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Charging-delay induced dust acoustic collisionless shock wave: Roles of negative ions

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The effects of charging-delay and negative ions on nonlinear dust acoustic waves are investigated. It has been found that the charging-delay induced anomalous dissipation causes generation of dust acoustic collisionless shock waves in an electronegative dusty plasma. The small but finite amplitude wave is governed by a Korteweg-de Vries Burger equation in which the Burger term arises due to the charging-delay. Numerical investigations reveal that the charging-delay induced dissipation and shock strength decreases (increases) with the increase of negative ion concentration (temperature). © *2006 American Institute of Physics*. [DOI: 10.1063/1.2374861]

I. INTRODUCTION

In modern technological applications[1–3] electronegative dusty plasmas are widely used in semiconductor materials processing,[4] in development of novel nanomaterials,[5] etc. The dust grains produced by the chemical reaction in gas or plasma-surface interaction in such plasmas can significantly affect the local as well as the global discharge characteristics for the efficient deposition of quality thin films.[6] The linear wave propagation characteristics of both dust acoustic and ion acoustic waves,[7,8] the spatial profiles of number densities of electrons, positive ions, negative ions, electron temperature, and dust charge[9] and also the ion drag forces on a dust grain[10] have been investigated in an electronegative dusty plasma. Recently, the floating potential acquired by a dust grain in a homogeneous, collisionless electronegative dusty plasma has been investigated assuming that all three sets of particle electrons, positive ions, and negative ions are Boltzmann distributed.[11] It has been found that due to their lower thermal velocity, the presence of negative ions reduces the dust surface potential. Moreover, such electronegative dusty plasmas can also support nonlinear phenomena (such as double layer, etc.).[12]

In a plasma, a dispersive shock wave is generated when wave breaking due to the nonlinearity is balanced by the combined action of dispersion and dissipation. Dissipation is often caused by viscosity which arises due to collision and is taken into account by the Burger term in the KdV Burger equation.[13] Landau damping and particle reflection may also cause dissipation leading to the generation of the so-called collisionless shock waves. In charge varying weakly coupled (electropositive) dusty plasmas without negative ions, it is well understood that the charging-delay ($\omega_{pd}/\nu_{\rm ch} \ll 1$ but $\neq 0$, ω_{pd} is the dust oscillation frequency and $\nu_{\rm ch}$ is the dust charging frequency) produces a dissipative effect for which the linear dust acoustic wave (DAW) (Ref. 14) suffers a collisionless non-Landau damping.[15] On the other hand in the nonlinear regime, this delay in the charging (which is known as *nonadiabatic* dust charge variation) causes an anomalous dissipation and this dissipation leads to the formation of collisionless shock waves in an electropositive dusty plasma.[16] Also, experimental observation[17] reveals that in inhomogeneous strongly coupled electropositive dusty plasmas, the dust acoustic waves becomes unstable due to the charging-delay. Thus the nonlinear feature of dust acoustic waves in charge varying electronegative dusty plasma is very important to understand the physics of localized dust acoustic waves.

Recently, the nonlinear characteristics of dust acoustic waves in homogeneous, collisionless, unbounded electronegative dusty plasmas have been investigated incorporating both high and low charging rates of dust grains compared to the dust oscillation frequency.[18] But, to the best of our knowledge no study of nonlinear dust acoustic waves including charging-delay effect has been done in an electronegative dusty plasma. The main objective of this paper is to show that when viscosity, Landau damping or collisional damping effects are not important, the charging-delay induced dissipation provides an alternate physical mechanism for the generation of shock waves in an electronegative dusty plasma. Thus, in this paper, we extend our previous work[18] on nonlinear dust acoustic waves in homogeneous, collisionless, unbounded electronegative dusty plasmas by incorporating the effect of charging-delay under the assumption $\omega_{pd}/\nu_{\rm ch} \ll 1$ but $\neq 0$.

The manuscript is organized in the following manner: The formulation of the problem including basic equations are given in Sec. II. Section III describes the physical assumptions, nonlinear evolution equations and solution of the problem. The numerical solution and the results of the present investigation are discussed in Sec. IV. Finally, a summary of the results is presented in Sec. V.

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II. FORMULATION OF THE PROBLEM

We consider a one-dimensional unmagnetized homogeneous, collisionless, unbounded electronegative dusty plasma whose constituents are electrons, singly charged positive and negative ions and charge fluctuating negatively charged cold dust grains. At far upstream $x \to -\infty$, there is a dust flow velocity V_0 and the charge on the dust grain surface is $-z_d e$, where the plasma is assumed to be in an undisturbed uniform state $\phi=0$ and $n_\alpha = n_{\alpha 0}$ $(\alpha = e, +, -, d)$ so that the charge neutrality condition becomes

$$n_{e0} + z_d n_{d0} + n_{-0} = n_{+0}. \tag{1}$$

In the very slow dust motion time scale, one can assume that the electrons, positive ions and negative ions are in approximately local thermodynamic equilibrium and their densities therefore follow a Boltzmann distribution:[11]

$$n_e = n_{e0} e^{\Phi}, \quad n_+ = n_{+0} e^{-\Phi/\sigma_+}, \quad n_- = n_{-0} e^{\Phi/\sigma_-}, \tag{2}$$

where $\Phi = e\phi/T_e$, $\sigma_+ = T_+/T_e$, and $\sigma_- = T_-/T_e$.

Thus the one-dimensional behavior of the DAW in unmagnetized, homogeneous, collisionless, unbounded electronegative dusty plasma is described by the following normalized fluid equations:[18]

$$\begin{aligned} &\partial_T N_d + \partial_X (N_d V_d) = 0, \\ &\partial_T V_d + V_d \partial_X V_d = -(-1+Q)\partial_X \Phi, \end{aligned} \tag{3}$$

and the Poisson's equation

$$\Delta \partial_X^2 \Phi = \delta e^{\Phi} + \delta_- e^{\Phi/\sigma_-} - e^{-\Phi/\sigma_+} - \Delta N_d (-1+Q), \tag{4}$$

where $N_d = n_d/n_{d0}$, $\delta = n_{e0}/n_{+0}$, $\delta_- = n_{-0}/n_{+0}$, $\Delta = 1 - \delta - \delta_-$, and $Q_d = q_d/z_d e = -1 + Q$, where Q is the fluctuating dust charge. The time (T) and space (X) scales are normalized in units of the inverse of the dust plasma frequency $\omega_{pd} (= \sqrt{n_{d0} z_d^2 e^2/\epsilon_0 m_d})$ and dust Debye length $\lambda_d (= \sqrt{\epsilon_0 T_e / z_d n_{d0} e^2})$, respectively. The dust relative velocity $V_d (= V - V_0)$ is normalized in units of the dust acoustic speed $c_d (= \sqrt{z_d T_e / m_d})$.

The normalized dust charging equation is[18]

$$\left(\frac{\omega_{pd}}{\nu_{ch}}\right)(\partial_T Q + V_d \partial_X Q) = \frac{(I_e + I_+ + I_-)}{z_d e \nu_{ch}}. \tag{5}$$

Also it is assumed that the dust flow velocity V_0 is much smaller than the thermal velocities of electron, positive ion, and negative ion. Thus the normalized expressions for the electron current (I_e), positive ion current (I_+), and negative ion current (I_-) for spherical dust grains of radius r_0 are as follows:[11]

$$\begin{aligned} &I_e = -J_e e^{(\Phi + z(-1+Q))}; \\ &I_+ = J_+ e^{-\Phi/\sigma_+}\left[1 - \frac{z(-1+Q)}{\sigma_+}\right]; \quad I_- = -J_- e^{(\Phi + z(-1+Q))/\sigma_-} \end{aligned} \tag{6}$$

where, $J_j = \pi r_0^2 e \sqrt{\frac{8T_j}{\pi m_j}} n_{j0}$, $j = e, +, -$, and $z = \frac{z_d e^2}{4\pi\epsilon_0 r_0 T_e}$. In the above, $\nu_{ch} \left(= \frac{r_0}{\sqrt{\pi}} \frac{\omega_{p+}^2}{V_{t+}} (1 + \sigma_+ + z + \mu_-)\right)$ is the dust charging frequency and $\mu_- = \delta_- \sqrt{m_+ T_+ / m_- T_-} (1 - \delta_-) e^{-z/\sigma_-}$.

III. NONLINEAR EVOLUTION EQUATIONS

To employ the reductive perturbation technique for the investigation of nonlinear characteristics of dust acoustic waves, we introduce the stretched coordinate $\xi = \sqrt{\epsilon}(X - \Lambda T)$; $\tau = \epsilon^{3/2} T$, where $\Lambda = V_{ph} - V_0$ and V_{ph} is the normalized linear phase velocity of the dust acoustic wave. The dynamical variables N_d, V_d, Q and the electrostatic potential Φ are expanded in powers of ϵ as follows:

$$f = f^{(0)} + \sum_{i=1}^{\infty} \epsilon^i f^{(i)}; \quad f = N_d, Q, V_d, \Phi \tag{7}$$

where $f^{(0)} = 1$ for N_d and $=0$ for Φ, Q and V_d, respectively, and ϵ is a small parameter, measuring the wave amplitude or weakness of the wave dispersion. Now applying the stretching and perturbation expansion (7) to the dynamical Eqs. (3) and (4), and equating the terms in the lowest powers of ϵ, we obtain

$$\begin{aligned} &V_d^{(1)} = \Lambda N_d^{(1)}; \quad \Phi^{(1)} = -\Lambda^2 N_d^{(1)}; \\ &\Delta N_d^{(1)} = \Delta Q^{(1)} - \left(\frac{1}{\sigma_+} + \frac{\delta_-}{\sigma_-} + \delta\right)\Phi^{(1)} \end{aligned} \tag{8}$$

and equating the terms of next higher order of ϵ and using (8) we obtain the following equations:

$$\begin{aligned} &\partial_\tau N_d^{(1)} + 2\Lambda N_d^{(1)} \partial_\xi N_d^{(1)} = \Lambda \partial_\xi N_d^{(2)} - \partial_\xi V_d^{(2)}, \\ &\Lambda \partial_\tau N_d^{(1)} + \Lambda^2 N_d^{(1)} \partial_\xi N_d^{(1)} - \Lambda^2 Q^{(1)} \partial_\xi N_d^{(1)} = \Lambda \partial_\xi V_d^{(2)} + \partial_\xi \Phi^{(2)}, \\ &\Lambda^2 \Delta \partial_\xi^2 N_d^{(1)} = \Delta (Q^{(2)} - N_d^{(2)}) - \frac{\Lambda^4}{2}\left(\frac{\delta_-}{\sigma_-^2} - \frac{1}{\sigma_+^2} + \delta\right) N_d^{(1)2} \\ &\qquad - \left(\frac{1}{\sigma_+} + \frac{\delta_-}{\sigma_-} + \delta\right)\Phi^{(2)} + \Delta N_d^{(1)} Q^{(1)}. \end{aligned} \tag{9}$$

A. Charging-delay effect: Nonadiabatic charge variation

To include the effect of delay in the dust charging time scale $(\sim \nu_{ch}^{-1})$ with respect to the dust oscillation time scale $(\sim \omega_{pd}^{-1})$, we assume that $\omega_{pd}/\nu_{ch} \ll 1$ but $\neq 0$ and also to make the nonlinear perturbation consistent with that of (6), we consider the following scaling:[16]

$$\frac{\omega_{pd}}{\nu_{ch}} = \nu_d \sqrt{\epsilon}, \tag{10}$$

where $\nu_d \approx O(1)$. Later in the numerical section, the validity of the above assumption and scaling are justified on the basis of plasma parameters relevant to laboratory electronegative dusty plasmas. Using the above scaling in Eq. (5) together with (6) and equating the terms in lowest and next to lowest powers of ϵ, we obtain

$$Q^{(1)} = -\beta_{ch}\Phi^{(1)}; \quad \beta_{ch} = \frac{(1+\sigma_+)(z+\sigma_+)}{z\sigma_+(1+z+\sigma_+ + \mu_-)} \tag{11}$$

and

$$Q^{(2)} = \nu_d \Lambda \partial_\xi Q^{(1)} + \frac{\beta_{ch}\sigma_+ A_2}{2(1+\sigma_+)}\Phi^{(1)2} - \frac{z^2\beta_{ch}\sigma_+ A_1}{2(1+\sigma_+)}Q^{(1)2} - \beta_{ch}\Phi^{(2)} + \frac{z\beta_{ch}A_3}{(1+\sigma_+)(z+\sigma_+)}Q^{(1)}\Phi^{(1)}, \quad (12)$$

where $A_1 = 1+\frac{\mu_-(1+\sigma_-)}{\sigma_-(z+\sigma_+)}$, $A_2=\frac{1}{\sigma_+^2}-A_1$ and $A_3 = 1-\sigma_+(z+\sigma_+) - \frac{\mu_-\sigma_+(1+\sigma_-)}{\sigma_-}$. The value of Λ self-consistently follows from (8) and (11):

$$\Lambda = -\sqrt{\Delta/\left(\beta_{ch}+\frac{1}{\sigma_+}+\frac{\delta_-}{\sigma_-}+\delta\right)} \Rightarrow V_{ph} = V_0 - \sqrt{\Delta/\left(\beta_{ch}+\frac{1}{\sigma_+}+\frac{\delta_-}{\sigma_-}+\delta\right)}. \quad (13)$$

Here the negative sign is taken because of the fact that for the generation of the shock waves the dust flow velocity (V_0) at far upstream is much greater than the wave phase velocity (V_{ph}).

Finally combining (9) and (10) and using (8) and (11), we derive the following KdV-Burger equation for nonlinear dust acoustic waves in unmagnetized, homogeneous, collisionless, unbounded electronegative dusty plasma:

$$\partial_\tau N_d^{(1)} + \alpha N_d^{(1)}\partial_\xi N_d^{(1)} + \beta \partial_\xi^3 N_d^{(1)} = \mu_{ch}\partial_\xi^2 N_d^{(1)}, \quad (14)$$

where

$$\alpha = \frac{\Lambda^3}{2}\left[\frac{3}{\Delta}\left(\frac{1}{\sigma_+}+\frac{\delta_-}{\sigma_-}+\delta\right) + \frac{\Lambda^2}{\Delta}\left(\left(\frac{\delta_-}{\sigma_-^2}-\frac{1}{\sigma_+^2}+\delta\right) + \frac{\Delta\beta_{ch}\sigma_+ A_4}{1+\sigma_+}\right)\right]; \quad \beta = \frac{\Lambda^3}{2} \quad (15)$$

with $A_4 = \left(1+\frac{\mu_-(1+\sigma_-)}{\sigma_-(z+\sigma_+)}\right)(z\beta_{ch}-1)^2 + \frac{2z\beta_{ch}}{\sigma_+(z+\sigma_+)} - \frac{1}{\sigma_+^2}$, and

$$\mu_{ch} = \frac{\nu_d\beta_{ch}\Delta^2}{2}\left(\beta_{ch}\Delta+\frac{1}{\sigma_+}+\frac{\delta_-}{\sigma_-}+\delta\right)^{-2}. \quad (16)$$

Using the expression for ν_d from Eq. (10), the Burger term μ_{ch} can be rewritten in the following form in terms of actual physical parameters:

$$\sqrt{\epsilon}\mu_{ch} = \frac{1}{2}\left(\frac{\omega_{pd}}{\nu_{ch}}\right)\frac{\beta_{ch}\left(\frac{z_d n_{d0}}{n_{+0}}\right)^2}{\left(\frac{z_d n_{d0}}{n_{+0}}\beta_{ch}+\frac{T_e}{T_+}+cn_{-0}n_{+0}\frac{T_e}{T_-}+\frac{n_{e0}}{n_{+0}}\right)^2} = \frac{1}{2}\left(\frac{\omega_{pd}}{\nu_{ch}}\right)\frac{\beta_{ch}\left(\frac{z_d n_{d0}}{n_{e0}}\right)^2}{\left(1+\frac{z_d n_{d0}}{n_{e0}}\beta_{ch}+\frac{n_{+0}}{n_{e0}}\frac{T_e}{T_+}+\frac{n_{-0}}{n_{e0}}\frac{T_e}{T_-}\right)^2}. \quad (17)$$

The expression (16) shows that the Burger term, i.e., the dissipative term μ_{ch} disappears if we put $\nu_d=0$, as it should be because $\nu_d=0$ implies that the left-hand side of Eq. (4) =0 i.e., $I_e+I_++I_-=0$. In this case the dust charge q_d is in local equilibrium state providing an approximate description of the charge state of a dust grain and produces no dissipative

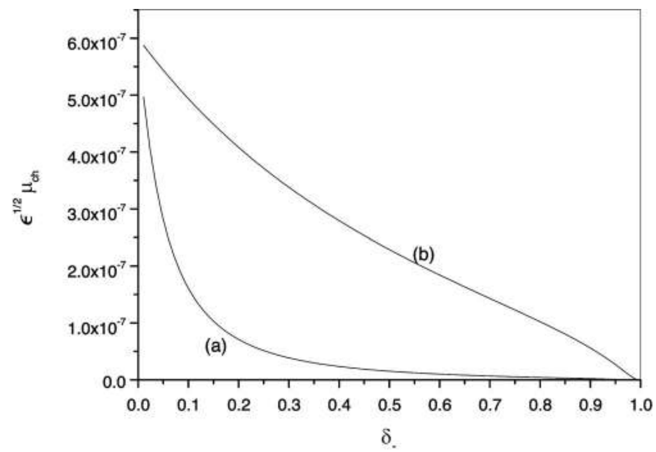

FIG. 1. Variation of coefficient of Burger term μ_{ch} [Eq. (17)] with $\delta_-(=n_{-0}/n_{+0})$ for $\sigma_+(=T_+/T_e)=0.1$ and for different $\sigma_-(=T_-/T_e)$. The different curves are: (a) $\sigma_-=0.01$; (b) $\sigma_-=0.1$.

effects.[19] Also note that $\mu_{ch}=0$ for $\Delta\left(=1-\delta-\delta_-=\frac{z_d n_{d0}}{n_{+0}}\right)=0$; this is expected as $\Delta=0$ implies the absence of charged dust grains and consequently no charge fluctuation. Hence, the Burger term or dissipation arises due to the charging-delay effect ($\nu_d \neq 0$) under the assumption $\omega_{pd}/\nu_{ch} \ll 1$ but $\neq 0$. The expression for μ_{ch} [Eq. (17)] also shows that the charging-delay induced dissipation decreases with an increase of negative ion concentration, but it increases with an increase of negative ion temperature. The variation of $\epsilon^{1/2}\mu_{ch}$ with negative ion concentration is shown in Fig. 1 for different negative ion temperature.

B. Solution

The KdV-Burger equation (14) is not analytically exactly solvable. To find the numerical solutions of (14), transforming this equation to the wave frame $\eta = V_f\tau+\xi$ and then integrating twice we obtain the following second order ordinary differential equation

$$d_\eta^2 N_d^{(1)} = (V_f/\beta)N_d^{(1)} - (\alpha/2\beta)N_d^{(1)^2} - (\mu_{ch}/\beta)d_\eta N_d^{(1)}. \quad (18)$$

Fixed point analysis of this equation shows that, this equation possesses two fixed points ($N_d^{(1)}=0, d_\eta N_d^{(1)}=0$) and ($N_d^{(1)}=2V_f/\alpha, d_\eta N_d^{(1)}=0$). The first one is a saddle point and second one is a stable focus or a stable node according as the parameter

$$\mu = \frac{\mu_{ch}}{2\sqrt{V_f\beta}}\langle\text{or}\rangle 1. \quad (19)$$

A stable focus corresponds to oscillatory shock and hence the condition for oscillatory shock is

$$\mu < 1 \Rightarrow \frac{\omega_{pd}}{\nu_{ch}} < \sqrt{2(M-1)}\frac{\left(\Delta\beta_{ch}+\delta+\frac{1}{\sigma_+}+\frac{\delta_-}{\sigma_-}\right)}{\Delta\beta_{ch}}, \quad (20)$$

where $M = 1+\epsilon\frac{V_f}{\Lambda}$ is the Mach number. On the other hand, a stable node corresponds to monotonic shock and hence the condition for monotonic shock is

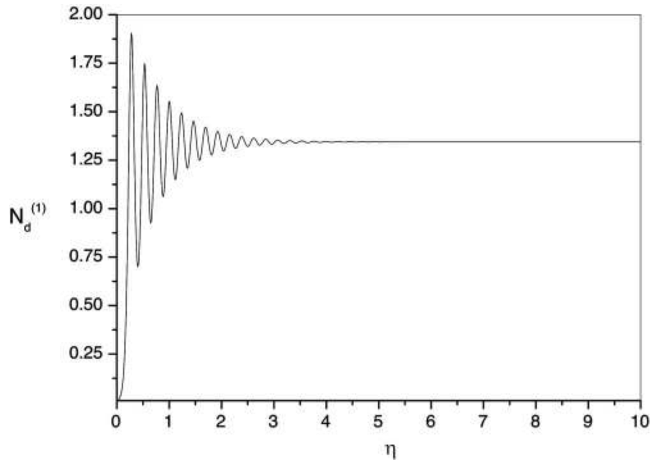

FIG. 2. Variation of $N_d^{(1)}$ [Eq. (18)] with η for weak dissipation $\nu_d \sim 0.1$: Oscillatory shock structure. The plasma parameters are $\sigma_+ = \sigma_- = 0.1$, $\delta_- = 0.4$.

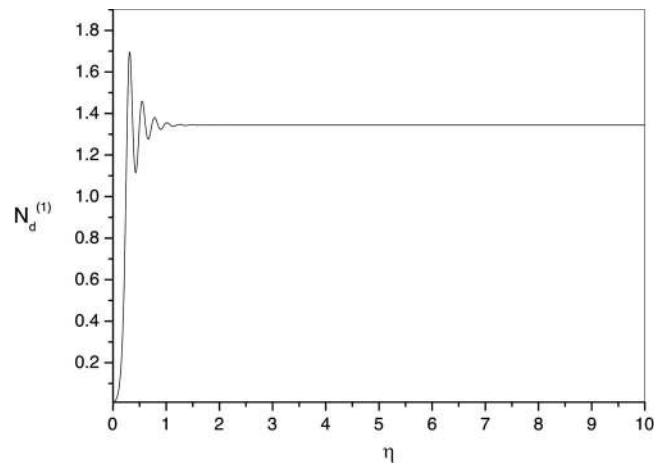

FIG. 3. Variation of $N_d^{(1)}$ [Eq. (18)] with η for moderate dissipation $\nu_d \sim 1$: Monotonic shock transition. The physical parameters are the same as in Fig. 2.

$$\mu > 1 \Rightarrow \frac{\omega_{pd}}{\nu_{\rm ch}} > \sqrt{2(M-1)} \frac{\left(\Delta\beta_{\rm ch} + \delta + \frac{1}{\sigma_+} + \frac{\delta_-}{\sigma_-}\right)}{\Delta\beta_{\rm ch}}. \quad (21)$$

The shock strength of the dust acoustic shock wave in an electronegative dusty plasma is given by

$$\begin{aligned}[\epsilon N_d^{(1)}]_{\max} &= \epsilon(2V_f/\alpha) \\ &= 2\Lambda(M-1)/\alpha \\ &= (2(M-1)/\alpha)\sqrt{\Delta\Big/\left(\beta_{\rm ch}\Delta + \frac{1}{\sigma_+} + \frac{\delta_-}{\sigma_-} + \delta\right)}.\end{aligned} \quad (22)$$

The variation of shock strength is plotted in Fig. 4 against $\delta_- = n_{-0}/n_{+0}$ for different $\sigma_- = T_-/T_e$. Since the potential is negative ($\Phi^{(1)} = -\Lambda^2 N_d^{(1)}; N_d^{(1)} > 0$), the negatively charged dust is energized by passing shock waves to

$$\begin{aligned}E &= -z_d e\phi = -z_d T_e \Phi = -z_d T_e[\epsilon\Phi^{(1)}]_{\max} \\ &= z_d T_e \Lambda^2 [\epsilon N_d^{(1)}]_{\max} = 2(M-1)\frac{z_d T_e \Lambda^3}{\alpha}.\end{aligned} \quad (23)$$

For laboratory electronegative dusty plasmas $E/(M-1)Z_d T_e$ is plotted in Fig. 5 against δ_- for different σ_-.

It should be noted that in the case of $\mu \gg 1$ i.e., if the dissipation due to charging-delay is much greater than the wave dispersion ($\mu_{\rm ch} \gg \beta$), the nonlinear dust acoustic wave is governed by the Burger equation

$$\partial_\tau N_d^{(1)} + \alpha N_d^{(1)} \partial_\xi N_d^{(1)} = \mu_{\rm ch} \partial_\xi^2 N_d^{(1)}. \quad (24)$$

Transforming the above Burger equation (24) to the wave frame $\eta = V_f \tau + \xi$, one can easily find the analytic solution of the Burger equation in the following form:

$$N_d^{(1)} = N[1 + \tanh(\eta/L)]; \quad L = 2\mu_{\rm ch}/V_f \quad (25)$$

subject to the boundary conditions $N_d^{(1)}$, $d_\eta N_d^{(1)} \to 0$ as $\eta \to -\infty$, which exhibits monotonic shock (double layer) solutions. In the above N is the shock amplitude, $V_f (= \alpha N)$ is the velocity of the shock and L is the shock width.

IV. NUMERICAL SOLUTION AND DISCUSSIONS

For numerical computations, we use the following representative plasma parameters. In modern technology, for $Ar^+ - F^-$ electronegative dusty plasmas,[8] which are often used for ultrafine and highly selective etching of polysilicon[4] ($n_{+0} \sim 4 \times 10^{17}$ m^{-3}, $n_{d0} \sim 4 \times 10^{13}$ m^{-3}, $T_e = 2$ eV, $T_- = 0.1$ eV, $T_+ = 0.2$ eV, $m_- \sim 3.2 \times 10^{-26}$ kg, $m_+ \sim 6.69 \times 10^{-26}$ kg, dust mass density $\rho_d \sim 1.5 \times 10^3$ kg m^{-3}) and also for $SiH_3^+ - SiH_3^-$ electronegative dusty plasmas,[9] which are often used for efficient deposition of quality thin films[6] ($n_{+0} \sim 2 \times 10^{17}$ m^{-3}, $n_{d0} \sim 5 \times 10^{13}$ m^{-3}, $T_e = 2$ eV, $T_- \approx T_+ = 0.035$ eV, $m_- \approx m_+ \sim 5.2 \times 10^{-26}$ kg, dust mass density $\rho_d \sim 1.5 \times 10^3$ kg m^{-3}). The negative ion-positive ion number density ratio $\delta_- (= n_{-0}/n_{+0})$ as a function of nondimensional dusty plasma parameter z is given by the following equilibrium current balance equation:

$$\delta_- = \frac{n_{-0}}{n_{+0}} = \frac{(1-\Delta)e^{-z} - \left(1 + \frac{z}{\sigma_+}\right)\sqrt{\frac{\sigma_+ m_e}{m_+}}}{e^{-z} - e^{-z/\sigma_-}\sqrt{\frac{\sigma_- m_e}{m_-}}}; \quad \Delta = zP \quad (26)$$

where $P = 4\pi r_0 n_{d0} \lambda^2$ and $\lambda = \sqrt{\epsilon_0 T_e / n_{i0} e^2}$. We observe that for $r_0 = 0.1$ μm and $\delta_- = 0.4$, ω_{pd} and $\nu_{\rm ch}$ are, respectively, $\sim 5.6 \times 10^4$ s^{-1}, 4.5×10^4 s^{-1}, and $\sim 4.1 \times 10^6$ s^{-1}, 4×10^6 s^{-1} so that $\omega_{pd}/\nu_{\rm ch}$ are, respectively, $\sim 1.4 \times 10^{-2}$ and $\sim 1.1 \times 10^{-2}$. Thus these values of $\omega_{pd}/\nu_{\rm ch}$ justify the scaling (10) on the basis of which the charging equation (5) is approximated.

Figure 1 shows that the coefficient of Burger term [Eq. (17)] decreases (increases) monotonically with the increase of $\delta_- = n_{-0}/n_{+0} (\sigma_- = T_-/T_e)$ [Fig. 1, curves (a) and (b)]. Thus the dissipation arising due to charging-delay decreases (increases) with the increase of negative ion density (temperature). Because of the lower thermal velocity of negative ions than electrons, the magnitude of the dust surface potential z in an electronegative dusty plasma is reduced which implies the reduction of dust charge magnitude $|q_d|$ and hence decreases the charging-delay induced dissipation. On the other

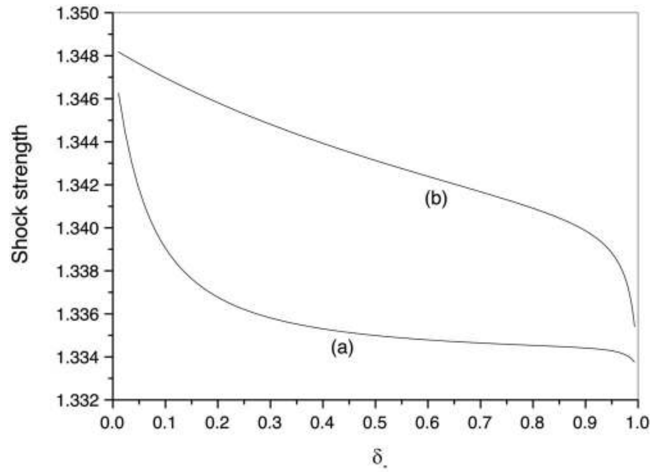

FIG. 4. Variation of shock strength as given by Eq. (22) with δ_- for $\sigma_+=0.1$, Mach number $M=2$, and for different σ_-. The different curves are: (a) $\sigma_-=0.01$; (b) $\sigma_-=0.1$.

hand with an increase of negative ion temperature, the thermal velocity of the negative ion increases which implies the increment of the magnitude of the dust surface potential ($\propto q_d$) and hence increase the dissipation.

Starting from a small perturbation of the boundary conditions $N_d^{(1)}$, $d_\eta N_d^{(1)} \to 0$ as $\eta \to -\infty$ at far upstream and upon numerical integration of Eq. (18) by the Runge-Kutta-Fehlberg method, it is seen that the perturbation develops into a shock wave. Prototypes of dust acoustic shock wave structures (oscillatory nature for weak dissipation in Fig. 2 and monotonic shock transition for moderate dissipation in Fig. 3) are shown in Figs. 2 and 3. The observed shock wave is of a compressional type. This shock wave occurs due to the fact that the delay in the charging process causes decay of the dust charge magnitude and consequently plays the role of dissipative mechanism on dust fluid by decreasing the driving force ($\propto q_d$) causing dust motion. This charging-delay induced anomalous dissipation and the dispersive effects tend to strike a balance with the wave breaking nonlinear forces and under suitable condition gives rise to a generation of shock waves in an electronegative dusty plasma.

The variations of shock strength [Eq. (22)] are shown in Fig. 4. It is seen that shock strength decreases with the increase of δ_-, i.e., negative ion concentration, whereas it increases with the increase of σ_-, i.e., negative ion temperature [Fig. 4, curves (a) and (b)].

Figure 5 shows the plot of the energy E (with suitable normalization) [Eq. (23)] to which the negatively charged dust grains are raised by the negative potential of the passing electrostatic shock. This figure shows that the energy E behaves in a qualitatively similar fashion as done by shock strength.

V. SUMMARY

The results of the present investigation can be summarized as follows:

(1) It is shown that the delay in the charging processes ($\omega_{pd}/\nu_{\rm ch} \ll 1$ but $\neq 0$) causes anomalous dissipation represented by the term $\mu_{\rm ch}\partial_\xi^2 N_d^{(1)}$ in the KdV-Burger equation (14) describing small but finite amplitude dust acoustic shock waves in electronegative dusty plasmas. It is collisionless shock in the sense that no viscous or damping effects resulting from collisions between dust and plasma particles are involved.

(2) The structure of steady small amplitude shocks given by Eq. (18) are shown in Figs. 1 and 2. The transition from upstream to far downstream state changes from being of oscillatory to monotonic nature as charging-delay induced dissipation $\mu_{\rm ch}$ increases. The observed shock wave is of the compressional type with dust density enhancement.

(3) The presence of negative ions reduce the charging-delay induced dissipation (Fig. 1), the shock strength (Fig. 4) and also the energy E to which dust grains are raised by passing shock waves (Fig. 5). Thus the dust acoustic wave in an electropositive (absence of negative ions) dusty plasma

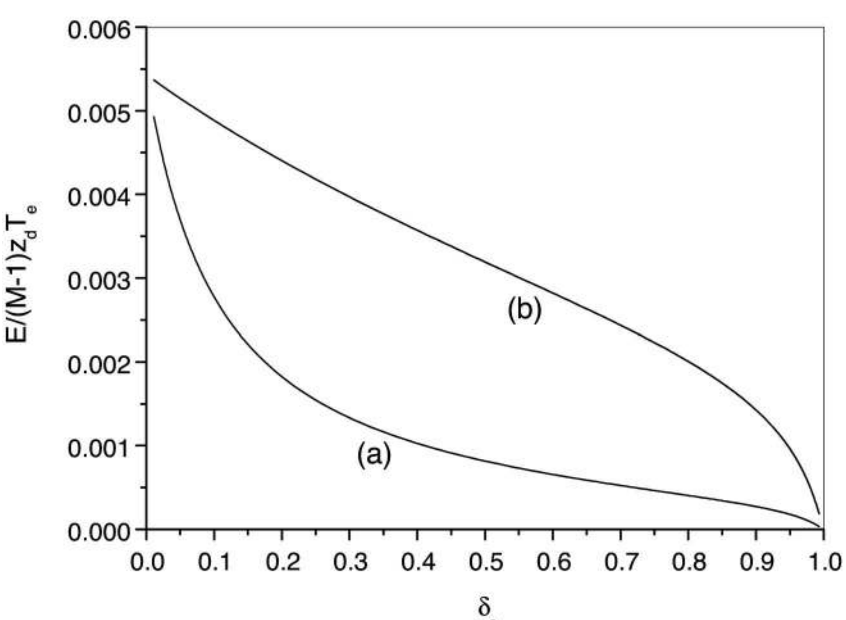

FIG. 5. Variations of E=Energy [as given by Eq. (23)] to which dust grains are raised by passing shock waves with δ_-. The physical parameters are the same as in Fig. 4.

suffers more collisionless damping due to delayed charging than in the electronegative (presence of negative ions) dusty plasma.

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