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Charging-delay effect on longitudinal dust acoustic shock wave in strongly coupled dusty plasma

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Taking into account the charging-delay effect, the nonlinear propagation characteristics of longitudinal dust acoustic wave in strongly coupled collisional dusty plasma described by generalized hydrodynamic model have been investigated. In the "hydrodynamic limit," a Korteweg–de Vries Burger (KdVB) equation with a damping term arising due to dust-neutral collision is derived in which the Burger term is proportional to the dissipation due to dust viscosity through dust-dust correlation and charging-delay-induced anomalous dissipation. On the other hand, in the "kinetic limit," a KdVB equation with a damping term and a nonlocal nonlinear forcing term arising due to memory-dependent strong correlation effect of dust fluid is derived in which the Burger term depends only on the charging-delay-induced dissipation. Numerical solution of integrodifferential equations reveals that (i) dissipation due to dust viscosity and principally due to charging delay causes excitation of the longitudinal dust acoustic shock wave in strongly coupled dusty plasma and (ii) dust-neutral collision does not appear to play any direct role in shock formation. The condition for the generation of shock is also discussed briefly. © *2005 American Institute of Physics*.
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I. INTRODUCTION

In a dusty plasma, due to higher electric charge $z_{d0}e$ and lower temperature T_d of dust grains, the coupling parameter $\Gamma[=(z_{d0}{}^2e^2/4\pi\epsilon_0 aT_d)e^{-a/\lambda_p}]$ (a is the interdust distance, and λ_p is the plasma Debye length) can easily become of order unity or larger. In strongly coupled dusty plasma ($\Gamma>1$), and for $\Gamma>\Gamma_c$ (critical value) the dust grains form crystalline structures in dusty plasma which supports a variety of dust lattice wave (DLW) modes,[1–3] whereas for $1\ll\Gamma<\Gamma_c$, i.e., when the system is in a quasicrystal state, the system supports both longitudinal[4–6] and transverse modes.[5,7–9]

A natural feature of a dusty plasma is that the charge (q_d) on a dust grain is not constant but changes in time according to the grain charging equation $(\omega_{pd}/\nu_{ch})[dq_d/d(\omega_{pd}t)]=I(\phi,q_d)/\nu_{ch}$, where $I(\phi,q_d)$ is the total current flowing to the grain surface, ω_{pd} is the dust oscillation frequency, and $\nu_{ch}(\sim\partial I/\partial q_d)$ is the dust charging frequency. In a weakly coupled dusty plasma ($\Gamma\ll1$) the linear dust acoustic wave[10] (DAW) suffers a collisionless, non-Landau damping when ω/ν_{ch} (ω is the DAW frequency) is not negligible,[11] whereas for non-negligible values of Ω/ν_{ch} (Ω is DLW frequency), the DLW becomes energywise unstable due to statistical charge fluctuations in the dusty plasma crystal state ($\Gamma>\Gamma_c$).[12] The nonlinear analysis shows that in the absence of charging delay ($\omega_{pd}/\nu_{ch}\approx0$, which is known as *adiabatic* dust charge variation) the DAW possesses solitary wave solution with negative or positive potential in a weakly coupled dusty plasma.[13] On the other hand, the charging delay ($\omega_{pd}/\nu_{ch}\ll1$, but $\neq0$, which is known as *nonadiabatic* dust charge variation) causes a dissipation because of the phase difference between the dust charge variation and the wave in a dusty plasma and this dissipation leads to the collisionless shock wave in a weakly coupled dusty plasma,[14] whereas in an inhomogeneous strongly coupled dusty plasma the transverse shear wave becomes unstable due to this charging-delay.[2]

In the dusty plasma crystal state ($\Gamma>\Gamma_c$), the nonlinear properties of DLW have been studied both theoretically[15] and experimentally,[16] considering fixed charged dust grain ($\nu_{ch}/\omega_{pd}\approx0$). The properties of nonlinear waves in the quasicryatal state ($1\ll\Gamma<\Gamma_c$) have not been studied as completely as for linear waves in a dusty plasma. There are two limiting cases of interest in studying the collective behavior of longitudinal dust-acoustic wave (LDAW) in the quasicrystal state ($1\ll\Gamma<\Gamma_c$) of dusty plasma described by the generalized hydrodynamic (GH) equation.[5,17] One is the "hydrodynamical limit" $\omega_{pd}\tau_m<1$ where the fluid behavior of dusty plasma persists, retaining approximate influence of memory and nonlocality through the dust viscosity and the linear LDAW suffers viscous damping in this case.[5] The other one is the "strong coupling limit" or the so-called "kinetic limit" $\omega_{pd}\tau_m>1$ (here the strain relaxation time $\tau_m>$the dust acoustic time ω_{pd}^{-1}). Recently in the hydrodynamical limit the LDAW turbulence has been observed experimentally in the quasicrystal state, assuming a fixed charged dust grain.[18] However, to the best of our knowledge, no study of nonlinear LDAW has been carried out including charging-delay effect in both "hydrodynamic regime" and "kinetic regime" of the quasicrystal state. In this paper we report on the theoretical

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investigation of longitudinal dust acoustic shock wave (LDASW) in both the regime of strongly coupled dusty plasma incorporating charging-delay effect. The dust-neutral collision is also incorporated. The principal results of this communication are (i) in the kinetic limit the Burger term which causes LDASW formation depends essentially on dissipation mechanism related to charging-delay effect and thus focuses its physical importance; and (ii) the dust-neutral collision does not contribute to shock wave formation.

The paper is organized in the following manner. The nonlinear evolution equations in both the limits have been derived using the well-known reductive perturbation technique in Sec. II. The numerical solution of these nonlinear equations and their graphical as well as physical explanations are given in Sec. III. Section IV summarizes the results and its possible importance in applications.

II. NONLINEAR ANALYSIS

For very low-frequency oscillation, the electrons and ions are modeled by Boltzmann distributions:

$$N_e(\Phi) = e^{\Phi}, \quad N_i(\Phi) = e^{-\Phi/\sigma_i}. \tag{1}$$

In terms of the normalized variables the dust fluid equation of continuity is

$$\partial_T N_d + \partial_X (N_d V_d) = 0. \tag{2}$$

The normalized GH equation,[5,17]

$$(1 + \omega_{\mathrm{pd}}\tau_m D_T)F = \eta_d \partial_X^2 V_d,$$

$$F = N_d D_T V_d + \sigma_d N_d Q \partial_X \Phi + \gamma_d \mu_d \partial_X N_d + \nu_{dn} N_d V_d, \tag{3}$$

where $D_T = \partial_T + V_d \partial_X$, $N_j = n_j / n_{j0}$ $(j=e,i,d)$, $Q = q_d / z_{d0} e$, $\sigma_i = T_i / T_e$, $\sigma_d = z_{d0} T_e / T_d$, $\Phi = e\phi / T_e$, the time scale T and ν_{dn} (dust-neutral collision frequency) are normalized by ω_{pd}, and the space scale X and the velocity V_d are normalized by λ_d (dust Debye length) and V_{td} (dust thermal velocity), respectively. In Eq. (3),

$$\eta_d = 3\eta^* \Gamma, \quad \omega_{\mathrm{pd}}\tau_m = 3\eta^* \Gamma \left[1 - \gamma_d \mu_d + \frac{4}{15} u(\Gamma) \right]^{-1}, \tag{4}$$

where $\eta^* = \left(\frac{4}{3}\eta + \zeta\right) / m_d n_{d0} \omega_{\mathrm{pd}} a_d^2 \approx 0.02\sqrt{\Gamma}$,[19] γ_d is the adiabatic index, η_d is the longitudinal viscosity coefficient normalized by λ_d^2 / a_d^2, ($a_d = a e^{-a/2\lambda_p}$ is the effective interaction distance of dust grains), $\mu_d = 1 + \frac{1}{3} u(\Gamma) + \Gamma / 9 \partial_\Gamma u(\Gamma)$ is the compressibility, $u(\Gamma) = E_c / (n_{d0} T_d)$ is a measure of the excess internal energy of the system, and E_c is the correlation energy. For weakly coupled plasmas $(\Gamma \ll 1)$ $u(\Gamma) \approx -\sqrt{3}/2\Gamma^{3/2}$, whereas in the range of $1 \leqslant \Gamma \leqslant 200$, $u(\Gamma) \approx -0.89\Gamma + 0.95\Gamma^{1/4} + 0.19\Gamma^{-1/4} - 0.81.$[20].

We assume that the dust grains suspended in a plasma are negatively charged; the plasma electron (I_e) and ion (I_i) currents given by

$$I_e = -eJ_j N_e(\Phi) e^{-zQ}, \quad I_i = eJ_j N_i(\Phi)(1 - zQ/\sigma_i), \tag{5}$$

where $J_j = \pi r_0^2 \sqrt{8T_j / \pi m_j} n_{j0}$, $j(=e,i)$, $z = z_{d0} e^2 / 4\pi\epsilon_0 r_0 T_e$ is the nondimensional dusty plasma parameter, and $4\pi\epsilon_0 r_0 T_e$ is the capacitance of the charged dust grain of radius r_0. The charge Q on the dust surface is determined by the following orbital-motion-limited (OML) normalized dust charging equation:

$$f(\Phi, Q) \equiv e^{(1+1/\sigma_i)\Phi} - [\sigma_i / (z + \sigma_i)](1 - zQ/\sigma_i) e^{-z(1+Q)} = -\beta_d^{-1} e^{[\Phi/\sigma_i - z(1+Q)]} (\omega_{\mathrm{pd}} / \nu_{\mathrm{ch}}) D_T Q, \tag{6}$$

where $\beta_d = (z + \sigma_i) / z(1 + z + \sigma_i)$. The dust density N_d at the same potential Φ is fixed up by the following Poisson equation:

$$h(\Phi, Q, N_d) \equiv e^{\Phi} - \delta e^{-\Phi/\sigma_i} - (\delta - 1) Q N_d = (\lambda_e / \lambda_d)^2 \partial_X^2 \Phi, \tag{7}$$

where $\delta = n_{i0} / n_{e0}$ and λ_e is the electron Debye length.

We consider small but finite amplitude perturbations ΔN_d, ΔV_d, $\Delta\Phi$, and ΔQ about the unperturbed steady state defined by $N_{d0} = 1$, $V_{d0} = V_0$, $\Phi_0 = 0$, and $Q_0 = -1$. To find the nonlinear evolution equation, the stretched variables and the perturbation expansions are

$$\xi = \epsilon^{1/2}(X - \lambda T), \quad \tau = \epsilon^{3/2} T,$$

$$\Delta G = \epsilon \Delta G^{(1)} + \epsilon^2 \Delta G^{(2)} + \cdots, \tag{8}$$

where $G = N_d$, V_d, Φ, Q and λ is the phase velocity of the normalized LDAW and ϵ measures the order of smallness of the perturbations. Also from (8) it follows that

$$D_T = \epsilon^{1/2}(\Delta V_d - \Lambda)\partial_\xi + \epsilon^{3/2}\partial_\tau, \quad \Lambda = \lambda - V_0, \tag{9}$$

To include the effect of delay in the dust charging time scale $(\sim \nu_{\mathrm{ch}}^{-1})$ with respect to the dust oscillation time scale $(\sim \omega_{\mathrm{pd}}^{-1})$ and the effect of dust-neutral collision under the assumption that the dust-neutral colission frequency is much smaller than that of the dust oscillation frequency and maintain consistency in the nonlinear perturbation expansion in powers of ϵ when ΔG is expanded as in (8), we introduce the scalings

$$\omega_{\mathrm{pd}} / \nu_{\mathrm{ch}} = \nu_d \epsilon^{1/2}, \quad \nu_{\mathrm{dn}} = \nu_c \epsilon^{3/2}. \tag{10}$$

Now to express $\Delta\Phi$ and ΔQ in terms of ΔN_d to $O(\epsilon^2)$, ΔG of (8), Eqs. (6) and (7) are rewritten in the following form using Eq. (9):

$$f(\Phi_0 + \Delta\Phi, Q_0 + \Delta Q) = -\Delta_{\mathrm{ch}},$$

$$h(\Phi_0 + \Delta\Phi, Q_0 + \Delta Q, N_{d0} + \Delta N_d) = \Delta_{\mathrm{dis}}, \tag{11}$$

where

$$\Delta_{\mathrm{ch}} = \nu_d \beta_d^{-1} \epsilon^{1/2} e^{[(\Phi/\sigma_i) - z\Delta Q]} [\epsilon^{3/2}\partial_\tau + \epsilon^{1/2}(\Delta V_d - \Lambda)\partial_\xi] \Delta Q,$$

$$\Delta_{\mathrm{dis}} = \epsilon (\lambda_e / \lambda_d)^2 \partial_\xi^2 \Delta\Phi. \tag{12}$$

Taylor expansion of (11) yields

$$f_{\Phi_0}\Delta\Phi + f_{Q_0}\Delta Q + A + \Delta_{\rm ch} = 0,$$

$$h_{\Phi_0}\Delta\Phi + h_{Q_0}\Delta Q + h_{N_{d0}}\Delta N_d + B - \Delta_{\rm dis} = 0,$$

$$A = \tfrac{1}{2}[f_{\Phi_0\Phi_0}(\Delta\Phi)^2 + f_{Q_0Q_0}(\Delta Q)^2],$$

$$B = \tfrac{1}{2}[h_{\Phi_0\Phi_0}(\Delta\Phi)^2 + h_{Q_0N_{d0}}\Delta Q\Delta N_d]. \tag{13}$$

Solving this equation, we get

$$\Delta\Phi = J^{-1}[f_{Q_0}(h_{N_{d0}}\Delta N_d + B - \Delta_{\rm dis}) - h_{Q_0}(A + \Delta_{\rm ch})],$$

$$\Delta Q = J^{-1}[-f_{\Phi_0}(h_{N_{d0}}\Delta N_d + B - \Delta_{\rm dis}) + h_{\Phi_0}(A + \Delta_{\rm ch})], \tag{14}$$

where $J=\partial(f,h)/\partial(\Phi,Q)||_0$ is the Jacobian and $f_{y_0}=(\partial f/\partial y)_0$, $f_{z_0}=(\partial f/\partial z)_0$, $f_{y_0z_0}=f_{z_0y_0}=(\partial^2 f/\partial y\partial z)_0$, etc., and y, $z=\Phi$, N_d, or Q.

On using (8) and (9) and equating the term $O(\epsilon)$ and the terms $O(\epsilon^2)$, from (14) we get the following equations:

$$\Delta\Phi^{(1)} = J^{-1}f_{Q_0}h_{N_{d0}}\Delta N_d^{(1)}, \quad \Delta Q^{(1)} = -J^{-1}f_{\Phi_0}h_{N_{d0}}\Delta N_d^{(1)}, \tag{15}$$

$$\begin{aligned}\Delta\Phi^{(2)} = J^{-1}\{&\Lambda\nu_d\beta_d^{-1}h_{Q_0}\partial_\xi\Delta Q^{(1)} - f_{Q_0}(\lambda_e/\lambda_d)^2\partial_\xi^2\Delta\Phi^{(1)}\\ &+ [(f_{Q_0}h_{\Phi_0\Phi_0} - h_{Q_0}f_{\Phi_0\Phi_0})/2]\Delta\Phi^{(1)^2}\\ &- (f_{Q_0Q_0}h_{Q_0}/2)\Delta Q^{(1)^2} + f_{Q_0}h_{Q_0N_{d0}}\Delta Q^{(1)}\Delta N_d^{(1)}\\ &+ f_{Q_0}h_{N_{d0}}\Delta N_d^{(2)}\},\end{aligned} \tag{16}$$

$$\begin{aligned}\Delta Q^{(2)} = J^{-1}\{&-\Lambda\nu_d\beta_d^{-1}h_{\Phi_0}\partial_\xi\Delta Q^{(1)} + f_{\Phi_0}(\lambda_e/\lambda_d)^2\partial_\xi^2\Delta\Phi^{(1)}\\ &+ [(h_{\Phi_0}f_{\Phi_0\Phi_0} - f_{\Phi_0}h_{\Phi_0\Phi_0})/2]\Delta\Phi^{(1)^2}\\ &+ (h_{\Phi_0\Phi_0}f_{Q_0Q_0}/2)\Delta Q^{(1)^2} - f_{\Phi_0}h_{Q_0N_{d0}}\Delta Q^{(1)}\Delta N_d^{(1)}\\ &- f_{\Phi_0}h_{N_{d0}}\Delta N_d^{(2)}\}.\end{aligned} \tag{17}$$

Also using (8) and (9) and equating the term $O(\epsilon^{3/2})$ and the terms $O(\epsilon^{5/2})$, from (2) we get the following equations:

$$\Delta V_d^{(1)} = \Lambda\Delta N_d^{(1)} \tag{18}$$

and

$$\partial_\tau\Delta N_d^{(1)} + 2\Lambda\Delta N_d^{(1)}\partial_\xi\Delta N_d^{(1)} = \partial_\xi[\Lambda\Delta N_d^{(2)} - \Delta V_d^{(2)}]. \tag{19}$$

A. Hydrodynamic limit ($\omega_{\rm pd}\tau_m<1$)

In this case we assume that the dust oscillation period ($\sim\omega_{\rm pd}$) is less than the dust viscoelastic relaxation period ($\sim\tau_m^{-1}$), i.e., $\omega_{\rm pd}\tau_m<1$ but $\neq 0$. Applying (8) and (9), the GH equation (3) can be reexpressed in the following form:

$$\begin{aligned}F = \frac{1}{\sqrt{\epsilon}}[&1 + \omega_{\rm pd}\tau_m(-\Lambda + \epsilon\Delta V_d^{(1)}\\ &+ \cdots)\partial_\xi]^{-1}\{\eta_d\epsilon\partial_\xi^2[\epsilon\Delta V_d^{(1)} + \epsilon^2\Delta V_d^{(2)} + \cdots]\\ &- \epsilon^{3/2}\partial_\tau F\}.\end{aligned} \tag{20}$$

Now to maintain consistency in equating terms involving different powers of ϵ, we consider the following scalings:

$$\omega_{\rm pd}\tau_m = \nu_h\epsilon^{3/2}, \quad \eta_d = \eta\epsilon^{1/2}. \tag{21}$$

Finally using (18) and (21) and taking the inversion of the operator [Eq. (20)] and equating the terms $O(\epsilon^{3/2})$, we get

$$\Delta N_d^{(1)} = -[\sigma_d/(\Lambda^2 - \gamma_d\mu_d)]\Delta\Phi^{(1)}. \tag{22}$$

In this "hydrodynamical limit" the value of λ self-consistently follows from Eqs. (15) and (22):

$$\begin{aligned}\Lambda &= \pm\sqrt{\gamma_d\mu_d - J^{-1}\sigma_d f_{Q_0}h_{N_{d0}}} \Rightarrow \lambda\\ &= V_0 \pm \sqrt{\gamma_d\mu_d - J^{-1}\sigma_d f_{Q_0}h_{N_{d0}}}.\end{aligned} \tag{23}$$

Equating the terms $O(\epsilon^{5/2})$ and using (15) and (22), we get from (20)

$$\begin{aligned}&\Lambda\partial_\tau\Delta N_d^{(1)} - J^{-1}\sigma_d f_{Q_0}h_{N_{d0}}(1 + J^{-1}f_{\Phi_0}h_{N_{d0}})\Delta N_d^{(1)}\partial_\xi\Delta N_d^{(1)}\\ &\quad + \Lambda\nu_c\Delta N_d^{(1)}\\ &\quad = \Lambda\eta\partial_\xi^2\Delta N_d^{(1)} + \Lambda\partial_\xi\Delta V_d^{(2)}\\ &\qquad - \Lambda\gamma_d\mu_d\partial_\xi\Delta N_d^{(2)} + \sigma_d\partial_\xi\Delta\Phi^{(2)}.\end{aligned} \tag{24}$$

Eliminating all the second-order quantities from Eqs. (16), (17), (19), and (24), we get the following Korteweg–de Vries Burger (KdVB) equation with a damping term:

$$\begin{aligned}&\partial_\tau\Delta N_d^{(1)} + \alpha\Delta N_d^{(1)}\partial_\xi\Delta N_d^{(1)} + \beta\partial_\xi^3\Delta N_d^{(1)} + \gamma_c\Delta N_d^{(1)}\\ &\quad = \mu\partial_\xi^2\Delta N_d^{(1)},\end{aligned}$$

$$\mu = \mu_{\rm ch} + \mu_{\rm vis}, \tag{25}$$

where α, β, γ, and μ can be written in the following simplified form (after using the values of partial derivatives at their equilibrium values):

$$\begin{aligned}\alpha = \frac{1}{2\Lambda}\Bigg[&2\gamma_d\mu_d + \left(\frac{\lambda_p}{\lambda_c}\right)^2\frac{3\Gamma l_p^2}{[1+(\lambda_p/\lambda_c)^2]^3}\Bigg(\frac{(\delta-1)(\sigma_i^2-\delta)}{\sigma_i^2}\\ &+ \left(\frac{\lambda_e}{\lambda_c}\right)^4\left\{\frac{z^2\beta_d}{(z+\sigma_i)^2} + 3\left[1 + \left(\frac{\lambda_p}{\lambda_c}\right)^2\right]\right\}\Bigg)\Bigg],\end{aligned} \tag{26}$$

$$\beta = \frac{9}{2\Lambda}\frac{\Gamma^2 l_p^4}{[1+(\lambda_p/\lambda_c)^2]^2}, \quad \gamma_c = \frac{\nu_c}{2}, \tag{27}$$

$$\mu_{\rm ch} = \frac{3\nu_d}{2}\left(\frac{\lambda_e}{\lambda_c}\right)^2\frac{\Gamma l_p^2}{[1+(\lambda_p/\lambda_c)^2]^2}, \quad \mu_{\rm vis} = \frac{\eta}{2}, \tag{28}$$

where $\lambda_c=\sqrt{\sigma_i(1+z+\sigma_i)/[4\pi r_0 n_{d0}(z+\sigma_i)(1+\sigma_i)]}$ is the dust space-charge self-shielding length arising due to the dust charge variation,[21] and $l_p=\lambda_p/a_d$. The damping term γ_c present in Eq. (26) is due to dust-neutral collision as $\gamma_c\propto\nu_c$. The Burger term $\mu(=\mu_{\rm ch}+\mu_{\rm vis})$ present in Eq. (26) is partly due to delay in the charging processes and partly due to the dust viscosity. In terms of actual parameters [by virtue of (4), (10), and (21)] the ratio $\mu_{\rm ch}$ to $\mu_{\rm vis}$ becomes

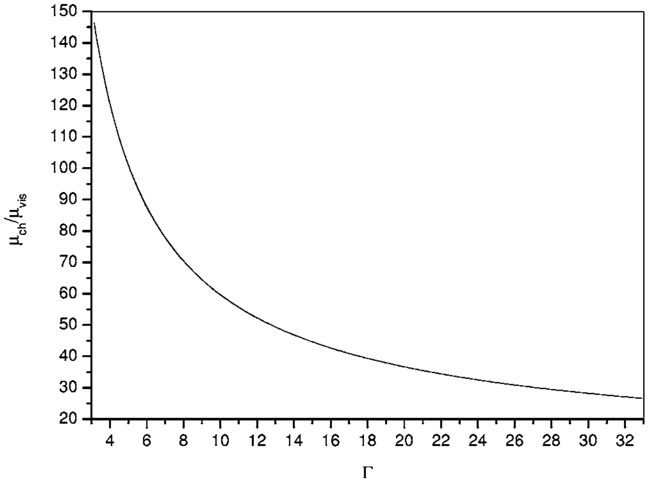

FIG. 1. Variation of $\mu_{\rm ch}/\mu_{\rm vis}$ [Eq. (29)] with Γ for $\delta<1.4$.

$$\frac{\mu_{\rm ch}}{\mu_{\rm vis}} \approx 50\frac{\omega_{\rm pd}}{\nu_{\rm ch}}\left(\frac{\lambda_e}{\lambda_c}\right)^2\frac{l_p^2}{[1+(\lambda_p/\lambda_c)^2]^2}\frac{1}{\sqrt{\Gamma}}. \tag{29}$$

This shows that with the increase of Γ the ratio decreases, as shown in Fig. 1. If the other plasma parameters remain constant, $\mu_{\rm ch}/\mu_{\rm vis}\propto 1/\sqrt{\Gamma}$. Next we consider the "kinetic limit" case.

B. Kinetic limit ($\omega_{\rm pd}\tau_m>1$)

In the strong coupling case, we assume that the dust oscillation period is greater than the dust viscoelastic relaxation period, i.e., $1/\omega_{\rm pd}\tau_m<1$ but $\neq 0$. Due to this assumption, applying (8) and (9), the GH equation (3) can be reexpressed in the following form:

$$F=\frac{1}{\sqrt{\epsilon}}\{[-\Lambda+\epsilon\Delta V_d^{(1)}+\cdots]\partial_\xi\}^{-1}\left\{l\epsilon\partial_\xi^2[\epsilon\Delta V_d^{(1)}+\epsilon^2\Delta V_d^{(2)}+\cdots]-\left(\frac{1}{\omega_{\rm pd}\tau_m}+\epsilon^{3/2}\partial_\tau\right)F\right\}, \tag{30}$$

where $l=\eta_d/\omega_{\rm pd}\tau_m=1-\gamma_d\mu_d\frac{4}{15}u(\Gamma)$. In fact, in this case instead of viscous dissipation, the viscosity coefficient η_d contributes to an elastic restoring force $\propto l$.[5] Under the assumption $\omega_{\rm pd}\tau_m>1$, the values of the plasma parameters quoted in Sec. III show that for $\delta\sim 1.6$, $\Gamma\sim 47$ and $\eta_d=3\eta^*\Gamma\sim 0.06\Gamma^{3/2}\sim 19$ or for $\delta\sim 2$, $\Gamma\sim 105$ and $\eta_d\sim 65$. Hence we can easily assume that η_d is large enough so that l is a finite quantity. Also to make the perturbation expansion consistent with that in (8) we consider the following scaling:

$$1/\omega_{\rm pd}\tau_m=\nu_k\epsilon^{3/2}. \tag{31}$$

Finally using (18) and (31) and taking the inversion of the operator [Eq. (30)] and equating the terms $O(\epsilon^{3/2})$, we get

$$\Delta N_d^{(1)}=-[\sigma_d/(\Lambda^2-l-\gamma_d\mu_d)]\Delta\Phi^{(1)}. \tag{32}$$

In this limit the value of λ self-consistently follows from the above Eqs. (15) and (32):

$$\Lambda=\pm\sqrt{l+\gamma_d\mu_d-J^{-1}\sigma_d f_{Q_0}h_{N_{d0}}}\Rightarrow\lambda=V_0\pm\sqrt{l+\gamma_d\mu_d-J^{-1}\sigma_d f_{Q_0}h_{N_{d0}}}. \tag{33}$$

Equating the terms $O(\epsilon^{5/2})$ and using (15) and (19), we get from (30):

$$(\Lambda^2+l)\partial_\tau\Delta N_d^{(1)}+\Lambda[l-J^{-1}\sigma_d f_{Q_0}h_{N_{d0}}(1+J^{-1}f_{\Phi_0}h_{N_{d0}})]\Delta N_d^{(1)}\partial_\xi\Delta N_d^{(1)}+(\Lambda^2\nu_c+l\nu_k)\Delta N_d^{(1)}=\partial_\xi((\Lambda^2-l)\Delta V_d^{(2)}-\Lambda\gamma_d\mu_d\Delta N_d^{(2)}+\Lambda\sigma_d\Delta\Phi^{(2)})+\Lambda l\int_{-\infty}^{\xi}(\partial_{\acute{\xi}}\Delta N_d^{(1)})^2 d\acute{\xi}. \tag{34}$$

Eliminating all the second-order quantities from Eqs. (16), (17), (19), and (34), we get the following KdVB equation with a damping and a nonlocal nonlinear forcing term:

$$\partial_\tau\Delta N_d^{(1)}+\alpha\Delta N_d^{(1)}\partial_\xi\Delta N_d^{(1)}+\beta\partial_\xi^3\Delta N_d^{(1)}+\gamma\Delta N_d^{(1)}=\mu_{\rm ch}\partial_\xi^2\Delta N_d^{(1)}+\mu_\Gamma\int_{-\infty}^{\xi}(\partial_{\acute{\xi}}\Delta N_d^{(1)})^2 d\acute{\xi}, \tag{35}$$

where

$$\alpha=\frac{1}{2\Lambda}\left[l+2\gamma_d\mu_d+\left(\frac{\lambda_p}{\lambda_c}\right)^2\frac{3\Gamma l_p^2}{[1+(\lambda_p/\lambda_c)^2]^3}\left(\frac{(\delta-1)(\sigma_i^2-\delta)}{\sigma_i^2}+\left(\frac{\lambda_e}{\lambda_c}\right)^4\left\{\frac{z^2\beta_d}{(z+\sigma_i)^2}+3\left[1+\left(\frac{\lambda_p}{\lambda_c}\right)^2\right]\right\}\right)\right], \tag{36}$$

$$\beta=\frac{9}{2\Lambda}\frac{\Gamma^2 l_p^4}{[1+(\lambda_p/\lambda_c)^2]^2},\quad \gamma=\frac{\nu_c}{2}+\frac{\nu_k}{2\Lambda^2}, \tag{37}$$

$$\mu_{\rm ch}=\frac{3\nu_d}{2}\left(\frac{\lambda_e}{\lambda_c}\right)^2\frac{\Gamma l_p^2}{[1+(\lambda_p/\lambda_c)^2]^2},\quad \mu_\Gamma=\frac{l}{2\Lambda}. \tag{38}$$

The damping term γ present in Eq. (37) arises partly due to the dust-neutral collision (under the assumption that the dust-nerutral collision frequency is less than the dust plasma frequency) [Eq. (10)] and partly due to finite strain relaxation time τ_m, i.e., $(\omega_{\rm pd}\tau_m)^{-1}<1$ but $\neq 0$ [Eq. (31)]. The Burger term present in Eq. (35) is entirely due to delay in the charging processes as the term $\mu_{\rm ch}\propto\nu_d$ [Eqs. (10) and (38)]. The coefficient μ_Γ of the second term on the right-hand side of (38) shows that the memory-dependent strong correlation effect introduces a nonlocal nonlinear forcing term in the nonlinear propagation of LDAW in the "kinetic regime" of strongly coupled dusty plasma through the occurence of the term $l=\eta_d/\omega_{\rm pd}\tau_m$ in the expression for the wave velocity λ given by Eq. (33).

This is in contrast with the situation in the hydrodynamical limit where the dissipation is partly due to charging delay and partly due to viscosity. Here there is no appreaciable viscous dissipation; the viscoelastic force plays the role of a restoring force. This is reflected through the appearence of the term l in the expressions for the wave velocity [Eq. (33)].

III. NUMERICAL SOLUTION

To estimate the numerical values of different plasma parameters we use the gas discharge laboratory plasma conditions:[9,18] $T_e \sim 9$ eV, $T_i = T_d = T_n \sim 0.026$ eV, $n_{i0} \sim 5 \times 10^{16}$ m^{-3}, $m_i = m_n \sim 6.69 \times 10^{-26}$ kg, $r_0 \sim 1$ μm, the dust mass density $\rho_d \sim 2.5 \times 10^3$ kg m^{-3}, and neutral gas pressure ~ 75 mtorr ($\sim$neutral density $n_n \sim 2.4 \times 10^{21}$ m^{-3}). For the above set of values of the plasma parameters $\omega_{\text{pd}}/\nu_{\text{ch}} \sim 10^{-3}$ and $\nu_{\text{dn}} \sim 10^{-4}$ which agrees with the scaling (10).

For later use we express the coupling parameter Γ in terms of $\delta = n_{i0}/n_{e0}$ and $z = z_{d0}e^2/4\pi\epsilon_0 r_0 T_e$,

$$\Gamma = \frac{z_{d0}{}^2 e^2}{4\pi\epsilon_0 a T_d} e^{-a/\lambda_p} = z(\delta)(\delta - 1)\frac{n_{i0}}{n_{d0}}\frac{r_0}{a} e^{-a/\lambda_p}, \tag{39}$$

where δ and $z(\delta)$ are related by $\delta = \sqrt{m_i/m_e}[\sqrt{\sigma}/(\sqrt{\sigma}+z)]e^{-z}$ in accordance with current balance $I_{i0} + I_{e0} = 0$.

The modified form of KdVB equations (25) and (35) is not analytically exactly solvable. To find the numerical solutions of (25) and (35), we transform these equations to the wave frame $\chi = V_0\tau + \xi$ and after integrating twice we obtain the following second-order ordinary integrodifferential equations, which in the hydrodynamical and kinetic cases are, respectively:

$$\{d_\chi^2 + (\mu/\beta)d_\chi - [V_0/\beta - (\alpha/2\beta)\Delta N_d^{(1)}]\}\Delta N_d^{(1)} = (\gamma_c/\beta)\int_{-\infty}^{\chi} \Delta N_d^{(1)} d\acute{\chi} \tag{40}$$

and

$$\left\{d_\chi^2 + \frac{\mu_{\text{ch}}}{\beta}d_\chi - \left[\frac{V_0}{\beta} - \frac{\alpha}{2\beta}\Delta N_d^{(1)}\right]\right\}\Delta N_d^{(1)} = \frac{\gamma}{\beta}\int_{-\infty}^{\chi}\Delta N_d^{(1)} d\acute{\chi} + \frac{\mu_\Gamma}{\beta}\int_{-\infty}^{\chi}\int_{-\infty}^{\acute{\chi}} (\partial_{\acute{\chi}}\Delta N_d^{(1)})^2 d\chi d\acute{\chi}, \tag{41}$$

where we have imposed the boundary conditions, viz., $\Delta N_d^{(1)} \to 0, d_\chi \Delta N_d^{(1)} \to 0, d_\chi^2 \Delta N_d^{(1)} \to 0$ at $\chi \to -\infty$.

A. Hydrodynamical case

Numerical calculations using Eq. (4) with η^* and μ_d given by Eq. (4) show that we are in the hydrodynamical regime for $\Gamma \leqslant 30$. In this case the scaling (21) is justified. Moreover Eq. (39) shows that for plasma parameters stated above the hydrodynamical regime corresponds to $\delta < 1.4$.

The variation of $\mu_{\text{ch}}/\mu_{\text{vis}}$, the ratio of the dissipation due to charging-delay effect and that due to dust fluid viscosity as given by Eq. (29), with respect to the dust coupling parameter Γ is shown in Fig. 1. The results indicate the predominance of charging-delay-induced dissipation effect.

In the hydrodynamical case $\Delta N_d^{(1)}$ is obtained from numerical integration of Eq. (40) subject to the stated boundary conditions. For any value of X and small values of γ/β, the number density builds up from a near-zero value at long past $T \to -\infty(\chi \to -\infty)$ to a steady value $\Delta N_d^{(1)} \approx 2(M-1)\lambda/\alpha$ as $T \to \infty$, where $M = 1 + \epsilon(V_0/\lambda)$ is the Mach number. Equation (23) shows that if the unperturbed dust fluid velocity V_0 at $\chi \to -\infty$ exceeds the wave velocity λ, the dust density $N_{d0} + \Delta N_d^{(1)}$ increases from its initial value with a concomitant decrease of the dust fluid velocity $V_0 + \Delta V_d^{(1)}$ since $\Delta V_d^{(1)}$ and $\Delta N_d^{(1)}$ related by Eq. (18) are of opposite algebraic sign when $\Lambda < 0$. This is a consequence of the conservation of dust fluid mass flow. The shock wave generated by imparting a velocity V_0 to the dust fluid at $\chi \to -\infty$ is found to lead to an oscillatory buildup of the dust number density, ultimately reaching a steady level at $\chi \to \infty$. This is shown in Fig. 2.

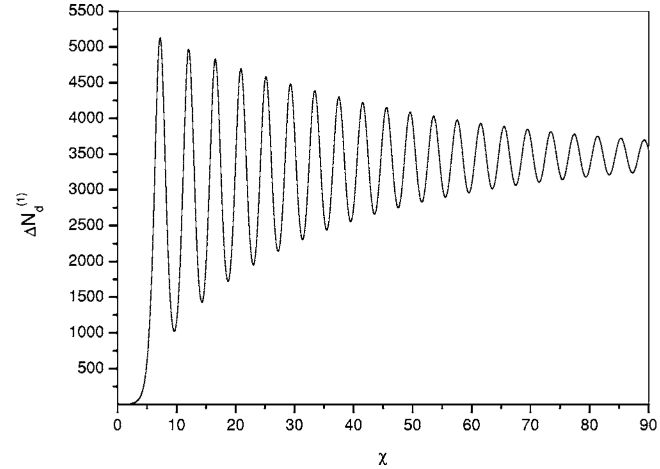

FIG. 2. Solution of $\Delta N_d^{(1)}$ for χ [Eq. (40)]. The curve is drawn for the following parameters: $\delta \sim 1.2$, $\Gamma \sim 3$, $\nu_d \sim 0.01$, $\nu_c \sim 0.01$, and $\eta \sim 0.01$.

B. Kinetic case

The kinetic regime defined by $\omega_{\text{pd}}\tau_m > 1$ corresponds according to Eq. (4) to $\Gamma > 30$ and consequently for the given plasma parameters to $\delta > 1.4$ [Eq. (39)].

In this case $\Delta N_d^{(1)}$ is obtained by numerical integration of Eq. (41) subject to the boundary conditions stated earlier. Here also as in the hydrodynamic regime shock wave results provided the unperturbed dust fluid velocity $V_0 >$ dust acoustic wave velocity λ. The dust number density flux conservation causes mounting of the dust number density if the operation of some braking mechanism can lead to the decrease of the dust fluid velocity. Up to this point the behavior of the dust fluid motion in the hydrodynamic and the kinetic regimes are similar. But there is one important point of difference. The braking effect on the dust fluid motion caused by the dissipative force represented by $(\mu/\beta)d_\chi \Delta N_d^{(1)}$ [Eqs. (40) and (41)]. In the hydrodynamic case $\mu = \mu_{\text{ch}} + \mu_{\text{vis}}$ so that dissipation is partly due to viscous drag and partly due to the charging-delay effect. In the kinetic regime, however, the dust acoustic wave does not experience viscous dissipation (except only a weak one) as in the hydrodynamic regime. Instead of dissipation the shear/bulk viscosity gives rise to a restoring force leading to possible generation of transverse mode/affecting longitudinal mode.[5] Thus in the kinetic regime dissipation is caused by charging delay alone so that $\mu = \mu_{\text{ch}}$ [Eq. (40)]. There is no shock generation in the kinetic case if $\mu_{\text{ch}} = 0$. This is illustrated by the dotted curve in Fig. 3. Note that we have set $\nu_d = 0$ (leading to $\mu_{\text{ch}} = 0$) but $\nu_c \neq 0$, i.e., dust-neutral collision is not neglected [Eqs. (3) and (10)]. There arises only a nonlinear wave train and no shock.

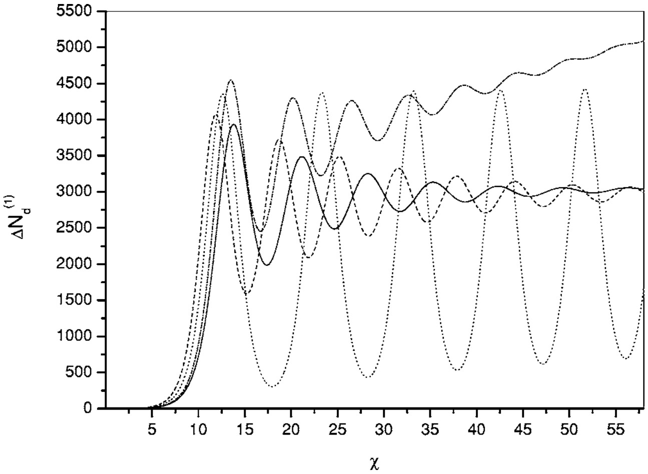

FIG. 3. Solution of $\Delta N_d^{(1)}$ for χ [Eq. (41)]. The curves are drawn for the following parameters. Solid curve: $\delta\sim 2$, $\Gamma\sim 105$, $\nu_d\sim 0.1$, $\nu_c\sim 0.01$, $\nu_k\sim 0.01$, and $\mu_\Gamma\sim 0.4$; dotted curve: $\delta\sim 2$, $\Gamma\sim 105$, $\nu_d\sim 0$, $\nu_c\sim 0.01$, and $\mu_\Gamma\sim 0.4$; dash-dotted curve: $\delta\sim 2$, $\Gamma\sim 105$, $\nu_d\sim 0.001$, ν_c=0.01, $\nu_k\sim 0.01$, and $\mu_\Gamma\sim 10$; and dash curve: $\delta\sim 1.6$, $\Gamma\sim 47$, $\nu_d\sim 0.1$, $\nu_c\sim 0.01$, $\nu_k\sim 0.01$, and $\mu_\Gamma\sim 0.1$.

This is in contrast with the situation in the hydrodynamic case where $\mu=\mu_{\text{vis}}\neq 0$ even when $\mu_{\text{ch}}=0$ and thus demonstrates the role of charging delay in the shock generation—one of the main conclusions of the present paper. A second important conclusion is that the dust-neutral collisions play no role in shock generation. This conclusion is emphasized also by the dotted curve in Fig. 3 where we have taken $\mu_{\text{ch}}=0$ but $\nu_c\neq 0$ (hence dust-neutral collision $\nu_{\text{dn}}\neq 0$). The other curves in Fig. 3 also support the above conclusion. The damping only compensates the possible growth at the expense of the activation energy (nonlocal and non-Markovian in nature) stored in the process of rearrangement of dust grains associated with correlation effect and represented by the term $\propto\mu_\Gamma$. Thus if the dust fluid be imported a velocity V_0 in excess dust acoustic wave velocity λ, the charging-delay-induced dissipation be sufficiently strong and the plasma parameters are in an approximate range, then shock wave generation in the kinetic regime may be observed in the laboratory.

IV. DISCUSSIONS

In this paper we have investigated the physical mechanism of the generation of shock wave in strongly coupled collisional dusty plasma using the GH model incorporating a charging-delay effect. It should be noted that in the hydrodynamical case the shock generating dissipations are due to the dust viscous damping and charging delay. The physical mechanism for the formation of shock structure found in the hydrodynamical case is somewhat similar to that of Ref. 22 where shock was collisional in a weakly coupled dusty plasma.[23] In the kinetic case it is completely different from that of Ref. 22 because of the fact that the longitudinal dust acoustic shock structure found in the kinetic case is only due to charging delay, which is collisionless in the sense that no viscous damping effects resulting from collisions between dust-dust or dust-plasma particles are involved since in the kinetic regime of strongly coupled dusty plasma the LDAW does not suffer any viscous damping.[5] In both the cases, the LDASW observed here is of compressional type showing a considerable increase of the dust density which is of significant importance in astrophysical plasma as in the presence of gravity field, it may lead to enhanced gravitational attraction considered as a viable process for star formation. Thus our present investigation has also important application in astrophysical plasma as many astrophysical plasmas are strongly coupled.[19]

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