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Bell-correlated activable bound entanglement in multiqubit systems

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We show that the Hilbert space of even number ($\geqslant 4$) of qubits can always be decomposed as a direct sum of four orthogonal subspaces such that the normalized projectors onto the subspaces are activable bound entangled (ABE) states. These states also show a surprising recursive relation in the sense that the states belonging to $2N+2$ qubits are Bell correlated to the states of $2N$ qubits; hence, we refer to these states as Bell-correlated ABE (BCABE) states. We also study the properties of noisy BCABE states and show that they are very similar to that of two qubit Bell-diagonal states.

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I. INTRODUCTION AND RESULTS

The quantum states that are not distillable [1] under local operations and classical communications (LOCC) despite being inseparable are said to be bound entangled (BE) [2–7]. Bound entangled states exhibit a new kind of irreversibility in physics where one has to spend finite amount of entanglement to prepare such states but one cannot extract any nonzero amount of entanglement from such states via LOCC. Thus the amount of entanglement of formation is irreversibly lost during the state preparation. Recent studies involving bound entangled states include characterization of such states [8–12], violation of Bell type inequalities [13–16] and possible practical applications [17,18].

For bipartite systems, bound entanglement is clearly defined as it involves only two spatially separated parties and a necessary and sufficient condition for distillability of bipartite quantum states is known [5]. In a multiparty setting, however, due to several distinct spatially separated configurations, the definition of bound entanglement is not unique. A multipartite quantum state is said to be bound entangled if there is no distillable entanglement between any subset as long as *all* the parties remain spatially separated from each other. When, however, one also allows some of the parties to group together and perform local operations collectively, two qualitatively different classes of bound entanglement arise: (a) activable bound entangled (ABE) states—the states that are not distillable when every party is separated from every other but becomes distillable, if certain parties decide to group together [19,20]. This implies that there is at least one bipartite partition/cut where the state is negative under partial transposition (NPT) [21]. Such states have been also referred to as unlockable bound entangled (UBE) states in the literature. (b) Nonactivable Bound Entangled states—states that are not distillable under any modified configuration as long as there are at least two spatially separated groups. In other words, such states are always positive under partial transposition across any bipartite partition [6].

Despite recent studies, the distribution and structure of such states in the Hilbert space have not been explicitly studied. In this work we show that bound entangled states have natural existence in the structure of the Hilbert space of even number, $2N+2$, of qubits (when $N \geqslant 1$). In particular, the Hilbert space of $2N+2$ qubits, $N \geqslant 1$, can be decomposed as a direct sum of four orthogonal subspaces such that the normalized projector onto each subspace is an activable bound entangled state. The set of four ABE states are shown to be unitarily related to each other via a local Pauli operator on one of the qubits. Surprisingly, the states exhibit a recursive property, i.e., each state of $2N+2$ qubits can be expressed as a convex combination (with equal weights) of four two-qubit Bell states correlated with the four ABE states of $2N$ qubits. The only exception occurs for four qubit states, where the Bell states of two qubits are correlated to Bell states of the other two qubits. It is interesting to note that one of these four ABE states for the four-qubit system has been previously discovered by Smolin [20]. We call these

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$(2N+2)$-qubit ABE states Bell-correlated activable bound entangled states (BCABE).

As noted before the bound entangled states that we present in this work are activable. In such an activable configuration we find that the distillable entanglement between any two parties is always one ebit and therefore independent of N. We also study properties of the noisy BCABE states. The noisy states are constructed by taking a convex combination of the four BCABE states. Remarkably, the entanglement properties of these noisy $2N+2$ qubit bound entangled states can be directly mapped onto that of two qubit Bell diagonal states.

II. HILBERT SPACE OF $2N+2$ QUBITS: DECOMPOSITION AND BOUND ENTANGLED STATES

Consider now a system of $2N$ qubits. Let $|p_i\rangle = |a_1^i a_2^i,\dots,a_{2N}^i\rangle$ where $a_1^i=0$, and $a_j^i \in \{0,1\}$, for all $j=2,\cdots,2N$ such that there is an even number of 0*s* in the string $a_1^i a_2^i,\dots,a_{2N}^i$. Likewise, let $|q_i\rangle = |b_1^i b_2^i \cdots b_{2N}^i\rangle$, where $b_1^i=0$, and $b_2^i,\dots,b_{2N}^i$ are either 0 or 1 with odd number of 0*s* in the string $b_1^i b_2^i,\dots,b_{2N}^i$. One can also define the states orthogonal to $|p_i\rangle$, $|q_i\rangle$ as: $|\overline{p_i}\rangle = |\overline{a_1^i a_2^i \cdots a_{2N}^i}\rangle$ and $|\overline{q_i}\rangle = |\overline{b_1^i b_2^i \cdots b_{2N}^i}\rangle$ where $\langle a_j^i | \overline{a_j^i}\rangle = 0 = \langle b_j^i | \overline{b_j^i}\rangle$, $\forall j=1,\dots,2N$ and $i=1,\dots,2^{2N-2}$. Note that the four sets of states, defined by $|p_i\rangle$'s, $|\overline{p_i}\rangle$'s, $|q_i\rangle$, and $|\overline{q_i}\rangle$'s, respectively, are nonoverlapping and all have same cardinality, and they together span the complete Hilbert space of $2N+2$ qubit systems.

Now we define the following four sets of states:

$$S_\Phi^\pm = \left\{ |\Phi_i^\pm\rangle = \frac{1}{\sqrt{2}}(|p_i\rangle \pm |\overline{p_i}\rangle), i=1,\dots,2^{2N-2} \right\}, \quad (1)$$

$$S_\Psi^\pm = \left\{ |\Psi_i^\pm\rangle = \frac{1}{\sqrt{2}}(|q_i\rangle \pm |\overline{q_i}\rangle), i=1,\dots,2^{2N-2} \right\}. \quad (2)$$

We can associate with every set S, a subspace of the complete Hilbert space where the states belonging to S span that subspace and all the subspaces are orthogonal to each other. In terms of Hilbert space decomposition we can write this as

$$H = H_\Phi^+ \oplus H_\Phi^- \oplus H_\Psi^+ \oplus H_\Psi^-. \quad (3)$$

Observe that together the states span the full Hilbert space and often this basis is referred to as the cat or GHZ basis.

We will use the notation $[\cdot]$ for pure state projector $|\cdot\rangle\langle\cdot|$. Let us now define the unnormalized projectors on to the subspaces spanned by the set of states given by Eqs. (1) and (2):

$$P_{2N}^\pm = \sum_{i=1}^{2^{2N-2}} [\Phi_i^\pm]; \quad Q_{2N}^\pm = \sum_{i=1}^{2^{2N-2}} [\Psi_i^\pm]. \quad (4)$$

The set of the above four projectors are connected to each other by one Pauli operation on one qubit. For instance, consider the unitary operators $U_i = I_1 \otimes \cdots \otimes I_{2N-1} \otimes \sigma_{2N}^i$, where $i \in \{z,x,y\}$, i.e., U_i applies the ith Pauli operator on the $(2N)$th qubit. Then one can verify that

$$P_{2N}^- = U_z P_{2N}^+ U_z^\dagger, \quad (5)$$

$$Q_{2N}^+ = U_x P_{2N}^+ U_x^\dagger, \quad (6)$$

$$Q_{2N}^- = U_y P_{2N}^+ U_y^\dagger. \quad (7)$$

We will now show how to generate the above set of four projectors in the case of $2N+2$ qubits starting from the set of $2N$ qubits. First one can write P_{2N+2}^+ as

$$P_{2N+2}^+ = \sum_{i=1}^{2^{2N-2}} \sum_{k=1}^{k=4} [\Omega_i^k], \quad (8)$$

where the Ω states are defined as

$$|\Omega_i^1\rangle = \frac{1}{\sqrt{2}}(|00\rangle|p_i\rangle + |11\rangle|\overline{p_i}\rangle)$$

$$|\Omega_i^2\rangle = \frac{1}{\sqrt{2}}(|11\rangle|p_i\rangle + |00\rangle|\overline{p_i}\rangle)$$

$$|\Omega_i^3\rangle = \frac{1}{\sqrt{2}}(|01\rangle|q_i\rangle + |10\rangle|\overline{q_i}\rangle)$$

$$|\Omega_i^4\rangle = \frac{1}{\sqrt{2}}(|10\rangle|q_i\rangle + |01\rangle|\overline{q_i}\rangle). \quad (9)$$

Now recall that,

$$|00\rangle = \frac{1}{\sqrt{2}}(|\Phi^+\rangle + |\Phi^-\rangle), \quad |11\rangle = \frac{1}{\sqrt{2}}(|\Phi^+\rangle - |\Phi^-\rangle),$$

$$|01\rangle = \frac{1}{\sqrt{2}}(|\Psi^+\rangle + |\Psi^-\rangle), \quad |10\rangle = \frac{1}{\sqrt{2}}(|\Psi^+\rangle - |\Psi^-\rangle) \quad (10)$$

where the two qubit Bell states are defined by

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \quad |\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle). \quad (11)$$

Substituting the above in the expression for Ω, and after some algebraic manipulations, one obtains

$$P_{2N+2}^+ = [\Phi^+] \otimes P_{2N}^+ + [\Phi^-] \otimes P_{2N}^- + [\Psi^+] \otimes Q_{2N}^+ + [\Psi^-] \otimes Q_{2N}^-. \quad (12)$$

This recursive form is particularly illuminating. However at this point let us normalize the above projector to make it a legitimate density matrix and write it as:

$$\rho_{2N+2}^+ = \frac{1}{4}([\Phi^+] \otimes \rho_{2N}^+ + [\Phi^-] \otimes \rho_{2N}^- + [\Psi^+] \otimes \sigma_{2N}^+ + [\Psi^-] \otimes \sigma_{2N}^-), \quad (13)$$

where

$$\rho_{2N}^\pm = \frac{1}{2^{2N-2}} P_{2N}^\pm, \sigma_{2N}^\pm = \frac{1}{2^{2N-2}} Q_{2N}^\pm. \quad (14)$$

Let us now look at the properties of the state ρ_{2N+2}^+ more closely.

(i) By construction, the state is invariant under interchange of parties. To see this, consider the unnormalized state P^{+}_{2N}. This projector is an equally weighted convex combination of the states belonging to the set S^{+}_{Φ}. Now if we interchange the qubits, then under any such permutation the states belonging to the set just map onto each other leaving the whole projector invariant. In other words if one denotes the jth party as A_j, then $\rho(\ldots,A_i,\ldots,A_k,\ldots)=\rho(\ldots,A_k,\ldots,A_i,\ldots)$ for all possible i,k.

(ii) The state is entangled. One way to see is that if any $2N$ parties come together and do a joint measurement to discriminate the states $\{\rho^{+}_{2N},\rho^{-}_{2N},\sigma^{+}_{2N},\sigma^{-}_{2N}\}$ (as they are mutually orthogonal), then this will result in a maximally entangled state between the remaining two. Or else, N parties could pair up and do sequential Bell measurements on their two qubits which will lead to distillation of a maximally entangled state between the remaining two who did not come together and remained spatially separated. Therefore the state must be entangled to begin with, otherwise no configuration could allow any entanglement to be distilled between separated parties.

(iii) When all the $2N+2$ parties remain spatially separated, then the state is not distillable as it is separable across every $2:2N$ bipartite cut. This is easily seen as the state itself is written in a $2:2N$ separable form. That it is separable across every such cut follows from the permutation symmetry. This makes every party separated from every other by at least one separable cut and hence no entanglement can be distilled.

As the state is entangled but not distillable when *all* the parties are separated from each other the state must be a bound entangled state. Since the state becomes distillable if a subset of the parties come together and perform collective LOCC, the state is activable. Hence the state is an ABE state.

For $2N+2$ qubits one can generate the other three states following the same prescription. However it is much simpler by noting that the states are all single Pauli connected. Explicitly the remaining three states can be written using Eqs. (5)–(7) as:

$$\rho^{-}_{2N+2}=\frac{1}{4}([\Phi^{+}]\otimes\rho^{-}_{2N}+[\Phi^{-}]\otimes\rho^{+}_{2N}+[\Psi^{+}]\otimes\sigma^{-}_{2N}+[\Psi^{-}]\otimes\sigma^{+}_{2N}), \tag{15}$$

and

$$\sigma^{\pm}_{2N+2}=\frac{1}{4}([\Psi^{+}]\otimes\rho^{\pm}_{2N}+[\Psi^{-}]\otimes\rho^{\mp}_{2N}+[\Phi^{+}]\otimes\sigma^{\pm}_{2N}+[\Phi^{-}]\otimes\sigma^{\mp}_{2N}). \tag{16}$$

The above results can now be summarized in the form of a theorem.

Theorem. The Hilbert space of $2N+2$ qubits, $N\geqslant 1$, can always be decomposed as a direct sum of four orthogonal subspaces such that the normalized projectors onto the subspaces are activable bound entangled states.

Let us note here that when $N=1$, the set of states $\{\rho^{\pm}_{4},\sigma^{\pm}_{4}\}$ are Bell correlated to the set of states $\{\rho^{\pm}_{2},\sigma^{\pm}_{2}\}$, which are not bound entangled but maximally entangled and hence distillable. However this case is the only exception when the set of bound entangled states $\{\rho^{\pm}_{2N+2},\sigma^{\pm}_{2N+2}\}$ is not Bell correlated to the set of bound entangled states $\{\rho^{\pm}_{2N},\sigma^{\pm}_{2N}\}$.

III. ILLUSTRATIONS

A. Hilbert space of four qubits

Consider the four sets of states as defined before in Eqs. (1) and (2).

$$S^{\pm}_{\Phi}=\left\{\frac{1}{\sqrt{2}}(|0000\rangle\pm|1111\rangle),\frac{1}{\sqrt{2}}(|0011\rangle\pm|1100\rangle),\frac{1}{\sqrt{2}}(|0101\rangle\pm|1010\rangle),\frac{1}{\sqrt{2}}(|0110\rangle\pm|1001\rangle),\right.$$

$$S^{\pm}_{\Psi}=\left\{\frac{1}{\sqrt{2}}(|0001\rangle\pm|1110\rangle),\frac{1}{\sqrt{2}}(|0010\rangle\pm|1101\rangle),\frac{1}{\sqrt{2}}(|0100\rangle\pm|1011\rangle),\frac{1}{\sqrt{2}}(|0111\rangle\pm|1000\rangle).\right. \tag{17}$$

The 16 states span the full Hilbert space. The four sets are all mutually orthogonal to each other. As before, we can assign a subspace to each of the four sets spanned by the members of the respective set and therefore get the desired decomposition. Consider now the normalized projector onto the first subspace spanned by the set S^{+}_{Φ}:

$$\rho^{+}=\frac{1}{4}([0000+1111]+[0011+1100]+[0101+1010]+[0110+1001]). \tag{18}$$

The permutation symmetry is obvious in the above form. Now replacing the first two and the last two qubit states by linear combination of Bell states in accordance to Eq. (10), one obtains,

$$\rho^{+}=\frac{1}{4}\sum_{k=\pm}([\Phi^{k}]\otimes[\Phi^{k}]+[\Psi^{k}]\otimes[\Psi^{k}]), \tag{19}$$

which we recognize as the unlockable bound entangled state presented by Smolin [20].

One can now generate three other mutually orthogonal activable bound entangled states by applying local pauli operators on any one of the qubits of ρ^{+} using Eqs. (5)–(7). For instance, ρ^{-} can be generated in the following way:

$$\rho^{-}=(I\otimes I\otimes I\otimes\sigma_z)\rho^{+}(I\otimes I\otimes I\otimes\sigma_z)=\frac{1}{4}([0000-1111]+[0011-1100]+[0101-1010]+[0110-1001]), \tag{20}$$

$$=\frac{1}{4}\sum_{k,l=\pm;k\neq l}([\Phi^{k}]\otimes[\Phi^{l}]+[\Psi^{k}]\otimes[\Psi^{l}]). \tag{21}$$

The remaining two states $\sigma^{\pm}$ can likewise be obtained from ρ^{+} by applying the appropriate Pauli operators $\sigma_{x/y}$.

B. Hilbert space of six qubits

Although by following our prescription, the six qubit BCABE states can be generated from the four qubit states in a straightforward manner, here we provide the construction from the first principles. First define the following four sets of states:

(a) S_Φ^+: States with even number of 0 and 1 with + sign in the superposition, such as $1/\sqrt{2}(|000000\rangle+|111111\rangle)$, $1/\sqrt{2}(|000011\rangle+|111100\rangle)$.

(b) S_Φ^-: States with even number of 0 and 1 with − sign in the superposition, such as $1/\sqrt{2}(|000000\rangle-|111111\rangle)$, $1/\sqrt{2}(|000011\rangle-|111100\rangle)$.

(c) S_Ψ^+: States with odd number of 0 and 1 with + sign in the superposition, such as $1/\sqrt{2}(|000001\rangle+|111110\rangle)$, $1/\sqrt{2}(|100011\rangle+|011100\rangle)$.

(d) S_Ψ^-: States with odd number of 0 and 1 with − sign in the superposition, such as $1/\sqrt{2}(|000001\rangle-|111110\rangle)$, $1/\sqrt{2}(|100011\rangle-|011100\rangle)$.

Note that every group consists of 16 members. Every group spans subspaces that are orthogonal to each other by construction and together they span the full Hilbert space. The decomposition is also clearly understood. Let us now consider the unnormalized projector on the subspace spanned by the states in the first group:

$$\begin{aligned}P^+ = {}&[000000+111111]+[000011+111100]+[000101\\&+111010]+[000110+111001]+[001001+110110]\\&+[001010+110101]+[001100+110011]+[001111\\&+110000]+[010001+101110]+[010010+101101]\\&+[010100+101011]+[010111+101000]+[011000\\&+100111]+[011011+100100]+[011101+100010]\\&+[011110+100001]. \end{aligned}\tag{22}$$

By construction the projector is invariant under permutation and the normalized projector can indeed be written in a Bell-correlated form using the Eq. (10):

$$\begin{aligned}\rho^+_{ABCDEF} = {}&\frac{1}{4}([\Phi^+]_{AB}\otimes\rho^+_{CDEF}+[\Phi^-]_{AB}\otimes\rho^-_{CDEF}+[\Psi^+]_{AB}\\&\otimes\sigma^+_{CDEF}+[\Psi^-]_{AB}\otimes\sigma^-_{CDEF}).\end{aligned}\tag{23}$$

It is now easy to construct the other three activable bound entangled states whose unnormalized forms are projectors on the orthogonal subspaces:

$$\begin{aligned}\rho^-_{ABCDEF} = {}&\frac{1}{4}([\Phi^+]_{AB}\otimes\rho^-_{CDEF}+[\Phi^-]_{AB}\otimes\rho^+_{CDEF}+[\Psi^+]_{AB}\\&\otimes\sigma^-_{CDEF}+[\Psi^-]_{AB}\otimes\sigma^+_{CDEF}).\end{aligned}\tag{24}$$

$$\begin{aligned}\sigma^\pm_{ABCDEF} = {}&\frac{1}{4}([\Psi^+]_{AB}\otimes\rho^\pm_{CDEF}+[\Psi^-]_{AB}\otimes\rho^\mp_{CDEF}+[\Phi^+]_{AB}\\&\otimes\sigma^\pm_{CDEF}+[\Phi^-]_{AB}\otimes\sigma^\mp_{CDEF}).\end{aligned}\tag{25}$$

IV. NOISY BELL CORRELATED ACTIVABLE BOUND ENTANGLED STATES

The noisy bound entangled states are constructed by taking a convex combination of the four BCABE states with different weights. For $2N+2$ qubits we construct the following state:

$$\rho^{\text{noisy}}_{2N+2} = \sum_{i=\pm} x_i\rho^i_{2N+2} + y_i\sigma^i_{2N+2},\tag{26}$$

where $\sum_{i=\pm}(x_i+y_i)=1$, $1\geqslant x_i, y_i\geqslant 0$.

Expanding the states $\rho^i_{2N+2},\sigma^i_{2N+2}$ using Eqs. (13), (15), and (16) one obtains,

$$\begin{aligned}\rho^{\text{noisy}}_{2N+2} = {}& x_+\frac{1}{4}\sum_{k=\pm}([\Phi^k]\otimes\rho^k_{2N}+[\Psi^k]\otimes\sigma^k_{2N})\\&+x_-\frac{1}{4}\sum_{k\neq l;k,l=\pm}([\Phi^k]\otimes\rho^l_{2N}+[\Psi^k]\otimes\sigma^l_{2N})\\&+y_+\frac{1}{4}\sum_{k=\pm}([\Psi^k]\otimes\rho^k_{2N}+[\Phi^k]\otimes\sigma^k_{2N})\\&+y_-\frac{1}{4}\sum_{k\neq l;k,l=\pm}([\Psi^k]\otimes\rho^l_{2N}+[\Phi^k]\otimes\sigma^l_{2N}),\end{aligned}\tag{27}$$

which can be further expressed as

$$\rho^{\text{noisy}}_{2N+2} = \frac{1}{4}\sum_{k=\pm}(\Pi^k\otimes\rho^k_{2N}+\Gamma^k\otimes\sigma^k_{2N}),\tag{28}$$

where Π,Γ are two qubit Bell diagonal density matrices defined as follows:

$$\Pi^\pm = x_+[\Phi^\pm]+x_-[\Phi^\mp]+y_+[\Psi^\pm]+y_-[\Psi^\mp],\tag{29}$$

$$\Gamma^\pm = x_+[\Psi^\pm]+x_-[\Psi^\mp]+y_+[\Phi^\pm]+y_-[\Phi^\mp].\tag{30}$$

The entanglement properties of such states are well known [23]. Let us note that the two qubit Werner states are special cases of the above class of Bell diagonal states. Let $w=\max\{x_\pm,y_\pm\}$. Then the states $\Pi^\pm,\Gamma^\pm$ are entangled as well as distillable if and only if $w>1/2$. With the aid of this result we can now state the following properties of the noisy states $\rho^{\text{noisy}}_{2N+2}$:

The state is an activable bound entangled state when $w>1/2$. The proof is as follows. First note that the state is invariant under interchange of parties. This is because the state is a convex combination of the states that are permutationally invariant. From Eq. (27) the state is written in a separable form across the $2:2N$ bipartite cut. By virtue of being symmetric under interchange of parties, the state is separable across every $2:2N$ bipartite cut. Hence, the state is not distillable if all the parties are separated from each other.

The state is entangled and distillable when $w>1/2$. This is also clear from Eq. (28). In this case, if $2N$ parties come together and do collective LOCC they can distill one of the $\Pi^\pm,\Gamma^\pm$ states between the remaining two parties. However these states are distillable if $w>1/2$.

The states further resemble other properties of mixture of Bell states. For instance, a mixture of two Bell states is always entangled as long as the weights are different. Similarly one can show here by putting any two of the coefficients $\{x_{\pm}, y_{\pm}\}$ equal to zero that the noisy state thus constructed is also entangled as long as the weight of the two nonvanishing coefficients are different. However the difference is that a mixture of Bell states is distillable while these states are only activable and not distillable when all the parties remain separated from each other.

V. DISCUSSION AND OPEN PROBLEMS

To summarize we showed that the Hilbert space of an even number of qubits (greater than equal to four) can always be decomposed as a direct sum of four orthogonal subspaces such that the normalized projectors onto the subspaces are activable bound entangled states. The states show a surprising recursive relation in the sense that the states belonging to $2N+2$ qubits are Bell correlated to the states of $2N$ qubits. It is also shown that in an activable configuration the distillable entanglement between any two qubits is always one ebit irrespective of the total number of qubits forming the state itself. We also studied the properties of noisy BCABE states and showed that they are very similar to that of two qubit Bell diagonal states.

One question is immediate: Can such a decomposition be observed in the case of odd number of qubits? Our strategy definitely does not work in the case of odd number of qubits because the states do not have the even–even symmetry with respect to the number of 0*s* or 1*s* in its cat/GHZ basis states. This lack of symmetry only allows two orthogonal decomposition following our strategy but they result into separable states.

A possible generalization of our states would be to extend to higher dimensions. Although one can possibly do that using general Pauli matrices, the structure of such states is not immediately clear. We suspect if such a decomposition is indeed possible then it would certainly be the number of generators of the pauli group in dimension d.

As a part of future research work, one could investigate several properties of these BCABE states. For example, one of the four qubit BCABE states (i.e., Smolin state) has been shown to be useful for secret key distillation [17], violation of Bell inequality [16], remote information concentration [18] and superactivation of bound entanglement [22]. We believe the results obtained in the case of the four-qubit BCABE state, could also be generalized using our states.

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