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A theory for Landau damping of ion acoustic waves in dope plasma

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Landau damping, free from the influence of other forms of damping, has been experimentally detected in ion acoustic waves in traces of a dope plasma of a light inert gas, when it is introduced into partially ionized plasma of a heavy inert gas. A theory has been worked out of Landau damping for longitudinal waves in Vlasov plasma for study of this and other familiar Landau dampings. © *2001 American Institute of Physics.* [DOI: 10.1063/1.1339834]

I. INTRODUCTION

The physics of plasma consisting of partially ionized and neutral particles is linked with the excitation of charge exchange, or collisional excitation of ion acoustic waves. These excitations have remained important since the time of Langmuir[1] and continue to be so.[2–11] For instance, the laser fusion community has many reports on experimental findings with partly ionized gold[11] in their plasmas.

The force of simple harmonic motion proportional to the field induced displacement of electrons about their ionic cores, in addition to the Lorentz force, acts on the bound electrons only. Actually, the conservative central Coulomb potential of an atomic nucleus reduces to the centrifugal force of rotation of bound electrons about their nuclei. The particle dynamics of bound electrons in the presence of applied wave fields in the classical limit exists and gives useful results in the Larmor precession effect, and in scattering theories of Rayleigh,[12] Thomson, and others. The nonlinear distortion of orbits of bound electrons contributes to the optical properties of atoms excited by strong electromagnetic (em) radiation.[13]

Since the time of Druds and Lorentz,[14] the linear polarization of a medium is determined by regarding the electrons as harmonically bound to their nuclei. The valence electrons of atoms are bound by the Coulomb force of ionic cores, and the anharmonicity of the electron oscillations for large deviation from their equilibrium positions had been used by Rayleigh[12] to explain nonlinearities in acoustic resonators. The nonlinear distortions in the path of electrons bound to their nuclei, when excited by strong em radiation, were used by Bloembergen[13] to determine the nonlinear optical properties of atoms in dielectric media.

Bonnedal and Willelmsson[15] investigated nonlinear effects in a model of partially ionized, collisionally damped, homogeneous magnetized plasma of infinite extent. This plasma is a mixture of a fully ionized plasma of free electrons, and a partially ionized plasma of active molecules, the energy levels of which have inverted population, that is, a maser system when the active molecules have a uniform drift velocity parallel to the magnetization direction. So, these authors replace the consequence of molecules with inverted population levels by a damped Lorentz oscillator model. The forces acting on these molecules are the same as on the population of bound electrons.

In plasmas some atoms remain neutral, but their valence electrons are weakly bound to their nuclei. Laser produced plasmas, consisting of multiply-ionized ions and a highly charged heavy ion, accept many electrons in their high-lying loosely bound orbitals. When the numbers of bound electrons becomes large, the line spectra may become very densely crowded. The assumption that electrons are bound classically to the core ions seems valid in the study of this dynamism.

Since the phase velocity $v_p(=\omega/k)$ of ion acoustic waves is independent of the plasma density, it is convenient to perform experiments with these waves in partially ionized plasma. For completely free electrons, their binding frequencies must vanish. Alexeff *et al.*[2] (1967) measured Landau damping length of ion acoustic waves in a dope plasma of a light inert gas without help of an ambient magnetic field along the direction of wave propagation. To eliminate the influence of collisional and other such losses of damping, these authors first produced a plasma of a heavy inert gas like argon or xenon, at a wave phase velocity v_p greater than the average ion thermal velocity $C_{\text{ih}}(=K_B T_{\text{ih}} M_h)$, where the subscript h stands for the plasma of the heavy gas. Similarly, the subscript l stands for the plasma of the light gas. Since $v_p > C_{\text{ih}}$, Landau damping of ion acoustic waves is not possible in this plasma. Actually, the direct measurement of Landau damping by a wave at phase velocity $v_p = C_{\text{ih}}$ is not possible because this noncollisional damping cannot be separated from collisional and other such losses. These authors,[2] therefore, introduced a small amount of a dope (helium or neon) plasma. Dope ions being much lighter, move at higher velocities and so are Landau damped. If concentration of the dope is increased considerably, this Landau damping ceases because v_p then exceeds C_{il}.

We have used the Boltzmann–Vlasov equation for distribution functions of a four-species plasma and the divergence law of Gauss for the electric field. Electrons of all

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species of the plasma are assumed to be bound loosely (classically) to their inner cores of positive charges, like particles experiencing simple harmonic motion about fixed points. A plasma species is specified by the binding frequency of its electrons. Parameters specifying partial ionization of inert gases in which Landau dampings of ion acoustic waves and electron acoustic waves are investigated are difficult to determine directly. So, the binding frequencies are assumed to represent these parameters.

II. FORMULATION OF THE PROBLEM

The four distribution functions of time, velocity components, and position coordinates for electrons and ions of a heavy inert gas and a light inert gas are

$$f_{\rm ih}(r,\nu,t),\quad f_{\rm il}(r,\nu,t),\quad f_{\rm eh}(r,\nu,t), f_{\rm el}(r,\nu,t). \tag{1}$$

The applied electric field for longitudinal waves is

$$E(z,t)=E_0\cos\theta,\quad \theta=kz-\omega t. \tag{2}$$

The normalized equilibrium velocity distribution functions are the Maxwellians

$$f^0_{\rm eh}(v^2)=\left(\frac{m}{2\pi k_B T_{\rm eh}}\right)^{1/2}\exp\left(-\frac{mv^2}{2k_B T_{\rm eh}}\right), \tag{3}$$

etc., where, as suggested by the form of the longitudinal electric field of (2), v is the v_z of (1). The perturbed state distribution functions are expressed as

$$f_{\rm eh}(r,v,t)=N_{\rm eh}f^0_{\rm eh}(v^2)+f_{\rm eh}(z,v,t), \tag{4}$$

etc., where

$$|f_{\rm eh}(r,v,t)|\gg|f_{\rm eh}(z,v,t)|, \tag{5a}$$

etc., The number densities in the equilibrium state and the per particle charge of the four species are $N_{\rm eh}$, $N_{\rm el}$, $N_{\rm ih}$, $N_{\rm il}$, $q_{\rm eh}$, $q_{\rm el}$, $q_{\rm ih}$, $q_{\rm il}$. The infinitesimally small perturbations in the distribution functions are

$$f_{\rm eh}(z,v,t),\quad f_{\rm el}(z,v,t),\quad f_{\rm ih}(z,v,t),\quad f_{\rm il}(z,v,t). \tag{5b}$$

In the linearized approximation, the Boltzmann–Vlasov equations for the four species of plasma are

$$\frac{\partial f_{\rm eh}}{\partial t}+v\frac{\partial}{\partial z}f_{\rm eh}+N_{\rm eh}\left(\frac{q_{\rm eh}}{m}E+\omega^2_{\rm oh}\xi_h\right)\frac{\partial f^0_{\rm eh}}{\partial v}=0, \tag{6}$$

$$\frac{\partial f_{\rm ih}}{\partial t}+v\frac{\partial}{\partial z}f_{\rm ih}+N_{\rm ih}\frac{q_{\rm ih}}{M_h}E\frac{\partial f^0_{\rm ih}}{\partial v}=0, \tag{7}$$

$$\frac{\partial f_{\rm el}}{\partial t}+v\frac{\partial}{\partial z}f_{\rm el}+N_{\rm en}\left(\frac{q_{\rm el}}{m}E+\omega^2_{\rm ol}\xi_l\right)\frac{\partial f^0_{\rm el}}{\partial v}=0, \tag{8}$$

$$\frac{\partial f_{\rm il}}{\partial t}+v\frac{\partial}{\partial z}f_{\rm il}+N_{\rm il}\frac{q_{\rm il}}{M_l}E\frac{\partial f^0_{\rm il}}{\partial v}=0. \tag{9}$$

Gauss's divergence law gives

$$\frac{\partial E}{\partial Z}=4\pi\int\,(q_{\rm ih}f_{\rm ih}+q_{\rm il}f_{\rm il}+q_{\rm eh}f_{\rm eh}+q_{\rm el}f_{\rm el})dv. \tag{10}$$

The prefield macroscopic charge neutrality ensures

$$q_{\rm il}N_{\rm il}+q_{\rm ih}N_{\rm ih}+q_{\rm el}N_{\rm el}+q_{\rm eh}N_{\rm eh}=0. \tag{11}$$

ξ_l and ξ_h are the wave-field-induced displacements of the bound electrons of the light element and heavy element, respectively; the frequencies of their oscillations are $\omega_{\rm ol}$ and $\omega_{\rm oh}$, respectively. Time derivatives of ξ_l and ξ_h give the field-induced average velocities v_l and v_h. So

$$\xi''_l=\frac{1}{N_{\rm el}}\int_v vf_{\rm el}\,dv,\quad \dot{\xi}_h=\frac{1}{N_{\rm eh}}\int_v vf_{\rm eh}\,dv. \tag{12}$$

Using (2) and (12) in (6) we find that

$$f_{\rm eh}(z,v,t)=-iN_{\rm eh}\left(\frac{q_{\rm eh}}{m}E+\frac{i\omega^2_{\rm eh}}{\omega N_{\rm eh}}\int_v vf_{\rm eh}\,dv\right)\frac{f^1_{\rm eh}(v^2)}{kv-\omega}, \tag{13}$$

where $f^{01}_{\rm eh}=df^0_{\rm eh}/dv$, etc.

Hence, the integral $\int_v vf_{\rm eh}\,dv$ is given by

$$\int_v vf_{\rm eh}dv=\frac{-iN_{\rm eh}\dfrac{q_{\rm eh}}{m}\dfrac{\omega}{k}E\displaystyle\int_v\frac{f^{01}_{\rm eh}(v^2)dv}{kv-\omega}}{1-\dfrac{\omega^2_{\rm oh}}{k}\displaystyle\int_v\frac{f^{01}_{\rm eh}dv}{kv-\omega}}. \tag{14}$$

Using this integral in (13) gives

$$f_{\rm eh}(z,v,t)=-\frac{\dfrac{iq_{\rm eh}}{m}N_{\rm eh}E\dfrac{f^{01}_{\rm eh}(v^2)}{kv-\omega}}{1-\dfrac{\omega^2_{\rm oh}}{k}\displaystyle\int_v\frac{f^{01}_{\rm eh}(v^2)dv}{kv-\omega}}. \tag{15}$$

Similarly, proceeding for $f_{\rm el}$, $f_{\rm ih}$, $f_{\rm il}$ we obtain

$$f_{\rm el}(z,v,t)=-\frac{\dfrac{iq_{\rm el}}{m}N_{\rm el}E\dfrac{f^{01}_{\rm el}(v^2)}{kv-\omega}}{1-\dfrac{\omega^2_{\rm ol}}{k}\displaystyle\int_v\frac{f^{01}_{\rm el}(v^2)dv}{kv-\omega}}, \tag{16}$$

$$f_{\rm ih}(z,v,t)=-\frac{iq_{\rm ih}N_{\rm ih}}{M_h}E\frac{f^{01}_{\rm ih}(v^2)}{kv-\omega}, \tag{17}$$

$$f_{\rm il}(z,v,t)=-\frac{iq_{\rm il}N_{\rm il}}{M_l}E\frac{f^{01}_{\rm il}(v^2)}{kv-\omega}. \tag{18}$$

Then Eq. (10) becomes the wave evolution equation

$$k=\omega^2_{\rm ih}\int_v\frac{f^{01}_{\rm ih}(v^2)dv}{kv-\omega}+\omega^2_{\rm il}\int_v\frac{f^{01}_{\rm il}(v^2)dv}{kv-\omega}+\frac{\omega^2_{\rm eh}\displaystyle\int_v\frac{f^{01}_{\rm eh}(v^2)dv}{kv-\omega}+}{1-\dfrac{\omega^2_{\rm oh}}{k}\displaystyle\int_v\frac{f^{01}_{\rm eh}(v^2)dv}{kv-\omega}}+\frac{\omega^2_{\rm el}\displaystyle\int_v\frac{f^{01}_{\rm el}(v^2)dv}{kv-\omega}}{1-\dfrac{\omega^2_{\rm ol}}{k}\displaystyle\int_v\frac{f^{01}_{\rm el}(v^2)dv}{kv-\omega}}. \tag{19}$$

This is the key equation for study of the problem of Landau damping discussed in Sec. I. It is a singular integro-differential equation relating ω and k in which the singularity occurs at the wave phase velocity $v_p(=\omega/k)$. Carrying out the contour integration over the singularity, we obtain

$$k=\omega_{\rm ih}^2 p\int_v \frac{f_{\rm ih}^{01}(v^2)dv}{kv-\omega}\pm\frac{\omega_{\rm ih}^2}{k}\pi i f_{\rm ih}^{01}(v_p^2)+\omega_{\rm il}^2 p\int_v \frac{f_{\rm il}^{01}(v^2)dv}{kv-\omega}$$

$$\pm\frac{\omega_{\rm il}^2}{k}\pi i f_{\rm il}^{0^1}(v_p^2)+\frac{\omega_{\rm eh}^2 p\int_v \frac{f_{\rm eh}^{01}(v^2)dv}{kv-\omega}\pm\pi i\frac{\omega_{\rm eh}^2}{k}f_{\rm eh}^{01}(v_p^2)}{1-\frac{\omega_{\rm oh}^2}{k}p\int_v \frac{f_{\rm eh}^{01}(v^2)dv}{kv-\omega}\pm\pi i\frac{\omega_{\rm eh}^2}{k^2}f_{\rm eh}^{0^1}(v_p^2)}$$

$$+\frac{\omega_{\rm el}^2 p\int_v \frac{f_{\rm el}^{01}(v^2)dv}{kv-\omega}\pm\pi i\frac{\omega_{\rm el}^2}{k}f_{\rm el}^{0^1}(v_p^2)}{1-\frac{\omega_{\rm ol}^2}{k}p\int_v \frac{f_{\rm el}^{01}(v^2)dv}{kv-\omega}\pm\pi i\frac{\omega_{\rm el}^2}{k^2}f_{\rm el}^{01}(v_p^2)}, \tag{20}$$

where P stands for the principal value of the singular integral, derivative with respect to v is denoted by prime, and v_p is the wave phase velocity. In the general case of large number of plasma constituents, the right side of (20) is replaced by its sum over all the constituents.

We write (15) and (16) as

$$f_{\rm eh}(z,v,t)=-\frac{iE\omega_{\rm eh}^2 f_{\rm eh}^{01}(v^2)}{4\pi(1-a_h)(kv-\omega)}, \tag{21}$$

$$f_{\rm el}(z,v,t)=-\frac{iE\omega_{\rm el}^2 f_{\rm el}^{01}(v^2)}{4\pi(1-a_l)(kv-\omega)}, \tag{22}$$

for determining the electric current density j_b and the space charge separation density ρ_b^c in the bound electron plasma. We obtain

$$\begin{aligned} j_b&=\int_v v(q_{\rm eh}f_{\rm eh}+q_{\rm el}f_{\rm el}+q_{\rm ih}f_{\rm ih}+q_{\rm il}f_{\rm il})dv\\ &=-\frac{iE}{4\pi}\left[\frac{\omega_{\rm eh}^2}{1-a_h}\int_v \frac{vf_{\rm eh}^{01}dv}{kv-\omega}+\frac{\omega_{\rm el}^2}{1-a_l}\int_v \frac{vf_{\rm el}^{01}dv}{kv-\omega}\right.\\ &\left.\quad+\omega_{\rm ih}^2\int_v \frac{vf_{\rm ih}^{01}dv}{kv-\omega}+\omega_{\rm il}^2\int_v \frac{vf_{\rm il}^{01}dv}{kv-\omega}\right], \end{aligned} \tag{23}$$

$$\begin{aligned} \rho_b^c=&-\frac{iE\omega_{\rm eh}^2}{4\pi(1-a_h)}\int_v \frac{f_{\rm eh}^{01}dv}{kv-\omega}-\frac{iE\omega_{\rm el}^2}{4\pi(1-a_l)}\int_v \frac{f_{\rm el}^{01}dv}{kv-\omega}\\ &-\frac{iE\omega_{\rm ih}^2}{4\pi}\int_v \frac{f_{\rm ih}^{01}dv}{kv-\omega}-\frac{iE\omega_{\rm il}^2}{4\pi}\int_v \frac{f_{\rm il}^{01}dv}{kv-\omega}, \end{aligned} \tag{24}$$

where

$$a_h=\frac{\omega_{\rm oh}^2}{k}\int_v \frac{f_{\rm eh}^{01}dv}{kv-\omega},\quad a_l=\frac{\omega_{\rm ol}^2}{k}\int_v \frac{f_{\rm el}^{01}dv}{kv-\omega}. \tag{25}$$

Then

$$j_b-j_f=-\frac{iE}{4\pi}\left[\frac{a_h\omega_{\rm eh}^2}{(1-a_h)}\int_v \frac{vf_{\rm eh}^{01}dv}{kv-\omega}+\frac{a_l\omega_{\rm el}^2}{(1-a_l)}\int_v \frac{vf_{\rm el}^{01}dv}{kv-\omega}\right], \tag{26}$$

and

$$\rho_b^c-\rho_f^c=-\frac{iE}{4\pi}\left[\frac{a_h\omega_{\rm eh}^2}{(1-a_h)}\int_v \frac{f_{\rm eh}^{01}dv}{kv-\omega}+\frac{a_l\omega_{\rm el}^2}{(1-a_l)}\int_v \frac{f_{\rm el}^{01}dv}{kv-\omega}\right], \tag{27}$$

where j_f is the electric current density in the corresponding free electron plasma, and ρ_f^c is the charge separation density in the same plasma. Since

$$j_b-j_f=\frac{\omega}{k}(\rho_b^c-p_f^c), \tag{28}$$

the current j_b-j_f satisfies the condition of continuity

$$\nabla\cdot(j_b-j_f)+\partial(\rho_b^c-\rho_f^c)/\partial t=0. \tag{29}$$

Actually, this conservation condition is satisfied by (j_b,ρ_b^c) as well as by (j_f,ρ_f^c).

The dimensionless quantities a_h and a_l are contributions to $f_{\rm eh}(z,v,t)$ and $f_{\rm el}(z,v,t)$ in the bound electron plasma. Following the mathematics of the solution of singular integro-differential equations in the subsequent sections for study of some cases of Landau damping of waves in plasma, for analysis of a_h of (25) we find that

$$a_h=\frac{\omega_{\rm oh}^2}{\omega^2}\pm i\sqrt{\frac{\pi}{2}}\frac{\omega_{\rm oh}^2}{K^2C_{\rm e_h}^2}\frac{v_p}{C_{\rm eh}}\exp\left(-\frac{v_p^2}{2C_{\rm eh}^2}\right). \tag{30}$$

Both the real and imaginary parts are much smaller than unity because, for wave processes to be effective in such plasmas, the bound electron frequency ω_{oh} has to be much smaller than the wave frequency ω. The imaginary part is small, moreover, for the existence in it of the exponential factor. So, the electron current and space charge separation due to bound electrons are small quantities.

III. ELECTRON ACOUSTIC WAVES IN FREE ELECTRON PLASMA

For electron acoustic waves in free electron plasma, Eq. (20) reduces to

$$\frac{k\omega}{\omega_{\rm pe}^2}=-p\int\frac{f_e^{01}(v^2)dv}{1-(kv/\omega)}\pm\pi i\frac{\omega}{k}f_e^{01}(v_p^2). \tag{31}$$

The real part of (31) gives the dispersion relation for the wave. The imaginary part yields γ (the damping rate) or d_e (the damping distance). Expanding $(1-(kv/\omega))^{-1}$ in positive integral powers of (kv/ω), carrying out the integration for the principal value, retaining terms up to k^3v^3/ω^3, and replacing k by $k+(i/d_e)$, we equate the real and the imaginary parts in both sides and approximately obtain

$$\frac{\omega^2}{\omega_{\rm pe}^2}=1+\frac{3k^2C_e^2}{\omega^2}, \tag{32}$$

$$\frac{1}{d_e}=\sqrt{\frac{\pi}{8}}\frac{\omega^5}{k^4C_e^5}\exp\left(-\frac{v_p^2}{2c_e^2}\right), \tag{33}$$

where $C_e^2=K_BT_e/m$. For finding γ we use the physical condition that $v<v_p$ because v_p is large, retain terms only up to

$1+kv/\omega$ of the expansion of $(1-(kv/\omega))^{-1}$ in the first term of (31), replace ω by $\omega+i\gamma$, and obtain the familiar results

$$\omega^2=\omega_{\rm pe}^2, \tag{34}$$

$$\gamma_e=-\sqrt{\frac{\pi}{8}}\,\frac{\omega_{\rm pe}^4}{k^3C_e^3}\exp\left(-\frac{v_p^2}{2C_e^2}\right), \tag{35}$$

where γ_e is the time rate of Landau damping of electron acoustic waves.

IV. ELECTRON ACOUSTIC WAVES IN PLASMA HAVING BOUND ELECTRONS

For electron acoustic waves in a plasma consisting of bound electrons, free electrons, and ions for macroscopic charge neutralization in the prefield state, Eq. (20) reduces to

$$k=-\frac{\omega_{\rm pe}^2}{\omega}p\int_v\frac{f_e^{01}(v^2)dv}{kv-\omega}\pm\pi i\frac{\omega_{\rm pe}^2}{k}f_e^{01}(v_p^2)-\frac{\dfrac{-\omega_{\rm pb}^2}{\omega}p\displaystyle\int\frac{f_b^{01}(v^2)dv}{(kv-\omega)}\pm\pi i\frac{\omega_{\rm pb}^2}{k}f_b^{01}(v_p^2)}{1-\dfrac{\omega_0^2}{k}p\displaystyle\int\frac{f_b^{01}(v^2)dv}{kv-\omega}\pm\pi i\frac{\omega_0^2}{k^2}f_b^{01}(v_p^2)}. \tag{36}$$

We ignore k in the real part of (36), and obtain

$$\frac{\omega_{\rm pe}^2}{\omega^2}+\frac{(\omega_{\rm pb}^2/\omega^2)}{1-\dfrac{\omega_0^2}{\omega^2}}=1. \tag{37}$$

The imaginary part gives

$$-\frac{\omega_{\rm pe}^2}{\omega^2}\frac{2\gamma}{\omega}\pm\sqrt{\frac{\pi}{2}}\,\frac{\omega_{\rm pe}^2v_p}{k^2C_e^3}\exp\left(-\frac{v_p^2}{2C_{\rm eb}^2}\right)+\frac{-\dfrac{\omega_{\rm pb}^2}{\omega^2}\dfrac{2\gamma}{\omega}\pm\sqrt{\dfrac{\pi}{2}}\,\dfrac{\omega_{\rm pb}^2}{k^2C_{\rm eb}^2}\exp\left(-\dfrac{v_p^2}{2C_{\rm eb}^2}\right)}{1-\dfrac{\omega_0^2}{\omega^2}}-\frac{(\omega_{\rm pb}^2/\omega^2)}{\pm\sqrt{\dfrac{\pi}{2}}\,\dfrac{\omega_O^2v_p}{k^2C_{\rm eb}^3}\exp\left(-\dfrac{v_p^2}{2C_{\rm eb}^2}\right)}=1, \tag{38}$$

where (38) determines γ. Equation (37) is quadratic in ω^2, both roots of which are positive. So, the density oscillation at the Langmuir frequency $\omega_{\rm pe}$ is split up into oscillations at two different frequencies, one of which is higher and the other lower than $\omega_{\rm pe}$. This frequency splitting appears at all admissible wave lengths. This split causes interference and standing wave patterns in the plasma.

For finding the Landau damping distance d_e, we keep terms up to k^3v^3/ω^3 in the expansion of $(1-(kv/\omega))^{-1}$ in positive integral powers of (kv/ω) in the principal value of the integral of (36), and in this case obtain the wave evolution equation

$$\frac{\omega_{\rm pe}^2}{\omega^2}+\frac{3k^2C_p^2}{\omega^2}\frac{\omega_{\rm pe}^2}{\omega^2}\pm i\sqrt{\frac{\pi}{2}}\,\frac{\omega_{\rm pb}^2v_p}{k^2C_e^3}\exp\left(-\frac{v_p^2}{2C_p^2}\right)+\frac{\dfrac{\omega_{\rm pb}^2}{\omega^2}+\dfrac{3k^2C_{\rm pb}^2}{\omega^2}\dfrac{\omega_{\rm pb}^2}{\omega^2}\pm i\sqrt{\dfrac{\pi}{2}}\,\dfrac{\omega_{\rm pb}^2\nu_p}{k^2C_{\rm eb}^3}\exp\left(-\dfrac{v_p^2}{2C_{\rm pb}^2}\right)}{1-\dfrac{\omega_0^2}{\omega^2}+\dfrac{3k^2C_{\rm eb}^2}{\omega^2}\dfrac{\omega_{\rm oe}^2}{\omega^2}\pm i\sqrt{\dfrac{\pi}{2}}\,\dfrac{\omega_0^2\nu_p}{k^2C_{\rm eb}^2}\exp\left(-\dfrac{v_p^2}{2C_{\rm pb}^2}\right)}=1, \tag{39}$$

for wave dispersion and Landau damping.

V. ION ACOUSTIC WAVES IN A TWO-COMPONENT PLASMA

For ion acoustic waves in a two-component plasma, the wave phase velocity $v_p\leqslant C_i$, where $C_i^2=k_BT_i/M$. So, Eq. (19) simplifies to

$$k=\omega_i^2p\int_v\frac{f_i^{01}(\nu^2)d\nu}{k\nu-\omega}\pm\pi i\frac{\omega_i^2}{k}f_i^{01}(v_p^2)+\frac{\omega_e^2p\displaystyle\int_v\frac{f_e^{01}(\nu^2)d\nu}{k\nu-\omega}\pm\pi i\frac{\omega_e^2}{k}f_e^{01}(v_p^2)}{1-\omega_0^2p\displaystyle\int_v\frac{f_e^{01}(\nu^2)d\nu}{k\nu-\omega}\pm\pi i\frac{\omega_0^2}{k^2}f_e^{01}(v_p^2)}. \tag{40}$$

Since $C_e\gg(\omega/k)$, for electrons $v\gg(\omega/k)$; so we ignore the residue term, assume $v\neq0$ and expand $(1-(\omega/kv))^{-1}$ in positive integral powers of (ω/kv). Thus (40) reduces to

$$1=\frac{\omega_i^2}{\omega^2}\pm\pi i\frac{\omega_i^2}{k^2}f_i^{01}(v_p^2)-\frac{(\omega_e^2/k^2C_e^2)}{1+(\omega_0^2/k^2C_e^2)}. \tag{41}$$

Since $(\omega_e^2/C_e^2)=(\omega_i^2/C_s^2)$, where $C_s^2=K_BT_e/M$, separating the real and imaginary parts from both sides for finding the wave dispersion relation and the Landau damping rate γ_i, and since $\omega_i^2/\omega^2>1$, we obtain

$$C_e^2\omega^2=(\omega_0^2+k^2C_e^2)C_s^2, \tag{42}$$

for wave dispersion and

$$\frac{\gamma_i}{\omega}=\sqrt{\frac{\pi}{8}}\,\frac{\omega^2v_p}{k^2C_e^3}\exp(-v_p^2/2C_e^2), \tag{43}$$

for time rate of wave damping. Keeping terms up to the integrals $\int_v v^2f^0\,dv$ and $\int_{|v|>|\omega/k|}1/v^2f^0\,dv$, which are necessary for determining the Landau damping length, in place of (41) we obtain

$$\frac{\omega_i^2}{\omega^2}+\frac{\omega_i^2}{\omega^2}\frac{k^23C_i^2}{\omega^2}\pm\pi i\frac{\omega_i^2}{k^2}f_i^{01}(v_p^2)-1=\frac{\omega_i^2(\omega^2+k^2C_{ei}^2)C_e^2}{(k^4C_e^2C_{e_1}^2+\omega_0^2(k^2C_{e_1}^2+\omega^2))C_e^2}, \tag{44}$$

because for physics it is enough to assume in this case that $f_e^0(v^2)=0$ when $|v|<(\omega/k)$; so

$$\frac{k_\beta T_e}{m}=C_e^2=\int_{-\infty}^{\infty} v^2 f_e^0(v^2)dv$$
$$\approx\int_{-\infty}^{-|\omega/k|} v^2 f_e^0(v^2)dv+\int_{|\omega/k|}^{\infty} v^2 f_e^0(v^2)dv, \tag{45}$$

because effectively, $f_e^0(v^2)\approx 0$ in $-|\omega/k|\leqslant v\leqslant|\omega/k|$ and

$$\frac{1}{C_{e_1}^2}=\left(\int_{|\omega/k|}^{\infty}+\int_{-\infty}^{-|\omega/k|}\right)\frac{1}{v^2}f_e^0(v^2)dv. \tag{46}$$

Since $\omega_i^2>\omega^2$, we ignore the last term (minus one) in left side of (44), and obtain

$$1+\frac{3k^2C_i^2}{\omega^2}\pm\pi i\frac{\omega^2}{k^2}f_i^{01}(v_p^2)$$
$$=\frac{C_e^2\omega^2(\omega^2+k^2C_{e_1}^2)}{C_s^2[k^4C_e^2C_{e_1}^2+\omega_0^2(k^2C_{e_1}^2+\omega^2)]}. \tag{47}$$

The dispersion relation and the Landau damping distance are given by

$$1+\frac{3k^2C_i^2}{\omega^2}=\frac{C_e^2\omega^2(\omega^2+k^2C_{e_1}^2)}{C_s^2[k^4C_e^2C_{e_1}^2+\omega_0^2(k^2C_{e_1}^2+\omega^2)]}, \tag{48}$$

$$\left(\frac{2k}{d}\right)\left[\frac{3C_i^2}{\omega^2}-\frac{(C_e^2C_{e_1}^2\omega^2/C_s^2)}{k^4C_e^2C_{e_1}^2+\omega_0^2(k^2C_{e_1}^2+\omega^2)}\right.$$
$$\left.+\frac{(C_e^2C_{e_1}^2\omega^2/C_s^2)(\omega^2+k^2C_{e_1}^2)(2k^2C_e^2+\omega_0^2)}{[k^4C_e^2C_{e_1}^2+\omega_0^2(k^2C_{e_1}^2+\omega^2)]^2}\right]$$
$$=\pi\frac{\omega^2}{k^2}f_i^{01}(v_p^2). \tag{49}$$

VI. ION ACOUSTIC WAVES IN A DOPE PLASMA

The wave is introduced in the plasma of the heavy inert gas at the wave phase velocity v_p greater than its ion thermal velocity C_i. Then Landau damping of ion acoustic waves is not possible. So, a trace of the dope plasma of a light inert gas is introduced, the ion thermal velocity of which is more than the thermal velocity of the ions of the heavy gas. The dope ions are Landau damped in this mixture of two plasmas. For electrons of argon and helium $C_{\mathrm{el}}>V_{p_1}C_{\mathrm{il}}>V_p$, so physically, $v>v_p$, and $v\neq 0$. Then the factor $(kv-\omega)^{-1}$ is expanded in the principal value of the singular integral involving $f(z,v,t)$ as an integrand, in positive integral powers of (ω/kv) and the residue at the singularity $v=0$ is not physically possible. For ions of the heavy gas $v<v_p$, the residue physically vanishes and the factor $(kv-\omega)^{-1}$ is expanded in positive integral powers of (kv/ω). For ions of the light gas since, $v\leqslant(\omega/k)$, in the singular integro-differential equation the principal value of the integral of $f_{\mathrm{il}}(z,v,t)$ containing $(kv-\omega)^{-1}$ is determined after expanding this factor in positive integral powers of (kv/ω). Also the residue of this integral is evaluated at the wave phase velocity v_p. Then Eq. (20) becomes

$$k=-\frac{\omega_{\mathrm{ih}}^2}{\omega}\left(-\frac{k}{\omega}-\frac{3k^3C_{\mathrm{ih}}^2}{\omega^3}\right)-\frac{\omega_{\mathrm{il}}^2}{\omega}\left(-\frac{k}{\omega}-\frac{3k^3C_{\mathrm{il}}^2}{\omega^3}\right)$$
$$\pm\pi i\frac{\omega_{\mathrm{il}}^2}{k}f_{\mathrm{il}}^{01}(v_p^2)$$
$$+\frac{\frac{\omega_{\mathrm{eh}}^2}{k}\left[\int_{v\neq 0}\frac{1}{v}\left(1+\frac{\omega}{kv}+\frac{\omega^2}{k^2v^2}\right)f_{\mathrm{eh}}^{01}(v^2)dv\right]}{1-\frac{\omega_{\mathrm{oh}}^2}{k^2}\int_{v\neq 0}\frac{1}{v}\left(1+\frac{\omega}{kv}+\frac{\omega^2}{k^2v^2}\right)f_{\mathrm{eh}}^{01}(v^2)dv}$$
$$+\frac{\frac{\omega_{\mathrm{el}}^2}{k}\left[\int_{v\neq 0}\frac{1}{v}\left(1+\frac{\omega}{kv}+\frac{\omega^2}{k^2v^2}\right)f_{\mathrm{el}}^{01}(v^2)dv\right]}{1-\frac{\omega_{\mathrm{ol}}^2}{k^2}\int_{v\neq 0}\frac{1}{v}\left(1+\frac{\omega}{kv}+\frac{\omega^2}{k^2v^2}\right)f_{\mathrm{el}}^{01}(v^2)dv}, \tag{50}$$

where $\int_{v\neq 0}=\int_{-\infty}^{-v_p}+\int_{v_p}^{\infty}$. We now define the electron thermal velocities $C_{\mathrm{e_1l}}$ and $C_{\mathrm{e_1h}}$ by the relations

$$\frac{1}{C_{\mathrm{e_1l}}^2}=\int_{-\infty}^{-v_p}+\int_{v_p}^{\infty}\frac{1}{v^2}f_{\mathrm{el}}^{01}(v^2)dv,$$
$$\frac{1}{C_{\mathrm{e_1h}}^2}=\int_{-\infty}^{-v_p}+\int_{v_p}^{\infty}\frac{1}{v^2}f_{\mathrm{eh}}^{01}(v^2)dv. \tag{51}$$

Using these, (40) reads

$$1=\frac{\omega_{\mathrm{ih}}^2}{\omega^2}+\left(\frac{3k^2C_{\mathrm{eh}}^2}{\omega^2}\right)\left(\frac{\omega_{\mathrm{ih}}^2}{\omega^2}\right)+\frac{\omega_{\mathrm{il}}^2}{\omega^2}+\left(\frac{3k^2C_{\mathrm{il}}^2}{\omega^2}\right)\left(\frac{\omega_{\mathrm{il}}^2}{\omega^2}\right)$$
$$\pm\pi i\frac{\omega_{\mathrm{il}}^2}{k^2C_{\mathrm{il}}^2}\frac{\nu_p}{\sqrt{2\pi}C_{\mathrm{il}}}\exp\left(-\frac{v_p^2}{2C_{\mathrm{il}}^2}\right)-\frac{\frac{\omega_{\mathrm{eh}}^2}{k^2C_{\mathrm{e_1h}}^2}\left(1+\frac{\omega^2}{C_{\mathrm{eh}}^2}\right)}{1+\frac{\omega^2}{C_{\mathrm{eh}}^2}\left(1+\frac{\omega^2}{C_{\mathrm{e_1h}}^2}\right)}$$
$$-\frac{\frac{\omega_{\mathrm{el}}^2}{\omega^2}\left(1+\frac{\omega^2}{k^2C_{\mathrm{e_1l}}^2}\right)}{1+\frac{\omega_{\mathrm{ol}}^2}{k^2C_{\mathrm{e_1l}}^2}\left(1+\frac{\omega^2}{k^2C_{\mathrm{e_1l}}^2}\right)}. \tag{52}$$

Since $\omega_e^2/C_e^2=\omega_i^2/C_s^2$ where $C_s^2=K_\beta T_e/M$, where M is the nuclear mass, (52) becomes

$$1=\frac{\omega_{\mathrm{ih}}^2}{\omega^2}\left(1+\frac{3k^2C_{\mathrm{ih}}^2}{\omega^2}\right)+\left(\frac{\omega_{\mathrm{il}}^2}{\omega^2}\right)\left(1+\frac{3k^2C_{\mathrm{il}}^2}{\omega^2}\right)\pm\pi i\frac{\omega_{\mathrm{il}}^2}{k^2C_{\mathrm{il}}^2}$$
$$\times\frac{\nu_p}{\sqrt{2\pi}C_{\mathrm{il}}}\exp\left(-\frac{v_p^2}{2C_{\mathrm{il}}^2}\right)-\frac{\frac{\omega_{\mathrm{ih}}^2}{k^2C_{\mathrm{sh}}^2}\left(1+\frac{\omega^2}{k^2C_{\mathrm{e_1h}}^2}\right)}{1+\frac{\omega_{\mathrm{oh}}^2}{k^2C_{\mathrm{e_{sh}}}^2}\left(1+\frac{\omega^2}{k^2C_{\mathrm{e_{sh}}}^2}\right)}$$
$$-\frac{\frac{\omega_{\mathrm{il}}^2}{k^2C_{\mathrm{el}}^2}\left(1+\frac{\omega^2}{k^2C_{\mathrm{e_1l}}^2}\right)}{1+\frac{\omega_{\mathrm{ol}}^2}{k^2C_{\mathrm{el}}^2}\left(1+\frac{\omega^2}{k^2C_{\mathrm{e_1l}}^2}\right)}. \tag{53}$$

The resulting simpler equation,

$$1=\frac{\omega_{\mathrm{ih}}^2}{\omega^2}+\frac{\omega_{\mathrm{il}}^2}{\omega^2}\pm\pi i\,\frac{\omega_{\mathrm{il}}^2}{k^2C_{\mathrm{il}}^2}\,\frac{\nu_p}{\sqrt{2n}C_{\mathrm{il}}}\exp\left(-\frac{v_p^2}{2C_{\mathrm{il}}^2}\right)-\frac{(\omega_{\mathrm{ih}}^2/k^2C_{\mathrm{sh}}^2)}{1+(\omega_{\mathrm{oh}}^2/k^2C_{\mathrm{eh}}^2)}-\frac{(\omega_{\mathrm{il}}^2/k^2C_{\mathrm{sl}}^2)}{1+(\omega_{\mathrm{ol}}^2/k^2C_{\mathrm{el}}^2)}, \tag{54}$$

determines the dispersion law and the damping rate:

$$1=\frac{\omega_{\mathrm{ih}}^2}{\omega^2}\left(1-\frac{\omega^2/k^2C_{\mathrm{sh}}^2}{1+\omega_{\mathrm{oh}}^2/k^2C_{\mathrm{eh}}^2}\right)+\frac{\omega_{\mathrm{il}}^2}{\omega^2}\left(1-\frac{\omega^2/k^2C_{\mathrm{sl}}^2}{1+\omega_{\mathrm{ol}}^2/k^2C_{\mathrm{el}}^2}\right), \tag{55}$$

$$\frac{\gamma}{\omega}=\sqrt{\frac{\pi}{8}}\,\frac{\omega^2\omega_{\mathrm{il}}^2\nu_p}{(\omega_{\mathrm{ih}}^2+\omega_{\mathrm{il}}^2)k^2C_{\mathrm{il}}^3}\exp(-v_p^2/2C_{\mathrm{il}}^2). \tag{56}$$

Since $\omega<\omega_{\mathrm{il}}$, the dispersion relation (55) reduces to

$$\omega_{\mathrm{ih}}^2+\omega_{\mathrm{il}}^2=\frac{\omega_{\mathrm{eh}}^2}{\omega_{\mathrm{oh}}^2+k^2C_{\mathrm{eh}}^2}+\frac{\omega_{\mathrm{el}}^2}{\omega_{\mathrm{ol}}^2+k^2C_{\mathrm{el}}^2}. \tag{57}$$

We replace k in (53) by $K+(i/d)$ and, since $\omega_{\mathrm{il}}>\omega$, obtain for wave dispersion the relation

$$\frac{(\omega_{\mathrm{ih}}^2+\omega_{\mathrm{il}}^2)}{\omega^2}+\frac{3k^2(C_{\mathrm{ih}}^2\omega_{\mathrm{ih}}^2+C_{\mathrm{il}}^2\omega_{\mathrm{il}}^2)}{\omega^4}=\frac{\omega_{\mathrm{eh}}^2(k^2C_{\mathrm{e_1h}}^2+\omega^2)k^4C_{\mathrm{eh}}^2C_{\mathrm{e_1h}}^2}{k^4C_{\mathrm{eh}}^2Ce_{\mathrm{ih}}^2+\omega_{\mathrm{oh}}^2(k^2C_{\mathrm{e_1h}}^2+\omega^2)}+\frac{\omega_{\mathrm{el}}^2(k^2C_{\mathrm{e_1l}}^2+\omega^2)}{k^4C_{\mathrm{el}}^2C_{\mathrm{e_1l}}^2+\omega_{\mathrm{lh}}^2(k^2C_{\mathrm{e_1l}}^2+\omega^2)}. \tag{58}$$

For finding the Landau damping at distance d we prepare (53) like (44), and obtain

$$\frac{6k}{\omega^4d}(C_{\mathrm{ih}}^2\omega_{\mathrm{ih}}^2+C_{\mathrm{il}}^2\omega_{\mathrm{il}}^2)\pm\sqrt{\frac{\pi}{2}}\,\frac{\omega_{\mathrm{il}}^2}{k^2C_{\mathrm{il}}^2}\,\frac{v_p}{C_{\mathrm{il}}}\exp\left(-\frac{v_p^2}{2C_{\mathrm{il}}^2}\right)=\frac{2kC_{\mathrm{e_1h}}^2\omega_{\mathrm{ih}}^2C_{\mathrm{eh}}^2}{C_{\mathrm{sh}}^2d(k^4C_{\mathrm{eh}}^2C_{\mathrm{eh}}^2+k^2\omega_{\mathrm{oh}}^2C_{\mathrm{e_1h}}^2+\omega^2\omega_{\mathrm{oh}}^2)}+\frac{2kC_{\mathrm{e_1l}}^2\omega_{\mathrm{il}}^2C_{s_1}^2}{C_{\mathrm{sl}}^2d(k^4C_{\mathrm{el}}^2C_{\mathrm{e_1l}}^2+k^2\omega_{\mathrm{el}}^2C_{\mathrm{e_1l}}^2+\omega^2\omega_{\mathrm{ol}}^2)}-\frac{\omega_{\mathrm{ih}}^2C_{\mathrm{eh}}^2\omega^2(4k^3C_{\mathrm{e_1h}}^2+2k\omega_{\mathrm{oh}}^2)}{C_{\mathrm{sh}}^2(P^2+Q^2)d}-\frac{\omega_{\mathrm{il}}^2C_{\mathrm{el}}^2\omega^2(4k^3C_{\mathrm{sh}}^2+2k\omega_{\mathrm{oh}}^2)}{C_{\mathrm{sl}}^2(R^2+S^2)d}, \tag{59}$$

where

$$P=k^4C_{\mathrm{eh}}^2C_{\mathrm{e_1h}}^2+k^2\omega_{\mathrm{oh}}^2C_{\mathrm{e_1h}}^2+\omega^2\omega_{\mathrm{oh}}^2, \tag{60}$$

$$Q=(4k^3/d)C_{\mathrm{eh}}^2C_{\mathrm{e_1h}}^2+(2k/d)\omega_{\mathrm{oh}}^2C_{\mathrm{e_1h}}^2, \tag{61}$$

$$R=k^4C_{\mathrm{el}}^2C_{\mathrm{e_1l}}^2+k^2\omega^2C_{\mathrm{e_1l}}^2+\omega^2\omega_{\mathrm{ol}}^2, \tag{62}$$

$$S=(4k^3/d)C_{\mathrm{el}}^2C_{\mathrm{e_1l}}^2+(2k/d)\omega_{\mathrm{ol}}^2C_{\mathrm{e_1l}}^2. \tag{63}$$

VII. CONCLUDING REMARKS

A theory is developed for study of Landau damping of ion acoustic waves in a dope plasma of a light inert gas like helium in the presence of plasma of a heavy inert gas like argon. This theory also determines other familiar cases of Landau damping in Vlasov plasma and is intended to be in support of the experimental investigation of Landau damping free from the influence of other noncollisional, as well as collisional damping. This theory can be further generalized for the study of Landau damping of various types of longitudinal waves in multispecies Vlasov plasma.

[1] L. Tonks and I. Langmuir, Phys. Rev. **33**, 195 (1929).
[2] I. Alexeff, W. D. Jones, and D. Montogomery, Phys. Rev. Lett. **19**, 422 (1967).
[3] Y. Nakamura, M. Nakamura, and T. Itoh, Phys. Rev. Lett. **37**, 209 (1976).
[4] N. Sato, A. Sasaki, K. Aoki, and Y. Hatta, Phys. Rev. Lett. **19**, 1174 (1967).
[5] R. W. Gould, Phys. Rev. **136**, A991 (1964).
[6] M. Nakamura, M. Ito, Y. Nakamura, and T. Itoh, Phys. Fluids **18**, 651 (1975).
[7] A. Y. Wong, Phys. Rev. Lett. **14**, 252 (1965).
[8] D. L. Mathews, E. M. Campbell, N. M. Ceglio *et al.*, J. Appl. Phys. **54**, 4260 (1983).
[9] P. G. Burkhalter, D. J. Nagel, and R. R. Whitlick, Phys. Rev. A **9**, 2331 (1974).
[10] D. J. Nagel, P. G. Burkhalter, C. M. Dozier *et al.*, Phys. Rev. Lett. **33**, 743 (1974).
[11] K. Honda, K. Mima, and F. Koika, Phys. Rev. E **55**, 4594 (1997).
[12] Lord Rayleigh, *The Theory of Sound* (Dover, New York, 1945) (reprint).
[13] N. Bloembergen, Proc. IEEE **51**, 124 (1963).
[14] H. Lorentz, *Theory of Electrons* (Dover, New York, 1953) (reprint).
[15] M. Bonnedal and H. Wilhelmsson, J. Plasma Phys. **12**, 81 (1974).