

# A nonsupersymmetric resolution of the anomalous muon magnetic moment

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## Abstract

The recent result from the E821 experiment at BNL on the anomalous magnetic moment of the muon shows a distinct discrepancy with the Standard Model predictions. We calculate the additional correction that the anomalous magnetic moment receives in a model with scalar leptoquarks. We find that such models can account for the deviation from the SM value even for small leptoquark couplings.

The recent measurement [1] of the magnetic dipole moment of the muon has set off a flurry of excitement amongst theorists [2–6]. The unprecedented precision achieved seems to imply that the experimentally observed value disagrees with the Standard Model (SM) expectations at more than  $2.6\sigma$  level. If this discrepancy is to be accepted at its face value, it seems to indicate the presence of new physics just round the corner. The exact nature of this new physics is a matter of intense speculation though. Since it is difficult to accommodate this deviation within a large class of models such as those with left-right symmetry or anomalous gauge boson couplings [2] or a world with large extra dimensions [7], it is natural that most practitioners favour supersymmetry as a solution [4–6, 8]. Even within the family of supersymmetric models, certain classes are less favoured than others, an example of the former being afforded by scenarios with anomaly mediated supersymmetry breaking [5, 9]. The favoured models, on the other hand, require that the superpartners be relatively light and within reach of the next run at the Fermilab Tevatron. It, thus, is of great importance to examine other possible extensions of the SM that can accommodate the measured value of the muon magnetic moment. In this article, we argue that scalar leptoquarks offer a perfectly viable solution to the problem.

Leptoquarks, as the name suggests, are particles that couple to a current comprising of a lepton and a quark. Arising naturally in many models with extended gauge symmetry (including but not limited to grand unification), they have been studied extensively in the literature [10, 11]. In this article, we shall confine ourselves to a discussion of scalar leptoquarks <sup>1</sup>, a class that includes the squarks in a  $R$ -parity violating supersymmetric

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<sup>1</sup>While vector leptoquarks corresponding to a gauge symmetry would tend to be superheavy, non-gauged vector particles generically imply a lack of renormalizability in the theory.

model. Relevance to the current context demands that these states be relatively light (certainly  $\lesssim \mathcal{O}(1 \text{ TeV})$  or so). While within supersymmetric models, their mass can be protected by nonrenormalizability theorems, in a generic model, additional discrete symmetries may ensure this. On account of the leptoquark coupling violating both baryon and lepton numbers, it might appear, at first sight, that such a light state might lead to rapid proton decay. However, it is easy to see that the proton would decay only if the said leptoquark either couples to a two-quark current or mixes with another scalar which does so. Phenomenological consistency thus demands that any such two-quark coupling (or mixing) be severely suppressed, a requirement that can be easily satisfied in most models.

Leptoquark Type	Coupling	$SU(3)_c \times SU(2)_L \times U(1)_Y$
$\Phi_1$	$[\lambda_{ij}^{(1)} \bar{Q}_{Lj} e_{Ri} + \tilde{\lambda}_{ij}^{(1)} \bar{u}_{Rj} L_{Li}] \Phi_1$	$(3, 2, \frac{7}{3})$
$\Phi_2$	$\lambda_{ij}^{(2)} \bar{Q}_{Lj}^c L_{Li} \Phi_2$	$(\bar{3}, 3, \frac{2}{3})$
$\Phi_3$	$[\lambda_{ij}^{(3)} \bar{Q}_{Lj}^c L_{Li} + \tilde{\lambda}_{ij}^{(3)} \bar{u}_{Rj}^c e_{Ri}] \Phi_3$	$(\bar{3}, 1, \frac{2}{3})$
$\Phi_4$	$\lambda_{ij}^{(4)} \bar{d}_{Rj} L_{Li} \Phi_4$	$(3, 2, \frac{1}{3})$
$\Phi_5$	$\lambda_{ij}^{(5)} \bar{d}_{Rj}^c e_{Ri} \Phi_5$	$(\bar{3}, 1, \frac{8}{3})$

Table 1: *Gauge quantum numbers and Yukawa couplings of scalar leptoquarks* ( $Q_{em} = T_3 + \frac{Y}{2}$ ).

Rather than confine ourselves to a particular scenario, let us start by considering a generic scalar leptoquark. In Table 1, we list all the possible states that can couple to a SM lepton and quark pair. Confining ourselves to terms involving the muon field, the relevant part of the Lagrangian can be parametrized as

$$\mathcal{L}_{\text{Yukawa}} = \bar{q}_i (\lambda_L^{(A)} P_L + \lambda_R^{(A)} P_R) \mu \phi_A + H.c., \quad (1)$$

where  $\phi_A$  is one of the leptoquarks in Table 1. For a given  $\phi_A$ , the structure of the chiral couplings  $\lambda_{L,R}$  is determined by its quantum numbers and can be easily read off from Table 1. It should be noted that, in eq.(1), the field  $q_i$  may refer either to one of the usual SM quarks or its charge conjugate. The above Lagrangian leads to diagrams as in Fig. 1 that may contribute to the muon dipole magnetic moment  $g_\mu$ . As the corresponding effective operator

$$\frac{eg_\mu}{2m_\mu} \bar{\mu} \sigma_{\alpha\beta} \mu F^{\alpha\beta}$$

is a chirality changing one, the corrections due to the diagrams of Fig. 1 would be proportional to some fermion mass. For a generic diagram, this chirality flip can occur either in the external legs or along the internal lines, the relative sizes of the contributions being determined by the chirality structure of the leptoquark coupling as well as the mass of the internal quark. It is easy to see, that, for a term proportional to  $m_q$  to be present, one needs to have *both* of  $\lambda_{L,R}$  in eq.(1) to be nonzero, a condition that can be satisfied

only for  $\Phi_{1,3}$ . As we shall see later, the size of the discrepancy implies that only such leptiquarks are relevant in this context.

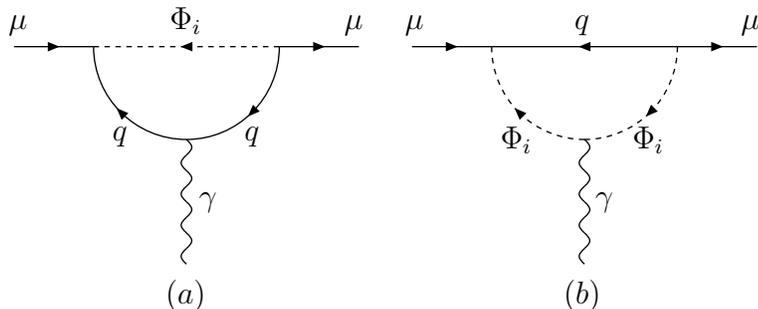


Figure 1: *Feynman diagrams that determine the leptiquark contribution to  $a_\mu$ .*

Defined as  $a_\mu \equiv (g_\mu - 2)/2$ , the anomalous magnetic dipole moment for the positively charged muon has been measured by the E821 Collaboration [1], to be

$$a_\mu^{\text{exp}} = (11\,659\,202 \pm 14 \pm 6) \times 10^{-10} \quad (2)$$

where the first uncertainty is statistical and the second systematic. This accuracy (1.3 ppm) has been achieved solely on the basis of the 1999 data. Analysis of last year's data, currently underway, should reduce the error to  $\sim 7 \times 10^{-10}$  (0.6 ppm), with the final targeted accuracy being  $4 \times 10^{-10}$  (0.35 ppm) [12]. When compared to the SM value [13]

$$a_\mu^{\text{SM}} = (11\,659\,159.7 \pm 6.7) \times 10^{-10}, \quad (3)$$

the new world average leads to a  $2.6\sigma$  discrepancy, *viz.*

$$\delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (42.6 \pm 16.5) \times 10^{-10}. \quad (4)$$

It might seem that the absolute magnitude of the deviation is small and that it should be easy to accommodate it by extending the SM. However, it should be noted that the bulk of the corrections are accounted for by QED loops [13] with hadronic vacuum polarization coming in a distant second [14]. Standard weak interactions account for [15] only  $(15.2 \pm 0.4) \times 10^{-10}$ , a contribution significantly smaller than the size of the discrepancy.

Reverting back to the diagrams of Fig. 1, we see that, to  $\mathcal{O}(m_\mu/m_q, m_\mu/m_\phi)$ , the leptiquark contribution to  $a_\mu$  is given by

$$a_\mu^{(\phi)} = \frac{-N_c m_\mu}{8\pi^2 m_\phi^2} \left[ m_q \lambda_L \lambda_R \{Q_\phi f_1(x) + Q_q f_2(x)\} + m_\mu (\lambda_L^2 + \lambda_R^2) \{Q_\phi f_3(x) + Q_q f_4(x)\} \right] \quad (5)$$

where  $x \equiv m_q^2/m_\phi^2$  and

$$\begin{aligned} f_1(x) &= \frac{1}{2(1-x)^3} [1 - x^2 + 2x \ln x] \\ f_2(x) &= \frac{1}{2(1-x)^3} [3 - 4x + x^2 + 2 \ln x] \\ f_3(x) &= \frac{1}{12(1-x)^4} [6x^2 \ln x - 2x^3 - 3x^2 + 6x - 1] \\ f_4(x) &= \frac{1}{12(1-x)^4} [6x \ln x + 2 + 3x - 6x^2 + x^3]. \end{aligned} \quad (6)$$

In eq.(5),  $N_c = 3$  is the color factor and we have suppressed both the generation indices and the superscript denoting the leptoquark type.

As we have already mentioned, an “explanation” of  $\delta a_\mu$  needs the term proportional to  $m_q$  to be nonzero. In other words, we need the leptoquark field to be of the types  $\Phi_{1,3}$  and the quark to be of the second or third generation. For the sake of concreteness, let us, for the time being, concentrate on the coupling to the top quark. In Fig. 2, we describe the region of the parameter space that is consistent with the data at different levels of confidence. Let us concentrate first on the case of  $\Phi_1$ . Since we can safely neglect the term proportional to  $m_\mu$ , the leptoquark contribution to  $\delta a_\mu$  is essentially proportional to the product  $\lambda_R \lambda_L$ . While the function  $f_1(x)$  is essentially positive (except for very small  $x$ ),  $f_2(x)$  is always negative and larger in magnitude compared to  $f_1(x)$ . Given the quantum numbers of  $\Phi_1$ , this results in  $a_\mu^{(\Phi_1)}$  having a sign opposite to that of the product  $\lambda_R \lambda_L$ . Since  $\delta a_\mu \gtrsim 0$ , this implies that a negative value of this product is preferred. For  $\Phi_3$ , the situation is the opposite. Here,  $a_\mu^{(\Phi_3)}$  has the same sign as the product, and consequently, the curves look upside down compared to those for  $\Phi_1$ . Note, however, that the scale on the product axis is quite different. This difference owes itself to the amount of cancellation between the contributions of the two Feynman diagrams in Fig. 1. Since the cancellation is more pronounced for  $\Phi_3$  than for  $\Phi_1$ , typically smaller values of  $a_\mu^{(\phi)}$  result. Consequently, an agreement with the data requires larger values for the couplings.

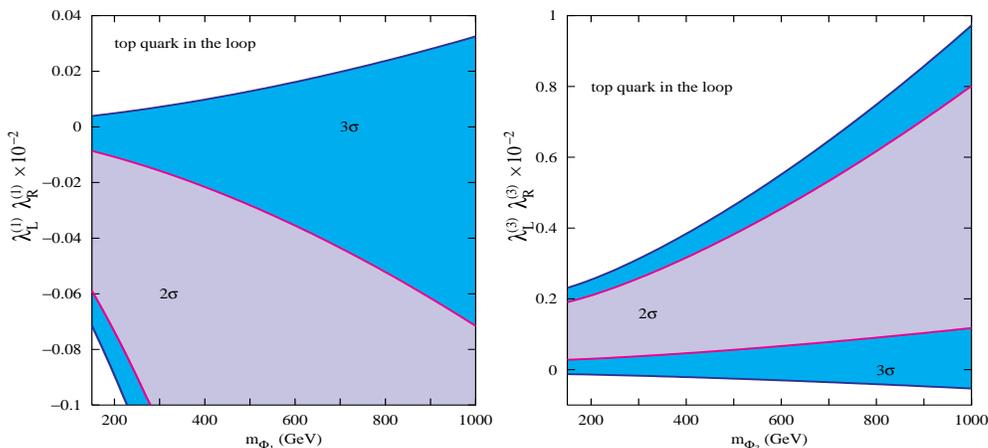


Figure 2: *The region of the parameter space consistent with the  $a_\mu$  measurement for (a)  $\Phi_1$ ; and (b)  $\Phi_3$ . It has been assumed that the only two non-zero couplings are those involving the top quark. The lightly shaded area agrees with the data at  $2\sigma$  level, whereas the encompassing darker region agrees at  $3\sigma$ .*

It appears, then, that the presence of either of  $\Phi_{1,3}$  can serve to explain  $\delta a_\mu$ . However, before we make such a claim, it is contingent upon us to examine the existing constraints on such a scenario. We proceed to do this next.

At the Tevatron, leptoquark production is dominated by strong interactions and proceeds primarily through  $q\bar{q}$  fusion. Subsequent decays into a quark (jet)-lepton pair has been extensively looked for by both the CDF and the D0 [16] collaborations. Nonobservation of such signals imply that a leptoquark decaying entirely<sup>2</sup> into a light quark and a

<sup>2</sup>For branching fractions less than unity, the bounds are understandably weaker

$e/\mu$  must be heavier than approximately 230 GeV. For a leptoquark decaying primarily into the top, the bounds would be weakened somewhat<sup>3</sup>. But what about the  $I_3 = -1/2$  partner, which would be produced as abundantly<sup>4</sup> and decays into a  $(b + \nu_\mu)$ -pair leading to an additional signal. The experimental efficiency for this channel is low though and the corresponding bound [16] is only  $m_\phi \gtrsim 150$  GeV. In fact, even combining the two individual bounds is unlikely to result in a constraint stronger than that for the ‘first-’ or ‘second-’ generation leptoquark as described above.

Apart from collider search experiments, low-energy data could, conceivably, also be used to constrain the parameter spaces for individual leptoquarks [11]. However, it is easy to see that the couplings to the top-quark are unconstrained by any such data. In fact, the strongest bound on such couplings come from LEP data [17] on  $Z \rightarrow \ell\bar{\ell}$ . Consistency with data typically requires that  $\lambda \lesssim g_{\text{Weak}}$ , a constraint that is easily satisfied by the entire parameter space of Fig. 2.

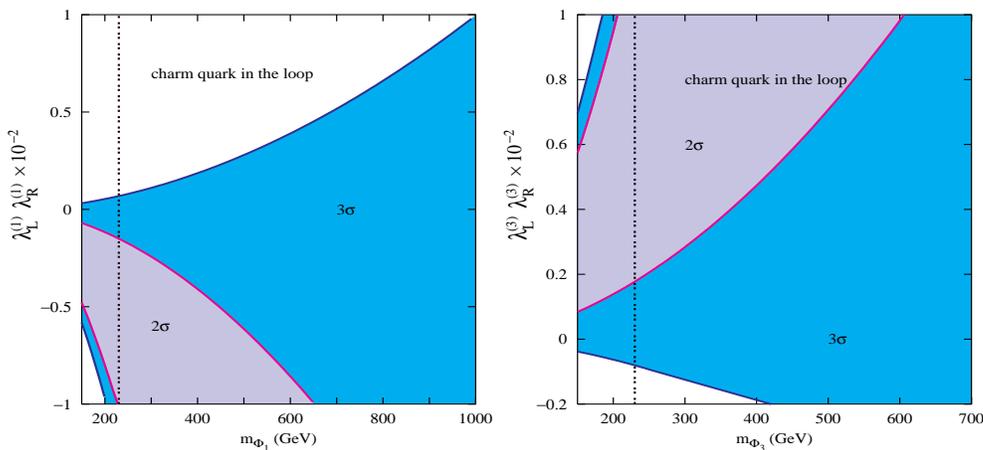


Figure 3: As in Fig. 2, but for couplings to the charm quark instead. The region to the left of the vertical lines are ruled out by direct search experiments at the Tevatron [16].

Having established that a  $\mu t\phi$  coupling can explain  $\delta a_\mu$  while respecting all other known constraints, let us now turn to another possibility, namely  $q = c$ . The suppression  $m_c/m_t$  immediately springs to mind. This, however, is ameliorated, to a significant extent, by the behaviour of  $f_2(x)$  as  $x \rightarrow 0$ . The corresponding results are exhibited in Fig. 3. Keeping in mind possible cancellation between terms, one might be tempted to question the neglect of the terms proportional to  $m_\mu$ . We have checked explicitly though that these continue to be numerically insignificant. Expectedly, somewhat larger values of the couplings are required although the suppression factor is not as large as  $m_t/m_c$ . Still, are such values of the couplings allowed, especially by low energy phenomenology? A search through literature yields nothing, except for a very weak constraint [11] from old measurements of  $a_\mu$  itself! Analysing all possible meson decays wherein the  $\mu c\Phi_{1,3}$  couplings could play a role, we find that the most significant constraint emanates from the helicity-suppressed decay  $D_s^+ \rightarrow \mu^+ \nu_\mu$ . If both  $\lambda_{L,R}$  be nonzero, the leptoquark contribution to the decay amplitude is no longer mass suppressed, and the branching

<sup>3</sup>To the best of our knowledge, this analysis has not yet been presented by either of the collaborations.

<sup>4</sup>Consideration of the  $\rho$ -parameter demands that the mass splitting between states in a multiplet be tiny.

fraction reads

$$Br(D_s^+ \rightarrow \mu^+ \nu_\mu) = \frac{1}{64\pi} f_{D_s}^2 m_{D_s}^3 \tau_{D_s} \left| \frac{4G_f V_{cs}}{\sqrt{2}} \frac{m_\mu}{m_{D_s}} - \frac{\lambda_L \lambda_R}{2m_\phi^2} \right|^2 \left( 1 - \frac{m_\mu^2}{m_{D_s}^2} \right)^2 \quad (7)$$

Comparing to the experimental number  $Br(D_s^+ \rightarrow \mu^+ \nu_\mu) = (4.6 \pm 1.9) \times 10^{-3}$  [18], leads, at the  $2\sigma$  level, to the constraint

$$-0.009 < \lambda_L \lambda_R \left( \frac{m_\phi}{100 \text{ GeV}} \right)^2 < 0.078 . \quad (8)$$

In Fig. 3, the lower limit would translate to a small parabolic curve at the extreme bottom left corner, a region already ruled out by direct searches [16]. The upper limit lies beyond the scale of the plot. It can thus be argued that, the direct search limit is the only relevant constraint for the part of the parameter space consistent with the  $a_\mu$  measurement.

Having seen that even couplings with the charm-quark can be instrumental in explaining  $\delta a_\mu$ , it is tempting to ask if a similar result obtains for the up-quark as well. In this case though, the growth in  $f_2(x)$  cannot compensate enough for the smallness of  $m_u$ , and, for moderate values of the couplings, the leptoquark contribution is too small to be of relevance.

In summary, we have investigated the corrections that a relatively light scalar leptoquark could wrought in coupling of the muon to the photon. We find that such a particle can indeed serve to reconcile the newly measured value of the magnetic dipole moment with theory. However, the leptoquark needs to have both left-handed and right-handed couplings to the muon field. This limits us to two species of leptoquarks out of a possible five. Experimental data prefers that these couple to the second or third generation quarks. The required magnitude of the couplings is on the smaller side and in consonance with all known bounds. The expected reduction in the error would serve to further constrain the region in parameter space allowed by the current data.

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