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A Novel and Accurate Approximation for the Fundamental Mode in Single-mode Fibers having Arbitrary Index Profiles

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Summary

A novel and extremely accurate near Gaussian approximation for the fundamental mode in single-mode optical fibers having arbitrary index profile is proposed and verified.The spot sizes calculated from present function matches exactly with the analytical and exact numerical results for step and parabolic index fibers respectively over a practical range of values of normalised frequency V. For low V region, where Gaussian approximation is not faithful, it will be suitable to predict accurately the propagation characteristics of devices like directional coupler involving evanescent field coupling.

1 Introduction

One of the important parameters characterising single-mode fibers is it's modal spot size. Considerable number of propagation characteristics of these fibers, such as bending loss, modal dispersion, splice loss and microbending loss can be predicted, from their direct relations to an appropriately defined spot size and it's spectral variations [1–7]. One can calculate the spot size if one knows the modal field accarately [8, 9]. Except in case of step index fibers where modal fields are analytically known, one has to approximate it for arbitrary index profiles. Initially, the Gaussian approximation was proposed in this regard [1, 10]. It is seen that it underestimates the evanescent field in the cladding, mainly, in low V-region. In recent years, tremendous interests have been generated in order to obtain a realistic approximation of the fundamental modal fields [11, 12] for accurate description of the fiber characteristics in the linear as well as non-linear regime.

In this paper, we propose a novel two-parameter near Gaussian approximation for the fundamental mode in single-mode fibers having arbitrary index profile. As there has been considerable interest in the estimation of propagation characteristics directly from near field measurements [3–7], we obtain the two parameters from least square minimisation [6, 7] of the exact near field and the proposed field. Our function is shown to be accurate in comparison to the earlier results, mainly, the recently proposed modified Gaussian approximation [11, 12] in describing the modal fields. Also spot sizes calculated by using the present approximation match identically with the analytical [13] and exact numerical [14] values for step and parabolic fibers, respectively.

2 Theory

For arbitrary refractive index profile

$$n^2(R)\begin{cases} = n_1^2 - \left(n_1^2 - n_2^2\right)f(R) & R \le 1 \\ = n_2^2 & R > 1 \end{cases} \tag{1}$$

n_1 and n_2 are the axial and cladding refractive indices, respectively, and $f(R)$ is the profile function defining profile shape for power law profiles given by

$$f(R)\begin{cases} = R^q & R \le 1 \\ = 1 & R > 1 \end{cases} \tag{2}$$

where q is the power law exponent. It is ∞ for step-index fiber and 2 for parabolic index fiber, R (= r/a) is the normalised radius, 'a' being the core radius.

Under weakly guiding approximation [15], the modal field $\psi(R)$ can be obtained in the core from the numerical solution of scalar wave equation given as

$$\frac{d^2\psi}{dR^2} + \frac{1}{R}\frac{d\psi}{dR} + n \cdot V^2\left(\frac{U^2}{V^2} - f(R)\right)\psi = 0, \quad R \le 1 \tag{3}$$

and in the cladding as

$$\psi(R) \approx K_o(WR), \quad R > 1$$

with the boundary conditions

$$\psi(R = 0) = 0$$

and

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$$\left[\frac{\psi'(R)}{\psi(R)}\right]_{R=1} = \frac{-WK_1(W)}{K_o(W)} \tag{4}$$

U,W and V being the conventional normalised fiber parameters.

We consider the exact fields of a step index fiber as our standard, which are easily and directly available from the analytical solution of (3) and are given by

$$\psi(R)\begin{cases} = J_o(UR) & R \le 1 \\ = \dfrac{J_o(U)K_o(WR)}{K_o(W)} & R > 1 \end{cases} \tag{5}$$

in terms of Bessel and modified Bessel functions.

However, as stated above, one tries to seek a suitable function for approximating the fundamental mode. Keeping in mind that the field behaves more or less like a Gaussian in the core and exponential in the cladding and that such functions are relatively much simpler for calculating various propagation characteristics like splice loss etc., we propose a novel function as our trial field approximating the fundamental mode in arbitrary index fibers [16]

$$\psi(R)\begin{cases} = e^{-\frac{1}{2}(R/R_0)^{(2+p)}} & R \le R_o \\ = \left(\dfrac{R_o}{R}\right)^{1/2} e^{-\frac{1}{2}(R/R_0)^{(1+p)}} & R > R_o \end{cases} \tag{6}$$

Here R_o and p are the two optimization parameters to be obtained by a simple least square fitting procedure. R_o $(= r_o/a)$ and r_o behave like the normalised and physical spot sizes, respectively.

We call this a 'near' Gaussian approximation because in the near core region the field is Gaussian for $p = 0$. The substitution $p = 0$ will make the field modified Gaussian as that proposed in [12], being a Gaussian for $R < R_o$ and modified exponential for $R > R_o$. Thus the inclusion of the parameter p is shown to give an increased accuracy over the previous approximation.The continuity of the proposed near Gaussian field and its derivatives at $R = R_o$ is also checked.

With the field function in (6), we compute the Petermann I [14] spot size from the expression given as

$$W_o^2 = \frac{\int_0^\infty \psi^2(R)R^3 dR}{\int_0^\infty \psi^2(R)R dR} \tag{7}$$

and obtain

$$W_o^2 = R_o^2 \left[\frac{\frac{1}{2+p}\gamma\left(\frac{4}{2+p},1\right) + \frac{1}{1+p}\Gamma\left(\frac{3}{1+p},1\right)}{\frac{1}{2+p}\gamma\left(\frac{2}{2+p},1\right) + \frac{1}{1+p}\Gamma\left(\frac{1}{1+p},1\right)}\right] \tag{8}$$

in terms of incomplete gamma functions.

In case of step index fiber, the spot size is analytically obtained from (7) by using the analytical fields in (5) and is given as

$$W_o^2 = \frac{2}{3}\left[\frac{J_o(U)}{UJ_1(U)} + 0.5 + \frac{1}{W^2} - \frac{1}{U^2}\right] \tag{9}$$

Then we compare the spot size obtained from (8) for $q = \infty$ with (9). The test and validity of our approximation is described in the next section.

3 Results and discussions

In order to study the feasibility and accuracy of the proposed field function, we take initially the step index fiber as a test case, as stated earlier. Since the Gaussian and similar other approximations are highly accurate in graded index fibers, we expect that, if the novel function works well in the case of step index profile, its applicability in other cases are obvious. Now for calculations, we simulate the values of the near field for the step index fiber from analytical expression of (5). To this data we apply the above fit procedure to obtain the parameters R_o and p and hence W_o. Figure 1 shows the simulated data (as crosses) and ψ (R) as obtained from near Gaussian fit (solid curve) along with a modified Gaussian fit [12] (broken lines) and purely Gaussian fit (dotted). The last two fits are obtained by applying the same fit procedure to the same simulated data. We present the curve for $V = 1.4$, a low V value where the field spread in the cladding behaves more like a non-Gaussian, as can be seen from the figure. The near Gaussian approximation gives the modal field identically coinciding with the analytical curve over the entire region and is superior to the other two approximations. The same

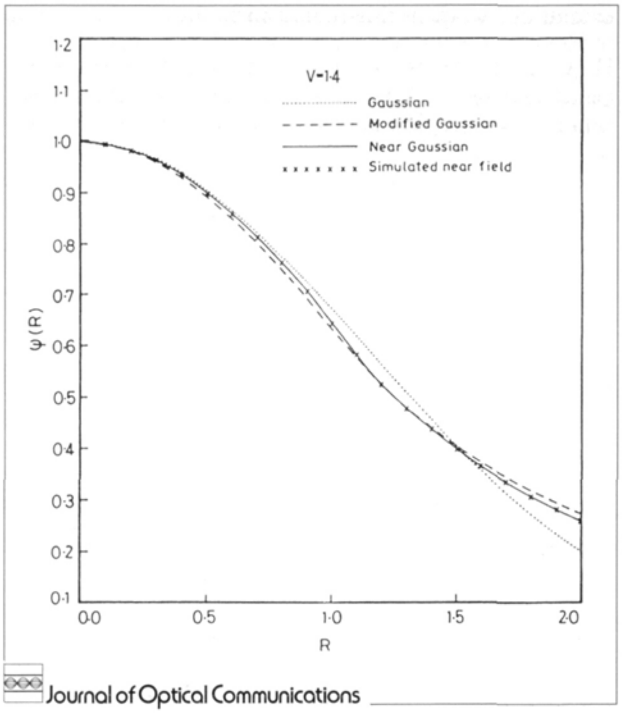

Figure 1: The modal field ψ as a function of normalised radius R for a step index fiber with $V = 1.4$

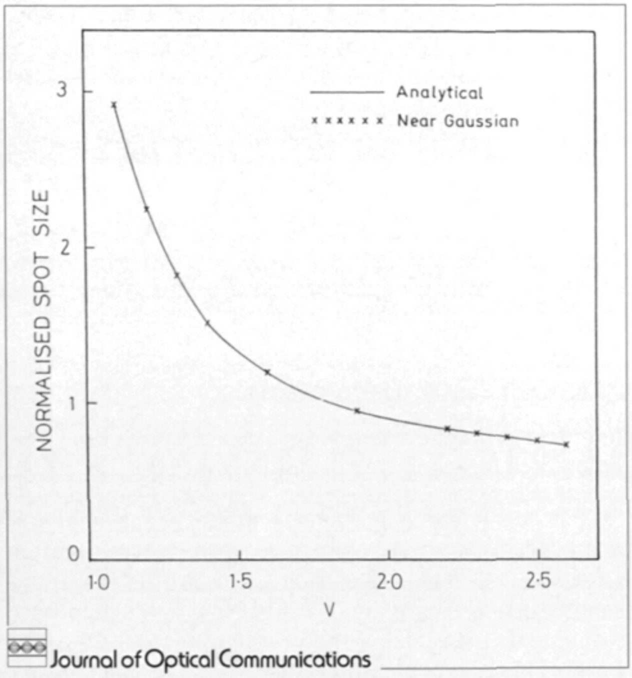

Figure 2: Normalised spot size W_o against the normalised frequency for step index fiber

agreement is found for the range of V lying within 2.5 and is also found to be similarly accurate in case of parabolic index fibers. In Figs. 2 and 3 the variation of W_o with V as obtained by the near Gaussian fit are shown (as crosses) and compared with the results obtained from analytical and exact numerical calculations (solid lines) [13, 14] for step and parabolic index fibers, respectively. For all other definitions of spot sizes [17] involving fields and it's derivative, the present function is expected to be as excellent as in describing Petermann I spot size since it describes the modal field accurately. We like to extend our work in this regard in future.

However as Gaussian approximation is sufficiently accurate in higher V region, the applicability of the function in low V region appears to make it more powerful in studying devices like directional couplers involving evanescent wave coupling. The proposed function is also simple in the sense that once R_o and p are obtained by applying least square fitting procedure to the near field points, one can use a pocket calculator to find the actual field at various values of the normalised radius using this simple function. We have also a plan in future to use the approximation in predicting different fiber characteristics in the linear and non-linear regime.

4 Conclusion

We propose a novel and accurate approximation for the fundamental mode in single-mode fibers having arbitrary index profile. The proposed function accurately describes the modal field in the core and a considerable region of the cladding. It is also shown to be superior to the other two approximations. The spot sizes calculated by this method are as accurate as the analytical and exact numerical spot sizes for step and parabolic index fibers, over a long and practical range of V values. Being highly accurate in the low V region, the approximation is supposed to faithfully describe the propagation characteristics of fiber based devices involving evanscent wave coupling.

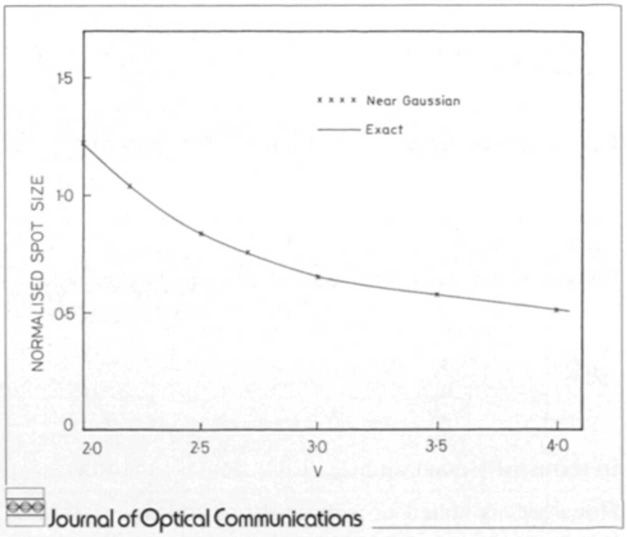

Figure 3: Normalised spot size W_o against the normalised frequency for a parabolic core fiber

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