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Analysis on Propagation Characteristics of Single-mode Step Index Linear and Nonlinear Optical Fiber with Revised Version of Improved Lorentzian Approximation for the Fundamental Mode

Abstract: In this paper, we approximate the fundamental mode of guided propagation by involving an extension of the previous version of Improved Lorentzian parameter for single mode step index liner and nonlinear optical fiber within the scalar variational framework. We show this version gives better performance in terms of field, propagation constants group delay and waveguide dispersion on wider scale of V-values in comparison to earlier ones and works better in low-V region.

Keywords: single mode fiber, V-parameter, step index fiber, propagation constants, group delay, wave guide dispersion

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1 Introduction

For processing and transmission of information through the fundamental mode we prefer using the single mode conventional and photonic crystal fiber. In single mode graded index fiber, if one knows the fundamental modal field one can find different propagation characteristics like spot size, splice loss and bending loss etc. and can predict the limitation of the optical fiber and helps the designer to construct the optical fiber properly. These are all possible provided one solves the Maxwell's wave equation utilized for guided electromagnetic wave propagation. By solving Maxwell's wave equation, one gets analytical field in terms of Bessel and modified Bessel's function for the fundamental mode in case of step index fiber. But to solve same Maxwell's equation for graded index fiber one has to take the help of mathematical methods-exact numerical solution or one may use other approximate methods like variational analysis or perturbation method. The variational approach demands a simple and accurate approximation for fundamental mode with one or two parameters with respect to which one optimizes the certain propagation constants and get a relation through which he can get the field distribution.

Different approximations for modal fields are still being proliferated in the literature to suitably select the expression to work with the optical fiber characteristics [1]. Many scientists are encourage to use Gaussian approximation of the fundamental mode [2, 3] to better judge the optical propagation characteristics. Though Gaussian approximation is much popular but it does not give much accurate results.

Many approximations modifying Gaussian expression [4, 5, 6] are used to get different propagation characteristics to test the optical fiber with the elaborate analysis to suitably help the designer to devise it but it needs a lot of computational time and complexities involved.

Very recently we have shown that Lorentzian approximation [7, 8] of the fundamental modal field for linear and nonlinear single moded step and graded index fiber that works excellently in low V-region in comparison to Gaussian approximation but with a limitation of range of V being less. Further, we adopt certain modification in the field of Lorentzian expression in the cladding region keeping the expression of core in tact, with the improved version of the Lorenztan [9] to get wider range of V in comparison to Lorentzian approximation.

In this paper, we formulate further modification in the same Lorentzian expression in the cladding and get better performance w.r.t. different linear as well as nonlinear characteristics of optical fiber with improved results depicted through graphical analysis through formulation and analysis presented in next section onwards.

2 Analysis

Our Improved Lorentzian field is as follows:

$$\psi = \frac{A}{1+\left(\frac{R^2}{\omega_0^2}\right)} \quad for\ 0 \le R \le 1 \tag{1a}$$

$$\psi = \frac{Ae^{1/R_0}}{1+\frac{1}{\omega_0^2}} e^{-R/R_0} \quad for\ 1 < R \le \infty \tag{1b}$$

with R being normalized radial distance $R = r/a$; a is the core radius. R_0, ω_0 are Lorentzian parameters and A is constant. We get the relation between the parameters by applying appropriate boundary condition taken for guided propagation by equating both equation at $R = 1$ and after differentiating w.r.t. R and equating both at the value of $R = 1$. This gives the relation between R_0 and ω_0 as follows:

$$\frac{d}{dR}\left(\frac{A}{1+R^2/\omega_0^2}\right)_{R=1} = \frac{d}{dR}\left(\frac{Ae^{1/R_0}}{1+1/\omega_0^2} e^{-R/R_0}\right)_{R=1} \tag{2}$$

after a little computation we get

$$R_0 = (\omega_0^2 + 1)/2 \tag{3}$$

The refractive index profile is as given:

$$\begin{aligned} n^2(R) &= n_1^2\left(1-\delta f(R)\right) \quad \text{for } R<1 \\ &= n_1^2\left(1-\delta\right) = n_0^2 \quad \text{for } R>1 \end{aligned} \tag{4}$$

where n_1 and n_0 are axial and cladding refractive indices and $\delta = (n_1^2 - n_0^2)/n_1^2$ and $f(R) = R^q$; in our case we use step profile where profile parameter $q \to \infty$ and $f(R) \to 0$.

The wave equation obtained from Maxwell's equations of electromagnetic wave propagation for the fundamental mode in the optical fiber with Kerr Non-linearity [4] is

$$\frac{1}{R}\frac{d}{dR}\left(R\frac{d\psi}{dR}\right) + a^2\left(k_0^2\{n^2(R)+n_0 n_2 I\} - \beta^2\right) \quad \psi = 0 \tag{5}$$

where n_2 is nonlinear refractive index. Equation (5) may be written as

$$\frac{1}{R}\frac{d}{dR}\left(R\frac{d\psi}{dR}\right) + a^2\left(k_0^2\{n_1^2(1-\frac{n_1^2-n_0^2}{n_0^2}R^q) + n_0 n_2 \psi^2) - \beta^2\right) \quad \psi = 0$$

Incorporating full nonlinearity and substituting other parameter as E.A.J. Marcatili et al. used [4] we get

$$\frac{1}{R}\frac{d}{dR}\left(R\frac{df}{dR}\right) + \left[u^2 - v^2 R^q + v^2 \gamma f^2\right] \quad f = 0 \tag{6}$$

where

$$u^2 = k^2 n_1^2 - \beta^2; \quad v^2 = a^2(n_1^2 - n_0^2);$$

$$\gamma = \frac{4 n_2 P}{c a^2 (n_1^2 - n_0^2)}; \quad f = \frac{a}{2}\sqrt{\frac{n_0 c}{P}}$$

and P = power for which Kerr nonlinearity may occur. As we consider step profile the value of $R^q \to 0$; after a little computation we get

$$u^2 = \frac{v^2 \int_1^\infty f^2(R)\rho dR + \int_0^\infty (\frac{df(R)}{dR})^2 R dR - \gamma v^2 \int_0^\infty f^4(R) R dR}{\int_0^\infty f^2(\rho)\rho d\rho} \tag{7}$$

The variational expression from above equation,

$$u^2 = \frac{v^2 \int_1^\infty \psi^2(R) R dR + \int_0^\infty (\frac{d\psi(R)}{dR})^2 R dR - \gamma v^2 \frac{a^2}{4}\frac{n_0 c}{P} \int_0^\infty \psi^4(R) R dR}{\int_0^\infty \psi^2(\rho)\rho d\rho} \tag{8}$$

Substituting ψ as given in equation 1a) and 1b) in the above expression (8) we find the variational relation (9) which is optimized with the Lorentzian parameter ω_0,

$$u^2 = \frac{I_1 + (I_2 + I_3) + (I_4 + I_5)}{I_6 + I_7} \tag{9}$$

where

$$I_1 = v^2 \int_1^\infty \psi^2(R) R dR = \frac{A^2 R_0^2 v^2}{4(1+\frac{1}{\omega})^2}(1+\frac{2}{R_0}) \tag{10}$$

$$I_2 = \int_0^1 (\frac{d\psi(R)}{dR})^2 R dR = A^2(\frac{1}{3} - \frac{1+\frac{3}{\omega_0^2}}{3(1+\frac{1}{\omega})^3}) \tag{11}$$

$$I_3 = \int_1^\infty (\frac{d\psi(R)}{dR})^2 R dR = A^2(1+\frac{2}{R_0})\frac{1}{4(1+\frac{1}{\omega_0^2})^2} \tag{12}$$

$$I_4 = -\gamma v^2 \frac{a^2 n_0 c}{4P}\int_0^1 (\psi(R))^4 R dR$$
$$= -A^4 v^2 \frac{\omega_0^2}{6}\left(1 - \frac{1}{(1+\frac{1}{\omega})^3}\right) \tag{13}$$

$$I_5 = -\gamma v^2 \frac{a^2 n_0 c}{4P}\int_1^\infty (\psi(R))^4 R dR$$
$$= -A^4 v^2 \frac{anc}{4P}\frac{R_0^2}{16(1+\frac{1}{\omega_0^2})^4}\left(1+\frac{4}{R_0}\right) \tag{14}$$

$$I_6 = \int_0^1 \psi^2(R) R dR = \frac{A^2\omega_0^2}{2(1+\omega_0^2)} \tag{15}$$

$$I_7 = \int_1^\infty \psi^2(R) R dR = \frac{A^2 R_0^2}{4(1+\frac{1}{\omega})^2}\left(1+\frac{2}{R_0}\right) \tag{16}$$

Above expression of u^2 as in equation (9) is optimized to obtain the relation between Lorentzian parameter and normalized frequency v by variational approach. Subsequently the results are well organized and presented ithrough discussion in the next section.

3 Results and discussion

Based on the above analysis, we compute the values related to different characteristics like field, normalized propagation constant, group delay and wave guide dispersion for improved Lorentzian both for linear and non linear regime then we compare the result with the analytical and exact numerical ones. They are depicted in the following figures. They are found excellently matching with the analytical ones with improved range of V in the low V region [7]. When we compare the results obtained from both approximations and those of the analytical field and exact numerical field in case of linear step profile we find improved Lorentzian shows better matching with that of analytical. We find the fundamental field for analytical by solving Bessel and modified Bessel equation and compare the fundamental field of Lorentzian and improved Lorentzian by the method of variational approach as shown in Fig. 1. Here, analytical field (black) is much coinciding with improved Lorentzian (red colour graph)than ordinary Lorentzian shown by blue colour graph. Fig. 1 represents field vs. radial distance with $V = 1.2$; Fig. 2 represents the same with v = 1.5. We observe that when the of v increases red curve comes nearer to black one than that of Lorentzian (blue). We would like to mention that Fig. 2 presents the linear regime with $\gamma = 0$; and Fig. 2a presents the non linear regime with the value of non linear term,

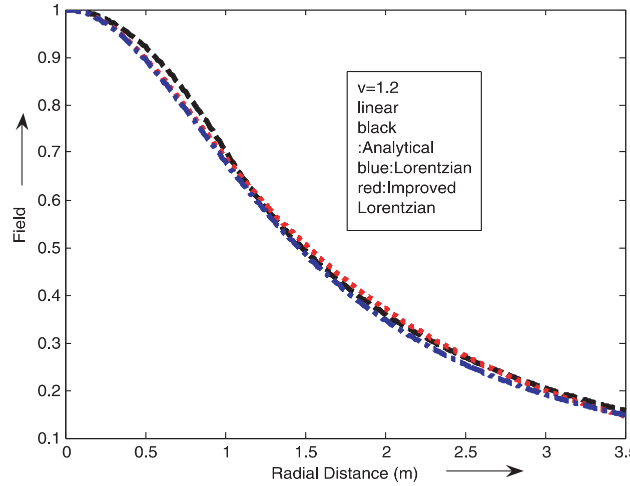

Fig. 1: Field vs. radial distance along the optical fiber at V = 1.2

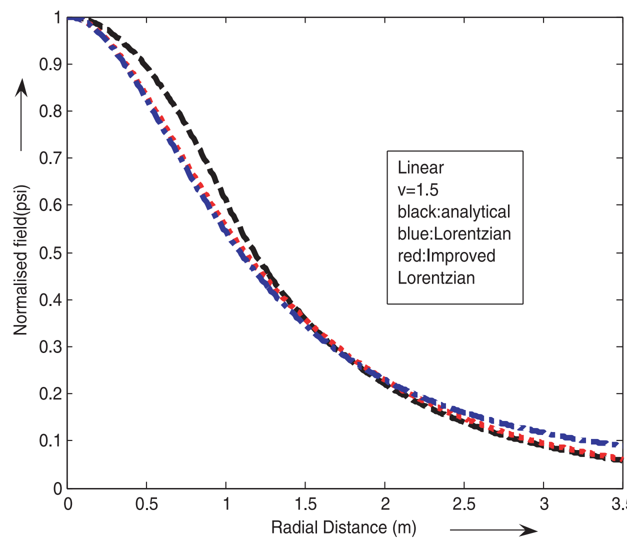

Fig. 2: Field vs. radial distance along the optical fiber at V = 1.5

$\gamma = 0.062$ corresponding to the power of the optical pulse at P = 200 kW. Next we represent the Figs. 3 and 4 which gives the plot of U (normalized propagation constant) vs. normlised frequency, V. Here also red colour graph which represent improved Lorentzian is much closer to analytical one n both linear and non linear cases. One interesting thing is to observe that in nonlinear case the curve extends for more values of U than that of linear one for a particular value of V. Similarly Figs. 5 and 6 give the variation of b(v) vs. normalised frequency (V). Here also in the low V region the red curve isexcellently matching with improved or wider values of V. Next in Figs. 7 and 8 we show the variations of group delay both for linear and nonlinear. Here also red curve representing improved Lorentzian is closer to analytical, black one but at higher values of normalized frequency, V, there is a lot of mismatch in both the cases,

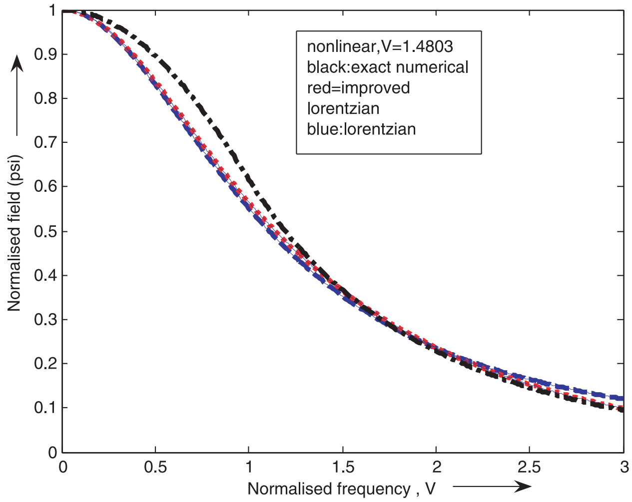

Fig. 2a: Field vs. radial distance along the optical fiber at V = 1.4803 with $\gamma = .062$

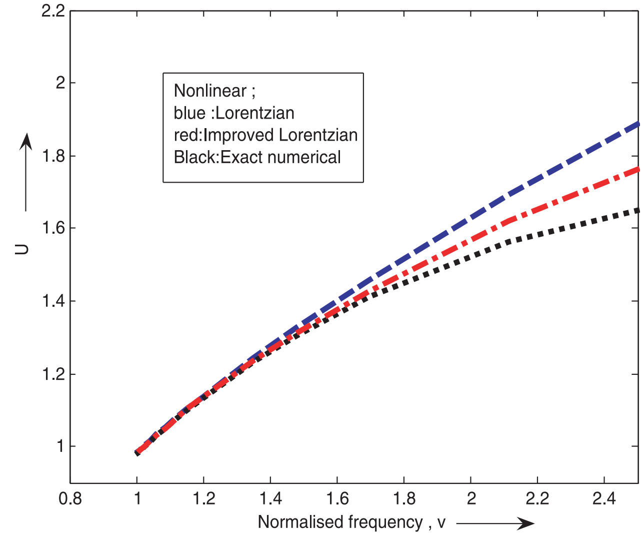

Fig. 4: U vs. V (normalised frequency) [$U^2 = u^2 = k^2 n_1^2 - \beta^2$]

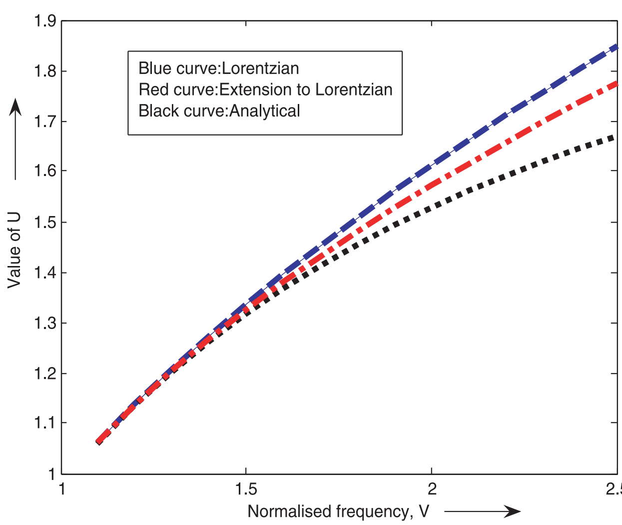

Fig. 3: U vs. V (normalised frequency) [$U^2 = u^2 = k^2 n_1^2 - \beta^2$]

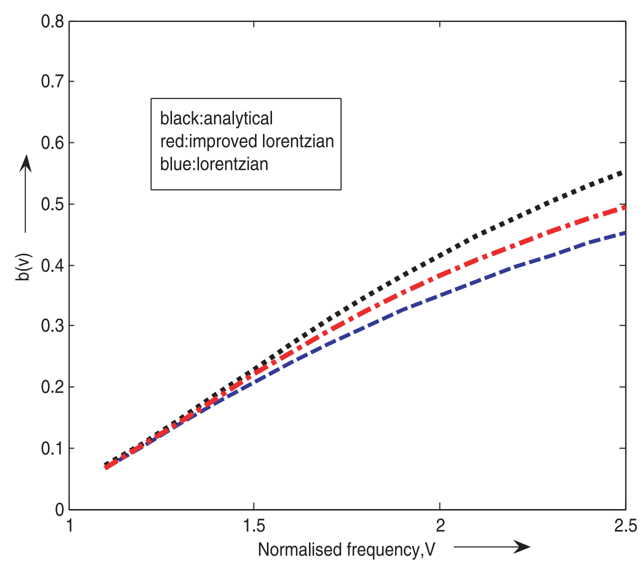

Fig. 5: b(V) (normalized propagation constant) vs. V (normalised frequency)

for linear and non linear graph. Now we show the wave guide dispersion curve both for linear and represented by Figs. 9 and 10. The trend of the dispersion curve of improved Lorentzian in both linear and nonlinear are similar but Lorentzian gives same trend but not like improved Lorentzian one. By all these curves showing different optical fiber characteristics we may indicate that improved Lorentzian is realy improved version of Lorentzian.

4 Conclusion

We propose for the first time a single parameter improved Lorentzian approximation of the fundamental mode in single mode linear and nonlinear step index fiber. With this approximation, we develop the variational analysis, present the approximate analytical formulae for various propagation characteristics and compare them with those based on calculation involving ordinary Lorentzian, inproved version and analytical or exact numerical ones. Our results show that improved version behaves better way in comparison to Lorentzian approximation and it is more superior to Gaussian [7] and ordinary Lorentzian.

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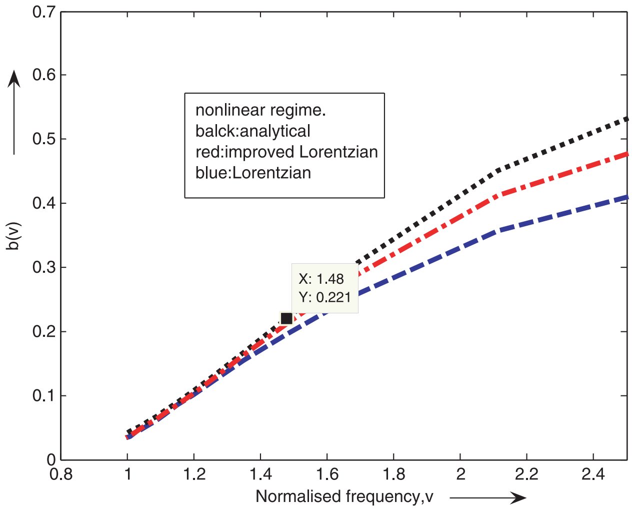

Fig. 6: b(V) (normalised propagation constant) vs. V (normalised frequency)

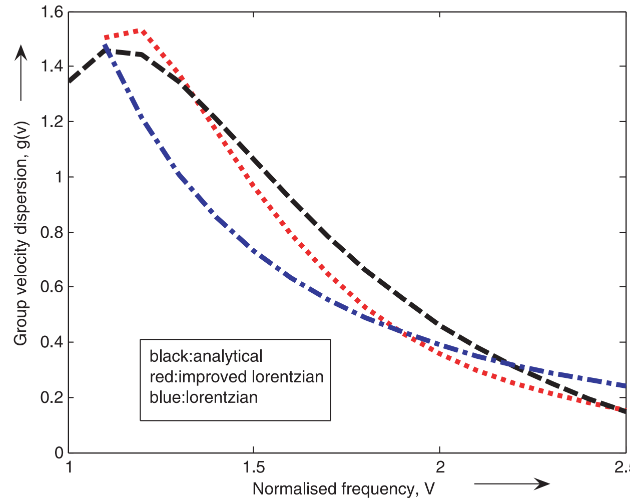

Fig. 9: g(V) (normalized wave guide dispersion) vs. V (normalised frequency)

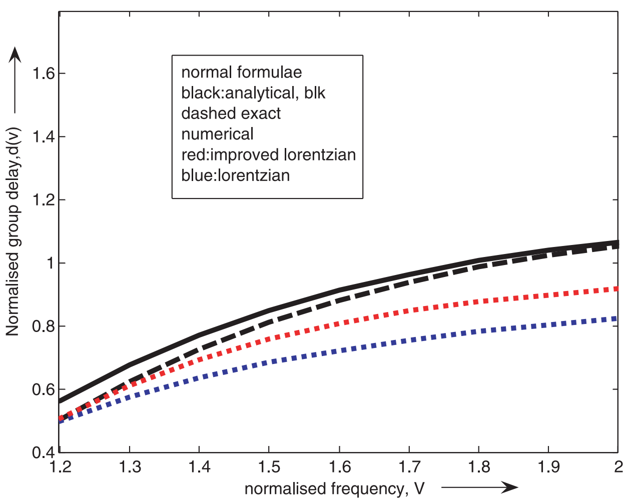

Fig. 7: d(V) (normalized group delay) vs. V (normalised frequency)

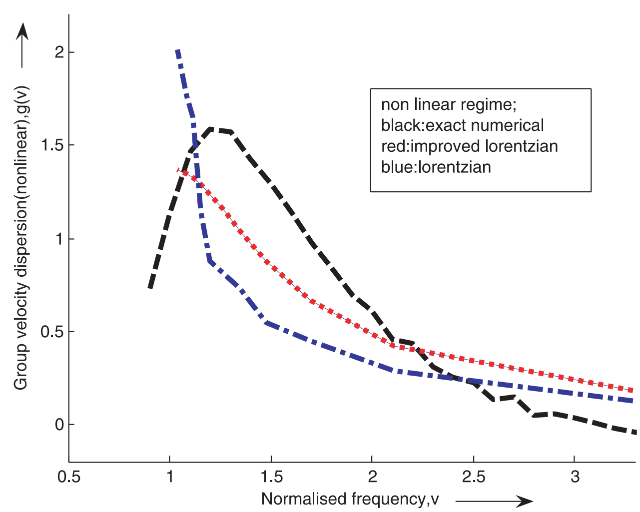

Fig. 10: g(V) (normalized wave guide dispersion) vs. V (normalised frequency)

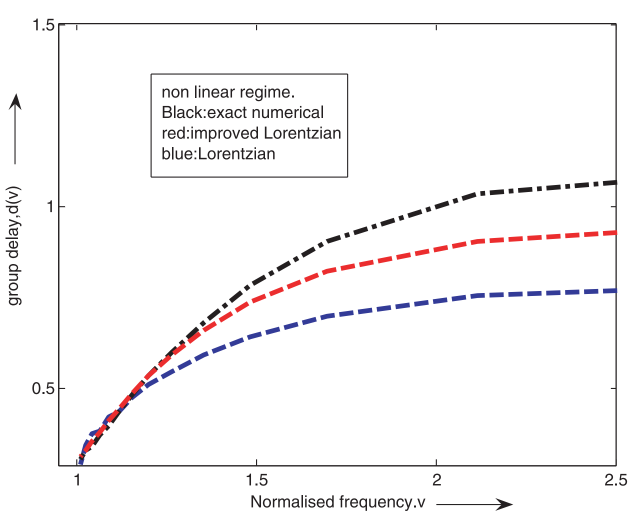

Fig. 8: d(V) (normalized group delay) vs. V (normalised frequency)

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