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Alternative ion-acoustic solitary waves in magnetized low beta plasma consisting of warm adiabatic ions, nonthermal electrons, and electrons having a vortexlike distribution

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The solitary wave structures of the ion-acoustic waves propagating obliquely to an external uniform magnetic field have been considered in a low beta plasma consisting of warm adiabatic ions, nonthermal electrons, due to Cairns *et al.* [Geophys. Res. Lett. **22**, 2709 (1995)], which generates the fast energetic electrons and electrons having a vortexlike distribution, due to Schamel [Plasma Phys. **13**, 491 (1971); **14**, 905 (1972)], taking care of both free and trapped electrons, immersed in a uniform static magnetic field. The nonlinear dynamics of ion-acoustic waves in such a plasma is shown to be governed by Schamel's modified Korteweg–de Vries–Zakharov–Kuznetsov equation. When the coefficient of the nonlinear term of this equation vanishes, the vortexlike velocity distribution function of electrons simply becomes the isothermal velocity distribution function of electrons and the nonlinear behavior of the same ion-acoustic wave is described by a Korteweg–de Vries–Zakharov–Kuznetsov (KdV-ZK) equation. A combined Schamel's modified Korteweg–de Vries–Zakharov–Kuznetsov (S-KdV-ZK) equation is shown to describe the nonlinear behavior of ion-acoustic wave when the vortexlike velocity distribution function of electrons approaches the isothermal velocity distribution function of electrons, i.e., when the contribution of trapped electrons tends to zero. This combined S-KdV-ZK equation admits an alternative solitary wave solution having profile different from sech^4 or sech^2. The condition for the existence of this alternative solitary wave solution has been obtained. It is found that this alternative solitary wave solution approaches the solitary wave solution (sech^2-profile) of the KdV-ZK equation when the contribution of trapped electrons tends to zero. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3073677]

I. INTRODUCTION

The observations of solitary structures with density depletion made by the Freja satellite[1] influenced Cairns *et al.*[2–4] to investigate how the presence of nonthermal electrons changes the properties of ion-acoustic waves for both positive and negative density perturbations. A model of the velocity distribution function of nonthermal electrons was considered for the first time by Cairns *et al.*[2] for the study of ion-acoustic solitary structures in the presence of the population of the fast energetic electrons together with a population of Maxwellian distributed electrons. This type of velocity distribution has been termed as nonthermal distribution and was considered by many authors in various studies of different collective processes in plasmas and dusty plasmas.[3–11] In a low beta plasma, the density function of nonthermal electrons has been shown by Cairns *et al.*[2] to be

$$n_{ce} = n_{c0}\left(1 - \beta_1\frac{e\varphi}{m_e v_{ce}^2} + \beta_1\frac{e^2\varphi^2}{m_e^2 v_{ce}^4}\right)\exp\left[\frac{e\varphi}{m_e v_{ce}^2}\right], \tag{1}$$

where

$$\beta_1 = 4\alpha_1/(1+3\alpha_1), \quad \text{with } \alpha_1 \geq 0. \tag{2}$$

It can be easily checked that $0 \leq \beta_1 < \frac{4}{3}$. Here α_1 and consequently β_1 is a parameter that determines the proportion of the fast energetic electrons, n_{c0} is the equilibrium number density of electrons, $v_{ce} = \sqrt{K_B T_{ce}/m_e}$ is the average thermal speed of electrons, and T_{ce} is the average temperature of electrons associated with the nonthermal velocity distribution of electrons as prescribed by Cairns *et al.*,[2] φ is the electrostatic potential, K_B is the Boltzmann constant, m_e is the mass of an electron, and e is the charge of an electron. For the electron population having a nonthermal distribution we take Eq. (1) as the density function of these electrons.

We have shown in the Appendix that if the length scales are the same in both parallel and perpendicular to the external magnetic field and if the characteristic time is very much greater than the electron Larmor period, then the electrons move essentially along the magnetic field. Therefore, assuming that these conditions are met in the plasma we are considering, then according to Schamel,[12] densities of free and trapped electrons for the second population of electrons can be taken as

$$n_{se} = I(\varphi) + \begin{cases} I_+(\varphi) & \text{for } \alpha > 0 \\ I_-(\varphi) & \text{for } \alpha < 0, \end{cases} \tag{3}$$

where

$$I(\varphi) = n_{so}\left(1 - \mathrm{Erf}\left[\sqrt{\frac{e\varphi}{m_e v_{\mathrm{sef}}^2}}\right]\right)\exp\left[\frac{e\varphi}{m_e v_{\mathrm{sef}}^2}\right], \tag{4}$$

$$I_+(\varphi) = \frac{n_{\mathrm{so}}}{\sqrt{\alpha}}\mathrm{Erf}\left[\sqrt{\frac{\alpha e\varphi}{m_e v_{\mathrm{sef}}^2}}\right]\exp\left[\frac{\alpha e\varphi}{m_e v_{\mathrm{sef}}^2}\right], \tag{5}$$

$$I_-(\varphi) = \frac{n_{\mathrm{so}}}{\sqrt{-\alpha}}\mathrm{Erfi}\left[\sqrt{\frac{-\alpha e\varphi}{m_e v_{\mathrm{sef}}^2}}\right]\exp\left[\frac{\alpha e\varphi}{m_e v_{\mathrm{sef}}^2}\right]. \tag{6}$$

Here the error functions $\mathrm{Erf}[z]$ and $\mathrm{Erfi}[z]$ are given by

$$(\mathrm{Erf}[z],\mathrm{Erfi}[z]) = \frac{2}{\sqrt{\pi}}\int_0^z (e^{-t^2},e^{t^2})dt, \tag{7}$$

and the parameter α associated with the number density of electrons as prescribed by Schamel[12] is given by the following equation:

$$\alpha^2 = (T_{\mathrm{sef}}/T_{\mathrm{set}})^2, \tag{8}$$

where T_{sef} and T_{set}, respectively, denote the free and trapped electrons' temperatures, $v_{\mathrm{sef}}=\sqrt{K_B T_{\mathrm{sef}}/m_e}$ is the average thermal speed for free electrons, φ is the electrostatic potential, and n_{s0} is the equilibrium number density of electrons for their vortexlike velocity distribution as prescribed by Schamel.[12]

Equations (3)–(6) (or other similar type of equations for electrons or ions) have been used by some authors[13] to describe the nonlinear behavior of the ion (electron or dust) acoustic waves in magnetized plasma (dusty plasma) by deriving Schamel's modified Korteweg–de Vries (S-KdV) equation. In the presence of a magnetic field, plasma is not homogeneous. But if we assume that plasma remains homogeneous in the presence of a magnetic field and if we take Boltzmann–Maxwell distribution of electrons in phase space as initial (unperturbed) distribution of electrons, instead of drifting Boltzmann–Maxwell distribution of electrons, then the solution of drift kinetic equation (1) of Jovanović and Shukla[14] in the stationary frame gives the density function of electrons as given by Eq. (3) of the present paper. Of course, in more general cases, if we consider the inhomogeneity of plasma and the unperturbed distribution of electrons in phase space is a drifting Boltzmann–Maxwellian distribution, then the perturbed electron distribution function with the usual Schamel model is given by Eq. (6) of Jovanović and Shukla[14] and the density function of resonant electrons is approximately given by Eq. (9) of Jovanović and Shukla.[14] Consequently the usual prescription is to use Eq. (9) of Jovanović and Shukla[14] instead of Eq. (3) of the present paper. Equation (9) of Jovanović and Shukla[14] reduces to Eq. (3) of the present paper under the assumptions mentioned earlier. Equation (3) of the present paper can also be obtained from Eq. (6) of Jovanović and Schamel[15] if we take $u_z=0$ in the expressions of a and b as given by Eqs. (7) and (8), respectively, of Jovanović and Schamel,[15] i.e., Eq. (3) of the present paper is the same as Eq. (6) of Jovanović and Schamel[15] if we set zeroth order drift velocity equal to zero.

Starting from the equation of continuity for ions, the equation of motion for ions together with the adiabatic law of pressure for ion fluid, and taking a superposition of two different populations of electrons whose number densities are given by Eqs. (1) and (3) we derive, after giving appropriate stretchings of space coordinates and time and appropriate perturbation expansions of the dependent variables, Schamel's modified Korteweg–de Vries–Zakharov–Kuznetsov (S-ZK) equation, which describes the oblique propagation of nonlinear ion-acoustic waves in such a plasma. This equation admits solitary wave solution (hereafter SWS) having a sech^4-profile. It is found that a factor B of the coefficient of the nonlinear term of this equation vanishes when $\alpha=1$. But for $\alpha=1$, Eq. (3) implies that the electron distribution function in this case becomes Boltzmann–Maxwell distribution, i.e., when $\alpha=1$, Eq. (3) reduces to

$$n_{se} = n_{so}\exp\left[\frac{e\varphi}{m_e v_{\mathrm{sef}}^2}\right],$$

and consequently, in this case, a long wavelength weakly nonlinear ion-acoustic wave is described by a Korteweg–de Vries–Zakharov–Kuznetsov (KdV-ZK) equation. This equation admits SWS having a sech^2-profile. It is found that neither S-ZK equation nor KdV-ZK equation can describe the nonlinear behavior of the ion-acoustic wave when $\alpha\neq 1$ but $\alpha\to 1$. In this case, i.e., when $\alpha\neq 1$ but $\alpha\to 1$, the appropriate evolution equation is the combined Schamel's modified Korteweg–de Vries–Zakharov–Kuznetsov (S-KdV-ZK) equation, which is derived here following Nejoh.[16]

One of the main motives of the present paper is to investigate theoretically the existence of an alternative SWS (hereafter ASWS) of the combined S-KdV-ZK equation, which fills the gap between sech^4-profile and sech^2-profile. Here it is found that ASWS of the combined S-KdV-ZK equation approaches the SWS (sech^2-profile) of the KdV-ZK equation as $\alpha\to 1$. It has also been shown here that an ASWS having profile different from sech^4 or sech^2 exists if and only if $L=MB^2\equiv B^2+\frac{75}{2}p^2B'(\cos^2\delta+D\sin^2\delta)>0$, where B, B' are the coefficients (except for a common factor) of the two nonlinear terms and D is the coefficient (except for a common factor) of the perpendicular dispersive term of the combined S-KdV-ZK equation. Here p is a positive real number dependent on the linear velocity of alternative solitary wave.

It is now well known that two electron Boltzmann–Maxwellian model is very common in space plasma, more specifically, in auroral plasma. Solitary waves in the frequency ranges of lower hybrid and drift waves, i.e., above and below the ion gyrofrequency, governed by the combined effects of fluid nonlinearities and trapped particles, have recently been described by Jovanović *et al.*[17] In the present paper, we have extended the work of Bandyopadhyay and Das[5] taking a superposition of two distinct populations of electrons, one due to Cairns *et al.*,[2] which generates the fast energetic electrons, and other due to Schamel,[12] which is a vortexlike distribution and takes care of both free and trapped electrons, immersed in a uniform static magnetic field. Therefore, the present work may be helpful for the understanding of different nonlinear structures of ion-acoustic waves in the space plasma.

The present paper is organized as follows. The basic equations are given in Sec. II. The evolution equations are given in Sec. III. The SWSs of the evolution equations are given in Sec. IV. Finally, a brief conclusion is given in Sec. V.

II. BASIC EQUATIONS

The following are the governing equations describing the nonlinear behavior of ion-acoustic waves propagating obliquely in fully ionized collisionless plasma consisting of warm adiabatic ions and two distinct populations of electrons, immersed in an uniform static magnetic field directed along z-axis. For the assumed densities of the two populations of electrons, the assumptions made are that the length scales are the same in both parallel and perpendicular to the external magnetic field, the characteristic time is very much greater than electron Larmor period, and the plasma beta is very much less than 1. Further assumptions are needed for long wavelength weakly dispersive waves, which have been explained in Sec. III. These assumptions are that the characteristic length is very much greater than the Debye length, electron cyclotron frequency, and ion plasma frequency are of the same order,

$$n_t + \nabla \cdot (n\mathbf{u}) = 0, \quad (9)$$

$$\mathbf{u}_t + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla\varphi + \omega_c(\mathbf{u}\times\hat{z}) - \frac{\sigma}{n}\nabla p, \quad (10)$$

$$\nabla^2\varphi = n_{ce} + n_{se} - n, \quad (11)$$

$$p = n^{\gamma}, \quad (12)$$

where n_{ce} and n_{se} are given by Eqs. (1) and (3), respectively.

Here n, $\mathbf{u}$, p, (x,y,z), and t are, respectively, the ion number density, ion fluid velocity, ion pressure, spatial variables, and time, and n, n_{ce}, n_{se}, $\mathbf{u}$, p, φ, (x,y,z), and t have been normalized by n_0 (unperturbed ion number density), n_0, n_0, $c_s(=\sqrt{K_BT_{\text{ef}}}/\sqrt{m})$ (ion-acoustic speed), $n_0K_BT_i$, K_BT_{ef}/e, $\lambda_D(=\sqrt{K_BT_{\text{ef}}}/\sqrt{4\pi n_0e^2})$ (Debye length), and ω_p^{-1} (ion plasma period), where $\sigma=T_i/T_{\text{ef}}$, ω_c is the ion cyclotron frequency normalized by $\omega_p(=\sqrt{4\pi n_0e^2}/\sqrt{m})$ and $\gamma(=5/3)$ is the ratio of two specific heats. Here K_B is the Boltzmann constant, T_i is the average ion temperature, m is the mass of an ion, and e is the electronic charge. The adiabatic law, Eq. (12), has been taken to form a closed and consistent system of equations on the basis of the assumption that the effect of viscosity, thermal conductivity, and the energy transfer due to collision can be neglected. T_{ef} is the effective electron temperature and is defined by the following equation:

$$\frac{n_{c0}+n_{s0}}{T_{\text{ef}}} = \frac{n_{c0}}{T_{ce}} + \frac{n_{s0}}{T_{\text{sef}}}. \quad (13)$$

Under the above normalization of the dependent and independent variables, the charge neutrality condition, expression of n_{ce}, and the expression of n_{se} as given by Eqs. (1) and (3), respectively, assume the following forms:

$$n_{c0} + n_{s0} = 1, \quad (14)$$

$$n_{ce} = n_{c0}(1 - \beta_1\sigma_c\varphi + \beta_1\sigma_c^2\varphi^2)e^{\sigma_c\varphi}, \quad (15)$$

$$n_{se} = I(\varphi) + \begin{cases} I_+(\varphi) & \text{for } \alpha > 0 \\ I_-(\varphi) & \text{for } \alpha < 0, \end{cases} \quad (16)$$

where

$$I(\varphi) = n_{s0}(1 - \text{Erf}[\sqrt{\sigma_s\varphi}])\exp(\sigma_s\varphi), \quad (17)$$

$$I_+(\varphi) = \frac{n_{s0}}{\sqrt{\alpha}}\text{Erf}[\sqrt{\alpha}\sqrt{\sigma_s\varphi}]\exp(\alpha\sigma_s\varphi), \quad (18)$$

$$I_-(\varphi) = \frac{n_{s0}}{\sqrt{-\alpha}}\text{Erfi}[\sqrt{-\alpha}\sqrt{\sigma_s\varphi}]\exp(\alpha\sigma_s\varphi), \quad (19)$$

with

$$\sigma_c = T_{\text{ef}}/T_{ce}, \quad \sigma_s = T_{\text{ef}}/T_{\text{sef}}. \quad (20)$$

From Eq. (14) we get

$$n_{c0} = 1/(1+n_{sc}), \quad n_{s0} = n_{sc}/(1+n_{sc}), \quad (21)$$

where

$$n_{sc} = (n_{s0}/n_{c0}). \quad (22)$$

Using Eqs. (14) and (21) we can write Eq. (13) as

$$n_{c0}\sigma_c + n_{s0}\sigma_s = 1, \quad (23)$$

or equivalently in the form

$$\sigma_c = \frac{1+n_{sc}}{1+n_{sc}\sigma_{sc}}, \quad \sigma_s = \frac{(1+n_{sc})\sigma_{sc}}{1+n_{sc}\sigma_{sc}}, \quad (24)$$

where

$$\sigma_{sc} = (\sigma_s/\sigma_c) = (T_{ce}/T_{\text{sef}}). \quad (25)$$

It can be easily verified that the following statements are always true. (1) $n_{sc}\geq 0$, $\sigma_{sc}>0$, (2) $\sigma_{sc}>1$ if and only if $\sigma_s>1>\sigma_c$, (3) $\sigma_{sc}<1$ if and only if $\sigma_s<1<\sigma_c$, and (4) $\sigma_{sc}=1$ if and only if $\sigma_s=\sigma_c=1$.

In view of Eq. (12), Eq. (10) can be written as

$$\mathbf{u}_t + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla\varphi + \omega_c(\mathbf{u}\times\hat{z}) - \tfrac{5}{3}\sigma n^{-1/3}\nabla n. \quad (26)$$

Expanding n_{se} and n_{ce} as given by Eqs. (16) and (15), respectively, in the small-amplitude limit and keep the terms up to φ^2, we can write Eq. (11) as follows:

$$\nabla^2\varphi = 1 + (1-\beta_1 n_{co}\sigma_c)\varphi + \frac{4}{3}\frac{\alpha-1}{\sqrt{\pi}}n_{so}(\sigma_s\varphi)^{3/2} + \frac{1}{2}[n_{so}\sigma_s^2 + n_{co}\sigma_c^2]\varphi^2 - n. \quad (27)$$

Equations (9), (26), and (27) are our basic equations.

III. EVOLUTION EQUATIONS

From the linearized form of the governing equations (9), (26), and (27) we get the following linear dispersion relation after assuming space-time dependence of the dependent variables to be of the form $\exp[i(k_xx+k_yy+k_zz-\omega t)]$:

$$\frac{k_\perp^2}{\omega^2-\omega_c^2}+\frac{k_z^2}{\omega^2}=\frac{k^2+(1-\beta_1 n_{c0}\sigma_c)}{1+(5/3)\sigma[k^2+(1-\beta_1 n_{c0}\sigma_c)]}, \quad (28)$$

where $k_\perp^2=k_x^2+k_y^2$ and $k^2=k_\perp^2+k_z^2$.

We now introduce a small parameter ε which measures the smallness of k_x,k_y,k_z indicating that the waves we are considering are of long wavelengths. In view of this, we replace k_x,k_y,k_z in Eq. (28), respectively, by $\varepsilon^{1/4}k_x$, $\varepsilon^{1/4}k_y$, and $\varepsilon^{1/4}k_z$ and also ω by $\varepsilon^{1/4}\omega$. Then solving Eq. (28) for ω^2 in a perturbative way, we get the following result:

$$\frac{\omega^2}{k_z^2}=V^2-\frac{\varepsilon^{1/2}k_\perp^2}{\omega_c^2}\lambda_1-\varepsilon^{1/2}k^2\lambda_2, \quad (29)$$

neglecting $\bigcirc(\varepsilon^{3/2})$ terms, where

$$V^2=\frac{1+\frac{5}{3}\sigma(1-\beta_1 n_{c0}\sigma_c)}{1-\beta_1 n_{c0}\sigma_c},$$

$$\lambda_1=\frac{\left[1+\frac{5}{3}\sigma(1-\beta_1 n_{c0}\sigma_c)\right]^2}{(1-\beta_1 n_{c0}\sigma_c)^2}, \quad (30)$$

$$\lambda_2=\frac{1}{(1-\beta_1 n_{c0}\sigma_c)^2}.$$

The last two terms in Eq. (29) are the lowest order dispersive terms. Dropping ε in these two terms, i.e., setting $\varepsilon=1$ and introducing a characteristic length of the problem, the dispersion relation (29) can be written as

$$\frac{\omega^2}{k_z^2}=V^2-\frac{L^2k_\perp'^2}{\omega_c'^2/\omega_p^2}\frac{\lambda_D^2}{L^2}\lambda_1-L^2k'^2\frac{\lambda_D^2}{L^2}\lambda_2,$$

where a prime denotes a dimensional quantity. Since $L^2k_\perp'^2=\bigcirc(1)$ and $L^2k'^2=\bigcirc(1)$, we must have $\lambda_D^2/L^2=\varepsilon^{1/2}\ll 1$ provided $\omega_c^2/\omega_p^2=\bigcirc(1)$. So characteristic length must be far greater than Debye length and electron cyclotron frequency and ion plasma frequency are of the same order. Here $\varepsilon=(\lambda_D/L)$ is a small parameter of the problem.

From Eq. (29) we get

$$\omega=Vk_z-\varepsilon^{1/2}Vk_z\lambda, \quad (31)$$

where $\lambda=k_\perp^2\lambda_1/2\omega_c^2V^2+k^2\lambda_2/2V^2$.

Equation (31) shows that ε is also a measure of weakness of dispersion.

By the use of the expression (31) for ω, the phase of the wave can be expressed as

$$\varepsilon^{1/4}k_xx+\varepsilon^{1/4}k_yy+\varepsilon^{1/4}k_zz-\varepsilon^{1/4}\omega t$$
$$=k_x(\varepsilon^{1/4}x)+k_y(\varepsilon^{1/4}y)+k_z\varepsilon^{1/4}(Z-Vt)+k_zV\lambda(\varepsilon^{3/4}t). \quad (32)$$

This expression for the phase guides us to choose the following stretched coordinates and time:

$$(\xi,\eta,\zeta)=\varepsilon^{1/4}(x,y,z-Vt), \qquad \tau=\varepsilon^{3/4}t, \quad (33)$$

where ε is a small parameter measuring the weakness of the dispersion and V is a constant. The stretchings of space coordinates and time as given by Eq. (33) were used for the first time by Schamel.[18]

A. S-ZK equation

With the help of the stretchings as given by Eq. (33) and following the method of Das *et al.*,[8] we have derived from Eqs. (9), (26), and (27) the following S-ZK equation for ion-acoustic waves in a fully ionized collisionless magnetized plasma consisting of warm adiabatic ions and two distinct populations of electrons,

$$\varphi_\tau^{(1)}+AB\sqrt{\varphi^{(1)}}\varphi_\zeta^{(1)}+(1/2)A\varphi_{\zeta\zeta\zeta}^{(1)}+(1/2)AD(\varphi_{\xi\xi}^{(1)}+\varphi_{\eta\eta}^{(1)})_\zeta=0, \quad (34)$$

where we have used the following perturbation expansions of the dependent variables so that a balance is produced between nonlinear and dispersive terms in the lowest order equation derived, which is Eq. (34),

$$(n,\varphi,w)=(1,0,0)+\varepsilon^{(i+1)/2}(n^{(i)},\varphi^{(i)},w^{(i)}),$$
$$(u,v)=\varepsilon^{(i+4)/4}(u^{(i)},v^{(i)}), \quad (35)$$

where $\mathbf{u}=(u,v,w)$ and repeated index means summation over that index on the set of natural numbers.

Here

$$A=V^{-1}[V^2-(5/3)\sigma]^2, \quad (36)$$

$$B=\frac{n_{sc}\sigma_{sc}}{1+n_{sc}\sigma_{sc}}\frac{(1-\alpha)}{\sqrt{\pi}}\sqrt{\frac{(1+n_{sc})\sigma_{sc}}{1+n_{sc}\sigma_{sc}}}, \quad (37)$$

$$D=1+\frac{V^4}{\omega_c^2}[V^2-(5/3)\sigma]^{-2}, \quad (38)$$

and the constant V is given by

$$[V^2-(5/3)\sigma]^{-1}-[1-\beta_1(1+n_{sc}\sigma_{sc})^{-1}]=0. \quad (39)$$

If we consider the limiting case that, $n_{sc}\to\infty$, i.e., if we consider a magnetized plasma consisting of warm adiabatic ions and only one species of electrons as prescribed by Schamel,[12] then Eq. (34) is exactly the same as Eq. (3.6) of Das *et al.*[8] if we set $\beta=0$ in the expression of B as given by Eq. (3.9) of Das *et al.*[8] Again considering the limiting case that $n_{sc}\to\infty$, $\beta_1\to 0$ and with the help of the transformation of the space coordinates $\varsigma=l\xi+m\eta+n\zeta$, where $l^2+m^2+n^2=1$, the S-ZK equation (34) of the present paper becomes exactly the same as Schamel's modified KdV equation (14) of Mamun[13] with slight modification of notations only.

From the expression of B as given by Eq. (37), we see that $B=0$ if either $n_{sc}=0$ or $\alpha=1$ and if $B=0$ the nonlinear behavior of the ion-acoustic wave can be described by a KdV-ZK equation.

B. KdV-ZK equation

When $B=0$, with the help of the same stretchings as given by Eq. (33), using the following perturbation expansions of the dependent variables:

$$(n,\varphi,w)=(1,0,0)+\varepsilon^{(i+1)/4}(n^{(i)},\varphi^{(i)},w^{(i)}),$$
$$(u,v)=\varepsilon^{(i+2)/4}(u^{(i)},v^{(i)}), \tag{40}$$

and following the same method of Das *et al.*,[8] we have derived the following KdV-ZK equation from the set of basic equations (9), (26), and (27):

$$\varphi_\tau^{(1)}+AB'\varphi^{(1)}\varphi_\zeta^{(1)}+(1/2)A\varphi_{\zeta\zeta\zeta}^{(1)}+(1/2)AD(\varphi_{\xi\xi}^{(1)}+\varphi_{\eta\eta}^{(1)})_\zeta=0. \tag{41}$$

Here A and D are the same as those given by Eqs. (36) and (38), respectively, and B' is given by the following equation:

$$B'=\frac{1}{2}\left[\frac{3V^2-\frac{5}{9}\sigma}{\left(V^2-\frac{5}{3}\sigma\right)^3}-\frac{(1+n_{sc})(1+n_{sc}\sigma_{sc}^2)}{(1+n_{sc}\sigma_{sc})^2}\right]. \tag{42}$$

The constant V appearing in A, B', and D is determined by Eq. (39).

If we set $n_{sc}=0$ ($\Rightarrow B=0$), i.e., if we consider a magnetized plasma consisting of warm adiabatic ions and only one species of nonthermal electrons as prescribed by Cairns *et al.*,[2] then Eqs. (39) and (42) assume the following forms:

$$[V^2-(5/3)\sigma]^{-1}-(1-\beta_1)=0,$$
$$B'=(1/2)\{[3V^2-(5/9)\sigma][V^2-(5/3)\sigma]^{-3}-1\}. \tag{43}$$

Consequently Eq. (41) is exactly the same as Eq. (10) of Bandyopadhyay and Das[5] with slight modification of the notation of the coefficient of the nonlinear term, which describes the nonlinear behavior of the ion-acoustic waves in a magnetized plasma consisting of warm adiabatic ions and only one species of nonthermal electrons as prescribed by Cairns *et al.*[2]

We shall see in Sec. IV A that it is also not possible to describe the nonlinear behavior of the ion-acoustic wave by the S-ZK equation (34) even when $B\neq 0$ but B tends to zero. Actually, for this case, the S-ZK equation (34) gives an unbounded SWS. In the case when $B\neq 0$ but $B\to 0$, the appropriate evolution equation is the combined S-KdV-ZK equation given in Sec. III C.

C. Combined S-KDV-ZK equation

Following Nejoh,[16] we assume that $B\neq 0$ but $B\approx \bigcirc(\varepsilon^{1/4})$ for the case $B\neq 0$ but $B\to 0$. Now giving the same stretchings and the same perturbation expansions of the dependent variables given by Eqs. (33) and (40), respectively, and following the same procedure of Das *et al.*,[8] we derive a combined S-KdV-ZK equation from the set of basic equations (9), (26), and (27) for ion-acoustic waves in a fully ionized collisionless plasma consisting of warm adiabatic ions and two distinct population of electrons, immersed in a uniform static magnetic field which remains valid in the neighborhood of $\alpha=1$. This combined S-KdV-ZK equation is the following:

$$\varphi_\tau^{(1)}+AB\sqrt{\varphi^{(1)}}\varphi_\zeta^{(1)}+AB'\varphi^{(1)}\varphi_\zeta^{(1)}+(1/2)A\varphi_{\zeta\zeta\zeta}^{(1)}+(1/2)AD(\varphi_{\xi\xi}^{(1)}+\varphi_{\eta\eta}^{(1)})_\zeta=0. \tag{44}$$

Here A, B, D, and B' are, respectively, given by Eqs. (36)–(38) and (42), and the constant V is determined by Eq. (39).

IV. SWSS

For SWS of Eqs. (34), (41), and (44), propagating at an angle δ with the external uniform static magnetic field, we make the following change of variables:

$$\left.\begin{array}{ll}\xi'=\xi\cos\delta-\zeta\sin\delta, & \eta'=\eta,\\ \zeta'=\xi\sin\delta+\zeta\cos\delta, & \tau'=\tau,\end{array}\right\}Z=\zeta'-U\tau', \tag{45}$$

and for the traveling wave solutions of Eqs. (34), (41), and (44), we set

$$\varphi^{(1)}=\varphi_0(Z), \tag{46}$$

and consequently Eqs. (34), (41), and (44) assume, respectively, the following forms:

$$-U\varphi_0'+a_1\sqrt{\varphi_0}\varphi_0'+a_3\varphi_0'''=0, \tag{47}$$

$$-U\varphi_0'+a_2\varphi_0\varphi_0'+a_3\varphi_0'''=0, \tag{48}$$

$$-U\varphi_0'+a_1\sqrt{\varphi_0}\varphi_0'+a_2\varphi_0\varphi_0'+a_3\varphi_0'''=0, \tag{49}$$

where we have used the notation Ψ' for $d\Psi/dZ$ and a_1, a_2, a_3 are given by the following equations:

$$a_1=AB\cos\delta,\quad a_2=AB'\cos\delta,$$

$$a_3=(1/2)A\cos\delta(\cos^2\delta+D\sin^2\delta).$$

Using the boundary conditions

$$\varphi_0,\varphi_0',\varphi_0''\to 0\ \text{as}\ |Z|\to\infty, \tag{50}$$

the SWS of Eqs. (47) and (48) can, respectively, be put in the following forms:

$$\varphi_0=a\ \text{sech}^4\ pZ:\ a=\frac{225U^2}{64a_1^2},\quad U=16p^2a_3, \tag{51}$$

$$\varphi_0=a\ \text{sech}^2\ pZ:\ a=\frac{3U}{a_2},\quad U=4p^2a_3. \tag{52}$$

We see from Eqs. (51) and (52) that there exists a relation between U and p and for both the SWSs (51) and (52), it can be easily checked that p is well defined if and only if $U\cos\delta$ is strictly positive. It can also be verified that for both the SWSs (51) and (52), p is independent of α. we shall now consider the existence of the SWSs (51) and (52) separately in the following subsections.

A. SWSS of the S-ZK equation

We have pointed out earlier that the width $p^{-1}=4\sqrt{a_3/U}$ of the SWS (51) is well defined if $U\cos\delta>0$ and if $U\cos\delta>0$, the amplitude of the SWS (51) is also well defined if $B\neq 0\Leftrightarrow\alpha\neq 1$ and consequently both the width and the amplitude of the SWS (51) are well defined if

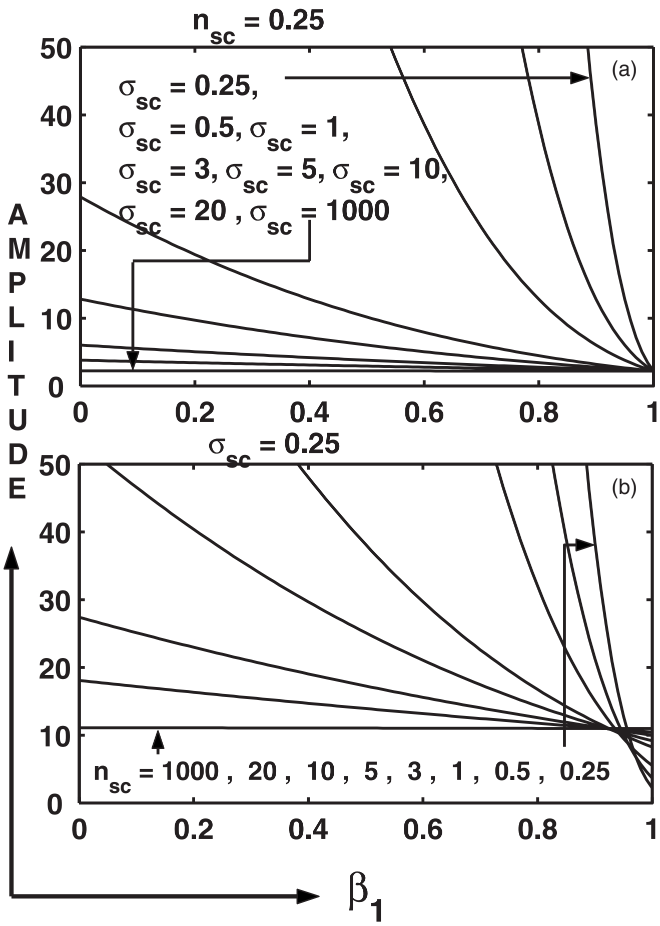

FIG. 1. (a) shows the variation in the amplitude of SWS (51) against β_1 for different σ_{sc} with n_{sc}=0.25 and (b) shows the variation in the amplitude of SWS (51) against β_1 for different n_{sc} with σ_{sc}=0.25.

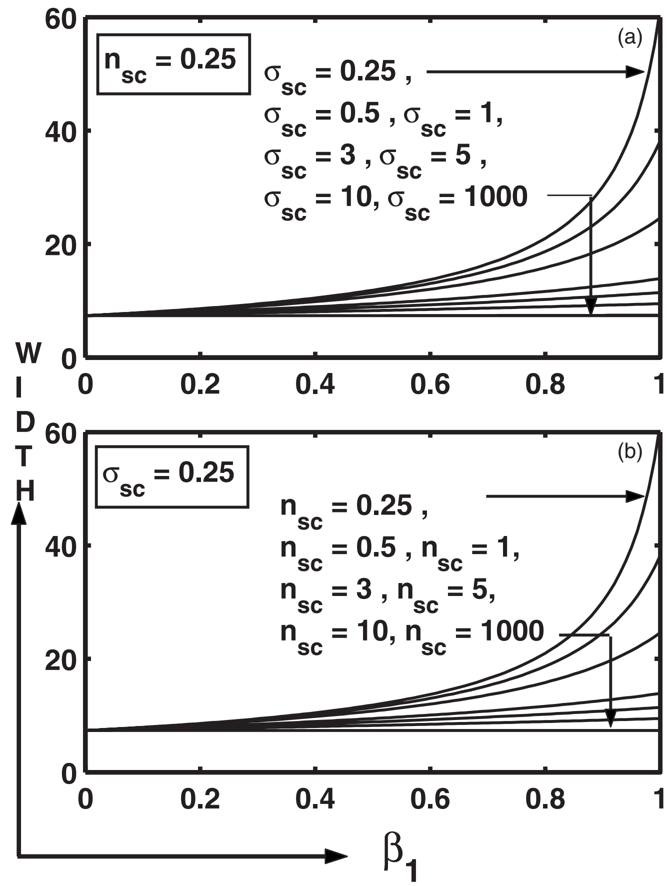

FIG. 2. (a) shows the variation in the width of SWS (51) against β_1 for different σ_{sc} with n_{sc}=0.25 and (b) shows the variation in the width of SWS (51) against β_1 for different n_{sc} with σ_{sc}=0.25.

$U\cos\delta>0$ and $B\neq 0 \Leftrightarrow \alpha\neq 1$. If the above two conditions hold then from the expression of a as given in Eq. (51) we see that $a>0$ and consequently Eq. (51) always defines a compressive solitary wave having a sech^4-profile. Now it can be easily checked that the amplitude of the SWS (51) is inversely proportional to $(\alpha-1)^2$ and consequently the amplitude of the SWS (51) goes to a large numerical value as $\alpha\rightarrow 1(\Leftrightarrow B\rightarrow 0)$ and consequently the nonlinear behavior of the ion-acoustic wave cannot be described by the S-ZK equation when $\alpha\rightarrow 1(\Leftrightarrow B\rightarrow 0)$. When $\alpha\neq 1(\Leftrightarrow B\neq 0)$ then the amplitude of the SWS (51) decreases if $|\alpha-1|>1$ and increases if $|\alpha-1|<1$ when the other parameters involved in the system remain fixed. We have already mentioned earlier that there is no effect of the parameter α on the width of the SWS (51). So, we shall consider here the effect of the nonthermal parameter β_1 on the amplitude and width of the SWS (51). Figure 1(a) shows the variation in the amplitude of SWS (51) against β_1 for different σ_{sc} with n_{sc}=0.25, σ=0.0001, α=2, $\delta=\pi/4$, and U=1.41. Figure 1(b) shows the variation in the amplitude of the SWS (51) against β_1 for different values of n_{sc} with n_{sc}=0.25, σ=0.0001, α=2, $\delta=\pi/4$, and U=1.41. From Figs. 1(a) and 1(b), we see that the amplitude of the SWS (51) decreases with increasing β_1 when the value of the other parameters involved in the system remain fixed. Again from Fig. 1(a), we see that for any fixed value of β_1, the amplitude of the SWS (51) decreases with increasing σ_{sc} and as σ_{sc} tends to be a large positive numerical value, the amplitude of the SWS (51) tends to a nonzero constant value if $n_{sc}>0$.

From Fig. 1(b), we see that for any fixed value of β_1, the amplitude of the SWS (51) decreases with increasing n_{sc}, and as n_{sc} tends to a large positive numerical value, the amplitude of the SWS (51) tends to a nonzero constant value.

Regarding the width of the SWS (51), from Figs. 2(a) and 2(b), we see that the width of the solitary wave increases with increasing β_1 for any fixed values of σ_{sc} as well as n_{sc} when the value of the other parameters involved in the system remains fixed. Again from Fig. 2(a), we see that for any fixed value of β_1, the width of the solitary wave decreases with increasing σ_{sc}, and as σ_{sc} tends to a large numerical value, the width of the solitary wave tends to a nonzero constant (with respect to σ_{sc} and n_{sc}) value. From Fig. 2(b), we see that for any fixed value of β_1, the width of the solitary wave decreases with increasing n_{sc}, and as n_{sc} tends to a large numerical value, the width of the solitary wave tends to a nonzero constant (with respect to σ_{sc} and n_{sc}) value.

B. SWSS of the KDV-ZK equation

In Sec. IV A we have already mentioned that the width $p^{-1}=4\sqrt{a_3/U}$ of the SWS (51) of the S-ZK equation is independent of α, i.e., $4\sqrt{a_3/U}$ is independent of α, i.e., a_3 is independent of α. Now the width of the SWS (52) of the KdV-ZK equation is equal to $2\sqrt{a_3/U}$ and consequently the width of the SWS (51) of the S-ZK equation is equal to twice that of the SWS (52) of the KdV-ZK equation, and consequently there is no qualitative change in the width of the SWS (52) of the KdV-ZK equation with respect to the width of the SWS (51) of the S-ZK equation. More precisely, with the help of simple algebra, one can easily check that

$$\frac{\text{width of the solitary wave as given by Eq. (46)}}{\text{width of the solitary wave as given by Eq. (45)}}=\frac{1}{2}.$$

The effects of different parameters involved in the system on the amplitude of the SWS (52) of the KdV-ZK equation have been extensively investigated by Islam *et al.*[19]

C. SWS of the combined S-KDV-ZK equation

Using Eq. (50), Eq. (49) can be written as

$$-U\varphi_0+(2/3)a_1\varphi_0^{3/2}+(1/2)a_2\varphi_0^2+a_3\varphi_0''=0. \quad (53)$$

Next we set

$$\varphi_0=\{a_0(b_0+c_0\,\mathrm{sech}^2\,pZ)^{-1}\mathrm{sech}^2\,pZ\}^2, \quad (54)$$

as a solution of Eq. (53). Following exactly the same method of Das *et al.*[8] (Sec. IV B), the SWS (54) of Eq. (53) can be written as

$$\varphi_0=a(S^2/\Psi^2), \quad (55)$$

where

$$S=\mathrm{sech}[2pZ], \quad (56)$$

$$\Psi=S+\lambda\sqrt{M}, \quad \lambda=\pm 1, \quad (57)$$

$$a=900p^4(\cos^2\delta+D\sin^2\delta)^2/B^2, \quad (58)$$

$$L=MB^2=B^2+\tfrac{75}{2}B'p^2(\cos^2\delta+D\sin^2\delta). \quad (59)$$

The SWS (55) exists if and only if

$$L\equiv B^2+(75/2)B'p^2(\cos^2\delta+D\sin^2\delta)>0 \quad (60)$$

and if the condition (60) holds good, U is given by

$$U=16p^2a_3. \quad (61)$$

Here $\lambda=1$ and $\lambda=-1$ give the two different modes of compressive solitary waves. In Fig. 3, L is plotted against β_1 for different δ and for $p=1$, $\alpha=1.0001$, $\omega_c=0.2$, $\sigma=0.0001$, $n_{sc}=0.25$, and $\sigma_{sc}=0.25$. This graph shows the range of β_1 where L is positive. From this graph we see that for any value of α lying in the neighborhood of $\alpha=1$, L is positive if $0\le\beta_1\le 0.412$. In Fig. 4, L is plotted against α for different β_1 and for $p=1$, $\delta=45°$, $\omega_c=0.2$, $\sigma=0.0001$, $n_{sc}=0.25$, and $\sigma_{sc}=0.25$. This graph also shows the range of β_1 where L is positive. From this figure, we infer that L is positive for any value of α lying in the neighborhood of $\alpha=1$ if $0\le\beta_1$

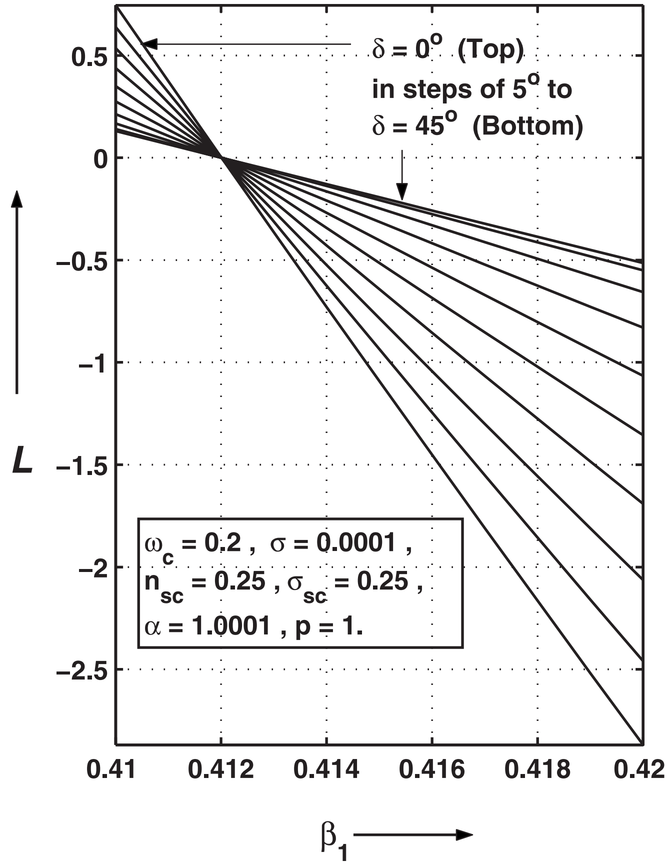

FIG. 3. L vs β_1 for different δ.

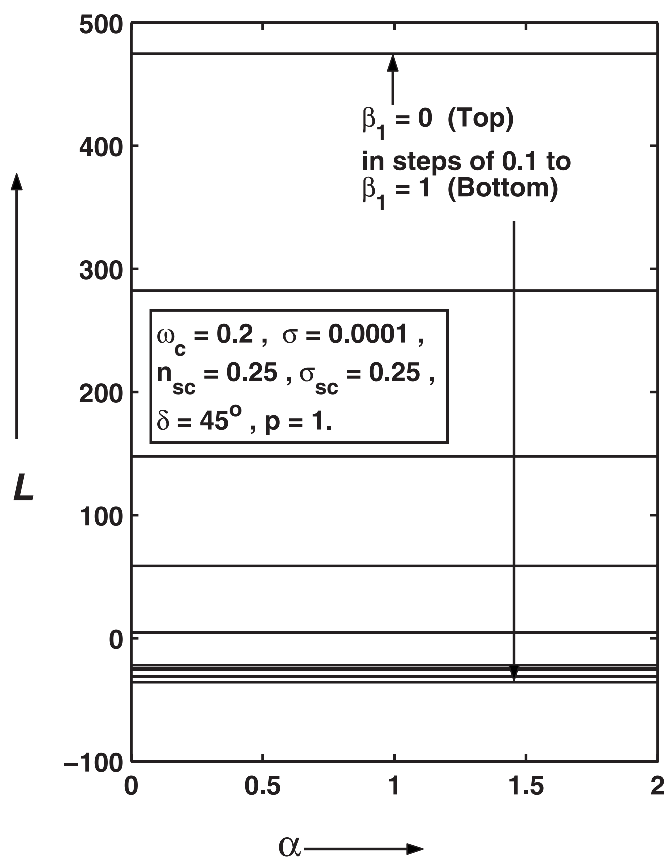

FIG. 4. L vs α for different β_1.

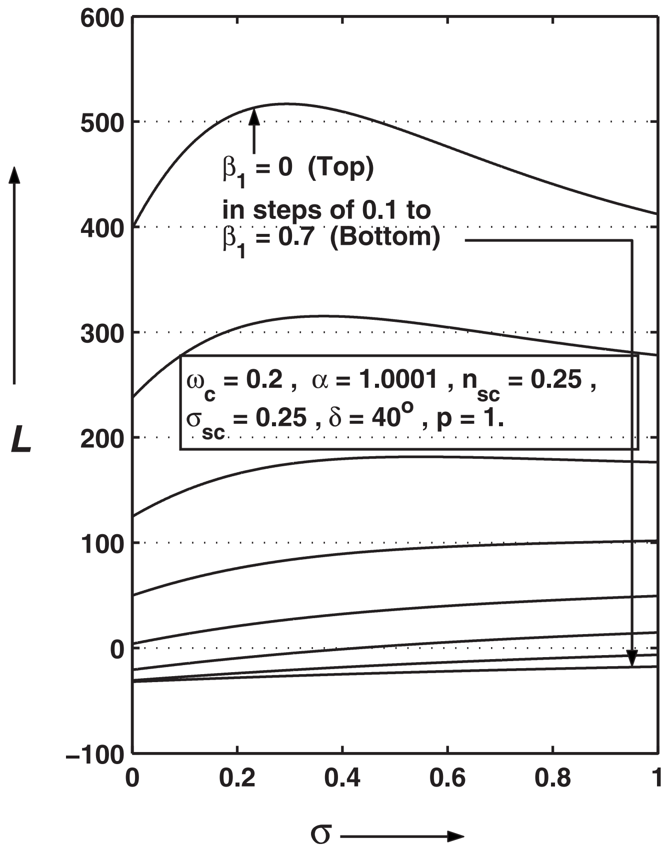

FIG. 5. L vs σ for different β_1.

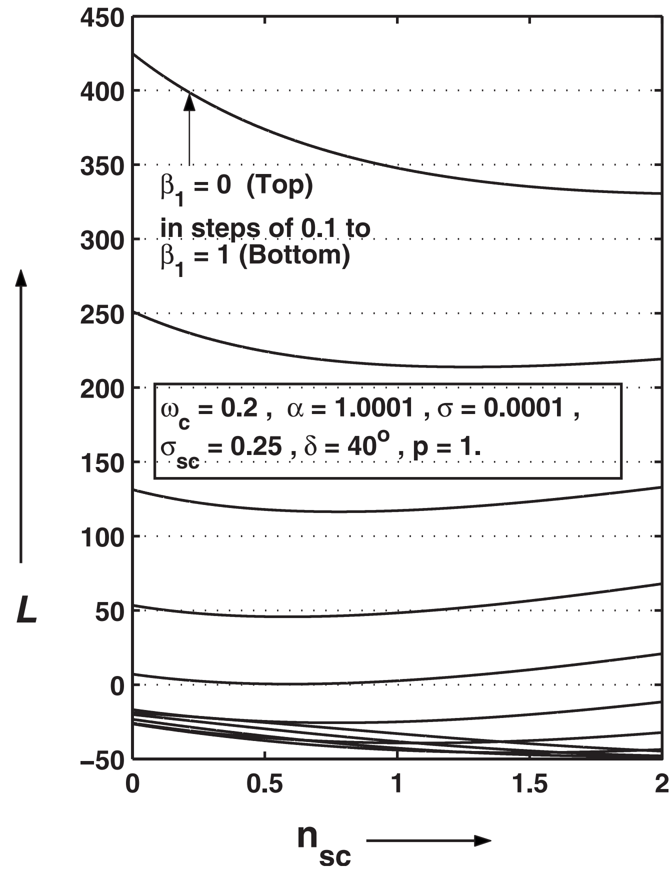

FIG. 6. L vs n_{sc} for different β_1.

≤ 0.4 (approximately). In Fig. 5, L is plotted against σ for different β_1 and for $p=1$, $\delta=40°$, $\omega_c=0.2$, $\alpha=1.0001$, $n_{sc}=0.25$, and $\sigma_{sc}=0.25$. From this figure, we derive that L is positive for any value of α lying in the neighborhood of $\alpha=1$ if $0\leq\beta_1\leq 0.4$ (approximately). In Fig. 6, L is plotted against n_{sc} for different β_1 and for $p=1$, $\delta=40°$, $\omega_c=0.2$, $\alpha=1.0001$, $\sigma=0.0001$, and $\sigma_{sc}=0.25$. From this figure, we find that L is positive for any value of α lying in the neighborhood of $\alpha=1$ and for all n_{sc} if $0\leq\beta_1\leq 0.4$ (approximately). In Fig. 7, L is plotted against σ_{sc} for different β_1 and for $p=1$, $\delta=40°$, $\omega_c=0.2$, $\alpha=1.0001$, $\sigma=0.0001$, and $n_{sc}=0.25$. This figure helps us to find that L is positive for any value of α lying in the neighborhood of $\alpha=1$ and for all σ_{sc} if $0\leq\beta_1\leq 0.3$ (approximately) and $L>0$ if $\sigma_{sc}>0.25$ if $0\leq\beta_1\leq 0.4$ (approximately). From these figures (Figs. 3–7), it is clear that for any value of α lying in the neighborhood of $\alpha=1$, there exists a finite interval of β_1 always containing the point $\beta_1=0$ such that $L>0$ for any fixed values of the parameters involved in the system. Again it can be easily checked that as α tends to 1 the range of β_1 cannot be a singleton set containing the point $\beta_1=0$. To prove this result we first of all note the following facts.

(1)

$$\lim_{\alpha\to 1} L=(75/2)B'p^2(\cos^2\delta+D\sin^2\delta), \tag{62}$$

and $\cos^2\delta+D\sin^2\delta>1$ for any admissible values of the parameters involved in the system.

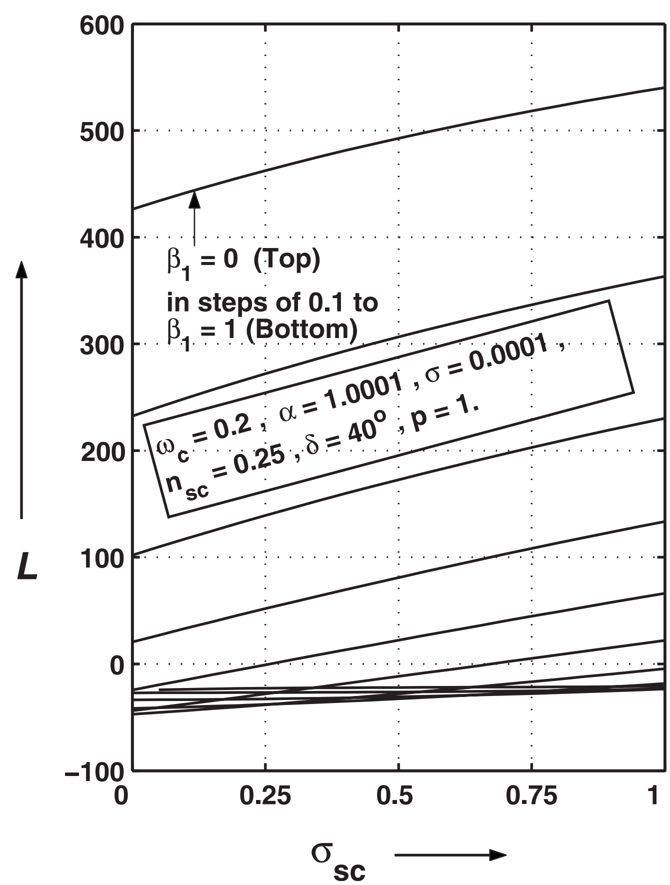

FIG. 7. L vs σ_{sc} for different β_1.

(2) Using Eq. (39), we can write the expression of B' [Eq. (42)] as

$$B' = \frac{20(1+n_{sc}\sigma_{sc}-\beta_1)^3}{9(1+n_{sc}\sigma_{sc})^3}(\sigma-\sigma_c), \quad (63)$$

where

$$\sigma_c = \frac{27(1+n_{sc}\sigma_{sc})(\beta_{2c}-\beta_1)(\beta_1-\beta_{1c})}{40(1+n_{sc}\sigma_{sc}-\beta_1)^3},$$

with

$$\beta_{1c} = 1+n_{sc}\sigma_{sc}-\beta_{3c},$$

$$\beta_{2c} = 1+n_{sc}\sigma_{sc}+\beta_{3c},$$

$$\beta_{3c} = \sqrt{\frac{(1+n_{sc})(1+n_{sc}\sigma_{sc}^2)}{3}}.$$

Therefore, from Eq. (62), we see that sign of L, as $\alpha \to 1$, mainly depends on the sign of B' and from the above expression of B' as given by Eq. (63), we find that B' (consequently L) is positive if $\sigma > \sigma_c$. Again, from Eq. (63), it can also be checked that for any value of the parameters involved in the system there exists a finite range of the nonthermal parameter β_1 such that $\sigma > \sigma_c$. Finally we can say that the SWS (55) always exists for $0 \leq \beta_1 \leq 0.3$ and when the value of α lies in the small neighborhood of $\alpha=1$. If $\sigma_{sc} > 0.25$, the SWS (55) always exists for $0 \leq \beta_1 \leq 0.4$ and when the value of α lies in the small neighborhood of $\alpha=1$. The profile of SWS (55) is shown in Fig. 8 for different β_1 and for $\lambda=1$, $p=1$, $\delta=40°$, $\omega_c=0.2$, $\alpha=1.0001$, $\sigma=0.0001$, $\sigma_{sc}=0.25$, and $n_{sc}=0.25$. A similar profile of the SWS (55) can also be obtained for $\lambda=-1$. From Fig. 8, we find that the amplitude of the solitary waves solution increases with the increasing β_1.

Now as $B \approx \bigcirc(\varepsilon^{1/4})$, M is large enough and consequently we can consider the limiting case where $\alpha \to 1 \Leftrightarrow B \to 0 \Leftrightarrow M \to +\infty$. For this limiting case, we get the following equation from Eq. (55):

$$\lim_{\alpha\to 1} \varphi_0 = \frac{3U}{AB' \cos\delta}\text{sech}^2\, 2pZ : U = 16p^2 a_3. \quad (64)$$

Again we can write Eq. (52) as

$$\varphi_0 = \frac{3U}{AB' \cos\delta}\text{sech}^2\, pZ : U = 4p^2 a_3. \quad (65)$$

Therefore, the equations in Eq. (64) are exactly the same as the equations in Eq. (65) if we replace $2p$ by p in Eq. (64). Thus the SWS (55) simply reduces to Eq. (52) when $B \to 0$, i.e., $\alpha \to 1$. Therefore, when the contribution of trapped electrons tends to zero, the ASWS of the combined S-KdV-ZK equation assumes the sech^2-profile of the KdV-ZK equation, which corresponds to the claim of Schamel[12] (p. 919, the nonlinear behavior of ion-acoustic wave is described by sech^2 profile only when $\alpha \to 1$). From Eq. (64), it is also clear that the amplitude of the ASWS (55) is always bounded (with respect to α) for any value of α lying in the neighborhood of $\alpha=1$ and for any value of the other parameters involved in the system because $[3U/AB' \cos\delta]$ is independent of α.

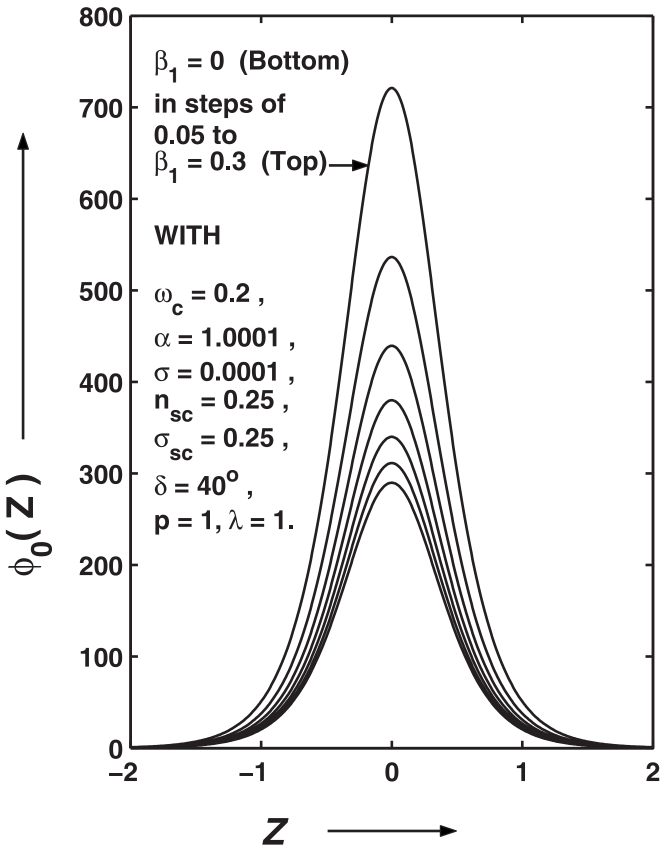

FIG. 8. Profile of SWS (55) for different β_1.

V. CONCLUSIONS

The S-ZK equation describes the behavior of long wavelength weakly nonlinear ion-acoustic wave propagating obliquely to an external uniform static magnetic field in a plasma which consists of warm adiabatic ions and two distinct populations of electrons. One due to Cairns *et al.*,[2] which generates the fast energetic electrons, and the other due to Schamel,[12] which is a vortexlike distribution and takes care of both free and trapped electrons. This equation admits SWS having a sech^4-profile. If the temperature of the trapped electrons is the same as that of the free electrons, the vortexlike distribution of electrons simply becomes isothermal distribution of electrons and the nonlinear dynamics of the same ion-acoustic wave is described by a KdV-ZK equation. This equation admits SWS having a sech^2-profile.

If the temperature of the trapped electrons approaches the temperature of the free electrons, the S-ZK equation fails to describe the nonlinear behavior of the same ion-acoustic wave. In this case, the evolution equation is a combined S-KdV-ZK equation. This equation admits ASWS having profile different from sech^4 or sech^2. The condition for the existence of such ASWS of the combined S-KdV-ZK equation has been extensively investigated. Here we find a definite (fixed) range of β_1 ($0 \leq \beta_1 \leq 0.3$), where ASWS (55) exists for admissible values of the other parameters involved in the system. For the limiting case, where the temperature of the trapped electrons approaches the temperature of the free electrons, the alternative solitary waves as expected are exactly the same as those of the solitary waves (sech^2-profile) of the KdV-ZK equation, although the algebraic manipulation involved to study the SWSs for these two evolution equations (KdV-ZK and the combined S-KdV-ZK) are completely different. This equivalence proves the correctness of the steady state solution of the combined S-KdV-ZK equation.

Graphically, it can easily checked that $B'=0$ along different family of curves ($\sigma=\sigma_c$) in the $\beta_1\sigma$-parametric plane for a large set of admissible values of the other parameters involved in the system. For this case, i.e., when $B'=0$ or $B'\to 0$, the KdV-ZK equation fails to describe the nonlinear behavior of the same ion-acoustic wave. The dynamics of the nonlinear behavior of the same ion-acoustic wave, when $B'=0$ or $B'\to 0$, can be considered as a separate problem. The stabilities of the SWSs of the different evolution equations are also important and can be considered as a separate problem.

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APPENDIX: CONDITIONS FOR THE MOVEMENT OF ELECTRONS ESSENTIALLY ALONG THE MAGNETIC FIELD

The drift kinetic equation for electrons is the following:

$$\frac{\partial \bar{f}}{\partial t} + v_z \frac{\partial \bar{f}}{\partial z} + \frac{c}{B^2}(\vec{E}\times\vec{B})\cdot\vec{\nabla}\bar{f} - \frac{e}{m_e}E_z\frac{\partial \bar{f}}{\partial v_z} = 0.$$

An estimate of the order of last two terms,

$$\left|\frac{c}{B^2}(\vec{E}\times\vec{B})\cdot\vec{\nabla}\bar{f}\right| = \left|\frac{c}{B^2}(\vec{\nabla}\varphi\times\vec{B})\cdot\vec{\nabla}\bar{f}\right| \sim \frac{c}{B}\frac{\varphi_0\bar{f}_0}{L^2},$$

$$\left|\frac{e}{m_e}E_z\frac{\partial \bar{f}}{\partial v_z}\right| = \left|\frac{e}{m_e}\frac{\partial \varphi}{\partial z}\frac{\partial \bar{f}}{\partial v_z}\right| \sim \frac{e}{m_e}\frac{\varphi_0\bar{f}_0}{LV},$$

where $\bar{f}_0$, φ_0 are some characteristic values of $\bar{f}$, φ, respectively, L and V are characteristic values of length and velocity in which length scales in both parallel and perpendicular directions are assumed to be the same. Therefore, an estimate of the ratio of the last two terms in the drift kinetic equation becomes

$$\frac{|\text{third term of drift kinetic equation}|}{|\text{fourth term of drift kinetic equation}|} \sim \frac{m_e c}{eB}\frac{V}{L} = \frac{1}{\Omega_e T} \ll 1, \quad \text{if } T \gg \frac{2\pi}{\Omega_e},$$

where $T(=L/V)$ is the characteristic time.

Hence if $T\gg 2\pi/\Omega_e$, we can neglect the third term and the drift kinetic equation for electrons assumes the following form:

$$\frac{\partial \bar{f}}{\partial t} + v_z\frac{\partial \bar{f}}{\partial z} - \frac{e}{m_e}E_z\frac{\partial \bar{f}}{\partial v_z} = 0.$$

This shows that the electrons move essentially along the magnetic field, if the characteristic time is very much greater than electron Larmor period and length scales in both parallel and perpendicular directions are the same.

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