A note on dimension-5 operators in GUTs and their impact

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ABSTRACT

Quantum gravity or string compactification can lead to effective dimension-5 operators in Grand Unified Theories which modify the gauge kinetic terms. We exhaustively discuss the group-theoretic nature of such operators for the popular SU(5), SO(10), and E(6) models. In particular, for SU(5) only a Higgs in the 200 representation can help bring the couplings to unification below the Planck scale and in consistency with proton decay limits while for a supersymmetric version 24, 75, or 200 representations are all acceptable. The results also have a direct application in non-universality of gaugino masses in a class of supersymmetric models where identical group-theoretic features obtain.

1. Introduction

The remarkable success of electroweak unification has been a motivation to seek a Grand Unified Theory (GUT) linking together the strong and electroweak interactions in a framework with quark–lepton unification [1]. The merits of this programme need no underscoring and rightfully it has been attracting continual attention over several decades. It has all along been also realised that this is but the penultimate step, unification of all interactions with gravity being the final objective.

The Standard Model (SM) is based on the gauge group $\mathcal{G}_{SM} \equiv SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ which has three independent couplings g_3, g_2, and g_1. The minimal scheme of grand unification envisions the placement of quarks and leptons in a common multiplet of the GUT group and the unification of the three SM couplings into one unified coupling g_{GUT} at high energies. The couplings evolve logarithmically with energy and so the unification, if achieved, is at a high scale of $\mathcal{O}(10^{15})$ GeV or more. The current low energy measured values of the couplings, in fact, are not consistent with unification in the minimal model. TeV-scale Supersymmetry (SUSY) is one much-discussed remedy for this. Another consequence of grand unification is the instability of the proton. Proton decay has so far not been experimentally observed, implying a scale of grand unification at least an order of magnitude higher than 10^{15} GeV.

A full quantum-theoretic treatment of gravity is not available currently. Nonetheless, it has been found useful to attempt to mimic some of its implications on grand unification through higher dimension effective contributions, suppressed by powers of the Planck mass, M_{Pl}. In a string theory setting, similar effective operators may also originate from string compactification, M_{Pl} being then replaced by the compactification scale M_c.

In this work we focus on the corrections to the gauge kinetic term:

$$\mathcal{L}_{kin} = -\frac{1}{4c} \text{Tr}(F_{\mu\nu} F^{\mu\nu}). \tag{1}$$

where $F^{\mu\nu} = \Sigma_i \lambda_i . F_i^{\mu\nu}$ is the gauge field strength tensor with λ_i being the matrix representations of the generators normalised to $\text{Tr}(\lambda_i \lambda_j) = c\delta_{ij}$. Conventionally, for SU(n) groups the λ_i are chosen in the fundamental representation with $c = 1/2$. In the following, we will often find it convenient to utilise other representations.

The lowest order contribution from quantum gravitational (or string compactification) effects, which is what we wish to consider here, is of dimension five and has the form:

$$\mathcal{L}_{dim-5} = -\frac{\eta}{M_{Pl}} \left[\frac{1}{4c} \text{Tr}(F_{\mu\nu} \Phi_D F^{\mu\nu}) \right] \tag{2}$$

where Φ_D denotes the D-component Higgs multiplet which breaks the GUT symmetry and η parametrises the strength of this interaction. In order for it to be possible to form a gauge invariant of the form in Eq. (2), Φ_D can be in any representation included in the symmetric product of two adjoint representations of the group. For example, in SU(5), $(24 \otimes 24)_{sym} = 1 \oplus 24 \oplus 75 \oplus 200$ and Φ_D may be in these representations only.

When Φ_D develops a vacuum expectation value (*vev*) v_D, which sets the scale of grand unification M_X, an effective gauge kinetic term is generated from Eq. (2). Depending on the structure of the *vev*, this additional contribution usually will not be the same for the different subgroups in $\mathcal{G}_{SM}$, leading, after an ap-

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propriate scaling of the gauge fields, to an alteration of the gauge coupling unification condition to:

$$g_1^2(M_X)(1 + \epsilon\delta_1) = g_2^2(M_X)(1 + \epsilon\delta_2) = g_3^2(M_X)(1 + \epsilon\delta_3), \qquad (3)$$

wherein the δ_i, $i = 1, 2, 3$, and $\epsilon = \eta v_D/2M_{Pl} \sim \mathcal{O}(M_X/M_{Pl})$ arise from Eq. (2). Thus, the presence of the dimension-5 terms in the Lagrangian modifies the usual boundary conditions on gauge couplings where they are expected to unify at M_X. It is indeed possible that this change will be just enough to entail the unification programme to succeed with the current low energy values of the coupling constants as a boundary condition. Here, we show that this is the case for SU(5) GUT.

In this work we work out the consequences of such dimension-5 operators for the unified theories based on SU(5), SO(10), and E(6). For ordinary SU(5) as well as its SUSY variant, using one- and two-loop renormalisation group evolution of the gauge couplings we examine the consistency of the low energy measurements with grand unification while remaining within the proton decay restrictions. We also make a brief remark about the applicability of these results to non-universality of gaugino masses in a class of SUSY models.

2. A relook at SU(5) GUTs

The simplest illustrative example is that of SU(5) with a Φ_{24} scalar. Such a scalar multiplet is customarily introduced in the theory to spontaneously break SU(5) $\rightarrow \mathcal{G}_{SM}$. If there is also a dimension-5 term as in Eq. (2) involving Φ_{24} then gauge couplings at the high scale get affected [2]. The *vev* of this field can be represented as a traceless 5×5 diagonal matrix ($c = 1/2$):

$$\langle\Phi_{24}\rangle = \frac{v_{24}}{\sqrt{15}} \text{diag}(1, 1, 1, -3/2, -3/2). \qquad (4)$$

The contributions to the δ_i in Eq. (3) can be simply read off from this expression and one finds: $\delta_1 = \delta_2/3 = -\delta_3/2 = 1/\sqrt{15}$.

For conveniently writing the *vev* for the 75-dimensional representation one uses the SU(5) relation: $10 \otimes \overline{10} = 1 \oplus 24 \oplus 75$. The *vev* $\langle\Phi_{75}\rangle$ must be so chosen that $\mathcal{G}_{SM}$ remains unbroken. Further, it must be orthogonal to $\langle\Phi_{24}\rangle$, which too can be expressed as a $10 \otimes 10$ diagonal matrix. Under $\mathcal{G}_{SM}$ the SU(5) $10 = (\bar{3}, 1)_{-\frac{4}{3}} + (3, 2)_{\frac{1}{3}} + (1, 1)_2$. This allows the identification of the generators of SU(5) in the 10-dimensional representation and, in particular, the U(1)$_Y$ generator corresponds to $\sqrt{\frac{1}{60}}$ diag($-4, -4, -4, 1, 1, 1, 1, 1, 1, 6$). Taking the above into consideration, $\langle\Phi_{75}\rangle$ can be expressed as the traceless 10×10 matrix ($c = 3/2$):

$$\langle\Phi_{75}\rangle = \frac{v_{75}}{\sqrt{12}} \text{diag}(1, 1, 1, -1, -1, -1, -1, -1, -1, 3). \qquad (5)$$

This results in $\delta_1 = -5\delta_2/3 = -5\delta_3 = 4/\sqrt{3}$.

Similarly, the relation $15 \otimes \overline{15} = 1 \oplus 24 \oplus 200$ permits the *vev* for Φ_{200} to be written as a (15×15) traceless diagonal matrix ($c = 7/2$). Noting that under $\mathcal{G}_{SM}$ the 15 of SU(5) is $(6, 1)_{-\frac{4}{3}} + (3, 2)_{\frac{1}{3}} + (1, 3)_2$ one has:

$$\langle\Phi_{200}\rangle = \frac{v_{200}}{\sqrt{12}} \text{diag}(1, 1, 1, 1, 1, 1, -2, -2, -2, -2, -2, -2, 2, 2, 2), \qquad (6)$$

which yields $\delta_1 = 5\delta_2 = 10\delta_3 = 1/\sqrt{21}$. These results for SU(5) are collected together in Table 1.

3. $\sin^2\theta_W$, gauge couplings

The shift in the gauge couplings as dictated by Eq. (3) leaves its mark at low energies through the Renormalization Group (RG)

Table 1
Effective contributions to gauge kinetic terms from different Higgs representations in Eq. (2) for SU(5). (See Eq. (3).)

SU(5) Representations	δ_1	δ_2	δ_3
24	$1/\sqrt{15}$	$3/\sqrt{15}$	$-2/\sqrt{15}$
75	$4/\sqrt{3}$	$-12/5\sqrt{3}$	$-4/5\sqrt{3}$
200	$1/\sqrt{21}$	$1/5\sqrt{21}$	$1/10\sqrt{21}$

equations. Moreover, besides the low energy value of the weak mixing angle $\sin^2\theta_W$, even its GUT-level prediction is affected.

The weak mixing angle $\sin^2\theta_W = g'^2/(g^2 + g'^2)$ is expressed in terms of the SM gauge couplings g' and $g \equiv g_2$. Of these, the U(1)$_Y$ coupling g' is related to the coupling g_1 arising in a unified theory through $g_1^2 = c^2 g'^2$ where $c^2 = \frac{5}{3}$. In the limit of unification of all couplings at a GUT-scale, M_X, this leads to the prediction $\sin^2\theta_W(M_X) = 3/8$. Now, due to the modified GUT relationship of Eq. (3) one has for the weak mixing angle $\hat{\theta}_W$:

$$\sin^2\hat{\theta}_W(M_X) = \frac{3}{8} + \frac{15}{64}\epsilon(\delta_2 - \delta_1). \qquad (7)$$

The experimentally determined value of $\sin^2\theta_W$ at low energies receives further RG-dependent corrections to which we now turn.

The RG equations governing gauge coupling evolution are:

$$\mu\frac{dg_i}{d\mu} = \beta_i(g_i, g_j), \qquad (8)$$

where at two-loop order [3]

$$\beta_i(g_i, g_j) = (16\pi^2)^{-1}b_ig_i^3 + (16\pi^2)^{-2}\sum_{j=1}^{3}b_{ij}g_j^2g_i^3. \qquad (9)$$

$i, j = 1, 2, 3$ for U(1)$_Y$, SU(2)$_L$, and SU(3)$_c$, respectively. The coefficients b_i and b_{ij} are:

$$b_1 = \frac{1}{10}n_H + \frac{4}{3}n_G, \qquad b_2 = \frac{1}{6}n_H + \frac{4}{3}n_G - 22/3,$$

$$b_3 = \frac{4}{3}n_G - 11, \qquad (10)$$

and

$$b_{ij} = n_H\begin{pmatrix} 9/50 & 9/10 & 0 \\ 3/10 & 13/6 & 0 \\ 0 & 0 & 0 \end{pmatrix} + n_G\begin{pmatrix} 19/15 & 3/5 & 44/15 \\ 1/5 & 49/3 & 4 \\ 11/30 & 3/2 & 76/3 \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & 0 & 0 \\ 0 & -136/3 & 0 \\ 0 & 0 & -102 \end{pmatrix} \qquad (11)$$

and n_H and n_G are respectively the number of Higgs doublets and the number of fermion generations in the theory. The RG equations must satisfy the boundary conditions set by Eq. (3) on the $g_i^2(M_X)$.

In our numerical analyses below we show the full two-loop RG equation results. For ease of discussion if only the lowest order contributions are retained, then in the absence of dimension-5 operators ($\alpha_i = g_i^2/4\pi$):

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M_X)} + \frac{2b_i}{2\pi}\ln\left[\frac{M_X}{\mu}\right] \quad (i = 1, 2, 3). \qquad (12)$$

$\alpha_i = g_i^2/4\pi$. These equations can be combined to yield:

$$\frac{\alpha}{2\pi}\ln\frac{M_X}{M_Z} = \left[\frac{3}{5b_1 + 3b_2 - 8b_3}\right]\left\{1 - \frac{8}{3}\frac{\alpha}{\alpha_3}\right\}, \qquad (13)$$

and therefrom

$$\sin^2\theta_W(M_Z) = \frac{3}{8} - \frac{15}{8}\left[\frac{b_1 - b_2}{5b_1 + 3b_2 - 8b_3}\right]\left\{1 - \frac{8}{3}\frac{\alpha}{\alpha_3}\right\}, \qquad (14)$$

Table 2
SU(5) dimension-5 interaction strength ϵ and the gauge unification scale, M_X, for different Φ representations.

SU(5) Representations	ϵ (from Eq. (17))	ϵ (using Eq. (8))	M_X (GeV)
24	0.087	0.088	5.01×10^{13}
75	−0.048	−0.045	4.79×10^{15}
200	−1.92	−1.40	1.05×10^{18}

Table 3
Effective contributions to gauge kinetic terms from different Higgs representations in Eq. (2) for SO(10). (see Eq. (3).)

SO(10) Representations	δ_{4c}	δ_{2L}	δ_{2R}
54	$-1/\sqrt{15}$	$3/2\sqrt{15}$	$3/2\sqrt{15}$
210	0	$1/\sqrt{2}$	$-1/\sqrt{2}$
770	$2/3\sqrt{5}$	$5/3\sqrt{5}$	$5/3\sqrt{5}$

where α − the fine structure constant − and α_3 are the couplings at the scale M_Z.

Inclusion of the boundary condition, Eq. (3), dictated by the dimension-5 interactions alters Eqs. (13) and (14) to:

$$\frac{\alpha}{2\pi} \ln \frac{\hat{M}_X}{M_Z} = \left[\frac{3}{5b_1 + 3b_2 - 8b_3} \right] \left\{ 1 - \frac{8}{3} \frac{\alpha}{\alpha_3} \right\} + \left(\frac{\epsilon}{5b_1 + 3b_2 - 8b_3} \right)$$
$$\times \left[\frac{-3(8\delta_3 - 3\delta_2 - 5\delta_1)b_3}{5b_1 + 3b_2 - 8b_3} - (5\delta_1 + 3\delta_2)\frac{\alpha}{\alpha_3} \right]$$
$$+ \mathcal{O}(\epsilon^2), \tag{15}$$

and

$$\sin^2 \hat{\theta}_W(M_Z)$$
$$= \frac{3(1 + \epsilon\delta_2)}{8 + \epsilon(3\delta_2 + 5\delta_1)} - \left(\frac{5(1 + \epsilon\delta_1)(1 + \epsilon\delta_2)}{8 + \epsilon(3\delta_2 + 5\delta_1)} \right)$$
$$\times \left[\frac{b_1}{1 + \epsilon\delta_1} - \frac{b_2}{1 + \epsilon\delta_2} \right]$$
$$\times \frac{3(1 + \epsilon\delta_3) - [8 + \epsilon(3\delta_2 + 5\delta_1)][\alpha/\alpha_3]}{(5b_1 + 3b_2)(1 + \epsilon\delta_3) - [8 + \epsilon(3\delta_2 + 5\delta_1)]b_3}, \tag{16}$$

which reduces to Eq. (14) in the appropriate limit. In fact,

$$\sin^2 \hat{\theta}_W(M_Z)$$
$$= \sin^2 \theta_W(M_Z) - \epsilon \left[\frac{5[\delta_1(b_3 - b_2) + \delta_2(b_1 - b_3) + \delta_3(b_2 - b_1)]}{(5b_1 + 3b_2 - 8b_3)^2} \right.$$
$$\left. \times \left\{ 3b_3 - (5b_1 + 3b_2)\frac{\alpha}{\alpha_3} \right\} \right] + \mathcal{O}(\epsilon^2). \tag{17}$$

The first term on the r.h.s. of Eq. (17) is fixed from Eq. (14). From the measured value of $\sin^2 \theta_W$ [4] one can extract the value of ϵ. These are presented for the different Φ representations in Table 2.

These $\mathcal{O}(\epsilon)$ one-loop analytic results can be cross-checked using the full RG equations in Eq. (8) with $n_G = 3$ and $n_H = 1$. Using the low energy ($\sim M_Z$) measured values [4], $\sin^2 \theta_W = 0.23119(14)$ and $\alpha_3 = 0.1176(20)$, the RG equations can be numerically integrated. The scale M_X is fixed through the requirement that the modified unification condition, Eq. (3), is satisfied there. From this analysis one can determine ϵ and M_X. The conclusions from one-loop RG running are shown in Table 2 and Fig. 1.

The two-loop results, shown as insets in Fig. 1, incorporate the proper matching conditions [5] as well as Eq. (3) at M_X, namely,

$$\frac{1}{\alpha_i(M_X)(1 + \epsilon\delta_i)} - \frac{C_i}{12\pi} = \text{constant, independent of } i$$
$$\text{for } i = 1, 2, 3, \tag{18}$$

where C_i is the quadratic Casimir for the ith subgroup. It is noteworthy that the results are not significantly affected and the coupling constants still unify. The unification scales, M_X, are found to be 5.01 $\times 10^{13}$, 2.09 $\times 10^{15}$, and 3.02 $\times 10^{17}$ GeV respectively for Φ_{24}, Φ_{75}, and Φ_{200}. Though unification is achieved within the Planck scale for all three choices, for Φ_{24} and Φ_{75} the results are not consistent with the existing limits from proton decay. Thus only a 5-dimensional operator with Φ_{200} yields a viable solution.

In [6], noting that the dimension-5 operator in Eq. (2) with Φ_{24} cannot by itself provide satisfactory gauge unification, it has been proposed that including gravitational contributions in the beta functions can help ameliorate this problem. Alternatively, within SUSY SU(5) it has been argued in [7] that one-loop (as well as two-loop) RG evolution with Φ_{24}-driven boundary conditions in Eq. (3) can yield satisfactory unification solutions provided the possible modification of the Planck scale itself due to the large number of GUT fields is given consideration.

4. SO(10) and E(6)

SU(5) is admittedly the GUT model which has been examined most in the literature by far. Nonetheless, there are also strong motivations for going to larger GUT groups like SO(10) and E(6). The minimal SU(5) model, as noted earlier, is in conflict with proton decay measurements. Further, the larger gauge groups offer the possibility of left–right symmetry. Here, we briefly discuss the effect of dimension-5 interactions, as in Eq. (2), on SO(10) and E(6) models. The detailed analysis of RG evolution of the gauge couplings for these cases is not presented in this Letter [8].

4.1. SO(10) GUT

SO(10) [9] is now the widely preferred model for grand unification, offering the option of descending to $\mathcal{G}_{SM}$ through a left–right symmetric route [10] − the intermediate Pati–Salam $\mathcal{G}_{PS} \equiv$ SU(4)$_c \otimes$ SU(2)$_L \otimes$ SU(2)$_R$ group − or via an SU(5) $\otimes$ U(1) theory. The latter option will not give any novel feature beyond what has already been discussed in the context of SU(5) GUTs. So, here we consider the breaking chain SO(10) $\rightarrow \mathcal{G}_{PS} \rightarrow \mathcal{G}_{SM}$.

In SO(10), one usually requires $g_{4c} = g_{2L} = g_{2R}$ at the unification scale. The presence of any dimension-5 effective interactions, of the form of Eq. (2), will affect this relation introducing corrections such that

$$g_{4c}^2(M_X)(1 + \epsilon\delta_{4c}) = g_{2L}^2(M_X)(1 + \epsilon\delta_{2L})$$
$$= g_{2R}^2(M_X)(1 + \epsilon\delta_{2R}). \tag{19}$$

The adjoint representation 45 of SO(10) satisfies $(45 \otimes 45)_{\text{sym}} = 1 \oplus 54 \oplus 210 \oplus 770$ implying that Φ_D can be chosen only in the 54, 210, and 770-dimensional representations. The vev $\langle\Phi_D\rangle$ must ensure that $\mathcal{G}_{PS}$ is unbroken.

Using the SO(10) relation $(10 \otimes 10) = 1 \oplus 45 \oplus 54$ one can see that $\langle\Phi_{54}\rangle$ can be expressed as a 10×10 diagonal traceless matrix. Under SU(4)$_c \otimes$ SU(2)$_L \otimes$ SU(2)$_R$, $10 \equiv (1, 2, 2) + (6, 1, 1)$. Thus ($c = 1$)

$$\langle\Phi_{54}\rangle = \frac{v_{54}}{2\sqrt{15}} \text{diag}(3, 3, 3, 3, -2, -2, -2, -2, -2, -2). \tag{20}$$

This leads to $\delta_{4c} = -\frac{1}{\sqrt{15}}$ and $\delta_{2L} = \delta_{2R} = \frac{3}{2\sqrt{15}}$. Notice that this correction to unification, through Eq. (19), ensures that $g_{2L}(M_X) = g_{2R}(M_X)$, i.e., D-parity [11] is preserved.

In SO(10) $(\overline{16} \otimes 16) = 1 \oplus 45 \oplus 210$; so one can represent $\langle\Phi_{210}\rangle$ as a 16-dimensional traceless diagonal matrix. Since $16 \equiv (4, 2, 1) + (\overline{4}, 1, 2)$ one can readily identify ($c = 2$)

$$\langle\Phi_{210}\rangle = \frac{v_{210}}{2\sqrt{2}} \text{diag}(1, 1, 1, 1, 1, 1, 1, 1,$$
$$-1, -1, -1, -1, -1, -1, -1, -1), \tag{21}$$

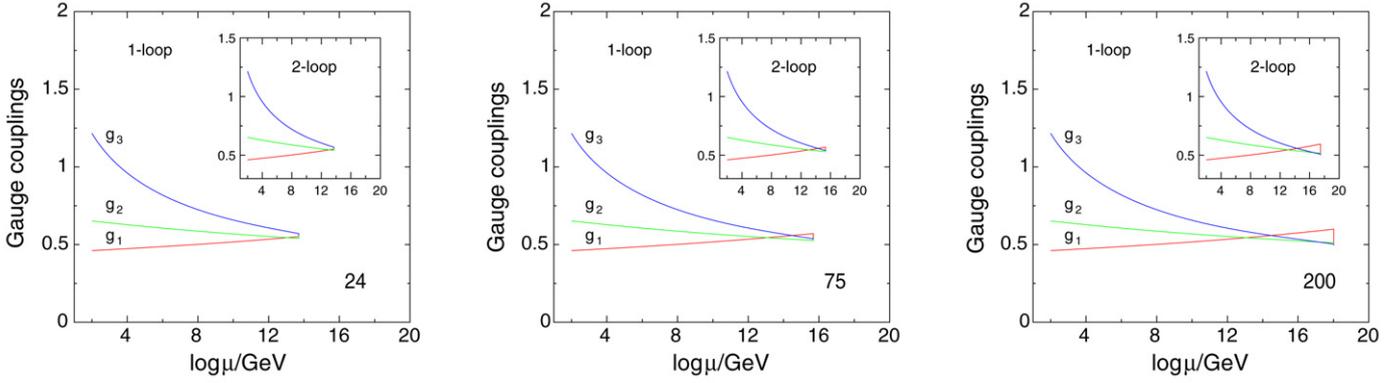**Fig. 1.** The evolution of gauge coupling constants for different choices of Φ for SU(5) GUTs: Φ_{24} (left), Φ_{75} (centre), Φ_{200} (right). In the inset the results for two-loop evolution are shown.

yielding $\delta_{4c} = 0$ and $\delta_{2L} = -\delta_{2R} = \frac{1}{\sqrt{2}}$. It is noteworthy that D-parity is broken through $\langle\Phi_{210}\rangle$ and $g_{2L}(M_X) \neq g_{2R}(M_X)$ though $SU(2)_L \otimes SU(2)_R$ remains unbroken at M_X. The effect of dimension-5 interactions arising from Φ_{54} and Φ_{210} on gauge coupling unification has been examined in the literature [12].

Finally, we turn to the last possibility for SO(10), namely Φ_{770}. Since $(45 \otimes 45)_{\text{sym}} = 1 \oplus 54 \oplus 210 \oplus 770$ with $45 \equiv (15, 1, 1) + (1, 3, 1) + (1, 1, 3) + (6, 2, 2)$ one can write the vev in terms of a 45×45 diagonal traceless matrix. The $\langle\Phi_{54}\rangle$ and $\langle\Phi_{210}\rangle$ can also be written in a similar form and care must be taken to ensure that $\langle\Phi_{770}\rangle$ is orthogonal to them. In this manner one arrives at ($c = 8$):

$$\langle\Phi_{770}\rangle = \frac{v_{770}}{\sqrt{180}} \text{diag}(\underbrace{-4, \ldots, -4}_{15 \text{ entries}}, \underbrace{-10, \ldots, -10}_{3+3 \text{ entries}}, \underbrace{5, \ldots, 5}_{24 \text{ entries}}). \quad (22)$$

From this one finds $\delta_{4c} = \frac{2}{3\sqrt{5}}$ and $\delta_{2L} = \delta_{2R} = \frac{5}{3\sqrt{5}}$.

All the results for SO(10) are collected together in Table 3.

4.2. E(6) GUT

The exceptional group E(6) has been proposed as a viable GUT alternative [13]. It offers $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$ as a subgroup besides $SO(10) \otimes U(1)$.

For E(6) the adjoint representation is 78-dimensional. Noting that $(78 \otimes 78)_{\text{sym}} = 1 \oplus 650 \oplus 2430$ it is clear that Φ_D can be only in the 650 and 2430-dimensional representations in this case.

In E(6), $(\overline{27} \otimes 27) = 1 \oplus 78 \oplus 650$ and under $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$ $27 = (1, \bar{3}, 3) + (3, 1, \bar{3}) + (\bar{3}, 3, 1)$. Therefore one can write $\langle\Phi_{650}\rangle$ as a 27×27 diagonal traceless matrix ($c = 3$). Further, the 650 representation has two fields which are singlet under $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$. Needless to say, any one of these fields or linear combinations thereof may be chosen to break the symmetry. In particular, two linear combinations may be identified which respect $\delta_{3L} = \pm\delta_{3R}$. These are:

$$\langle\Phi_{650}\rangle = \frac{v_{650}}{\sqrt{18}} \text{diag}(\underbrace{-2, \ldots, -2}_{9 \text{ entries}}, \underbrace{1, \ldots, 1}_{9 \text{ entries}}, \underbrace{1, \ldots, 1}_{9 \text{ entries}}). \quad (23)$$

This results in $\delta_{3c} = -\frac{1}{\sqrt{2}}$ and $\delta_{3L} = \delta_{3R} = \frac{1}{2\sqrt{2}}$, and

$$\langle\Phi'_{650}\rangle = \frac{v'_{650}}{\sqrt{6}} \text{diag}(\underbrace{0, \ldots, 0}_{9 \text{ entries}}, \underbrace{1, \ldots, 1}_{9 \text{ entries}}, \underbrace{-1, \ldots, -1}_{9 \text{ entries}}). \quad (24)$$

From this $\delta_{3c} = 0$ and $\delta_{3L} = -\delta_{3R} = \frac{3}{2\sqrt{6}}$.

Finally, using $78 = (8, 1, 1) + (1, 8, 1) + (1, 1, 8) + (3, 3, \bar{3}) + (\bar{3}, \bar{3}, 3)$ and maintaining consistency with $\langle\Phi_{650}\rangle$ and $\langle\Phi'_{650}\rangle$ one can write ($c = 12$)

Table 4
Effective contributions to gauge kinetic terms from different Higgs representations in Eq. (2) for E(6). (see Eq. (3).) Note that there are two $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$ singlet directions in 650.

E(6) Representations	δ_{3c}	δ_{3L}	δ_{3R}
650	$-1/\sqrt{2}$	$1/2\sqrt{2}$	$1/2\sqrt{2}$
650′	0	$3/2\sqrt{6}$	$-3/2\sqrt{6}$
2430	$-\frac{3}{\sqrt{26}}$	$-\frac{3}{\sqrt{26}}$	$-\frac{3}{\sqrt{26}}$

$$\langle\Phi_{2430}\rangle = \frac{v_{2430}}{3\sqrt{26}} \text{diag}(\underbrace{9, \ldots, 9}_{8 \text{ entries}}, \underbrace{9, \ldots, 9}_{8 \text{ entries}}, \underbrace{9, \ldots, 9}_{8 \text{ entries}},$$
$$\underbrace{-4, \ldots, -4}_{27 \text{ entries}}, \underbrace{-4, \ldots, -4}_{27 \text{ entries}}). \quad (25)$$

One can readily read off $\delta_{3c} = \delta_{3L} = \delta_{3R} = -\frac{3}{\sqrt{26}}$.

In Table 4 we collect the findings for the different representations of E(6).

5. Supersymmetric GUTs

5.1. Unification, neutrino mass

In a supersymmetric theory, the fermions, Higgs scalars, and gauge bosons of the GUT model are endowed with superpartners constituting chiral and vector supermultiplets. It is well known that gauge coupling unification is possible if SUSY is manifested at the TeV scale [14]. If dimension-5 interactions are also present then that will further affect this unification. In fact, it was shown within SUSY SU(5) that if the δ_i ($i = 1, 2, 3$) in Eq. (3) are fixed as determined (see Table 1) by the 24-dimensional representation [15] or permitted to vary arbitrarily [16] then unification, at the one-loop level, is always possible.

Here we perform a one-loop as well as a two-loop analysis. Above the SUSY scale (chosen as 1 TeV) this entails the replacement of Eqs. (10) and (11) by (for $n_G = 3$, $n_H = 2$)

$$b_1 = \frac{33}{5}, \qquad b_2 = 1, \qquad b_3 = -3, \quad (26)$$

and

$$b_{ij} = \begin{pmatrix} 199/25 & 27/5 & 88/5 \\ 9/5 & 25 & 24 \\ 11/5 & 9 & 14 \end{pmatrix}. \quad (27)$$

We find that unification is allowed for all three choices of Φ — namely, 24, 75, and 200 — when the δ_i ($i = 1, 2, 3$) are appropriately identified. The results are presented in Table 5. Unlike the non-SUSY alternative in Table 2, now for every case one gets $M_X \sim 10^{16}$ GeV. In line with expectation, the size of ϵ is reduced

Table 5
SU(5) dimension-5 interaction strength ϵ and the gauge unification scale, M_X, for different Φ representations in a supersymmetric theory.

SU(5) Representations	1 loop		2 loop	
	ϵ	M_X (GeV)	ϵ	M_X (GeV)
24	0.017	1.10×10^{16}	−0.003	1.38×10^{16}
75	−0.007	1.92×10^{16}	0.001	1.24×10^{16}
200	−0.204	3.16×10^{16}	0.071	1.10×10^{16}

in this SUSY case as the couplings tend to unify even without these interactions. The trend of agreement between the one-loop and two-loop results is gratifying.

The degenerate fermionic SUSY partner of Φ might play a role in the generation of realistic neutrino masses through the see-saw mechanism. Continuing with the SU(5) model for the purpose of illustration, if Φ is in the 24 representation then its fermion partner multiplet contains fields transforming as $(1, 1, 0)$ and $(1, 3, 0)$ under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. Through a Yukawa coupling of the form $5_F 24_F \bar{5}_H$ they can serve as the heavy exchanged fermionic mode of type I [17] and type III [18] see-saw models, respectively.[1] These fermions reside at the M_X scale and will lead to neutrino masses typically at the 10^{-2}–10^{-3} eV level. In fact, the above Yukawa coupling will result in simultaneous type I and type III see-saw contributions which together will lead to an enhancement. A similar situation obtains if Φ is chosen in the 200 representation. In the 75 of SU(5) the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ $(1, 3, 0)$ field is not present. So only type I see-saw is feasible for this alternative. For SO(10) and E(6) SUSY GUTs, over and above these options, there is even room for the type II see-saw [19] through heavy scalars transforming as $(1, 3, 0)$.

5.2. Non-universality of gaugino masses

The results presented in the earlier sections have a direct application — specifically in the generation of non-universal gaugino masses — in SUSY models which emerge from a GUT where supersymmetry breaking and GUT symmetry breaking are tied together. These arise from the gauge kinetic term which can be schematically written as

$$\mathcal{L} = \int d^2\theta \, f_{\alpha\beta}(\Phi) W^\alpha W^\beta + \text{h.c.}, \qquad (28)$$

where $f_{\alpha\beta}$ is a function of the chiral superfield Φ whose scalar component is responsible for the GUT symmetry breaking. $f_{\alpha\beta}$ is symmetric in the GUT gauge indices α, β and W^α are the GUT gauge superfields. When the F-component of the chiral superfield, F_Φ, gets a non-zero vev at the GUT scale, the gauginos λ^α develop an effective mass term

$$\mathcal{L}_{\text{mass}} \propto \frac{\langle F_\Phi \rangle_{\alpha\beta}}{M} \lambda^\alpha \lambda^\beta, \qquad (29)$$

where M is the mass scale of the GUT symmetry breaking. This scenario, including detailed phenomenological implications, has been widely discussed in the context of SUSY SU(5) [15,16]. Some cases have also been examined for SUSY SO(10) [20].

It is obvious that the group-theoretic structures of Eqs. (2) and (29) are identical. Hence the results discussed in the previous sections can be taken over *mutatis mutandis*. For example, for SU(5) breaking to $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ we should get for the gaugino masses M_i:

$$M_{3c} : M_{2L} : M_Y = \delta_3 : \delta_2 : \delta_1. \qquad (30)$$

This is to be compared with Eq. (3). The $\delta_{1,2,3}$ can be found in Table 1. The non-universality of gaugino masses has far-reaching implications for SUSY phenomenology [21].

Similarly, for $SO(10) \rightarrow SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$ one has

$$M_{4c} : M_{2L} : M_{2R} = \delta_{4c} : \delta_{2L} : \delta_{2R}, \qquad (31)$$

where $\delta_{4c,2L,2R}$ can be found in Table 3. Of course, at an intermediate scale there is the further breaking of $SU(4)_c \otimes SU(2)_R$ to $SU(3)_c \otimes U(1)_Y$ and for phenomenology at low energies the $SU(3)_c$, $SU(2)_L$, and $U(1)_Y$ gauginos are of relevance [22].

Finally, for E(6) breaking to $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$:

$$M_{3c} : M_{3L} : M_{3R} = \delta_{3c} : \delta_{3L} : \delta_{3R}. \qquad (32)$$

The deviation from universality for different Higgs representations is controlled by the $\delta_{3c,3L,3R}$ which are listed in Table 4.

6. Conclusions

Non-perturbative effects arising from quantum gravity or string compactification can be mimicked through higher dimensional non-renormalisable interactions. We have considered one class of such dimension-5 interactions which modify the unification condition of gauge coupling constants in grand unified theories. For SU(5), SO(10), and E(6) GUTs we have exhaustively worked out their implications for gauge coupling unification conditions. For the case of SU(5), we have shown how low energy physics can constrain these interaction strengths and the manner in which this would affect the unification scale. We also briefly examine the status of gauge unification in an SU(5) SUSY GUT model with the dimension-5 interactions and discuss a possible route to generate realistic neutrino masses through the see-saw mechanism. A corollary of the exercise is an application to SUSY models where non-universal gaugino mass relations obtain from a similar group-theoretic structure.

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