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Alloy-disorder-induced intrasubband scattering in a quantum well under an electric field

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The scattering rates in the lowest subband in a quantum well are calculated for alloy-disorder scattering when an electric field is applied perpendicular to the layer plane. Calculations for the InGaAs quantum well indicate that the scattering rate increases with increase in the electric field.

I. INTRODUCTION

In recent years, there has been considerable interest in the study of intersubband transitions in a quantum well (QW) both in the presence and in the absence of an electric field applied perpendicular to the QW layer plane.[1–14] The reason behind the interest is practical device application. It has been predicted[5–8] that stimulated emission arising out of the transition from the second subband to the first one in a QW can be achieved and that a large amount of optical gain is possible. An infrared detector in the 8–10 μm range has been proposed and fabricated[9,10] that relies on the intersubband transitions and resonant tunneling between adjacent QWs.

Recent theoretical studies[11–14] on intersubband optical absorption under an electric field have pointed out an increase in the oscillator strength. The nonlinearity of the absorption coefficient[11,12] has also been the subject of theoretical investigation. The study points out the feasibility of growing electro-optical modulators and photodetectors in the infrared region. In all the theories[11–14] the intra- and intersubband relaxation rates come into the calculation of the absorption coefficient. Various scattering events determine the inter- and intrasubband relaxation rates, of which polar optic phonon scattering has been found to be the dominant one at room temperature. The rates for such scattering have already been calculated.[14]

Ternary semiconductors like $In_xGa_{1-x}As$ have also been studied[15] in this connection. An important scattering mechanism in ternaries is the alloy-disorder scattering.[16–19] The mobility and relaxation rates in QWs made of ternary semiconductors have been formulated previously in the absence of an applied electric field.[16,17] In the present work we report similar calculations when an electric field is applied perpendicular to the QW layer,and investigate how the scattering rates are modified. As a first step, we consider intrasubband relaxation for the lowest subband.

II. CALCULATIONS

The electrons are assumed to be quantized along the z direction and the wave function is taken to be of the following form[1]:

$$\psi = A\exp\left[-\beta\left(\frac{z}{L}+\frac{1}{2}\right)\right]\sin\left[\frac{\pi}{L}\left(z+\frac{L}{2}\right)\right]\times\exp(i\mathbf{k}\cdot\mathbf{r}), \quad -L/2<z<L/2, \tag{1}$$

where

$$A^2=\frac{4(\beta^2+\pi^2)}{\beta L\,[1-\exp(-2\beta)]}. \tag{2}$$

L is the thickness of the well, and $\mathbf{k}$ and $\mathbf{r}$ are, respectively, the two-dimensional (2D) wave vector and position vector of the electron in the plane of free motion (x-y plane). In Eq. (1) β is the variational parameter related to the electric field F as[1]

$$\begin{aligned}\beta &= \tfrac{1}{2}\overline{F}\,[(\pi^2/6)-1], \quad \overline{F}<1,\\ &= (\tfrac{3}{4}\pi^2\overline{F})^{1/3}, \quad \overline{F}\gg 1,\end{aligned} \tag{3}$$

$$\overline{F}=|e|FL/E_1^0, \quad E_1^0=(\hbar^2\pi^2)/(2m^*L^2).$$

Following earlier work,[16–19] the scattering potential is assumed to be a spherically symmetric square well of height ΔE and radius r_0. The 2D Fourier transform of the scattering potential is expressed as[19]

$$V=\sum_{\mathbf{q}}\pi\Delta E\,[rJ_1(qr)/q]\exp(i\mathbf{q}\cdot|\mathbf{r}-\mathbf{r}_i|), \tag{4}$$

$$r^2=r_0^2-|z-z_i|^2.$$

In Eq. (4), $(\mathbf{r}_i,z_i)$ is the coordinate of the center of the ith scattering site, and J_1 is the Bessel function of first order of the first kind. Using the above, the matrix element for transition from an electronic state of wave vector $\mathbf{k}$ to another state of wave vector $\mathbf{k}'$, is expressed as

$$m(\mathbf{k},\mathbf{k}')_i=A^2\exp(-i\mathbf{q}\cdot\mathbf{r}_i)\left\{\int_{z_i-r_0}^{z_i+r_0}\pi\Delta E\times\left(\frac{rJ_1(qr)}{q}\right)\exp\left[-2\beta\left(\frac{z}{L}+\frac{1}{2}\right)\right]\times\sin^2\left[\frac{\pi}{L}\left(z+\frac{L}{2}\right)\right]dz\right\}\delta_{\mathbf{k}',\mathbf{k}+\mathbf{q}}. \tag{5}$$

We assume as before that r_0 is small so that $J_1(x)=x/2$, and

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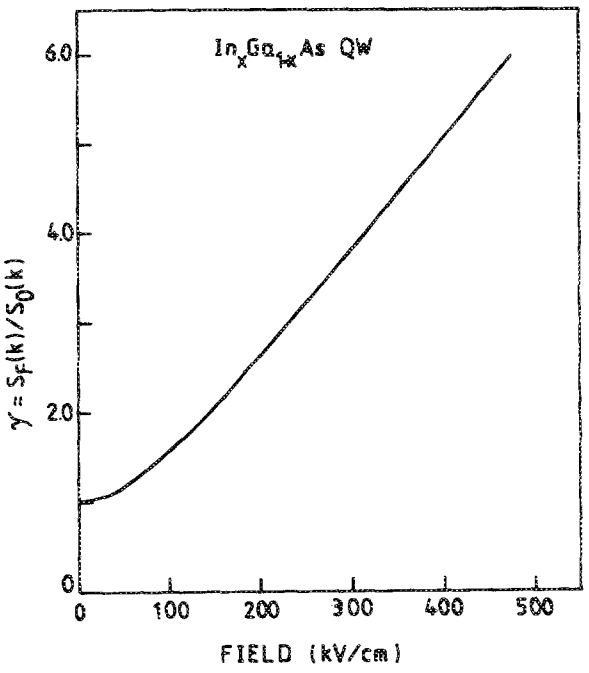

FIG. 1. Variation of the scattering rate, $S_F(\mathbf{k})$, with applied electric field. The rate S_F is normalized by the rate $S_0(\mathbf{k})$ in the absence of field, and the ratio is denoted by γ.

also that the variations of the exponential and sine terms in the range $z_i - r_0 \leqslant z \leqslant z_i + r_0$ are negligible. We may then put $z = z_i$ in the arguments and take the terms outside the integral. The z integration is then performed analytically to give

$$m(\mathbf{k},\mathbf{k}')_i = \exp(-i\mathbf{q}\cdot\mathbf{r}_i)A^2\left(\tfrac{4}{3}\pi r_0^3 \Delta E\right) \times \exp\left[-\left(\beta + \frac{2\beta z_i}{L}\right)\right]\cos^2\left(\frac{\pi z_i}{L}\right). \quad (6)$$

Now considering all the alloy sites to be randomly distributed, one may write for the matrix element as

$$|M(\mathbf{k},\mathbf{k}')|^2 = N_0\left(\tfrac{4}{3}\pi r_0^3 \Delta E\right)^2 A^4 \exp(-2\beta)x(1-x) \times \int_{-L/2}^{L/2} \exp(-4\beta z_i/L) \times \cos^4(\pi z_i/L)\,dz_i, \quad (7)$$

The transition probability from a state k to all other states k' is then given by

$$S_F(\mathbf{k}) = \frac{2\pi}{\hbar}\sum_{\mathbf{k}'}|M(\mathbf{k},\mathbf{k}')|^2\delta(E_{k'} - E_k) = [16\pi r_0^6(\Delta E)^2 N_0 x(1-x)m^* L f(\beta)]/(9\hbar^3), \quad (8)$$

where

$$f(\beta) = A^4[1 - \exp(-4\beta)]/[a(a^2+16)(a^2+4)], \quad a = 4\beta/\pi. \quad (9)$$

In order to facilitate comparison, a similar expression for the transition probability or scattering rate without an applied field is needed. For this purpose, the wave function is obtained by taking $\beta = 0$ in Eq. (1). Following exactly the similar procedure, we may write for the scattering rate without a field, $S_0(\mathbf{k})$, as

$$S_0(\mathbf{k}) = [8\pi^2 r_0^6(\Delta E)^2 m^* N_0 x(1-x)]/(3\hbar^3 L). \quad (10)$$

From Eqs. (8)–(10) the ratio is expressed as

$$\gamma = \frac{S_F(\mathbf{k})}{S_0(\mathbf{k}')} = \frac{16L^2 f(\beta)}{\pi}. \quad (11)$$

It appears therefore that the scattering rates are independent of the energy of the electrons.

III. RESULTS

We have calculated the scattering rates in $In_{0.53}Ga_{0.47}As$-InP QWs by using the following parameters[19]: $L = 10$ nm, $m^* = 0.042m_0$, $\Delta E = 0.53$ eV, $r_0 = \sqrt{3}\,a/4$, $N_0 = 4/a^3$, $a = 0.587$ nm, with a the lattice constant.

The values of the ratio γ calculated from Eq. (11) are plotted against the electric field F in Fig. 1. The scattering rate in the absence of the electric field is $1.0655 \times 10^{13}\,s^{-1}$. It is found from the figure that the scattering rate increases with an increase in the electric field. We believe that this trend will also be exhibited even if a more accurate wave function considering leakage into the barrier is used instead of that in Eq. (1). A similar trend will also be found in the case of hole scattering.

IV. CONCLUSION

We have demonstrated that there is an increase of the scattering rate of electrons in the first subband of a QW made of a ternary alloy semiconductor, with increase in the perpendicular electric field, when the scattering is due to alloy disorder.

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