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Amplification Factor and Perveance of an Elliptic Triode

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An attempt has been made to derive, in terms of valve geometry, theoretical expressions for the constants of an elliptic triode, the shape of the electrodes of which belongs to a family of confocal ellipses. The amplification factor (μ) is derived by following the method of conformal transformation. Result shows that in general it varies with the parametric angle θ of the ellipse—decreasing from the direction of minor to that of the major axis. The average value of μ increases with increasing value of the eccentricity of the grid and with decreasing value of the eccentricity of the anode. Curves for ready evaluation of μ for any value of θ are given. The problem of design of an elliptic triode having a constant μ for all values of θ is also considered and its method of solution indicated. An expression for the perveance (P) has also been obtained by adopting the procedure followed by O'Neill for determining the perveance of a plane diode having filamentary cathode. The expression shows that the value of perveance is determined mainly by the eccentricity of the grid and the focal distance.

To illustrate the practical applications of the various deductions made, a numerical example, involving the design of a 6C5-GT/G type triode, is given.

1. INTRODUCTION

A KNOWLEDGE, in terms of the electrode geometry, of the amplification factor, the interelectrode capacitances, and the perveance of an electronic valve is of great importance for the design and manufacture such valves. Many investigations have, therefore, been carried out to derive theoretical expressions for these parameters. The cases studied most are those of triodes and multigrid valves having planar and cylindrical geometry. Expressions are now available for calculating theoretically the various constants of these types of triodes with arbitrary electrode dimensions and spacings.[1–3] However, many commercial triodes have a geometry which is neither plane nor cylindrical. Of these, those possessing approximately an elliptic geometry are perhaps the most numerous. But it appears that no theoretical analysis has yet been attempted to obtain expressions for the constants of such triodes. In the existing design practice, therefore, such valves are treated more or less as of cylindrical or planar structure having electrode distances equal to the average distances of the valve electrodes, use being made of certain empirical relations for estimating the valve constants. Such a procedure is obviously unsatisfactory. In the present paper an attempt will be made to develop expressions for the amplification factor, interelectrode capacitances, and perveance of a triode of elliptic geometry. In order that the problem may be amenable to mathematical treatment, the case considered will be that for which the electrode surfaces constitute a system of confocal ellipses. Subject to certain reasonable restrictions, the analysis will be shown to yield expressions useful for practical valve design.

[1] K. R. Spangenberg, *Vacuum Tubes* (McGraw-Hill Book Company, Inc., New York, 1948).
[2] W. B. Walker, Proc. Inst. Elec. Engrs. (London), **98**(**III**), 57 (1951).
[3] M. Wada, Scientific Rept. Research Insts. Tohoku Univ. **B-1**, 2, 399 (1950).

2. TRIODE OF ELLIPTIC GEOMETRY

As mentioned earlier, the special case considered will be that of a valve for which the cathode, the grid, and the anode surfaces are confocal elliptic cylinders. Let the eccentricities of these be e_c, e_g, and e_a, respectively. The grid is supposed to be made of straight wires, all of the same radius r_g, and disposed parallel to the common axis of the cylinders at equal intervals over the grid surface. A transverse section of the above structure consisting of the anode (A), the grid (G), and the cathode (C) is shown in Fig. 1. The directions of the axes and the center of the confocal elliptic system have been taken, respectively, as the directions and the origin of a rectangular coordinate system (x,y). The two foci of the ellipses are the points $(\pm K,0)$ on the (x,y) coordinate system. The centers of the grid wires divide the elliptic grid section into a number of equal arcs. The length D of each of these arcs is related to the total number of grid wires ($2N$) by

$$D=2KE/(Ne_g), \tag{1}$$

where $\delta_g=\sin^{-1}e_g$ and $E=E(\delta_g,\pi/2)$ represents the complete elliptic integral of the second kind.

3. CONFORMAL TRANSFORMATION OF THE TRIODE GEOMETRY

In order to investigate analytically the foregoing problem, it is necessary first to reduce the elliptic triode to a simpler geometry for which solutions are easily obtainable. For this purpose, we restrict our interest to the region of the triode lying above the x axis (Fig. 1) and apply the transformation given by

$$Z=K\sin(W/K), \tag{2}$$

where

and

$$\left.\begin{aligned} W&=\varphi+j\psi \\ Z&=x+jy \end{aligned}\right\}. \tag{3}$$

From Eqs. (2) and (3), we obtain

$$x = K\cosh(\psi/K)\sin(\varphi/K) \tag{4a}$$

$$y = K\sinh(\psi/K)\cos(\varphi/K). \tag{4b}$$

As a result of the aforementioned transformation, a system of confocal ellipses with their foci located at $(\pm K, 0)$ in the Z plane transform in the W plane into a series of lines parallel to the φ axis. It follows, therefore, that the elliptic arc EAF representing the upper half of the anode in Fig. 1 becomes in the W plane a straight line given by

$$\psi = K\cosh^{-1}(a_a/K)$$
$$= K\cosh^{-1}(1/e_a) = \psi_a, \text{ say} \tag{5}$$

where a_a is the semimajor axis of the elliptic anode A.

Putting $e_a = \sin\delta_a$ and using the identity $\cosh^{-1}u = \ln[u + (u^2-1)^{\frac{1}{2}}]$, Eq. (5) reduces to

$$\psi = \psi_a = K\ln\cot(\delta_a/2). \tag{6}$$

Similarly, the transformed grid and cathode arcs are the straight lines, given, respectively, by

$$\psi = K\ln\cot(\delta_g/2) = \psi_g, \text{ say} \tag{7}$$

and

$$\psi = K\ln\cot(\delta_c/2) = \psi_c, \text{ say} \tag{8}$$

where $\delta_c = \sin^{-1}e_c$.

It also follows from Eqs. (4a) and (4b) that the three transformed lines would extend from $+K\pi/2$ to $-K\pi/2$. The transformed electrode surfaces constitute a plane triode having the following electrode spacings:

Cathode-to-grid distance d_{cg}

$$= \psi_g - \psi_c = K\ln[\cot(\delta_g/2)/\cot(\delta_c/2)]. \tag{9}$$

Grid-to-anode distance d_{ga}

$$= \psi_a - \psi_g = K\ln[\cot(\delta_a/2)/\cot(\delta_g/2)]. \tag{10}$$

If, now, Eqs. (4a) and (4b) are compared with the parametric equation of an ellipse, *viz.* $x = a\sin\theta$, $y = b\cos\theta$, a and b being the semimajor and semiminor diameter, respectively, it is seen that

$$\theta = \varphi/K. \tag{11}$$

Equation (11) shows that points on any of the confocal ellipses which correspond to equal intervals of the angle θ in the Z plane are spaced at equal intervals of φ in the W plane. Now, equal arc lengths of an ellipse correspond to increasing intervals in θ as one goes away from the semiminor axis. It, therefore, follows from Eq. (11) that the spacings of the transformed images of the grid wire centers would increase with the increasing values of φ. If the θ values for any two consecutive grid wire centers be denoted by θ_1 and

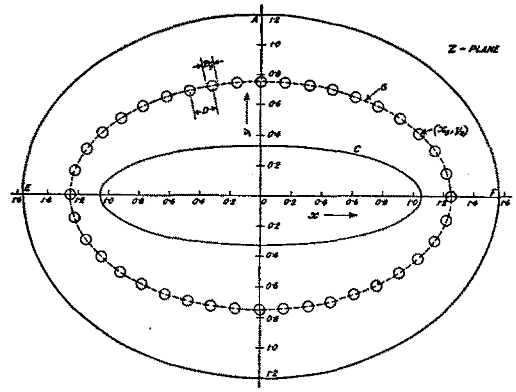

FIG. 1. Transverse section of the elliptic triode in the Z plane ($K=1$).

θ_2, then their spacing in the W plane is given by

$$c_\varphi = \varphi_2 - \varphi_1 = f(\varphi)$$
$$= K(\theta_2 - \theta_1) = Kf(\theta). \tag{12}$$

We now derive some approximate expressions which hold good closely for the case where the grid-wire radius (r_g) is small compared to (K/e_g)—a case usually met with in practice.

Firstly, as shown in the Appendix, for such case, a circle of radius r_g with its center lying on the ellipse G, has for its image in the W plane a figure which is very nearly a circle. This image circle has a radius given by

$$r_\varphi = r_g e_g / [1 - e_g^2 \sin^2(\varphi/K)]^{\frac{1}{2}}, \tag{13}$$

and its center is located at the point (φ, ψ_g).

Next we note that in the W plane, ψ_g is the same for all the grid-wire centers but φ varies. Thus the radii of the transformed grid-wire circles are larger for larger φ and hence for larger values of θ in the Z plane.

In the W plane, therefore, not only the spacing c_φ between consecutive grid-wire centers but also the grid-wire radius, r_φ increases as one goes away from the ψ axis. This is illustrated in Fig. 2(a). But, nevertheless, as shown below, the screening fraction (defined by $s = 2r_\varphi/c_\varphi$ remains very approximately a constant.

Consider a section $ABCD$ of the W plane [Fig. 2(a)] bounded by the lines $\varphi = \varphi' = K\theta'$ and $\varphi = \varphi'' = K\theta''$ ($\theta'' > \theta'$) with the transformed grid-wire center located at $\varphi = (\varphi' + \varphi'')/2 = K(\theta' + \theta'')/2 = K\theta$ and $\psi = \psi_g$. For this section, $c_\varphi = K(\theta'' - \theta')$ and the screening fraction of the W plane triode geometry is, from Eq. (13),

$$s = 2r_g e_g / [K(\theta'' - \theta')(1 - e_g^2 \sin^2\theta)^{\frac{1}{2}}]$$
$$= \frac{2r_g e_g}{K(\theta'' - \theta')[(\partial/\partial\theta)E(\delta_g, \theta)]},$$

where $E(\delta_g, \theta)$ is the incomplete elliptic integral of the second kind. Assuming that the interval $\theta'' - \theta' = \Delta\theta$ is so small that $E(\delta_g, \theta)$ may be taken to vary linearly with θ in the foregoing interval, the foregoing equation

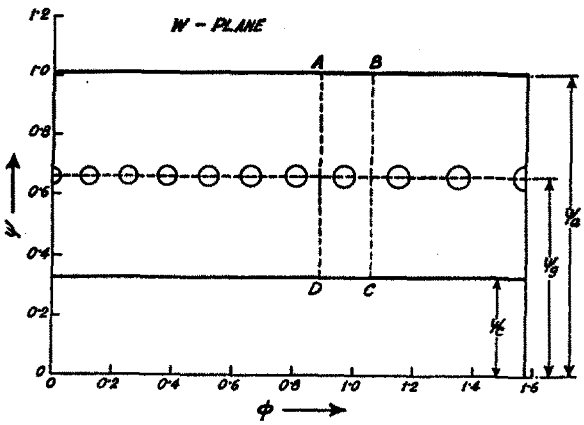

FIG. 2(a). Transverse section of the W plane representation of one quadrant of the elliptic triode ($K=1$).

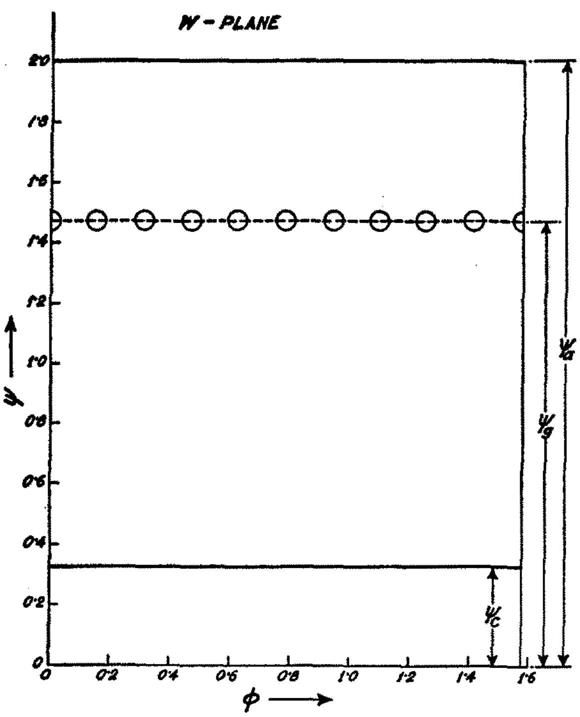

FIG. 2(b). Same as in Fig. 2(a) with small value of e_g.

reduces to

$$\begin{aligned} s &\approx 2r_g e_g/[K\Delta E(\delta_g,\theta)] \\ &= 2r_g/[(K/e_g)\{E(\delta_g,\theta'')-E(\delta_g,\theta')\}] \\ &\approx 2r_g/D \\ &= r_g e_g N/[KE], \quad \text{from Eq. (1).} \end{aligned} \tag{14}$$

It can be easily shown by actual numerical calculation that Eq. (14) holds good to a high degree of accuracy even for $e_g \simeq 1$, provided that $N \geqslant 6$. For such cases, therefore, the screening fraction of the W plane triode is constant and is given by Eq. (14).

Finally, from Eqs. (13) and (14) it follows that the grid-wire spacing should vary with φ as follows:

$$c_\varphi \approx c_0/[1-e_g^2 \sin^2(\varphi/K)]^{\frac{1}{2}}, \tag{15}$$

where $c_0 = 2KE/N = e_g D$.

It may be easily shown that for $e_g < 0.45$ the following relations hold valid within 10 percent;

$$c = K\pi/N, \quad r = r_g e_g, \quad s = 2Nr_g e_g/(K\pi).$$

Under this condition the transformed tube geometry in the W plane reduces to that of an ideal plane triode as shown in Fig. 2(b).

4. INTERELECTRODE CAPACITANCES AND AMPLIFICATION FACTOR

As pointed out in Sec. 3, the W plane image of the elliptic triode [Fig. 2(a)] can be looked upon as a plane triode of constant screening fraction but of variable pitch. No direct analytical solution for such a triode is known. It may, however, be noted that the W plane triode of Fig. 2(a) may be divided into $N/2$ symmetrical sections each containing a single grid wire. Each one of the sections, say $ABCD$, may be taken as a part of an ordinary plane triode whose screening fraction and pitch are the same as those of the section chosen. One can, therefore, apply the known results for a plane triode individually to these $N/2$ sections and obtain expressions for their interelectrode capacitances and amplification factors. The total interelectrode capacitances of the triode would then be given by the sum of these individual capacities and these, in their turn, will give the mean amplification factor of the valve.

Let us, therefore, consider one of the $N/2$ sections mentioned previously ($ABCD$) of unit height in a direction perpendicular to the plane of the figure and let the grid-wire center for this section be (φ,ψ_g). This section represents a system of electrostatic capacities which constitute a Δ network as shown in Fig. 3(a). This, again, is equivalent to the star network of Fig. 3(b). Here the common terminal represents a hypothetical equipotential plane whose electrostatic field near the cathode and the anode is the same as that of the actual grid and is called the equivalent grid sheet. Provided that $s<1/6$, $d_{cg}>c_\varphi$, $d_{ga}>c_\varphi$ elements of this star network are obtained from the theory of plane triodes as[1,4]

$$C_{1\varphi} = \frac{c_\varphi}{4\pi d_{ga} - 2c_\varphi \ln\cosh(2\pi r_\varphi/c_\varphi)} \tag{16}$$

$$C_{2\varphi} = \frac{c_\varphi}{4\pi d_{cg} - 2c_\varphi \ln\cosh(2\pi r_\varphi/c_\varphi)} \tag{17}$$

$$C_{3\varphi} = \frac{1}{2\ln\coth(2\pi r_\varphi/c_\varphi)}. \tag{18}$$

[4] W. G. Dow, *Fundamentals of Engineering Electronics* (John Wiley and Sons, Inc., New York, 1952).

Using Eqs. (9), (10), (14), and (15) and referring to the Z plane

$$C_{1\theta}=\frac{E}{[2\pi N(1-e_g^2\sin^2\theta)^{\frac{1}{2}}\ln\{\cot(\delta_a/2)/\cot(\delta_g/2)\}-2E\ln\cosh\{\pi r_g e_g N/(KE)\}]} \quad (19)$$

$$C_{2\theta}=\frac{E}{[2\pi N(1-e_g^2\sin^2\theta)^{\frac{1}{2}}\ln\{\cot(\delta_g/2)/\cot(\delta_c/2)\}-2E\ln\cosh\{\pi r_g e_g N/(KE)\}]} \quad (20)$$

$$C_{3\theta}=\frac{1}{2\ln\coth\{\pi r_g e_g N/(KE)\}}. \quad (21)$$

The capacitances C_1 and C_3 determine the relative effectiveness of the anode and the actual grid in fixing the potential of the equivalent grid sheet and hence the off-cathode field. Amplification factor of the valve is, therefore, defined as

$$\begin{aligned}\mu_\theta&=C_{3\theta}/C_{1\theta}\\&=[1/\ln\coth\{\pi r_g e_g N/(KE)\}]\\&\quad\times[(\pi N/E)(1-e_g^2\sin^2\theta)^{\frac{1}{2}}\ln\{\cot(\delta_a/2)/\cot(\delta_g/2)\}\\&\quad-\ln\cosh\{\pi r_g e_g N/(KE)\}]. \quad (22)\end{aligned}$$

In valves as used in practice the second term in the numerator of Eq. (22) is always less than one-tenth of the value of the first. This term may, therefore, be omitted. We thus get

$$\mu_\theta/\mu_0=(1-e_g^2\sin^2\theta)^{\frac{1}{2}}, \quad (23)$$

where

$$\mu_0=(\pi N/E)\frac{\ln\{\cot(\delta_a/2)/\cot(\delta_g/2)\}}{\ln\coth\{\pi r_g e_g N/(KE)\}}, \quad (24)$$

is the amplification factor for the central region of the valve. The r.h.s. of Eq. (23) can be expressed in terms of Jacobi's elliptic function dnu, which is readily available in graphical and tabular form.[5] Here the variable u is the incomplete elliptic integral of the first kind $F(\delta,\theta)$. For practical handling, however, it is desirable to have a plot of the ratio (μ_θ/μ_0) directly against θ. Such a set of curves is given in Fig. 4(a). In Fig. 4(b) is given a graphical representation of Eq. (24).

In order to compute the total interelectrode capacitances of the valve, we note that the second term in the denominator of Eqs. (16) and (17) does not exceed one-tenth of the value of the first. Neglecting this term and summing up for all the $2N$ sections, we get from Eq. (16),

$$C_1=\sum\frac{c_\varphi}{4\pi K\ln\{\cot(\delta_a/2)/\cot(\delta_g/2)\}}.$$

[5] E. Jahnke, and F. Emde, *Tables of Functions* (Dover Publications, New York, 1945).

Use of Eq. (13) gives $\sum c_\varphi=2K\pi$. Thus

$$C_1=1/[2\ln\{\cot(\delta_a/2)/\cot(\delta_g/2)\}]. \quad (25)$$

Similarly, from Eqs. (17) and (18)

$$C_2=1/[2\ln\{\cot(\delta_g/2)/\cot(\delta_c/2)\}] \quad (26)$$

$$C_3=N/\ln\coth\{\pi r_g e_g N/(KE)\}. \quad (27)$$

The average value of the amplification factor is therefore given by

$$\begin{aligned}\mu&=C_3/C_1\\&=\frac{2N\ln\{\cot(\delta_a/2)/\cot(\delta_g/2)\}}{\ln\coth\{\pi r_g e_g N/(KE)\}}. \quad (28)\end{aligned}$$

It is interesting to note from Eqs. (24) and (28) that μ_0 approaches μ as δ_g becomes small.

From Eqs. (25) to (27) one obtains expressions for the total Δ-network capacitances:

$$C_{ac}=\frac{C_2}{\mu(1+1/\mu+1/\mu')},\quad C_{gc}=\frac{C_2}{1+1/\mu+1/\mu'},\quad C_{ga}=\frac{C_1}{1+1/\mu+1/\mu'}, \quad (29)$$

where

$$\begin{aligned}\mu'&=C_3/C_2\\&=\frac{2N\ln\{\cot(\delta_g/2)/\cot(\delta_c/2)\}}{\ln\coth\{\pi r_g e_g N/(KE)\}}. \quad (30)\end{aligned}$$

5. EQUIVALENT GRID SHEET POTENTIAL AND PERVEANCE

The action of a triode valve is conveniently visualized in terms of the equivalent grid sheet potential. This potential may be obtained from the equivalent star network of the valve capacities by determining the potential of the neutral point. This potential (V_0)

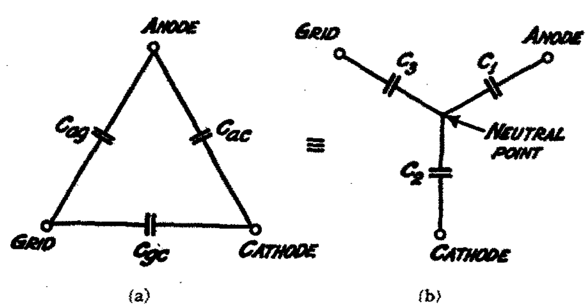

FIG. 3(a). Delta network representing the interelectrode capacitances of a symmetric section of the triode containing a single grid wire. (b) Equivalent star network of (a).

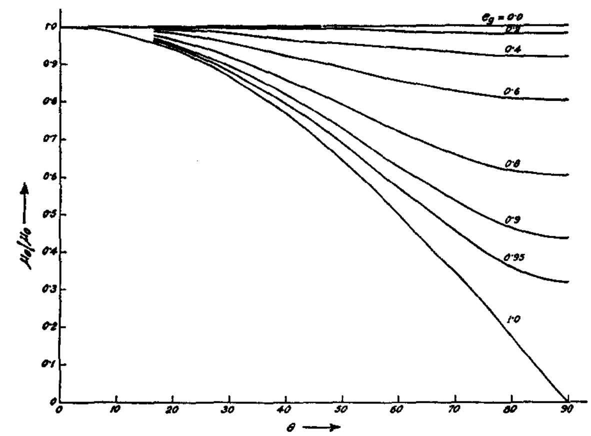

FIG. 4(a). Curve showing the variation of amplification factor with the parametric angle θ.

satisfies the condition that the net charge induced in the electrode system under purely electrostatic condition is zero, i.e.,[1,4]

$$C_1(V_a-V_0)+C_2(V_c-V_0)+C_3(V_g-V_0)=0,$$

where V_a, V_g, and V_c are the potentials of the anode, grid, and cathode, respectively.

Putting $V_c=0$, we get the equivalent grid sheet potential

$$V_0=\frac{V_g+V_a(C_1/C_3)}{1+(C_1/C_3)+(C_2/C_3)}=\frac{V_g+V_a/\mu}{1+(1/\mu)+(1/\mu')}. \quad (31)$$

We can now apply Child's law and write

$$i=PV_0^{\frac{3}{2}}, \quad (32)$$

where i is the cathode current and P is a constant of the valve. In conformity with the nomenclature adopted for the cases of plane and cylindrical triodes, P may be called the perveance of the elliptic triode. It remains, however, to express P in terms of the geometrical constants of the valve.

A rigorous analysis of the problem is difficult as it involves the solution of a two-dimensional Poisson's equation. An approximate solution may, however, be obtained by following the method adopted by O'Neill and by Matricon and Trouve for the case of a filamentary cathode between two equidistant plane anodes.[4,6,7] The method is based on the assumption

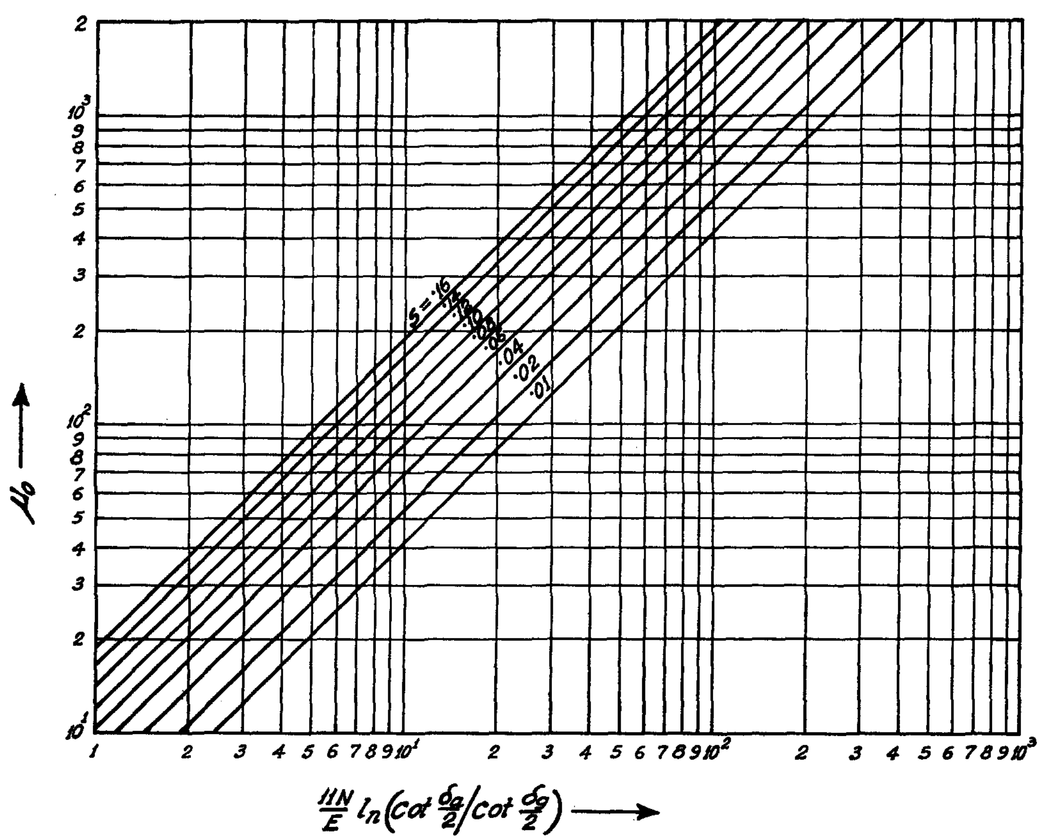

FIG. 4(b). Graphical representation of Eq. (24).

[6] G. D. O'Neill, Sylvania Technologist 3, 22 (1950).
[7] M. Matricon and S. Trouve, Onde. Elect. 30, 510 (1950).

that the space charge flow equations of two diodes, having the same value of electrostatic capacitance between their electrodes, are the same.

The system composed of the cathode and the equivalent grid sheet plane may be considered to act as a diode as shown in Fig. 5. From purely electrostatic consideration this represents a plane parallel capacity of value

$$\pi K/(4\pi d_{cg}) = C, \text{ say}$$

per unit length in the direction normal to the plane of the figure. Using Eq. (9)

$$C = 1/[4 \ln\{\cot(\delta_g/2)/\cot(\delta_c/2)\}] = \frac{1}{2\ln\left[\dfrac{K\cot^2(\delta_g/2)}{K\cot^2(\delta_c/2)}\right]}.$$

Thus, the capacity system composed of the semielliptic cathode and grid is equivalent to a cylindrical capacity having for its inner and outer radii

$$\left.\begin{aligned} R_i &= K\cot^2(\delta_c/2) \\ R_0 &= K\cot^2(\delta_g/2) \end{aligned}\right\}. \quad (33)$$

The perveance of this cylindrical diode may, therefore, be taken as the perveance of the actual triode. Remembering that the new cylindrical diode, as obtained above, represents only the upper half of the actual triode, as shown in Fig. 1, we obtain from the well-known equation of space charge flow in a cylindrical diode the perveance[1,4]

$$P = \frac{29.32\times 10^{-6}\times l}{\beta^2 R_0} = \frac{29.32\times 10^{-6}\times l}{K\beta^2\cot^2(\delta_g/2)}, \quad (34)$$

where

$$\beta^2 = 2\ln\{\cot(\delta_g/2)/\cot(\delta_c/2)\} - (8/5)[\ln\{\cot(\delta_g/2)/\cot(\delta_c/2)\}]^2 + (11/15)[\ln\{\cot(\delta_g/2)/\cot(\delta_c/2)\}]^3 + \text{etc.}, \quad (35)$$

and l is the length of the anode.

An expression for the transconductance g_m of the elliptic triode may now be obtained by evaluating $(\partial i/\partial V_g)_{V_a}$ from Eq. (32) and making use of Eqs. (28), (30), (34), and (35).

6. DISCUSSION

The expression for amplification factor of an elliptic triode shows that, in general, μ would depend upon the value of θ and hence would vary from section to section. The extent of this variation is determined mainly by the eccentricity of the grid ellipse. Even for $e_g = 0.45$, a 10 percent variation in μ will occur. As is well known, however, the value of μ as realized under actual operating condition of a valve may differ from the theoretical value by as much as ± 20 percent because of the unavoidable simplifying assumptions in course of the analysis. As such, it is reasonable to take the value of μ as given by Eq. (28) as the average value of μ, at least up to $e_g = 0.85$. It may also be noted that according to Eqs. (24) and (28) the values of both amplification factors, i.e., μ_0 and the average μ increase with

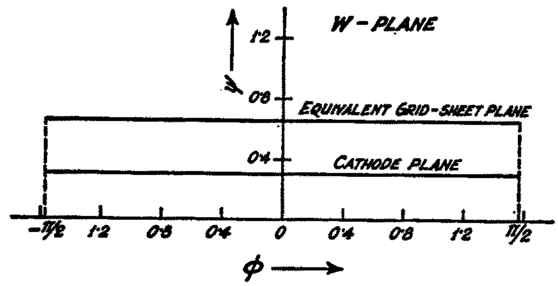

FIG. 5. Equivalent diode representation in the W plane of the upper half of the elliptic triode.

(i) the increase of the number of grid wires ($2N$),
(ii) the decrease in the eccentricity of the anode ellipse (e_a),
(iii) the increase in the eccentricity of the grid ellipse (e_g),
(iv) the increase in the diameter of the grid wires (r_g).

Of these, the roles of N and e_g appear to be more critical. We further note that the value of μ is independent of the eccentricity of the cathode ellipse. This is because of the assumption that the pitch of the grid-wire is small compared to the distance between the cathode and the grid surface.

The variation of μ with θ can be minimized by making the screening fraction (s) variable. This may be achieved by varying either the grid-wire radius or the grid pitch.

If the grid-wire radius is varied then one finds from Eqs. (13), (15), (16), and (18) that for μ to remain constant for all values of θ, the following condition should be satisfied:

$$(1-e_g^2\sin^2\theta)^{\frac{1}{2}}/\ln\coth(2\pi r_\theta/D) = (\mu E/\pi N)/\ln\{\cot(\delta_a/2)/\cot(\delta_g/2)\} = -1/B, \text{ say},$$

where r_θ is the grid-wire radius at θ. This readily gives

$$r_\theta = (D/4\pi)\ln\coth[(1/2)Bdnu], \quad (36)$$

where values of dnu are obtained from Fig. 4(a) and those of ln coth are obtainable from Smithsonian tables.[8]

If, however, the grid pitch is varied to make μ constant, then the variation of grid pitch with θ,

[8] G. F. Becker and C. E. Van Orstrand, *Smithsonian Matematic Tables, Hyperbolic Functions* (The Smithsonian Institution, 1949).

according to Eqs. (13), (15), (16), and (18), is given by the transcendental relation

$$\frac{D_\theta \ln \coth(2\pi r_g/D_\theta)}{(1-e_g^2 \sin^2\theta)^{\frac{1}{2}}} = [2\pi K/(\mu e_g)] \ln\{\cot(\delta_a/2)/\cot(\delta_g/2)\} = A, \quad \text{say},$$

or,

$$[\ln \coth f'(\theta)]/f'(\theta) = f(\theta), \tag{37}$$

where D_θ is the grid pitch at θ, $f(\theta)=\{A/(2\pi r_g)\}dnu$ and $f'(\theta)=2\pi r_g/D_\theta$. The solution of the above equation is given graphically in Fig. 6.

Coming to the expression for perveance one notices that it varies inversely as the focal distance K which otherwise plays the role of a mere scaling factor. Further, it varies inversely as the square of $\cot(\delta_g/2)$ which points to the crucial role played by the eccentricity of the grid ellipse in determining the performance of the valve. However, more critical discussion of perveance will be a little speculative at this stage, since it is to be remembered that all deductions regarding the same are based on a number of very tentative assumptions (originally introduced by O'Neill and by Matricon and Trouve).

In order to illustrate the practical applications of some of the deductions made let us consider the design of a 6C5-GT/G triode. This has the following electrode dimensions:

Cathode—cylindrical;	diameter	$=0.15$ cm
Grid—elliptic;	focal distance	$=0.225$ cm
	eccentricity	$=0.9$
	grid wire diameter	$=0.0075$ cm
	grid pitch	$=0.06$ cm

Anode—combination of a cylinder (diameter $=2.0$ cm) with an elliptic grid (focal distance $=0.225$ cm, eccentricity $=0.6$). Length $=1.7$ cm.

With the above dimensions the following constants are obtained under typical working conditions:

$$\mu=20, \quad P=9\times10^{-4} \text{ amp (volt)}^{\frac{3}{2}}.$$

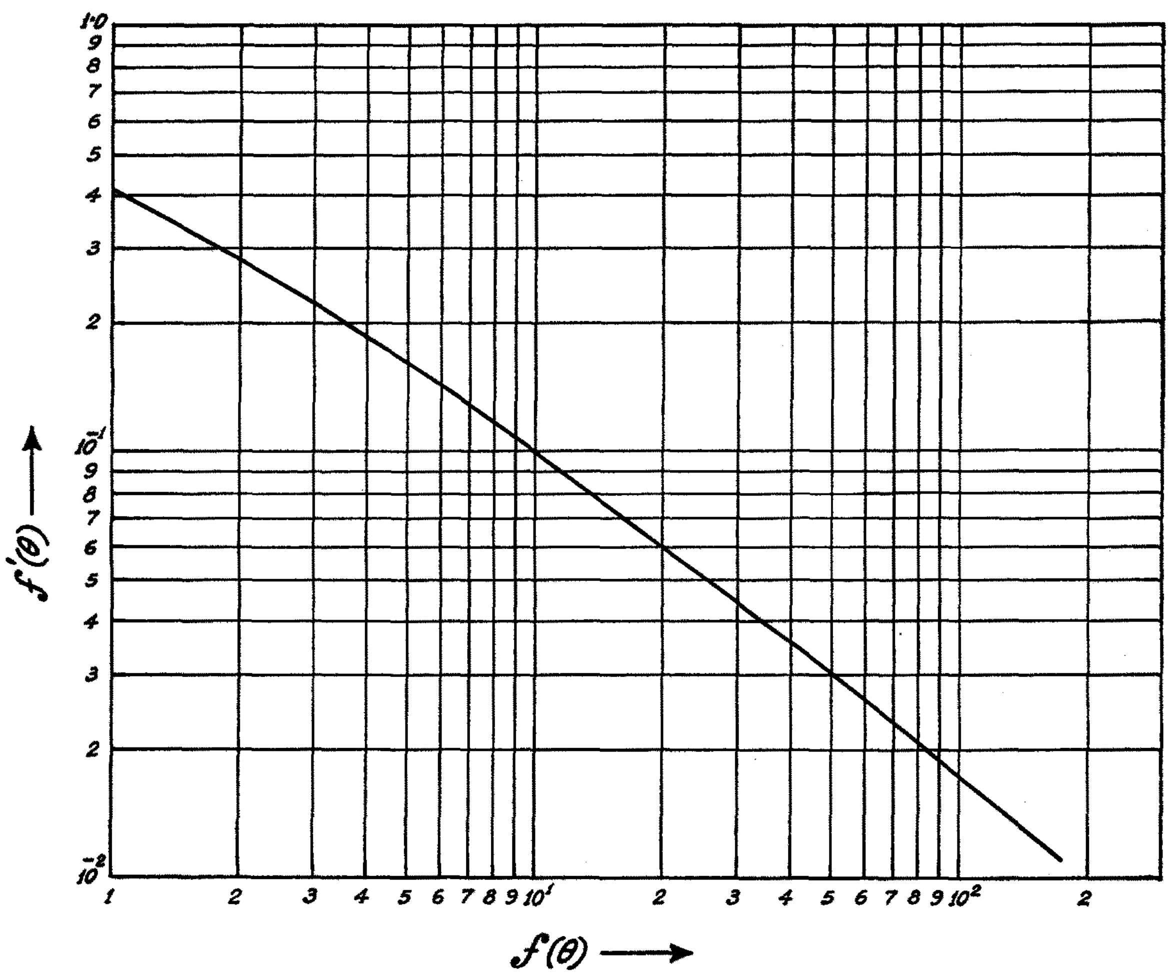

FIG. 6. Graphical solution of Eq. (37).

To realize the above valve constants from a triode whose electrodes belong to a system of confocal ellipses, we note that since we have only two equations [Eqs. (28) and (34)] at our disposal, the design of the electrode structure will not be unique. If, however, the grid structure and the active length of the elliptic triode are assumed to be the same as that of 6C5-GT/G, the dimensions of the cathode and anode, as determined from Eqs. (28) and (34), are found to be

Cathode—elliptic;	focal distance	=0.225 cm
	eccentricity	=0.96
	major axis	=0.47 cm
	minor axis	=0.13 cm
Anode—elliptic;	focal distance	=0.225 cm
	eccentricity	=0.45
	major axis	=1.0 cm
	minor axis	=0.9 cm.

The dimensions of the elliptic cathode as found in the foregoing is reasonable, for the diameter of the actual cathode which is cylindrical is found to be intermediate between the major and the minor axis of the ellipse. Further, the eccentricity 0.45 of the anode ellipse is also according to expectation, because, the combined effect of the elliptic grid and the cylinder as used in the practical valve would be to give rise to an effective anode which is approximately an ellipse occupying an intermediate position between the two.

APPENDIX. TRANSFORMATION OF THE GRID-WIRE CIRCLES FROM THE Z TO W PLANE

Let us consider a circle of radius r_g with its center located at the point (x_0,y_0) on an ellipse in the Z plane (Fig. 7). The locus of this circle, after transformation through the use of Eq. (1), in the W plane can be derived by substituting for x and y from the relations (4a) and (4b) in the equation

$$(x-x_0)^2+(y-y_0)^2=r_g^2. \quad \text{(i)}$$

Thus

$$\begin{aligned}\cosh^2(\psi_g/K)\sin^2(\varphi_g/K)&+\sinh^2(\psi_g/K)\cos^2(\varphi_g/K)\\ &+\cosh^2(\psi/K)\sin^2(\varphi/K)+\sinh^2(\psi/K)\cos^2(\varphi/K)\\ &-2[\cosh(\psi/K)\cosh(\psi_g/K)\sin(\varphi/K)\sin(\varphi_g/K)\\ &+\sinh(\psi/K)\sinh(\psi_g/K)\cos(\varphi/K)\cos(\varphi_g/K)\\ &=(r_g/K)^2, \quad \text{(ii)}\end{aligned}$$

where (φ,ψ) is any point on the locus in the W plane, and (φ_g,ψ_g) is a point in the same plane corresponding to the center of the circle in the Z plane.

Equation (ii) may be reduced to

$$\begin{aligned}[\cosh((\psi-\psi_g)/K)&-\cos((\varphi-\varphi_g)/K)]\\ &\times[\cosh((\psi+\psi_g)/K)+\cos((\varphi+\varphi_g)/K)]\\ &=(r_g/K)^2. \quad \text{(iii)}\end{aligned}$$

The locus represented by Eq. (iii) is a closed curve and approximates a circle when r_g is small. Thus putting

$$\psi+\psi_g\approx 2\psi_g \quad \text{and} \quad \varphi+\varphi_g\approx 2\varphi_g,$$

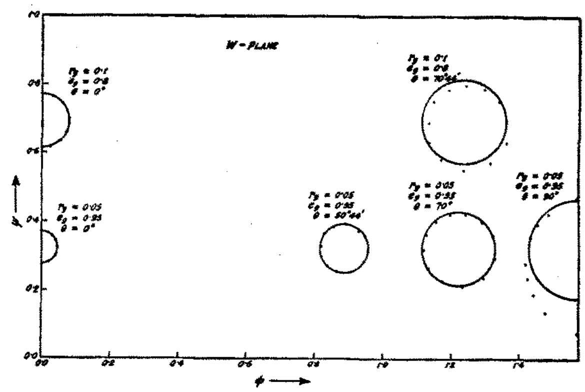

FIG. 7. W plane images of grid-wire circles ($K=1$). Solid line curves from Eq. (iv). Points obtained from Eq. (vi) are also shown.

and expanding $\cosh((\psi-\psi_g)/K)$ and $\cos((\varphi-\varphi_g)/K)$ by Maclaurin's theorem and neglecting higher-order terms, the equation of the locus in the W plane becomes

$$(\psi-\psi_g)^2+(\varphi-\varphi_g)^2=\left[\frac{r_g}{[\cosh^2(\psi_g/K)-\sin^2(\varphi_g/K)]^{\frac{1}{2}}}\right]^2.$$

Substituting for $\psi_g=K\cosh^{-1}(1/e_g)$,

$$(\psi-\psi_g)^2+(\varphi-\varphi_g)^2=\frac{r_g e_g}{[1-e_g^2\sin^2(\varphi_g/K)]^{\frac{1}{2}}}. \quad \text{(iv)}$$

Equation (iv) represents a circle in the W plane with its center located at (φ_g,ψ_g) and having a radius given by

$$r=r_g e_g/[1-e_g^2\sin^2(\varphi_g/K)]^{\frac{1}{2}}. \quad \text{(v)}$$

To test the range of validity of Eq. (iv) a point-by-point transformation of the circle given by (i) was carried out from the Z to W plane. The required relations are given explicitly by

$$\left.\begin{aligned}\psi&=K\cosh^{-1}\{(1/(2K))[(x+K)^2+y^2]^{\frac{1}{2}}\\ &\qquad+(1/(2K))[(x-K)^2+y^2]^{\frac{1}{2}}\}\\ \varphi&=K\sin^{-1}\{(1/(2K))[(x+K)^2+y^2]^{\frac{1}{2}}\\ &\qquad-(1/(2K))[(x-K)^2+y^2]^{\frac{1}{2}}\}\end{aligned}\right\}. \quad \text{(vi)}$$

The resultant loci obtained by such a procedure together with those obtained from Eq. (iv) are shown in Fig. 7 for several values of e_g and r_g. It is seen that the agreement between the two is quite satisfactory, provided that r_g is small in comparison to K/e_g.

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