

**CHARACTERIZATIONS OF QUIET QUASI-UNIFORMITY
AND ALMOST QUIET QUASI-UNIFORMITY
IN TERMS OF COVERS**

BY

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Abstract. In this paper quiet quasi-uniformity and almost quiet quasi-uniformity are characterized in terms of strong quasi-uniform covers. Also some applications of this type of covers are given in the last section.

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Key words: Quiet quasi-uniform space, almost quiet quasi-uniform space, strong quasi-uniform cover.

1. Introduction. In [5] PERVIN proved that every topological space is quasi-uniformizable in the sense that τ is a topology on a set X if and only if there exists a quasi-uniformity \mathcal{V} on X , which induces τ . Later in [4] MUKHERJEE and DEBRAY introduced strong quasi-uniform covers and characterized quasi-uniformity in terms of strong quasi-uniform covers.

In this paper, our aim is to characterize quiet quasi-uniformity, introduced by DOITCHINOV in [1], and almost quiet quasi-uniformity introduced by GANGULY, DUTTA and CHATTOPADHYAY in [3] in terms of quasi-uniform covers. To characterize both quiet quasi-uniformity and almost quiet quasi-uniformity we use strong quasi-uniform covers suitably. We shall also derive characterizations of some topological concepts in terms of covers for quasi-uniform spaces.

Throughout the paper, for $\text{int}(cl(A))$ where $A \subseteq X$, we shall use the notation \bar{A} .

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Definition 1.1 ([1]). A quasi-uniform space (X, \mathcal{U}) is called a quiet quasi-uniform space if the following property hold : for any $U \in \mathcal{U}$, there exists a $V \in \mathcal{U}$ such that, if $x', x'' \in X$ and $\{x'_\alpha : \alpha \in A\}$ and $\{x''_\beta : \beta \in B\}$ are two nets in X , then from $(x', x'_\alpha) \in V$ for $\alpha \in A$, $(x''_\beta, x'') \in V$ for $\beta \in B$ and $(x''_\beta, x'_\alpha) \rightarrow 0$ [i.e., for any $W \in \mathcal{U}$, there exist α_W and β_W belonging to A and B respectively such that $(x''_\beta, x'_\alpha) \in W$ for $\alpha \geq \alpha_W$, $\beta \geq \beta_W$] it follows that $(x', x'') \in U$. When a $V \in \mathcal{U}$ is connected with some $U \in \mathcal{U}$ by the above property, we then say that V is Q -subordinated to U .

Definition 1.2 ([3]). A topological space (X, τ) is said to be almost quiet quasi-uniformizable if and only if there exists a compatible quasi-uniformity \mathcal{U} with the following properties : for $U \in \mathcal{U}$ and $x \in X$, there exists $V_x \in \mathcal{U}$ for which the following conditions hold : if $\{x_\alpha : \alpha \in A\}$ & $\{y_\beta : \beta \in B\}$ be two nets such that $(x, x_\alpha) \in V_x$ for $\alpha \in A$, $(y_\beta, y) \in V_x$ (for some $y \in X$), for $\beta \in B$, and $(y_\beta, x_\alpha) \rightarrow 0$, then $y \in \overline{U(x)}$, where the closure and the interior of $U(x)$ and $\overline{U(x)}$ respectively are taken under the topology τ ; we call V_x subordinated to U with respect to x .

Definition 1.3 ([4]). A cover \mathcal{U} of a quasi-uniform space (X, \mathcal{V}) is said to be a strong quasi-uniform cover, if there exists some $V \in \mathcal{V}$ such that for each $x \in X$, $V(x) \subseteq \cap\{H \in \mathcal{U} : x \in H\}$, where as usual $V(x) = \{y \in X : (x, y) \in V\}$.

2. Characterization of quiet quasi-uniformity in terms of strong quasi-uniform covers. In this article we shall use strong quasi-uniform covers to characterize quiet quasi-uniformity.

Theorem 2.1. Let (X, \mathcal{V}) be a quiet quasi-uniform space and \mathcal{C} be the collection of all strong quasi-uniform covers of X . Then the followings hold:

(i) If $\mathcal{U} \in \mathcal{C}$ and \mathcal{U}_1 is a cover of X such that for each $x \in X$, $\cap\{H \in \mathcal{U} : x \in H\} \subseteq \cap\{K \in \mathcal{U}_1 : x \in K\}$, then $\mathcal{U}_1 \in \mathcal{C}$.

(ii) If $\mathcal{U}_1, \mathcal{U}_2 \in \mathcal{C}$, then there is a $\mathcal{U} \in \mathcal{C}$ such that $\cap\{H \in \mathcal{U} : x \in H\} \subseteq \cap\{K \in \mathcal{U}_1 : x \in K\}$ and $\cap\{H \in \mathcal{U} : x \in H\} \subseteq \cap\{K \in \mathcal{U}_2 : x \in K\}$, for each $x \in X$.

(iii) For $\mathcal{U} \in \mathcal{C}$, if $\{x_\alpha : \alpha \in A\}$ and $\{y_\beta : \beta \in B\}$ be two nets in (X, \mathcal{V}) such that $x_\alpha \in \cap\{H \in \mathcal{U} : x \in H\}$, $y_\beta \in \cap\{K \in \mathcal{U} : y \in K\}$ for $\alpha \in A$, $\beta \in B$ and $(y_\beta, x_\alpha) \rightarrow 0$, then $y \in \cap\{H \in \mathcal{U} : x \in H\}$.

Conversely, let X be a set and \mathcal{C} be the collection of all covers of X satisfying conditions (i), (ii) and (iii). Then there exists some quiet quasi-

uniformity \mathcal{V} on X w.r.t. which \mathcal{C} is precisely the collection of all strong quasi-uniform covers of X .

Proof. Let \mathcal{C} be the collection of all strong quasi-uniform covers of a quiet quasi-uniform space (X, \mathcal{V}) .

(i) If $\mathcal{U} \in \mathcal{C}$, then there exists $V \in \mathcal{V}$ such that

$$V(x) \subseteq \cap\{H \in \mathcal{U} : x \in H\} \subseteq \cap\{K \in \mathcal{U}_1 : x \in K\}, \text{ for each } x \in X.$$

Hence $\mathcal{U}_1 \in \mathcal{C}$.

(ii) Let $\mathcal{U}_1, \mathcal{U}_2 \in \mathcal{C}$. Then there exist $V, W \in \mathcal{V}$ such that $V(x) \subseteq \cap\{H \in \mathcal{U}_1 : x \in H\}$ and $W(x) \subseteq \cap\{K \in \mathcal{U}_2 : x \in K\}$ for each $x \in X$. Consider $\mathcal{U} = \mathcal{U}_1 \cup \mathcal{U}_2$. Then we get, $(V \cap W)(x) \subseteq \cap\{H \in \mathcal{U} : x \in H\}$, for each $x \in X$.

For, $y \in (V \cap W)(x) \Rightarrow y \in V(x)$ and $y \in W(x) \Rightarrow y \in \cap\{H \in \mathcal{U}_1 : x \in H\}$ and $y \in \cap\{K \in \mathcal{U}_2 : x \in K\} \Rightarrow y \in \cap\{H \in \mathcal{U}_1 \cup \mathcal{U}_2 : x \in H\}$. Now, \mathcal{V} being a quasi-uniformity on X , $V \cap W = U$ (say) $\in \mathcal{V}$. Thus \mathcal{U} is a strong quasi-uniform cover. Now, $\cap\{H \in \mathcal{U}_1 \cup \mathcal{U}_2 : x \in H\} \subseteq \cap\{K \in \mathcal{U}_1 : x \in K\}$ and $\cap\{H \in \mathcal{U}_1 \cup \mathcal{U}_2 : x \in H\} \subseteq \cap\{K \in \mathcal{U}_2 : x \in K\}$ for each $x \in X$.

(iii) Given that $\mathcal{U} \in \mathcal{C}$ and $\{x_\alpha : \alpha \in A\}$, $\{y_\beta : \beta \in B\}$ are two nets in (X, \mathcal{V}) such that $x_\alpha \in \cap\{H \in \mathcal{U} : x \in H\}$, $y_\beta \in \cap\{K \in \mathcal{U} : y \in K\}$ for $\alpha \in A$, $\beta \in B$ with $(y_\beta, x_\alpha) \rightarrow 0$. Since \mathcal{U} is a strong quasi-uniform cover, there exists $U \in \mathcal{V}$ such that $U(x) \subseteq \cap\{H \in \mathcal{U} : x \in H\}$ for each $x \in X$. Choose $V \in \mathcal{V}$ subordinated to U . Then $(x, x_\alpha) \in V$, $(y_\beta, y) \in V$ with $(y_\beta, x_\alpha) \rightarrow 0$ will give $(x, y) \in U \Rightarrow y \in U(x) \subseteq \cap\{H \in \mathcal{U} : x \in H\}$, i.e., $y \in \cap\{H \in \mathcal{U} : x \in H\}$.

For the converse part, let \mathcal{C} be the collection of all covers of X satisfying conditions (i), (ii) and (iii). For each $\mathcal{U} \in \mathcal{C}$, define $V(\mathcal{U}) = \cup\{(\{x\} \times [\cap\{H \in \mathcal{U} : x \in H\}]) : x \neq x_\alpha, y_\beta \text{ for } \alpha \in A, \beta \in B\}$ and $W(\mathcal{U}) = \cup\{([\cap\{K \in \mathcal{U} : y \in K\}] \times \{y\}) : y \neq x_\alpha, y_\beta \text{ for } \alpha \in A, \beta \in B\}$.

We first show that the collection $\mathcal{B} = \{V(\mathcal{U}) \cup W(\mathcal{U}) : \mathcal{U} \in \mathcal{C}\}$ forms a base for some quiet quasi-uniformity on X . Obviously $\Delta \subseteq V(\mathcal{U}) \cup W(\mathcal{U})$ for each $\mathcal{U} \in \mathcal{C}$, where Δ is the diagonal in $X \times X$. Next let, $V(\mathcal{U}_1) \cup W(\mathcal{U}_1)$, $V(\mathcal{U}_2) \cup W(\mathcal{U}_2) \in \mathcal{B}$. Then $\mathcal{U}_1, \mathcal{U}_2 \in \mathcal{C}$. By condition (ii), there exists $\mathcal{U} \in \mathcal{C}$ such that $\cap\{H \in \mathcal{U} : x \in H\} \subseteq \cap\{K \in \mathcal{U}_1 : x \in K\}$ and $\cap\{H \in \mathcal{U} : x \in H\} \subseteq \cap\{K \in \mathcal{U}_2 : x \in K\}$ for each $x \in X$. Let $(x, y) \in V(\mathcal{U}) \cup W(\mathcal{U})$. Then $(x, y) \in V(\mathcal{U})$ or $(x, y) \in W(\mathcal{U})$. If $(x, y) \in V(\mathcal{U})$, then $(x, y) \in \{x\} \times [\cap\{H \in \mathcal{U} : x \in H\}] \Rightarrow y \in \cap\{H \in \mathcal{U} : x \in H\} \subseteq \cap\{K \in \mathcal{U}_1 : x \in K\}$ and $y \in \cap\{H \in \mathcal{U} : x \in H\} \subseteq \cap\{K \in \mathcal{U}_2 : x \in K\}$. Thus $(x, y) \in V(\mathcal{U}_1) \cap V(\mathcal{U}_2)$.

If $(x, y) \in W(\mathcal{U})$ then $(x, y) \in [\cap\{H \in \mathcal{U} : y \in H\}] \times \{y\} \Rightarrow x \in \cap\{H \in \mathcal{U} : y \in H\} \subseteq \cap\{K \in \mathcal{U}_1 : y \in K\}$ and $x \in \cap\{H \in \mathcal{U} : y \in H\} \subseteq \cap\{K \in \mathcal{U}_2 : y \in K\}$. Thus $(x, y) \in W(\mathcal{U}_1) \cap W(\mathcal{U}_2)$. Thus $(x, y) \in (V(\mathcal{U}_1) \cap V(\mathcal{U}_2)) \cup (W(\mathcal{U}_1) \cap W(\mathcal{U}_2)) \subseteq (V(\mathcal{U}_1) \cup W(\mathcal{U}_1)) \cap (V(\mathcal{U}_2) \cup W(\mathcal{U}_2))$. Hence $V(\mathcal{U}) \cup W(\mathcal{U}) \subseteq (V(\mathcal{U}_1) \cup W(\mathcal{U}_1)) \cap (V(\mathcal{U}_2) \cup W(\mathcal{U}_2))$.

Next we prove that \mathcal{B} forms a base for some quiet quasi-uniformity on X . Let $\{x_\alpha : \alpha \in A\}$, $\{y_\beta : \beta \in B\}$ be two nets in X with $(x, x_\alpha) \in V(\mathcal{U}) \cup W(\mathcal{U})$, $(y_\beta, y) \in V(\mathcal{U}) \cup W(\mathcal{U})$ and $(y_\beta, x_\alpha) \rightarrow 0$. Then $x_\alpha \in \cap\{H \in \mathcal{U} : x \in H\}$, $y_\beta \in \cap\{K \in \mathcal{U} : y \in K\}$ with $(y_\beta, x_\alpha) \rightarrow 0$. Using condition (iii), we get, $y \in \cap\{H \in \mathcal{U} : x \in H\}$, i.e., $(x, y) \in V(\mathcal{U}) \cup W(\mathcal{U})$. Thus our assertion is proved.

Now, for any $\mathcal{U} \in \mathcal{C}$ and any $x \in X$ we have, $V(\mathcal{U})(x) = \{H \in \mathcal{U} : x \in H\}$ so that \mathcal{U} is a strong quasi-uniform cover of X . For the converse, let \mathcal{U} be any strong quasi-uniform cover. Then there is $V(\mathcal{U}_1) \cup W(\mathcal{U}_1) \in \mathcal{B}$ for some $\mathcal{U}_1 \in \mathcal{C}$ such that $(V(\mathcal{U}_1) \cup W(\mathcal{U}_1))(x) \subseteq \cap\{H \in \mathcal{U} : x \in H\}$, for each $x \in X$, i.e., $V(\mathcal{U}_1)(x) \subseteq \cap\{H \in \mathcal{U} : x \in H\}$ for each $x \in X$. Now as $\mathcal{U}_1 \in \mathcal{C}$, we have by condition (i), $\mathcal{U} \in \mathcal{C}$. Same is the case for $W(\mathcal{U})(x)$. This completes the proof. \square

3. Characterization of almost quiet quasi-uniformity in terms of strong quasi-uniform covers. Here we characterize almost quiet quasi-uniformity in terms of strong quasi-uniform covers.

Theorem 3.1. *Let (X, \mathcal{V}) be an almost quiet quasi-uniform space and \mathcal{C} be the collection of all strong quasi-uniform covers of X . Then the followings hold:*

(i) *If $\mathcal{U} \in \mathcal{C}$ and \mathcal{U}_1 is a cover of X such that for each $x \in X$, $\cap\{H \in \mathcal{U} : x \in H\} \subseteq \cap\{K \in \mathcal{U}_1 : x \in K\}$, then $\mathcal{U}_1 \in \mathcal{C}$.*

(ii) *If $\mathcal{U}_1, \mathcal{U}_2 \in \mathcal{C}$, then there exists $\mathcal{U} \in \mathcal{C}$ such that $\cap\{H \in \mathcal{U} : x \in H\} \subseteq \cap\{K \in \mathcal{U}_1 : x \in K\}$ and $\cap\{H \in \mathcal{U} : x \in H\} \subseteq \cap\{K \in \mathcal{U}_2 : x \in K\}$, for each $x \in X$.*

(iii) *For $\mathcal{U} \in \mathcal{C}$, if $\{x_\alpha : \alpha \in A\}$ and $\{y_\beta : \beta \in B\}$ be two nets in (X, \mathcal{V}) such that $x_\alpha \in \cap\{H \in \mathcal{U} : x \in H\}$, $y_\beta \in \cap\{K \in \mathcal{U} : y \in K\}$ for $\alpha \in A$, $\beta \in B$ and $(y_\beta, x_\alpha) \rightarrow 0$, then $y \in \cap\{H \in \mathcal{U} : x \in H\}$.*

Conversely, let X be a set and \mathcal{C} be the collection of all covers of X satisfying conditions (i), (ii) and (iii). Then there exists some almost quiet quasi-uniformity \mathcal{V} on X w.r.t. which \mathcal{C} is precisely the collection of all strong quasi-uniform covers of X .

Proof. The proof of the necessity part can be done similarly as for Theorem 2.1.

For the sufficient part, we define for each $\mathcal{U} \in \mathcal{C}$, $V(\mathcal{U}) = \cup\{(\{x\} \times [\cap\{H \in \mathcal{U} : x \in H\}]) : x \neq x_\alpha, y_\beta \text{ for } \alpha \in A, \beta \in B\}$ and $W(\mathcal{U}) = \cup\{([\cap\{K \in \mathcal{U} : x \in K\}] \times \{y\}) : y \neq x_\alpha, y_\beta \text{ for } \alpha \in A, \beta \in B\}$.

Define $\mathcal{B} = \{V(\mathcal{U}) \cup W(\mathcal{U}) : \mathcal{U} \in \mathcal{C}\}$. Then as in Theorem 2.1, it can be proved that \mathcal{B} is a base for some quasi-uniformity \mathcal{V} on X . We only prove that \mathcal{V} is an almost quiet quasi-uniformity on X . For this, let $\{x_\alpha : \alpha \in A\}$, $\{y_\beta : \beta \in B\}$ be two nets in X with $(x, x_\alpha) \in V(\mathcal{U}) \cup W(\mathcal{U})$, $(y_\beta, y) \in V(\mathcal{U}) \cup W(\mathcal{U})$ and $(y_\beta, x_\alpha) \rightarrow 0$. Then $(x, x_\alpha) \in \{x\} \times [\cap\{H \in \mathcal{U} : x \in H\}]$ and $(y_\beta, y) \in [\cap\{K \in \mathcal{U} : y \in K\}] \times \{y\}$, i.e., $x_\alpha \in \cap\{H \in \mathcal{U} : x \in H\}$, $y_\beta \in \cap\{K \in \mathcal{U} : y \in K\}$ with $(y_\beta, x_\alpha) \rightarrow 0$. Using condition (iii) we get, $y \in \cap\{H \in \mathcal{U} : x \in H\}$, i.e., $(x, y) \in V(\mathcal{U}) \cup W(\mathcal{U}) \Rightarrow y \in (V(\mathcal{U}) \cup W(\mathcal{U}))(x) = \overline{V(\mathcal{U})(x) \cup W(\mathcal{U})(x)}$. Thus our assertion is proved. \square

4. Applications. In this section we use the concept of strong quasi-uniform covers to characterize some topological concepts.

Definition 4.1 ([2]). *A collection \mathcal{C} of subsets of a topological space is interior preserving if $\mathcal{C}' \in \mathcal{C}$, then $\text{int} \cap \{C : C \in \mathcal{C}'\} = \cap \{\text{int} C : C \in \mathcal{C}'\}$.*

Remark 4.2 ([2]). If \mathcal{C} is a collection of subsets of a set X and $x \in X$, then $\mathcal{C}_x = \{C \in \mathcal{C} : x \in C\}$, so that $\cap \mathcal{C}_x = \cap \{C \in \mathcal{C} : x \in C\}$. In terms of this notation, a collection \mathcal{C} of open subsets of a topological space is interior preserving iff for each $x \in X$, $\cap \mathcal{C}_x \in \tau$.

Definition 4.3 ([2]). *A topological space is orthocompact provided that every open cover has an interior preserving open refinement.*

Definition 4.4. *A covering $\{A_\alpha : \alpha \in \Lambda\}$ of a topological space (X, τ) is called point-finite if for each $x \in X$, there exists at most finitely many indices $\alpha \in \Lambda$ such that $x \in A_\alpha$.*

Definition 4.5 ([2]). *A topological space (X, τ) is metacompact if each open covering has a point-finite open refinement.*

Definition 4.6 ([2]). *Let \mathcal{A} be a nonempty family of covers of a set X and for each $\mathcal{C} \in \mathcal{A}$, let $U_{\mathcal{C}} = \{(x, y) : x \in X \text{ and } y \in \cap\{C \in \mathcal{C} : x \in C\}\}$. It can be then checked that each such $U_{\mathcal{C}}$ is reflexive and transitive relation and that the family $\{U_{\mathcal{C}} : \mathcal{C} \in \mathcal{A}\}$ is a subbase for a quasi-uniformity on X , to be denoted by $\mathcal{U}_{\mathcal{A}}$.*

Theorem 4.7. *A topological space (X, τ) is orthocompact iff for every open cover \mathcal{C} of (X, τ) , there exists a strong quasi-uniform cover \mathcal{C}^* of (X, \mathcal{U}_A) such that \mathcal{C}^* refines \mathcal{C} , where A is the collection of all interior preserving open covers of X .*

Proof. First let (X, τ) be orthocompact and let \mathcal{C} be an open cover of (X, τ) . Then there is an interior preserving open refinement \mathcal{C}^* of \mathcal{C} . Now \mathcal{C}^* being an interior preserving open cover of X , $U_{\mathcal{C}^*} \in \mathcal{U}_A$ where $U_{\mathcal{C}^*} = \{(x, y) \in X \times X : x \in X \text{ and } y \in \cap\{C \in \mathcal{C}^* : x \in C\}\}$. Thus for each $x \in X$, $U_{\mathcal{C}^*}(x) = \cap\{C \in \mathcal{C}^* : x \in C\}$. Hence \mathcal{C}^* is a strong quasi-uniform cover of (X, \mathcal{U}_A) .

Conversely, let \mathcal{C} be an open cover of (X, τ) . By the given condition, there is a strong quasi-uniform cover \mathcal{C}^* of (X, \mathcal{U}_A) such that \mathcal{C}^* refines \mathcal{C} . Since \mathcal{C}^* is a strong quasi-uniform cover of (X, \mathcal{U}_A) , there exists finitely many $U_{\mathcal{C}_1}, U_{\mathcal{C}_2}, \dots, U_{\mathcal{C}_n} \in \mathcal{U}_A$ such that $U(x) \subseteq \cap\{C \in \mathcal{C}^* : x \in C\}$, for each $x \in X$, where $U = U_{\mathcal{C}_1} \cap U_{\mathcal{C}_2} \cap \dots \cap U_{\mathcal{C}_n}$, i.e., $\bigcap_{i=1}^n [\cap\{C \in \mathcal{C}_i : x \in C\}] \subseteq \cap\{C \in \mathcal{C}^* : x \in C\}$... (i), i.e., $\cap\{C \in \mathcal{C}_1 \cup \mathcal{C}_2 \cup \dots \cup \mathcal{C}_n : x \in C\} \subseteq \cap\{C \in \mathcal{C}^* : x \in C\}$.

Let $\mathcal{C}' = \{\cap\{C \in \mathcal{C}_1 \cup \mathcal{C}_2 \cup \dots \cup \mathcal{C}_n : x \in C\} : x \in X\}$. As $\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_n \in \mathcal{A}$, $U_{\mathcal{C}_i}(x) \in \tau$ for $i = 1, 2, \dots, n$ and for all $x \in X$. Hence $\bigcap_{i=1}^n U_{\mathcal{C}_i}(x) \in \tau$, for all $x \in X$. Thus each set of \mathcal{C}' is open. Also $\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_n$ being interior preserving, \mathcal{C}' is also so. Again by (i) \mathcal{C}' refines \mathcal{C}^* and \mathcal{C}^* also refines \mathcal{C} . Hence \mathcal{C}' refines \mathcal{C} . Thus there is an interior preserving open refinement \mathcal{C}' of \mathcal{C} of (X, τ) . Hence (X, τ) is orthocompact. \square

Theorem 4.8. *A topological space (X, τ) is metacompact iff for every open cover \mathcal{C} of (X, τ) , there exists a strong quasi-uniform cover \mathcal{C}^* of (X, \mathcal{U}_A) such that \mathcal{C}^* refines \mathcal{C} , where A is the collection of all point-finite open covers of X .*

Proof. Let (X, τ) be metacompact and let \mathcal{C} be an open cover of (X, τ) . Then there exists a point-finite open refinement \mathcal{C}^* of \mathcal{C} . Now \mathcal{C}^* being a point-finite open cover of X , $U_{\mathcal{C}^*} \in \mathcal{U}_A$ where $U_{\mathcal{C}^*} = \{(x, y) \in X \times X : x \in X \text{ and } y \in \cap\{C \in \mathcal{C}^* : x \in C\}\}$. Then for each $x \in X$, $U_{\mathcal{C}^*}(x) = \cap\{C \in \mathcal{C}^* : x \in C\}$. Hence \mathcal{C}^* is a strong quasi-uniform cover of (X, \mathcal{U}_A) .

Conversely, let \mathcal{C} be an open cover of (X, τ) . By the given condition, there is a strong quasi-uniform cover \mathcal{C}^* of (X, \mathcal{U}_A) such that \mathcal{C}^* refines \mathcal{C} . Since \mathcal{C}^* is a strong quasi-uniform cover of (X, \mathcal{U}_A) , there exists finitely many $U_{\mathcal{C}_1}, U_{\mathcal{C}_2}, \dots, U_{\mathcal{C}_n} \in \mathcal{U}_A$ such that $U(x) \subseteq \cap\{C \in \mathcal{C}^* : x \in C\}$, for each

$x \in X$, where $U = U_{\mathcal{C}_1} \cap U_{\mathcal{C}_2} \cap \dots \cap U_{\mathcal{C}_n}$, i.e., $\bigcap_{i=1}^n [\bigcap\{C \in \mathcal{C}_i : x \in C\}] \subseteq \bigcap\{C \in \mathcal{C}^* : x \in C\}$... (i), i.e., $\bigcap\{C \in \mathcal{C}_1 \cup \mathcal{C}_2 \cup \dots \cup \mathcal{C}_n : x \in C\} \subseteq \bigcap\{C \in \mathcal{C}^* : x \in C\}$.

Let $\mathcal{C}' = \{\bigcap\{C \in \mathcal{C}_1 \cup \mathcal{C}_2 \cup \dots \cup \mathcal{C}_n : x \in C\} : x \in X\}$. As $\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_n$ are point-finite, $\bigcap\{C \in \mathcal{C}_1 \cup \mathcal{C}_2 \cup \dots \cup \mathcal{C}_n : x \in C\}$ is a finite intersection for each $x \in X$. Hence each set $\bigcap\{C \in \mathcal{C}_1 \cup \mathcal{C}_2 \cup \dots \cup \mathcal{C}_n : x \in C\}$ is an open set in (X, τ) . Now $\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_n$ being point-finite, \mathcal{C}' is also so. By (i) \mathcal{C}' is a refinement of \mathcal{C}^* which in turn is a refinement of \mathcal{C} . Thus \mathcal{C}' is an open point-finite refinement of \mathcal{C} . Hence (X, τ) becomes metacompact. \square

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