ARE NEUTRON–ANTINEUTRON OSCILLATIONS ALLOWED IN SUPERSYMMETRIC GRAND UNIFIED THEORIES?

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We demonstrate that in supersymmetric grand unified theories, detectable n–n̄ oscillations are inconsistent with perturbative unification.

The experimental observation of neutron–antineutron (n–n̄) oscillations would signify effective $\Delta B = 2$ interactions [1] which may be mediated at the tree level by colour sextet diquark Higgs scalars with masses in the intermediate range ($10^5 - 10^6$ GeV). Such Higgs intermediaries are known to exist in non-minimal extensions of the SU(5) [1–3] grand unified theory (GUT), as well as in left–right symmetric models embeddable in SO(10) and E(6) [1]. However, in ordinary GUTs, their masses are constrained by the so-called extended survival hypothesis (ESH) (which follows from the principle of minimal fine tuning) to be large [4], thereby suppressing the n–n̄ oscillation frequency well below experimentally accessible limits [$\tau_{n\bar{n}}^{-1}$(expt.) $\lesssim 10^{-6}\ \mathrm{s}^{-1}$] [5].

In supersymmetric (SUSY) GUTs, naturalness obviates the need to invoke any fine tuning principle [6], removing the restrictions on Higgs masses imposed by the ESH. One is motivated to consider choosing these masses suitably, so as to yield detectable n–n̄ oscillations. The sole constraint on the Higgs masses now comes from the boundary conditions that one imposes on the coupling constant evolution equations, which are expressed in terms of the experimentally measured values of (i) the weak mixing angle [$\sin^2\theta_W(M_W)$] and (ii) the strong coupling parameters $\alpha_s(M_W)$. Recall also that the perturbative calculations are valid as long as $\alpha(\mu) \equiv g^2/4\pi < 1$ for all $M_W \leqslant \mu \leqslant M_X$, where g is any gauge coupling constant of the theory and M_X is the grand unification scale, i.e. $\alpha_s(M_X) = \alpha_{\mathrm{weak}}(M_X) = \alpha_{\mathrm{GUT}}$.

In the following, we analyse, in the context of SUSY GUTs, the restrictions imposed by the above conditions on the masses of the Higgs bosons that mediate n–n̄ oscillations. We consider separately the two popular chains of symmetry breaking

(i) $$G \xrightarrow[M_X]{} SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \xrightarrow[M_W]{} SU(3)_c \otimes U(1)_{EM},$$

(ii) $$G \xrightarrow[M_X]{} SU(4)_c \otimes SU(2)_L \otimes SU(2)_R \xrightarrow[M_c]{} SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$$
$$\xrightarrow[M_R]{} SU(3)_c \otimes SU(2)_L \otimes SU(1)_R \otimes U(1)_{B-L} \xrightarrow[M_{B-L}]{} SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \xrightarrow[M_W]{} SU(3)_c \otimes U(1)_{EM}.$$

For the SU(5) GUT only chain (i) is possible, but many bigger GUT groups which include as a subgroup the Pati–Salam $SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$ group,

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admit both alternatives.

Irrespective of the mechanism of SUSY breaking, we assume that the fermion–boson mass splittings in the effective low energy sector are characterised by M_W, beyond which entire supermultiplets with degenerate fermionic and bosonic components contribute to the appropriate evolution equations. This assumption would be valid for the geometric hierarchy scenaries [7] as well as models in which the hierarchy is generated by coupling to $N = 1$ supergravity [8], provided the intermediate Higgs bosons can be treated as "low energy" (in contrast to "superheavy") particles.

Our conclusions are the following. When the contributions to the evolution equations from the minimal set of chiral supermultiplets (necessary for n–n̄ oscillations) are included, we find that neither of the two chains of symmetry breaking mentioned above are consistent with perturbative unification. Essentially all the GUT models that have been considered in the literature come under one or the other of these two categories. *Our result therefore implies that detectable n–n̄ oscillations are not* allowed in any SUSY GUT.

Chain (i). The diquark scalar multiplets Δ, responsible for n–n̄ oscillations, transform under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ as $(\bar{6}, 3, -1/\sqrt{15})$. In addition to these, one requires another Higgs multiplet $\phi \equiv (1, 3, \sqrt{3/5})$ whose neutral member acquires a nonzero vacuum expectation value $\langle\phi\rangle$. The quartic coupling of the form $\lambda(\Delta\Delta\Delta\phi)$ ensures n–n̄ oscillations with a period

$$\tau_{n-\bar{n}} = [m_\Delta^6/\lambda h^3\langle\phi\rangle]\,|\psi(0)|^{-4}. \tag{1}$$

Here h is the Yukawa coupling constant of the Δ with the quarks and the factor $|\psi(0)|^{-4}$ takes into account wavefunction overlap effects. $\tau_{n-\bar{n}}(\text{expt.}) \geqslant 10^6$ s implies $m_\Delta \geqslant 10^3$ GeV for the normal choice of parameters.

In the supersymmetric version of this model some changes have to be incorporated. Firstly, the SUSY partners of all the particles in the model must be included, giving rise to chiral Higgs and matter (quark/lepton) supermultiplets and vector gauge supermultiplets. Moreover, in order that the fermions in the $(\bar{6}, 3, -1/\sqrt{15})$ and $(1, 3, \sqrt{3/5})$ supermultiplets may acquire a Dirac mass, we must also include additional multiplets transforming as $(\bar{6}, 3, 1/\sqrt{15})$ and $(1, 3, \sqrt{3/5})$. Also, as is well known, in order to give mass to quarks of both charge +2/3 and −1/3 one needs two (*not* one as in the ordinary case) Higgs multiplets transforming as $(1, 2, \pm\sqrt{3/20})$.

It is sufficient for our purpose to consider the evolution equations for the $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ coupling parameters over the ranges (a) M_X to M_I and (b) M_I to M_W. Here M_I is the intermediate mass scale which is to be identified with the mass m_Δ characterising n–n̄ oscillations. The chiral supermultiplets that contribute in the renormalisation group equations in the range M_X to M_I are

$$(\bar{6}, 3, -1/\sqrt{15}),\quad (6, 3, 1/\sqrt{15}),\quad (1, 3, \pm\sqrt{3/5}),$$

$$(1, 2, \pm\sqrt{3/20})\quad \text{(Higgs)},$$

$$(3, 2, \sqrt{1/60}),\quad (\bar{3}, 1, \sqrt{4/15}),\quad (\bar{3}, 1, -1/\sqrt{15}),$$

$$(1, 2, -\sqrt{3/20}),\quad (1, 1, -\sqrt{3/5})\quad \text{(matter)}. \tag{2}$$

At M_I, the colour sextet Higgs multiplets responsible for n–n̄ oscillations become massive and drop out from the renormalisation group equations in the range M_I to M_W.

For SUSY SU(N), the coefficient of the one loop beta function is given by

$$b_N = (16\pi^2)^{-1}\left(-3N + \sum_{\text{matter}} T_M + \sum_{\text{Higgs}} T_H\right), \tag{3}$$

where T_M (T_H) is the quadratic SU(N) Casimir invariant for the matter (Higgs) supermultiplets. (All scalar bosons are assumed to be complex throughout tis work.)

Using eq. (3) and the chiral supermultiplets in (2), we find for the strong coupling constant α_s at $M > M_I$

$$\alpha_s^{-1}(M) = \alpha_s^{-1}(M_W) - (2\pi)^{-1}\,[8\ln(M/M_I) - 7\ln(M_I/M_W)]. \tag{4}$$

In order for this symmetry breaking chain to be consistent with perturbative unification we must have $\alpha_s(M_X) \leqslant 1$. From eq. (4) it is apparent that $\alpha_s(M)$ increases with M. Moreover, choosing a typical low energy value of $\alpha_s(M_W) = 0.14$, and $M_I = 10^5$ GeV (the result is insensitive to minor modification of these choices) we find that $\alpha_s(M) = 1$ for $M \simeq 10^{9.7}$ GeV. Thus values of $M_X \geqslant 10^{14}$ GeV as required by limits on proton decay are inconsistent with perturbative unification and n–n̄ oscillations for this chain of symmetry breaking.

Before turning to the left–right symmetric chain (ii) we consider an alternative posibility of detectable n–n̄ oscillations that also exists in chain (i). The usual discussion of n–n̄ oscillation in this case assumes that interactions among left-chiral components of the quarks drive the oscillations. This requires that the colour sextet scalars, Δ, also be SU(2) triplets. The alternative to which we now turn uses interactions among right-chiral components of quarks to give rise to n–n̄ oscillations [3,9]. In this case the multiplets $(6, 3, -1/\sqrt{15})$, $(\bar{6}, 3, 1/\sqrt{15})$ and $(1, 3, \pm\sqrt{3/5})$ in eq. (2) should be replaced by $(6, 1, 2/\sqrt{15})$, $(\bar{6}, 1, -2/\sqrt{15})$, $(6, 1, 4/\sqrt{15})$ and $(\bar{6}, 1, -4/\sqrt{15})$. The number of colour sextets are now reduced from six to four and the strong interaction fine structure constant grows at a slower rate. It turns out that $\alpha_s(M) \simeq 1$ for $M \simeq 10^{17}$ GeV in this case. Thus $M_X \sim 10^{15}$ GeV is consistent with perturbative calculations in α_s. But for perturbative unification to be valid, *none* of the coupling constants must become comparable to unity in the range M_X to M_W. The evolution equation of the $U(1)_Y$ coupling constant is

$$\alpha_Y(M)^{-1} = \alpha_Y(M_W)^{-1} - (2\pi)^{-1}\left[\tfrac{93}{5}\ln(M/M_I) + \tfrac{13}{5}\ln(M_I/M_W)\right]. \qquad (5)$$

Choosing $\alpha(M_W) = 1/128$ and $\sin^2\theta_W = 0.22$, we get $\alpha_Y(M_W) = 0.017$. Substituting this into eq. (5) and using $M_I \sim 10^5$ GeV we find that perturbation theory in α_Y is valid only upto $M \sim 10^{13}$ GeV [i.e. $\alpha_Y(10^{13}\text{ GeV}) \simeq 1$]. Thus even this alternative possibility of n–n̄ oscillations allowed within chain (i) is inconsistent with perturbative unification.

Chain (ii). n–n̄ oscillations were first discussed in the context of left–right symmetric models as in chain (ii). We find that when such GUT models are supersymmetrised, detectable n–n̄ oscillations become inconsistent with perturbative unification. As discussed before, the colour sextet scalars must have a mass $(M_I) \sim 10^5$ GeV in order that the oscillations be detectable in current and upcoming experiments. We distinguish between two cases: case (a) $M_{B-L} > M_I > M_W$ and case (b) $M_R > M_I > M_{B-L}$. Since the Higgs field contributions to the coupling constant beta functions are different in the two cases, they are best considered separately. In both cases we include only the minimal set of scalar necessary for n–n̄ oscillations. Enriching the models by adding extra multiplets (pseudo Goldstone bosons, scalars to mediate H–H̄ oscillations etc.) makes the situation worse as regards perturbative unification.

Our calculations indicate that in both cases (a) and (b), the coupling constant of the $SU(2)_R$ gauge group, $\alpha_{2R}(M)$, becomes of the order of unity at $M \sim 10^{12-13}$ GeV. To obtain the above result, we have to do a little more calculation than in the case of chain (i). This is because in that case the low energy value of $\alpha_Y(M_W)$ was fixed by the values of $\alpha(M_W)$ and $\sin^2\theta_W$. In the left–right symmetric case, $\alpha_Y(M_{B-L})$ determines only a linear combination of $\alpha_{B-L}(M_{B-L})$ and $\alpha_{1R}(M_{B-L})$,

$$\alpha_Y^{-1} = \tfrac{3}{5}\alpha_{1R}^{-1} + \tfrac{2}{5}\alpha_{B-L}^{-1}. \qquad (6)$$

To extract α_{1R} from this equation we use the fact that $\alpha_{B-L}(M_c) = \alpha_s(M_c)$. Using $\alpha_s(M_W) = 0.14$ and the α_s renormalisation group equation we can determine $\alpha_s(M_c)$ and hence $\alpha_{B-L}(M_c)$. Using this value as a boundary conditions the α_{B-L} evolution equation can be utilised to obtain $\alpha_{B-L}(M_{B-L})$ and finally from (6) one obtains $\alpha_{1R}(M_{B-L})$.

Case (a). In this case $M_{B-L} > M_I$. Recall that M_I is the scale corresponding to the mass of the diquark Higgs scalars and must be $\sim 10^5$ GeV for detectable oscillations. The minimum multiplets of Higgs scalars that one required for n–n̄ oscillations as well as matter multiplets are listed in table 1. Note that up until the left–right symmetry is broken at M_R one must keep extra Higgs multiplets to ensure this L–R symmetry in the Higgs sector. The matter multiplet in the left–right symmetric case includes the right-handed neutrino. Using eq. (3) we get

$$\alpha_{1R}^{-1}(M) = \alpha_{1R}^{-1}(M_{B-L}) - (2\pi)^{-1}\left[39\ln(M/M_c) + 27\ln(M_c/M_R) + 29\ln(M_R/M_{B-L})\right], \qquad (7)$$

$$\alpha_{B-L}^{-1}(M_c) = \alpha_s^{-1}(M_c) = \alpha_s^{-1}(M_W) - (2\pi)^{-1}\left[23\ln(M_c/M_R) + 3\ln(M_R/M_{B-L}) + 3\ln(M_{B-L}/M_I) - 7\ln(M_I/M_W)\right], \qquad (8)$$

Table 1
Higgs multiplets contributing at different mass ranges for the case (a) of chain (ii) as discussed in the text. $3_c 2_L 2_R 1_{B-L}$ signifies the irreducible representation under $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$; U(1) quantum numbers are normalised.

M_X–M_c			M_c–M_R				M_R–M_{B-L}				M_{B-L}–M_I			M_I–M_w		
4_c	2_L	2_R	3_c	2_L	2_R	1_{B-L}	3_c	2_L	1_R	1_{B-L}	3_c	2_L	1_Y	3_c	2_L	1_Y
Higgs																
15	1	1	1	3	1	0	$\bar{6}$	1	±1	$-1/\sqrt{6}$	$\bar{6}$	1	$-2/\sqrt{15}$	1	2	$\pm\sqrt{3/26}$
							(twice)				(twice)					
1	3	1	1	1	3	0	1	1	1	$\pm\sqrt{3/2}$	$\bar{6}$	1	$4/\sqrt{15}$			
											(twice)					
1	1	3	1	3	1	$-\sqrt{3/2}$	1	2	±1/2	0	1	1	0			
			(twice)													
$\overline{10}$	3	1	1	1	3	$\sqrt{3/2}$					1	2	$\pm\sqrt{3/20}$			
(twice)																
10	1	3	6	3	1	$1/\sqrt{6}$										
(twice)			(twice)													
1	2	2	$\bar{6}$	1	3	$-1/\sqrt{6}$										
			(twice)													
			1	2	2	0										
Matter																
4	2	1	3	2	1	$1/\sqrt{24}$	3	2	0	$1/\sqrt{24}$	3	2	$1/\sqrt{60}$	3	2	$1/\sqrt{60}$
$\bar{4}$	1	2	$\bar{3}$	1	2	$-1/\sqrt{24}$	$\bar{3}$	1	±1/2	$-1/\sqrt{24}$	3	1	$\sqrt{4/15}$	$\bar{3}$	1	$\sqrt{4/15}$
			1	2	1	$-\sqrt{3/8}$	1	2	0	$-\sqrt{3/8}$	$\bar{3}$	1	$-1/\sqrt{15}$	$\bar{3}$	1	$-1/\sqrt{15}$
			1	1	2	$\sqrt{3/8}$	1	1	±1/2	$\sqrt{3/8}$	1	2	$-\sqrt{3/20}$	1	2	$-\sqrt{3/20}$
											1	1	$-\sqrt{3/5}$	1	1	$-\sqrt{3/5}$
											1	1	0	1	1	0

$$\alpha_{B-L}^{-1}(M_{B-L}) = \alpha_{B-L}^{-1}(M_c) + (2\pi)^{-1}\,[32\ln(M_c/M_R) + 9\ln(M_R/M_{B-L})]\,, \qquad (9)$$

$$\alpha_Y^{-1}(M_{B-L}) = \alpha_Y^{-1}(M_w) - (2\pi)^{-1}\,[\tfrac{93}{5}\ln(M_{B-L}/M_I) + \tfrac{13}{5}\ln(M_I/M_w)]\,. \qquad (10)$$

Using eqs. (6)–(10) and the standard value of $\sin^2\theta_w(M_w) = 0.22$, $\alpha(M_w) = 1/128$ and $\alpha_s(M_w)$ we find $\alpha_R(M) \sim 1$ for $M \lesssim 10^{13}$ GeV. In this case we get an upper bound on M because it is possible to vary the intermediate mass scales M_c and M_R leading to lower values of the mass scale M, at which perturbation theory breaks down.

Case (b). This case corresponds to $M_I > M_{B-L}$. The chiral supermultiplets that contribute to the renormalisation group equations at any stage of the symmetry breaking can again be read off from table 1 with minor modifications. These are as follows: the first two columns remain unchanged. The third and last columns also do not change except that the mass ranges are over $M_R - M_I$ and $M_{B-L} - M_w$. The fourth column ($M_{B-L} - M_I$) in table 1 has to be replaced by a new one. The mass range is from $M_I - M_{B-L}$ and the unbroken group is $SU(3)_c \otimes SU(2)_L \otimes U(1)_R \otimes U(1)_{B-L}$. The multiplets that contribute to the evolution equations are the same as over the range $M_R - M_I$ (column three) except that the colour sextet fields decouple at M_I. Instead of eqs. (7)–(10) we now have

$$\alpha_{1R}^{-1}(M) = \alpha_{1R}^{-1}(M_{B-L}) - (2\pi)^{-1} [39 \ln(M/M_c) + 27 \ln(M_c/M_R) + 29 \ln(M_R/M_I) + 5 \ln(M_I/M_{B-L})] , \quad (11)$$

$$\alpha_{B-L}^{-1}(M_c) = \alpha_s^{-1}(M_c) = \alpha_s^{-1}(M_w) - (2\pi)^{-1} [23 \ln(M_c/M_R) + 3 \ln(M_R/M_I) - 7 \ln(M_I/M_{B-L}) - 7 \ln(M_{B-L}/M_w)] , \quad (12)$$

$$\alpha_{B-L}^{-1}(M_{B-L}) = \alpha_{B-L}^{-1}(M_c) + (2\pi)^{-1} [32 \ln(M_c/M_R) + 9 \ln(M_R/M_I) + 5 \ln(M_I/M_{B-L})] , \quad (13)$$

$$\alpha_Y^{-1}(M_{B-L}) = \alpha_Y^{-1}(M_w) - (2\pi)^{-1} [\tfrac{13}{5} \ln(M_{B-L}/M_w)] . \quad (14)$$

Using these equations in conjunction with eq. (6) we get, in this case, $\alpha_{1R} \sim 1$ for $M \lesssim 10^{12}$ GeV. Therefore even for this alternative, perturbative unification is inconsistent with detectable n–n̄ oscillations.

To summarise, we find that within supersymmetric grand unified theories, detectable n–n̄ oscillations are inconsistent with perturbative unification. The reason behind this is that in supersymmetric theories, every matter and Higgs multiplet has to be extended to the corresponding supermultiplet. The consequent increase in the fermionic and scalar contributions to the renormalisation group equations leads to a coupling constant increasing with energy (the opposite of asymptotic freedom). Our calculations show that in every possible case, at least one of the gauge coupling constants becomes of order unity before the grand unification scale is reached. We have considered only one generation of quarks and leptons. Inclusion of extra generations will cause perturbation theory to break down at an even lower mass scale ar 1 hence make our conclusions stranger.

It has previously been noted by several authors [10,11] that n–n̄ oscillations are not allowed in specific SUSY GUT models. Our result is in agreement with, and a generalisation of, these findings. There has been a claim in the literature [12] of detectable n–n̄ oscillations in SUSY SO(10) GUTs. Eventhough in this work operators (allowed only in SUSY) of dimension less than nine have been used to generate n–n̄ oscillations, yet the large number of scalar and pseudogoldstones (and their superpartners) in this model make it obvious that it does not satisfy the restriction imposed by perturbative unification.

Finally, SUSY GUT models have been discussed in the literature in which the Higgs sector is enriched by adding colour triplet fields to produce an acceptably low value of $\sin^2\theta_w$ [13]. Since our discussion indicates that the expanded Higgs sector necessary for n–n̄ oscillations contradicts perturbative unification in SUSY GUTs, it may well be asked whether the above models also suffer from the same disease. This is not so; the reason being the large differences in the quadratic Casimir invariants that enter in eq. (3) in the two cases. For example, the contribution of a colour sextet (respectively triplet) to the $SU(3)_c$ beta function is proportional to 5/2 (1/2) and hence the effect of ten extra triplets is the same as that of only two sextets. Since n–n̄ oscillations depend on these colour sextets Higgs multiplets, they are especially sensitive to the restrictions of perturbative unification.

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