}e_i^2dy} \quad (5)$$

Now in order to calculate normalized propagation constant b for optical planar waveguide as for TE and TM modes using Eqs. (3) and (5), respectively. The normalized propagation constant [8] is expressed by

$$b=\frac{n_{eff}^2-n_2^2}{n_1^2-n_2^2} \quad (6)$$

where $n_{eff}=\beta/k_0$, n_1 and n_2 being the refractive indices of film and substrate of the optical planar waveguide. Also the normalized frequency of optical planar waveguide [8] is expressed by

$$V=k_0d\sqrt{(n_1^2-n_2^2)} \quad (7)$$

2.2 Modelling of refractive index structures

Now in order to apply FD method, we consider some interesting refractive index structures for which we will apply FD method first time to compute b vs V as follows.

Case 1: Step index profile waveguide

We consider the waveguide studied in [8], which has a step-index profile shown in Fig. 2(a) and expressed by:

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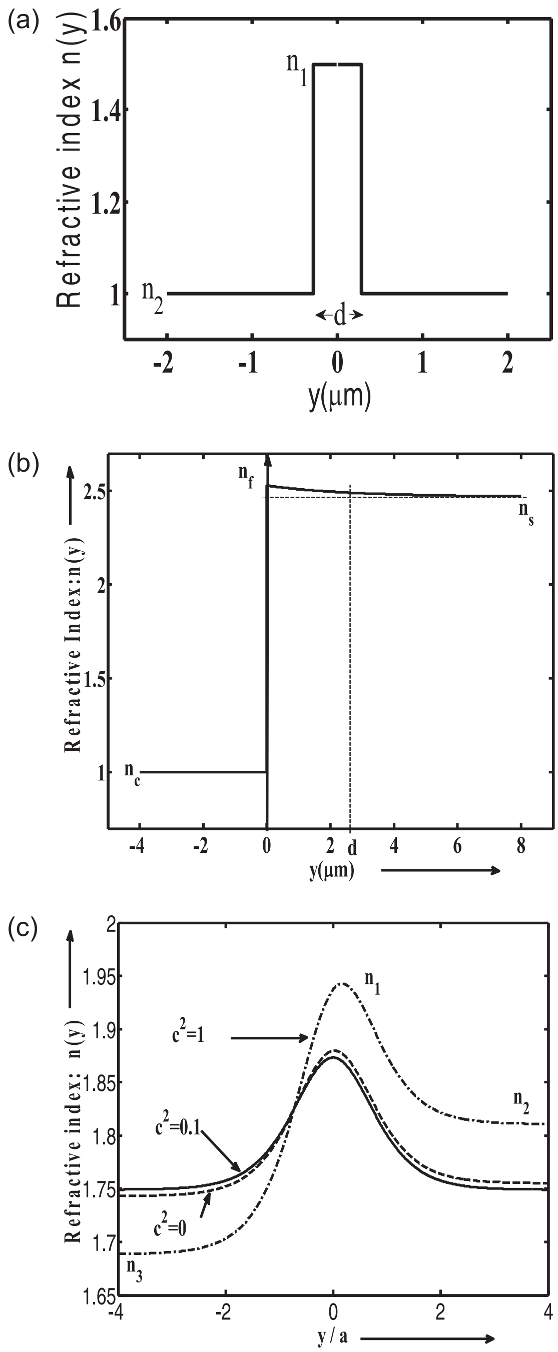

Fig. 2: Refractive index profile of (a) step index profile waveguide, (b) exponential index profile waveguide, and (c) general Epstein-layer profile for three values of the asymmetric factor c is given.

$$n(y) = n_1; \quad |y| < d/2$$
$$= n_2; \quad |y| > d/2 \qquad (8)$$

where n_1 is the film index, n_2 is the substrate index and d is the film width.

Case 2: Exponential index profile waveguide

Then for the same purpose we take the waveguide studied in [9], which has a graded refractive-index profile following a small exponential variation as depicted in Fig. 2(b). This type of refractive index profile is configured by diffusion with species that survive in two different forms occupying two different sites not in equilibrium, as Se in CdS substrate [9]. The index profile is given by

$$n^2(y) = n_s^2 + 2n_s\Delta n.\exp(-\frac{y}{d_y}), \quad \text{for } y \geq 0$$
$$= n_c^2, \quad \text{for } y < 0 \qquad (9)$$

where n_s is the substrate index, n_c is the refractive index of air, Δn is the maximum index change, and d_y is the depth of diffusion.

Case 3: Epstein-layer model profile

Lastly we consider Epstein-layer model profile [10], as shown in Fig. 2(c) which represents a wide range of refractive index profiles from symmetric to strongly asymmetric waveguide using its continuous nature. The most general form of the profile is expressed by,

$$n^2(y) = n_3^2 + 2\Delta n_1^2\{\frac{c^2e^{2y/a}}{(1+e^{2y/a})} + \frac{2(2+c^2)e^{2y/a}}{(1+e^{2y/a})^2}\} \qquad (10)$$

where

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \quad \text{and} \quad c^2 = \frac{n_2^2 - n_3^2}{n_1^2 - n_2^2}$$

where n_1, n_2 and n_3 are refractive indices of core, cover and substrate regions of the waveguide, respectively.

The values of parameters are given in the result section.

The normalized propagation constant b for the fundamental TE mode is given by the following eigenvalue equation [10] in terms of V:

$$b = \frac{c^4V^2}{4[(1+2V^2(2+c^2))^{1/2} - (2N+1)]^2} + \frac{[(1+2V^2(2+c^2))^{1/2} - (2N+1)]^2}{4V^2} - \frac{c^2}{2} \qquad (11)$$

where $N = 0, 1, 2, \ldots$.

The last two profiles are chosen for their realistic nature of practical importance [12, 13]. In the next section

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we take these profiles to compute b for various values of V as stated above.

3 Results and discussions

Now as stated above we proceed to crosscheck our formalism for known symmetric step index waveguide whether our FD based calculation presents results as excellent as those of [7, 8]. We have used $n_1 = 1.5$, $n_2 = 1.0$, and $d = 0.555$ µm. The domain width of the calculated part is 4 µm and the region is equally divided into mesh grids with a grid size 7 nm. The plot of normalized propagation constant b as a function of the normalized frequency V for the profile for fundamental TE and TM modes are shown in Fig. 3(a) and 3(b).

For this refractive index profile, we see an excellent agreement with the available results [8] showing the applicability of FD method of the entire b-V range. Further we have seen that our electric field represents exactly that presented by [7] to make us sure about rigour of our calculation which follows that of [7].

Then with our FD scheme, we try to predict the b-V curves for some interesting profiles like exponential and Epstein-layer model profiles for which we have available results from other methods for comparison. The parameters used for exponential index profile are $n_s = 2.47$, $\Delta n = 0.06$, $n_c = 1$ and $d = 2.5$ µm. The domain width of the calculated part is 9 µm and the region is equally divided into mesh grids with a grid size 19 nm. The plot of normalized propagation constant (b) as a function of the normalized frequency (V) for the profile for fundamental TE and TM modes are shown in Fig. 4(a) and 4(b). For this specially graded refractive index profile, an excellent agreement with [9] is obtained.

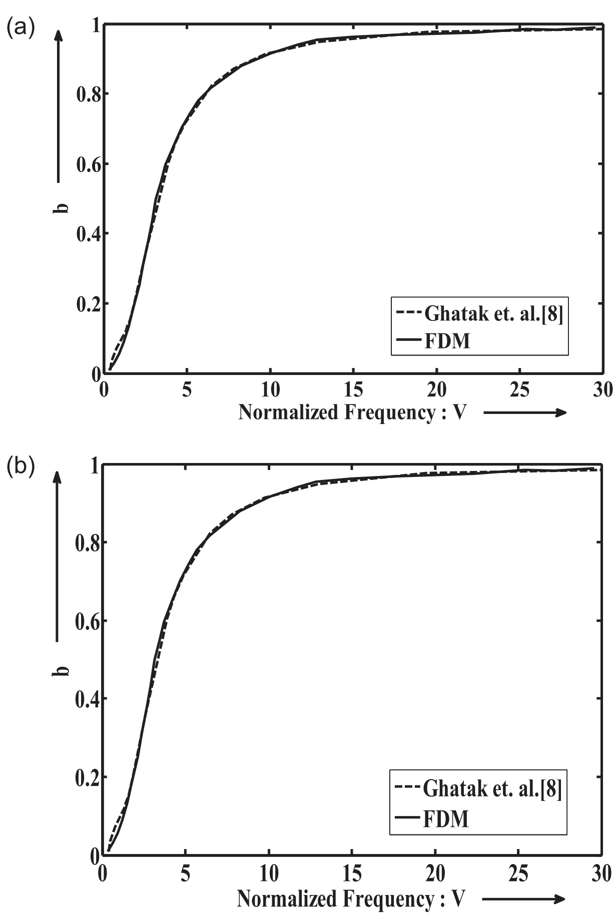

Fig. 3: Normalized b-V plots for the step index profile for (a) fundamental TE mode and (b) fundamental TM mode

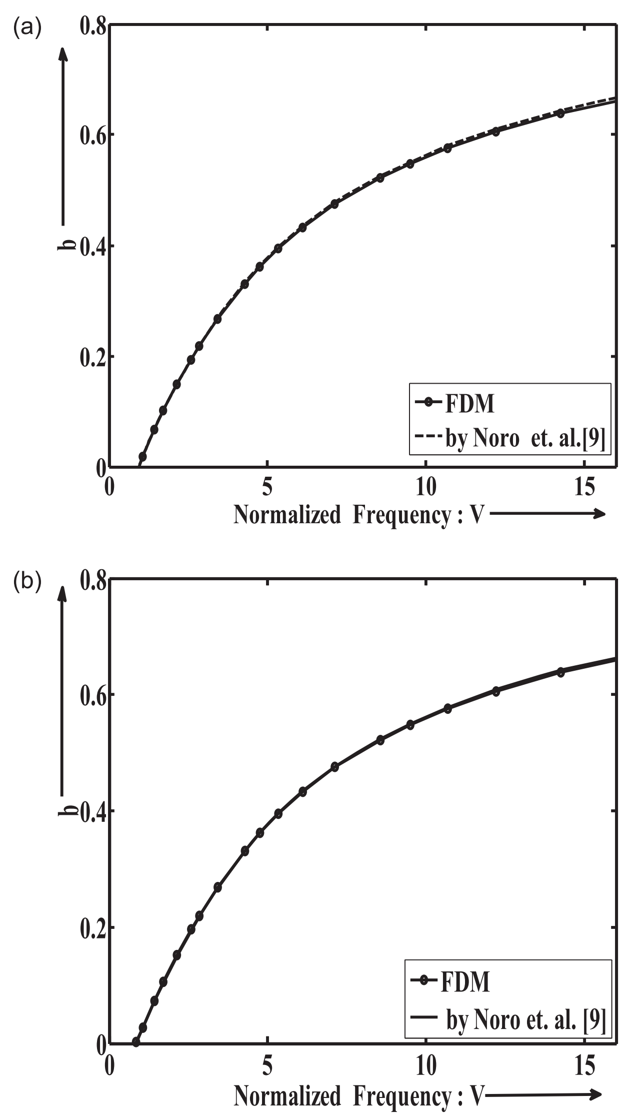

Fig. 4: Normalized b-V plots for the exponential index profile for (a) fundamental TE mode and (b) fundamental TM mode

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The parameters chosen for Epstein-layer profile are $n_1 = 1.16$, $n_2 = 1.15$ and $d = 1$ µm. The values of n_3 are 1.15, 1.149 and 1.1399 for c^2 equal to 0, 0.1 and 1 respectively. We have calculated the normalized propagation constant (b) for a grid size of 9 nm over a domain width of 8 µm. The plot of b as a function of the normalized frequency (V) for the profile for fundamental TE mode is shown in Fig. 5. Evidently, the results are same as those in [10], which may be calculated from the eigenvalue Eq. (11).

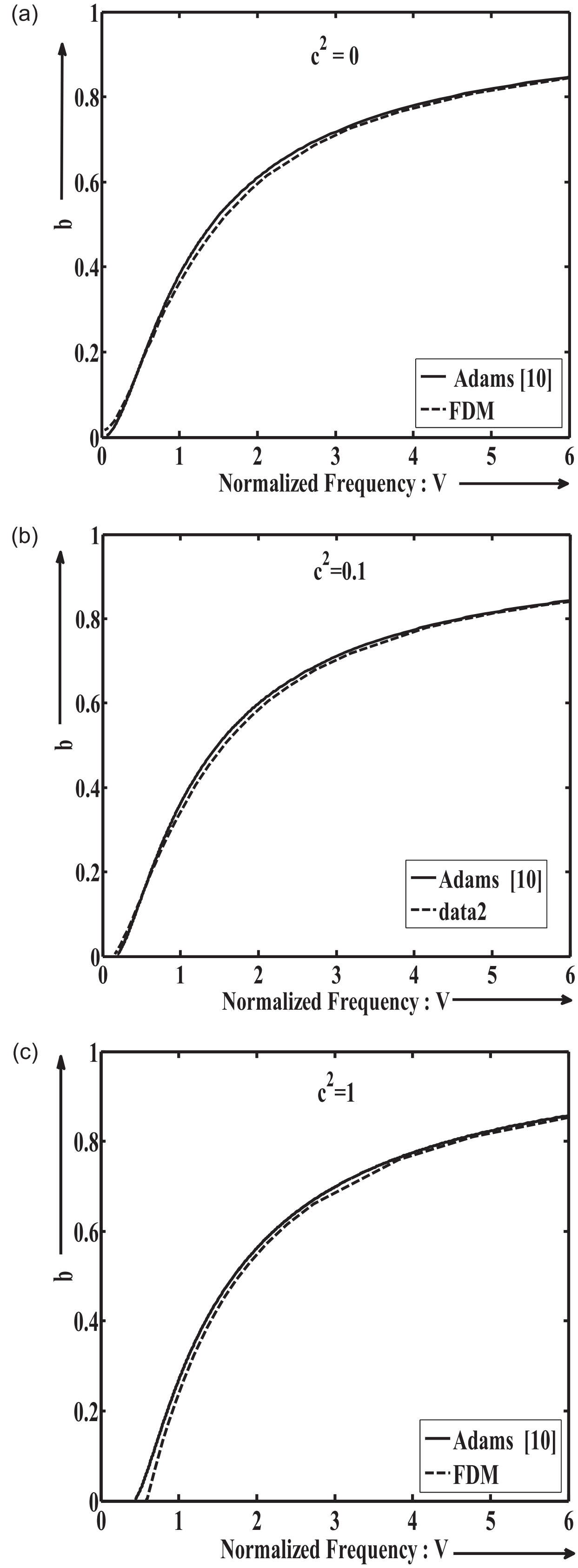

Fig. 5: Normalized b-V plots for the general Epstein-layer profile for the asymmetric factor (a) $c^2 = 0$, (b) $c^2 = 0.1$, and (c) $c^2 = 1$

4 Conclusion

The well-known finite difference method involves the conversion of Helmholtz equation into a finite difference equation and possesses the merit of transformation of an arbitrary field into a proper mode of a tailored planar waveguide structure by repeated use of the difference equation without any dependence on initial choice of the field. Based on finite difference (FD) method recently applied to investigate propagation characteristics of optical waveguides, we verify its applicability to predict the electric field and propagation parameter of planar waveguide for the entire range of normalized frequencies. As a test case, we crosscheck our FD result for symmetric step index planar waveguide and then predict the propagation characteristics of crucial planar waveguide structures like exponential index and Epstein layer model profile. Our findings show high degree of accuracy when compared with published results.

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