

Tripartite entanglement detection through tripartite quantum steering in one-sided and two-sided device-independent scenarios

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In the present work, we study tripartite quantum steering of quantum correlations arising from two local dichotomic measurements on each side in the two types of partially device-independent scenarios: 1-sided device-independent scenario where one of the parties performs untrusted measurements while the other two parties perform trusted measurements and 2-sided device-independent scenario where one of the parties performs trusted measurements while the other two parties perform untrusted measurements. We demonstrate that tripartite steering in the 2-sided device-independent scenario is weaker than tripartite steering in the 1-sided device-independent scenario by using two families of quantum correlations. That is these two families of quantum correlations in the 2-sided device-independent framework detect tripartite entanglement through tripartite steering for a larger region than that in the 1-sided device-independent framework. It is shown that tripartite steering in the 2-sided device-independent scenario implies the presence of genuine tripartite entanglement of $2 \times 2 \times 2$ quantum system, even if the correlation does not exhibit genuine nonlocality or genuine steering.

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I. INTRODUCTION

Multipartite entanglement is a resource for quantum information and computation when quantum networks are considered. Therefore, detecting the presence of multipartite entanglement in quantum networks is an important problem in quantum information science. In particular, a genuinely multipartite entangled state (which is not separable with respect to any partitions) [1] is important not only for quantum foundational research but also in various quantum information processing tasks, for example, in the context of extreme spin squeezing [2], high sensitive metrology tasks [3, 4]. Generation and detection of this kind of resource state is found to be difficult as the detection process deals with tomography and evaluation via constructing entanglement witness which require precise experimental control over the system subjected to measurements. But there is an alternative way to certify the presence of entanglement by observing the violation of Bell inequality [5] as entanglement is necessary ingredient to observe the violation. Motivated by this fact, a number of multipartite Bell type inequalities [6–10, 12] have been proposed to detect the genuine multipartite entanglement. To be specific, if the value of any Bell expression, in a Bell experiment, exceeds the value of the same expression obtained due to measurements on biseparable quantum states, then the pres-

ence of genuine entanglement is guaranteed. This kind of research was first initiated in [6, 7] but it took a shape by Bancal *et. al.* [10] where they have constructed device-independent entanglement witness (DIEW) of genuine multipartite entanglement for such Bell expressions.

The concept of quantum steering was first pointed out by Schrodinger [13] in the context of Einstein-Podolsky-Rosen paradox (EPR) [11], which has no classical analogue. Quantum steering as pointed out by Schrodinger occurs when one of the two spatially separated observers prepares genuinely different ensembles of quantum states for the other distant observer by performing suitable quantum measurements on her/his side. Wiseman *et. al.* [14] gave the formal definition of quantum steering from the foundational as well as quantum information perspective. Quantum steering is certified by the violation of steering inequalities. A number of steering inequalities have been proposed to observe steering [15]. Violation of such steering inequalities certify the presence of entanglement in a one-sided device-independent way.

In Refs. [17, 18], the notion of steering has been generalized for multipartite scenarios and multipartite steering inequalities have been derived to detect multipartite entanglement in asymmetric networks where some of the parties' measurements are trusted while the other parties' measurements are uncharacterized. These studies did not examine genuine multipartite steering, in which the nonlocality, in the form of steering, is necessarily shared among all observers. Genuine multipartite steering has been proposed in [19, 20]. In Refs. [21, 22], genuine tripartite steering inequalities have been derived to detect genuine tripartite entanglement in a par-

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tially device-independent way. Characterization of multipartite quantum steering through semidefinite programming has also been performed [21–23].

In the present work, we study tripartite steering (which is analogous to standard Bell nonlocality) and genuine tripartite steering of quantum correlations arising from two local measurements on each side in the two types of partially device-independent scenarios: 1-sided device-independent scenario where one of the parties performs untrusted measurements while the other two parties perform trusted measurements and 2-sided device-independent scenario where one of the parties performs trusted measurements while the other two parties perform untrusted measurements.

In the 1-sided device-independent framework, we study tripartite steering and genuine tripartite steering of two families of quantum correlations in the following scenarios: one of the parties performs two dichotomic black-box measurements and the other two parties perform incompatible qubit measurements that demonstrate Bell nonlocality [16] in one of the types or perform incompatible measurements that demonstrate EPR steering without Bell nonlocality [17, 24] in the other type. The first family of quantum correlation considered by us is called Svetlichny family as it can be obtained by performing the non-commuting measurements that lead to the violation of Svetlichny inequality and it violates Svetlichny inequality in a particular region. On the other hand, the second family of quantum correlation considered by us is called Mermin family as it can be obtained by performing the non-commuting measurements that lead to the violation of Mermin inequality and it violates Mermin inequality in a particular region, but it does not violate Svetlichny inequality in any region. We demonstrate in which range these two families detect tripartite and genuine tripartite steering in the aforementioned 1SDI scenarios, respectively.

We also explore in which range the Svetlichny family and Mermin family detect tripartite steering and genuine tripartite steering in the 2-sided device-independent framework.

Our study demonstrates that tripartite steering in the 2-sided device-independent framework is weaker than tripartite steering in the 1-sided device-independent framework. In other words, tripartite steering in the context of 2-sided device-independent framework detect tripartite entanglement for a larger region than that in the context of 1-sided device-independent framework. We demonstrate that tripartite steering in the 2-sided device-independent scenario implies the presence of genuine tripartite entanglement of $2 \times 2 \times 2$ quantum system, even if the correlation does not exhibit genuine nonlocality or genuine steering.

The plan of the paper is as follows. In Sections II and III the fundamental ideas of tripartite nonlocality and that of tripartite EPR steering in 1-sided device-independent scenario as well as in 2-sided device-independent scenario, respectively, are presented. In Sections IV and V tripartite steering and genuine tripartite steering in 1-sided device-independent scenario as well as in 2-sided device-independent scenario for Svetlichny family and Mermin family, respectively, are discussed. Certifying genuine tripartite entanglement of $2 \times 2 \times 2$ quantum system through tripartite steering inequality in 2-sided device-

independent scenario is also demonstrated in Sections IV and V. Finally, in the concluding Section VI, we discuss summary of the results obtained.

II. TRIPARTITE NONLOCALITY

We consider a tripartite Bell scenario where three spatially separated parties, Alice, Bob and Charlie, perform two dichotomic measurements on their subsystems. The correlation is described by the conditional probability distributions: $P(abc|A_x B_y C_z)$, here $x, y, z \in \{0, 1\}$ and $a, b, c \in \{0, 1\}$. The correlation exhibits standard tripartite nonlocality (i.e., Bell nonlocality) if it cannot be explained by a fully local hidden variable (LHV) model,

$$P(abc|A_x B_y C_z) = \sum_{\lambda} p_{\lambda} P_{\lambda}(a|A_x) P_{\lambda}(b|B_y) P_{\lambda}(c|C_z), \quad (1)$$

for some hidden variable λ with probability distribution p_{λ} ; $\sum_{\lambda} p_{\lambda} = 1$. The Mermin inequality (MI) [25],

$$\langle M \rangle := \langle A_0 B_0 C_1 + A_0 B_1 C_0 + A_1 B_0 C_0 - A_1 B_1 C_1 \rangle_{LHV} \leq 2, \quad (2)$$

is a Bell-type inequality whose violation implies that the correlation cannot be explained by a fully local hidden variable model as in Eq. (1). Here $\langle A_x B_y C_z \rangle = \sum_{abc} (-1)^{a \oplus b \oplus c} P(abc|A_x B_y C_z)$.

If a correlation violates a MI, it does not necessarily imply that it exhibits genuine tripartite nonlocality [6, 10]. In Ref. [6], Svetlichny introduced the strongest form of genuine tripartite nonlocality (see Ref. [10] for the other two forms of genuine nonlocality). A correlation exhibits Svetlichny nonlocality if it cannot be explained by a hybrid nonlocal-LHV (NLHV) model,

$$P(abc|A_x B_y C_z) = \sum_{\lambda} p_{\lambda} P_{\lambda}(a|A_x) P_{\lambda}(bc|B_y C_z) + \sum_{\lambda} q_{\lambda} P_{\lambda}(ac|A_x C_z) P_{\lambda}(b|B_y) + \sum_{\lambda} r_{\lambda} P_{\lambda}(ab|A_x B_y) P_{\lambda}(c|C_z), \quad (3)$$

with $\sum_{\lambda} p_{\lambda} + \sum_{\lambda} q_{\lambda} + \sum_{\lambda} r_{\lambda} = 1$. The bipartite probability distributions in this decomposition can have arbitrary nonlocality.

Svetlichny derived Bell-type inequalities to detect the strongest form of genuine tripartite nonlocality [6]. For instance, one of the Svetlichny inequalities (SI) reads,

$$\langle S \rangle := \langle A_0 B_0 C_1 + A_0 B_1 C_0 + A_1 B_0 C_0 - A_1 B_1 C_1 \rangle + \langle A_0 B_1 C_1 + A_1 B_0 C_1 + A_1 B_1 C_0 - A_0 B_0 C_0 \rangle \leq 4. \quad (4)$$

Quantum correlations violate the SI up to $4\sqrt{2}$. A Greenberger-Horne-Zeilinger (GHZ) state [26] gives rise to the maximal violation of the SI for a different choice of measurements which do not demonstrate GHZ paradox [27].

In the seminal paper [25], the MI was derived to demonstrate standard tripartite nonlocality of three-qubit correlations arising from the genuinely entangled states. For this

purpose, noncommuting measurements that do not demonstrate Svetlichny nonlocality was used. Note that when a Greenberger-Horne-Zeilinger (GHZ) state [26] maximally violates the MI, the measurements that give rise to it exhibit the GHZ paradox [27].

III. DEFINITIONS OF TRIPARTITE EPR STEERING

Before we define tripartite EPR steering, let us review the definition of bipartite EPR steering in the following 1-sided device-independent scenario. Two spatially separated parties, Alice (who is the trusted party) and Bob (who is the untrusted party) share an unknown bipartite system described by the density matrix ρ_{AB} in $\mathbb{C}^{d_A} \otimes \mathbb{C}^{d_B}$ with the dimension of Alice d_A is known and the dimension of Bob d_B is unknown. On this shared state, Bob performs black-box measurements (positive operator valued measurement, or in short, POVM) with the measurement operators $\{M_{by}\}_{b,y}$ ($M_{by} \geq 0 \forall b, y; \sum_b M_{by} = \mathbb{I} \forall y$), here y and b denote the measurement choices and measurement outcomes of Bob, respectively, to prepare the set of conditional states on Alice's side. The above steering scenario is characterized by the set of unnormalized conditional states on Alice's side $\{\sigma_{by}^A\}_{b,y}$, which is called an assemblage. Each element in this assemblage is given by $\sigma_{by}^A = \text{Tr}_B(\mathbb{I} \otimes M_{by} \rho_{AB})$.

Wiseman *et al.* [14] provided an operational definition of steering. According to this definition, Bob's measurements in the above scenario demonstrates steerability to Alice iff the assemblage certifies entanglement. The assemblage which does not certify entanglement, i.e., does not imply steerability from Bob to Alice has a local hidden state (LHS) model as follows: for all b, y , each element σ_{by}^A in the assemblage admits the following decomposition:

$$\sigma_{by}^A = \sum_{\lambda} q_{\lambda} P_{\lambda}(b|B_y) \rho_A^{\lambda}, \quad (5)$$

where λ denotes classical random variable which occurs with probability q_{λ} ; $\sum_{\lambda} q_{\lambda} = 1$; $P_{\lambda}(b|B_y)$ are some conditional probability distributions and the quantum states ρ_A^{λ} are called local hidden states which satisfy $\rho_A^{\lambda} \geq 0$ and $\text{Tr} \rho_A^{\lambda} = 1$. Suppose Alice performs positive operator valued measurements (POVM) with measurement operators $\{M_{a|x}\}_{a,x}$ ($M_{a|x} \geq 0 \forall a, x; \sum_a M_{a|x} = \mathbb{I} \forall x$) on the assemblage to detect steerability through the violation of a steering inequality. Then the scenario is characterized by the set of conditional probability distributions,

$$P(ab|A_x B_y) = \text{Tr}(M_{a|x} \sigma_{by}^A). \quad (6)$$

The above quantum correlation $P(ab|A_x B_y)$ detects steerability if and only if it cannot be explained by a LHS-LHV model of the form,

$$P(ab|A_x B_y) = \sum_{\lambda} q_{\lambda} P(a|A_x, \rho_A^{\lambda}) P_{\lambda}(b|B_y) \quad \forall a, x, b, y, \quad (7)$$

with $\sum_{\lambda} q_{\lambda} = 1$. Here $P(a|A_x, \rho_A^{\lambda})$ are the distributions arising from the local hidden states ρ_A^{λ} .

On the other hand, the quantum correlation $P(ab|A_x B_y)$ demonstrates Bell nonlocality if and only if it cannot be explained by a LHV-LHV model of the form,

$$P(ab|A_x B_y) = \sum_{\lambda} q_{\lambda} P_{\lambda}(a|A_x) P_{\lambda}(b|B_y) \quad \forall a, x, b, y, \quad (8)$$

with $\sum_{\lambda} q_{\lambda} = 1$. The quantum correlation that does not have a LHV-LHV model also implies steering, on the other hand, the quantum correlation that does not have a LHS-LHV model may not imply Bell nonlocality since certain local correlations may also detect steering in the given 1-sided device-independent scenario.

Let us now focus on the definition of tripartite steering. In the tripartite scenario, there are two types of partially device-independent scenarios where one can generalize bipartite EPR steering. These two scenarios are called 1-sided device-independent (1SDI) and 2-sided device-independent (2SDI) scenarios [23].

A. Tripartite steering in 1SDI scenario

We will consider the following 1-sided device-independent (1SDI) scenario (depicted in FIG. 1): Three spatially separated parties share an unknown tripartite quantum state ρ^{ABC} in $\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^d$ on which Charlie performs black-box measurements (POVMs). Suppose $M_{c|z}$ denote the unknown measurement operators of Charlie ($M_{c|z} \geq 0 \forall c, z; \sum_c M_{c|z} = \mathbb{I} \forall z$). Then, the scenario is characterized by the set of (unnormalized) conditional two-qubit states on Alice and Bob's side $\{\sigma_{c|z}^{AB}\}_{c,z}$, each element of which is given as follows:

$$\sigma_{c|z}^{AB} = \text{Tr}_C(\mathbb{I} \otimes \mathbb{I} \otimes M_{c|z} \rho^{ABC}). \quad (9)$$

Alice and Bob can do local state tomography to determine the above assemblage prepared by Charlie.

Analogous to the operational definition of bipartite EPR steering, we will now provide the operational definition of tripartite steering in the above 1SDI scenario. The assemblage $\sigma_{c|z}^{AB}$ given by Eq. (9) is called steerable if

i) the assemblage prepared on Alice and Bob's side cannot be reproduced by a fully separable state, in $\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^d$, of the form,

$$\rho^{ABC} = \sum_{\lambda} p_{\lambda} \rho_A^{\lambda} \otimes \rho_B^{\lambda} \otimes \rho_C^{\lambda}, \quad (10)$$

with $\sum_{\lambda} p_{\lambda} = 1$; and

ii) entanglement between Charlie and Alice-Bob is detected.

In the genuine steering scenario, Charlie demonstrates genuine tripartite EPR steering to Alice and Bob if the assemblage prepared on Alice and Bob's side cannot be reproduced by a biseparable state in $\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^d$,

$$\rho^{ABC} = \sum_{\lambda} p_{\lambda} \rho_A^{\lambda} \otimes \rho_{BC}^{\lambda} + \sum_{\lambda} q_{\lambda} \rho_{AC}^{\lambda} \otimes \rho_B^{\lambda} + \sum_{\lambda} r_{\lambda} \rho_{AB}^{\lambda} \otimes \rho_C^{\lambda}, \quad (11)$$

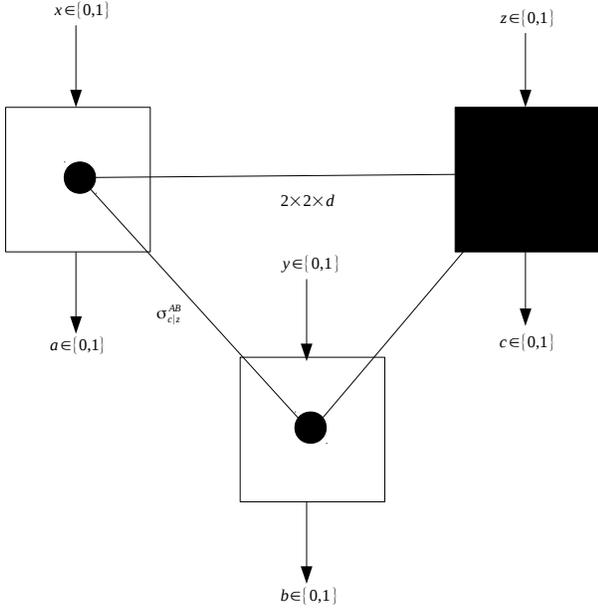


FIG. 1. Schematic diagram of our 1SDI tripartite steering scenario: Alice, Bob and Charlie share a $2 \times 2 \times d$ quantum state. Charlie performs two dichotomic black-box measurements to produce assemblages $\sigma_{c|z}^{AB}$ (9) on Alice and Bob's side. On this assemblage, Alice and Bob perform two dichotomic measurements producing the joint probability distributions $P(abc|A_x B_y C_z)$ (here a, b, c denotes the outcomes and x, y, z denotes the measurement choices) to check whether Charlie demonstrates steerability to them through the violation of a steering inequality by $P(abc|A_x B_y C_z)$. In case of the scenario considered in Section IV, Alice and Bob perform incompatible qubit measurements that demonstrate Bell nonlocality of certain two-qubit states [16]; for instance, the singlet state. On the other hand, in case of the scenario considered in Section V, they perform incompatible qubit measurements that demonstrate EPR steering without Bell nonlocality of certain two-qubit states [17, 24]; for instance, the singlet state.

with $\sum_\lambda p_\lambda + \sum_\lambda q_\lambda + \sum_\lambda r_\lambda = 1$.

Suppose in our tripartite 1SDI scenario, the trusted parties Alice and Bob perform POVMs having elements $\{M_{a|x}\}_{a,x}$ and $\{M_{b|y}\}_{b,y}$, respectively, for detecting tripartite steering. Here $M_{a|x} \geq 0 \forall a, x$; $\sum_a M_{a|x} = \mathbb{I} \forall x$; and $M_{b|y} \geq 0 \forall b, y$; $\sum_b M_{b|y} = \mathbb{I} \forall y$. Then the scenario is characterized by the set of conditional probability distributions,

$$P(abc|A_x B_y C_z) = \text{Tr}(M_{a|x} \otimes M_{b|y} \sigma_{c|z}^{AB}), \quad (12)$$

where $M_{a|x}$ and $M_{b|y}$ are the measurement operators of Alice and Bob, respectively. Suppose the above quantum correlation $P(abc|A_x B_y C_z)$ detects tripartite steerability. Then, it cannot be explained by a fully LHS-LHV model of the form,

$$P(abc|A_x B_y C_z) = \sum_\lambda q_\lambda P(a|A_x, \rho_A^\lambda) P(b|B_y, \rho_B^\lambda) P_\lambda(c|C_z), \quad (13)$$

with $\sum_\lambda q_\lambda = 1$. Here $P(a|A_x, \rho_A^\lambda)$ and $P(b|B_y, \rho_B^\lambda)$ are the distributions arising from the local hidden states ρ_A^λ and ρ_B^λ which are in \mathbb{C}^2 , respectively. It should be noted that if a

quantum correlation does not have a fully LHS-LHV model (13), then it does not necessarily imply that it detects tripartite steering from Charlie to Alice-Bob [17]. The correlation $P(abc|A_x B_y C_z)$ detects tripartite steerability if and only if

- i) $P(abc|A_x B_y C_z)$ does not have a fully LHS-LHV model as in Eq. (13); and
- ii) entanglement between Charlie and Alice-Bob is detected.

The quantum correlation $P(abc|A_x B_y C_z)$ that detects tripartite steering also detects genuine tripartite steering if it cannot be explained by the following steering LHS-LHV (StLHS) model:

$$\begin{aligned} P(abc|A_x B_y C_z) &= \sum_\lambda r_\lambda P(ab|A_x B_y, \rho_{AB}^\lambda) P_\lambda(c|C_z) \\ &+ \sum_\lambda p_\lambda P(a|A_x, \rho_A^\lambda) P_\lambda^Q(bc|B_y C_z) \\ &+ \sum_\lambda q_\lambda P(b|B_y, \rho_B^\lambda) P_\lambda^Q(ac|A_x C_z), \end{aligned} \quad (14)$$

with $\sum_\lambda p_\lambda + \sum_\lambda q_\lambda + \sum_\lambda r_\lambda = 1$. Here, $P(a|A_x, \rho_A^\lambda)$ and $P(b|B_y, \rho_B^\lambda)$ are the distributions arising from the qubit states ρ_A^λ and ρ_B^λ on Alice's side and Bob's side, respectively, $P_\lambda(c|C_z)$ is the distribution on Charlie's side arising from black-box measurements performed on a d dimensional quantum state and $P_\lambda^Q(bc|B_y C_z)$ and $P_\lambda^Q(ac|A_x C_z)$ are the distributions that can be produced from a $2 \times d$ quantum states; and $P(ab|A_x B_y, \rho_{AB}^\lambda)$ can be reproduced by two-qubit quantum states ρ_{AB}^λ shared between Alice and Bob. Note that in the model given in Eq. (14), the bipartite distributions at each λ level may have Bell nonlocality or steering without Bell nonlocality [17, 24]. Equivalently, the quantum correlation that detects genuine tripartite steering cannot be reproduced by a biseparable state in $\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^d$.

B. Tripartite steering in 2SDI scenario

We will consider the following 2-sided device-independent (2SDI) scenario (depicted in FIG. 2): Three spatially separated parties share an unknown tripartite quantum state ρ^{ABC} in $\mathbb{C}^2 \otimes \mathbb{C}^d \otimes \mathbb{C}^d$ on which Bob and Charlie performs local black-box measurements (POVMs). Suppose $\{M_{b|y}\}_{b,y}$ and $\{M_{c|z}\}_{c,z}$ denote the unknown measurement operators of Bob and Charlie, respectively. Here $M_{b|y} \geq 0 \forall b, y$; $\sum_b M_{b|y} = \mathbb{I} \forall y$ and $M_{c|z} \geq 0 \forall c, z$; $\sum_c M_{c|z} = \mathbb{I} \forall z$. Then, the scenario is characterized by the set of (unnormalized) conditional qubit states on Alice's side $\{\sigma_{bc|yz}^A\}_{b,c,y,z}$. The each element in this assemblage is given as follows:

$$\sigma_{bc|yz}^A = \text{Tr}_{BC}(\mathbb{I} \otimes M_{b|y} \otimes M_{c|z} \rho^{ABC}). \quad (15)$$

Alice can do local state tomography to determine the above assemblage prepared by Charlie.

We will now provide the operational definition of tripartite steering in the above 2SDI scenario. The assemblage $\{\sigma_{bc|yz}^A\}_{b,c,y,z}$ is called steerable if it cannot be reproduced by

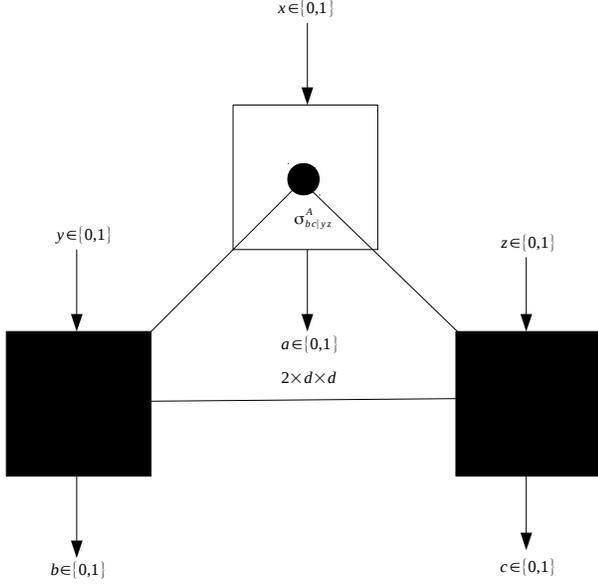


FIG. 2. Schematic diagram of our 2SDI tripartite steering scenario: Alice, Bob and Charlie share a $2 \times d \times d$ quantum state. Bob and Charlie perform two dichotomic black-box measurements to produce assemblages $\sigma_{bc|yz}^A$ (15) on Alice's side. On this assemblage, Alice performs two dichotomic measurements producing the joint probability distributions $P(abc|A_x B_y C_z)$ (here a, b, c denotes the outcomes and x, y, z denotes the measurement choices) to check whether the assemblages $\sigma_{bc|yz}^A$ prepared by Bob and Charlie demonstrate steerability through the violation of a steering inequality by $P(abc|A_x B_y C_z)$.

a fully separable state in $\mathbb{C}^2 \otimes \mathbb{C}^d \otimes \mathbb{C}^d$ of the form,

$$\rho^{ABC} = \sum_{\lambda} p_{\lambda} \rho_A^{\lambda} \otimes \rho_B^{\lambda} \otimes \rho_C^{\lambda}, \quad (16)$$

with $\sum_{\lambda} p_{\lambda} = 1$ in the given steering scenario. In our 2SDI scenario, even if entanglement is not certified between Alice and Bob-Charlie, tripartite steering can still occur by the presence of Bell nonlocality between Charlie and Bob [17]. When entanglement between Alice and Bob-Charlie is detected, our 2SDI scenario demonstrates genuine tripartite steering if the assemblage $\sigma_{bc|yz}^A$ cannot be reproduced by a biseparable state as given by Eq. (11) in $\mathbb{C}^2 \otimes \mathbb{C}^d \otimes \mathbb{C}^d$.

Suppose in our tripartite 2SDI scenario, the trusted party Alice performs POVMs having elements $\{M_{a|x}\}_{a,x}$ for detecting tripartite steering. Here $M_{a|x} \geq 0 \forall a, x$; $\sum_a M_{a|x} = \mathbb{I} \forall x$. Then the scenario is characterized by the set of conditional probability distributions,

$$P(abc|A_x B_y C_z) = \text{Tr}(M_{a|x} \sigma_{bc|yz}^A), \quad (17)$$

where $M_{a|x}$ are the measurement operators of Alice. Suppose the above quantum correlation $P(abc|A_x B_y C_z)$ cannot be explained by a fully LHS-LHV model of the form,

$$P(abc|A_x B_y C_z) = \sum_{\lambda} q_{\lambda} P(a|A_x, \rho_A^{\lambda}) P_{\lambda}(b|B_y) P_{\lambda}(c|C_z), \quad (18)$$

with $\sum_{\lambda} q_{\lambda} = 1$ (Here, $P(a|A_x, \rho_A^{\lambda})$ are the distributions arising from the local hidden states ρ_A^{λ} which are in \mathbb{C}^2). Then, it detects tripartite steerability.

The quantum correlation $P(abc|A_x B_y C_z)$ that detects tripartite steering in our 2SDI scenario also detects genuine tripartite steering if it cannot be explained by the following steering LHS-LHV (StLHS) model:

$$\begin{aligned} P(abc|A_x B_y C_z) &= \sum_{\lambda} r_{\lambda} P_{\lambda}^Q(ab|A_x B_y) P_{\lambda}(c|C_z) \\ &+ \sum_{\lambda} p_{\lambda} P(a|A_x, \rho_A^{\lambda}) P_{\lambda}(bc|B_y C_z) \\ &+ \sum_{\lambda} q_{\lambda} P_{\lambda}(b|B_y) P_{\lambda}^Q(ac|A_x C_z), \quad (19) \end{aligned}$$

with $\sum_{\lambda} p_{\lambda} + \sum_{\lambda} q_{\lambda} + \sum_{\lambda} r_{\lambda} = 1$. Here, $P(a|A_x, \rho_A^{\lambda})$ are the distributions arising from the qubit states ρ_A^{λ} and, $P_{\lambda}(b|B_y)$ and $P_{\lambda}(c|C_z)$ are the distribution on Bob's and Charlie's sides, respectively, arising from black-box measurements performed on a d dimensional quantum state and $P_{\lambda}^Q(ab|A_x B_y)$ and $P_{\lambda}^Q(ac|A_x C_z)$ are the distribution that can be produced from a $2 \times d$ quantum state; and $P_{\lambda}(bc|B_y C_z)$ can be reproduced by a $d \times d$ quantum state. Note that in the model given in Eq. (19), the bipartite distributions at each λ level may have Bell nonlocality or steering without Bell nonlocality [17, 24]. Equivalently, the quantum correlation that detects genuine tripartite steering in our 2SDI cannot be reproduced by a biseparable state in $\mathbb{C}^2 \otimes \mathbb{C}^d \otimes \mathbb{C}^d$.

In the next Section we study in which range two one-parameter families of quantum correlations obtained from local dichotomic measurements on tripartite quantum states detect tripartite quantum steering in our 1SDI and 2SDI scenarios.

IV. DETECTION OF TRIPARTITE STEERING WITH SVETLICHNY FAMILY

The Svetlichny family of tripartite correlations is defined as:

$$P_{SVF}^V(abc|A_x B_y C_z) = \frac{2 + (-1)^{a \oplus b \oplus c \oplus x \oplus y \oplus z \oplus xz} \sqrt{2} V}{16}, \quad (20)$$

where $0 \leq V \leq 1$, which can be obtained from the noisy three-qubit GHZ state, $\rho = V|\Phi_{GHZ}\rangle\langle\Phi_{GHZ}| + (1-V)\mathbb{I}/8$, where $|\Phi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$, for the measurements that give rise to the maximal violation of the SI; for instance, $A_0 = \sigma_x$, $A_1 = \sigma_y$, $B_0 = \frac{\sigma_x - \sigma_y}{\sqrt{2}}$, $B_1 = \frac{\sigma_x + \sigma_y}{\sqrt{2}}$, $C_0 = \sigma_x$ and $C_1 = \sigma_y$. The noisy three-qubit GHZ state is genuinely entangled iff $V > 0.429$ [29]. The Svetlichny family certifies genuine entanglement in a fully device independent way for $V > \frac{1}{\sqrt{2}}$, as it violates the SI in this range. The Svetlichny family has a fully local hidden variable (LHV) model when $V \leq \frac{1}{\sqrt{2}}$ [10]. This implies that in this range, it can also arise from a separable state in the higher dimensional space [28].

A. 1SDI scenario

We consider a tripartite 1SDI steering scenario where Charlie performs two dichotomic black-box measurements to pre-

pare conditional two-qubit states on Alice and Bob's side on which Alice and Bob perform pair of incompatible qubit measurements that demonstrate Bell nonlocality of certain two-qubit states; for instance, the singlet state. Now we are going to present a Lemma which is useful to find out in which ranges the Svetlichny family detects genuine tripartite steering and tripartite steering in the context of the above 1SDI scenario.

Lemma 1. *In our 1SDI scenario mentioned above the Svetlichny family has a steering LHS-LHV model as in Eq. (14) in the range $0 < V \leq \frac{1}{\sqrt{2}}$ and has a fully LHS-LHV model as in Eq. (13) iff $0 < V \leq \frac{1}{2\sqrt{2}}$.*

Proof. See Appendix A. \square

The above Lemma implies the following two propositions.

Proposition 1. *The Svetlichny family detects genuine tripartite steering iff $V > \frac{1}{\sqrt{2}}$ in the context of our 1SDI scenario.*

Proof. Since the Svetlichny family violates the Svetlichny inequality for $V > \frac{1}{\sqrt{2}}$, it certifies genuine tripartite entanglement in a fully device independent way in that range. Hence, it is followed that the Svetlichny family certifies genuine tripartite entanglement in our 1SDI scenario as well for $V > \frac{1}{\sqrt{2}}$. The Svetlichny family, therefore, does not have a steering LHS-LHV model as in Eq.(14) in our 1SDI scenario for $V > \frac{1}{\sqrt{2}}$. On the other hand, following Lemma 1 we can state that the Svetlichny family has a steering LHS-LHV model as in Eq. (14) in our 1SDI scenario in the range $0 < V \leq \frac{1}{\sqrt{2}}$. Hence, the Svetlichny family detects genuine tripartite steering iff $V > \frac{1}{\sqrt{2}}$ in the context of our 1SDI scenario. \square

Proposition 2. *The Svetlichny family detects tripartite steering iff $V > \frac{1}{\sqrt{2}}$ in the context of our 1SDI scenario.*

Proof. Svetlichny family detects entanglement between Charlie and Alice-Bob for $V > \frac{1}{\sqrt{2}}$ as it violates the Svetlichny inequality in this range. Moreover, the steering LHS-LHV model given in the proof of Lemma 1 for the Svetlichny family implies that for $V \leq \frac{1}{\sqrt{2}}$, it can be reproduced by a $2 \times 2 \times d$ biseparable state of the form,

$$\rho^{ABC} = \sum_{\lambda=0}^3 r_{\lambda} \rho_{AB}^{\lambda} \otimes |\lambda\rangle\langle\lambda|, \quad (21)$$

with $\sum_{\lambda} r_{\lambda} = 1$. Therefore, the Svetlichny family detects entanglement between Charlie and Alice-Bob iff $V > \frac{1}{\sqrt{2}}$. On the other hand, in the context of our 1SDI scenario, the Svetlichny family does not have a fully LHS-LHV model as in Eq. (13) following Lemma 1 for $V > \frac{1}{2\sqrt{2}}$. Combining these two facts we can state that the Svetlichny family detects entanglement between Charlie and Alice-Bob and does not have a fully LHS-LHV model as in Eq. (13) in the range $V > \frac{1}{\sqrt{2}}$ following Lemma 1. Hence, in the context of our 1SDI scenario, the Svetlichny family detects tripartite steering iff $V > \frac{1}{\sqrt{2}}$. \square

From the Propositions 1 and 2 we observe the following two salient features: 1) in our 1SDI scenario, the Svetlichny family does not detect tripartite steering in the range $\frac{1}{2\sqrt{2}} < V \leq \frac{1}{\sqrt{2}}$ despite it does not have a fully LHS-LHV model in this range and 2) the ranges in which the Svetlichny family detects tripartite steering and genuine tripartite steering in our 1SDI scenario are the same.

B. 2SDI scenario

We now consider a tripartite 2SDI steering scenario where Bob and Charlie perform two dichotomic black-box measurements to prepare conditional single qubit states on Alice's side on which Alice performs two mutually unbiased qubit measurements. We are now interested in which ranges the Svetlichny family detects genuine tripartite steering and tripartite steering in the context of this 2SDI scenario.

Proposition 3. *The Svetlichny family detects genuine tripartite steering in our 2SDI scenario iff $V > \frac{1}{\sqrt{2}}$.*

Proof. Note that the Svetlichny family can be reproduced by a $2 \times 2 \times d$ dimensional biseparable state of the form given in Eq. (21) for $V \leq \frac{1}{\sqrt{2}}$. This implies that it does not detect genuine tripartite entanglement in the range $V \leq \frac{1}{\sqrt{2}}$ in our 2SDI scenario. On the other hand, the Svetlichny family detects genuine tripartite entanglement for $V > \frac{1}{\sqrt{2}}$ in the fully device independent scenario as it violates the Svetlichny inequality in this range. Hence, the Svetlichny family detects genuine tripartite entanglement for $V > \frac{1}{\sqrt{2}}$ in our 2SDI scenario as well. The Svetlichny family, therefore, detects genuine tripartite steering in our 2SDI scenario iff $V > \frac{1}{\sqrt{2}}$. \square

Proposition 4. *The Svetlichny family detects tripartite steering in our 2SDI scenario for $V > \frac{1}{2}$.*

Proof. In Ref. [17], it has been shown that the violation of the following inequality (Eq. (22) in [17] with N (Number of parties) = 3 and T (Number of trusted parties) = 1):

$$\langle S \rangle_{2 \times 2 \times 2}^{\text{LHS}} \leq 2\sqrt{2}, \quad (22)$$

detects tripartite steering in our 2SDI scenario. Here, S is the Svetlichny operator given in the Svetlichny inequality (4), $2 \times 2 \times 2$ indicates that Alice performs qubit measurements while Bob and Charlie perform black-box measurements. Note that the Svetlichny family violates the above steering inequality for $V > \frac{1}{2}$. Thus, the Svetlichny family detects tripartite steering for $V > \frac{1}{2}$ in the 2SDI scenario. \square

From the aforementioned Propositions we observe the following two salient features: 1) the ranges in which the Svetlichny family detects tripartite steering and genuine tripartite steering in our 2SDI scenario are different and 2) Svetlichny family detects more tripartite entangled states to be tripartite steerable in the 2SDI scenario than in the case of 1SDI scenario. Now we are going to make the following important observation.

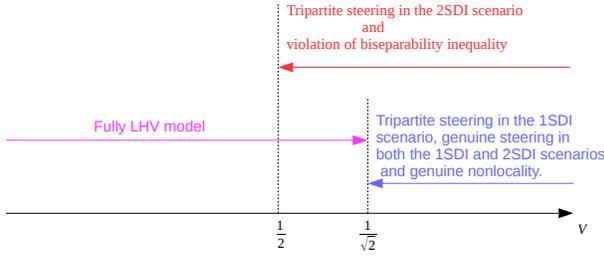


FIG. 3. Regions of the parameter V in which the Svetlichny family is genuinely nonlocal, detects genuine steering and tripartite steering, has a fully LHV model and violates biseparability inequality.

Observation 1. *Quantum violation of the tripartite steering inequality (22) by $2 \times 2 \times 2$ systems certifies genuine entanglement in that $2 \times 2 \times 2$ systems, even if genuine nonlocality or genuine steering is not detected.*

Proof. We consider the following Svetlichny biseparability inequality:

$$\langle S \rangle_{2 \times 2 \times 2}^{\text{Bi-sep}} \leq 2\sqrt{2}, \quad (23)$$

whose violation detects genuine tripartite entanglement in $2 \times 2 \times 2$ systems (for derivation see the appendix B). Here, $\langle S \rangle_{2 \times 2 \times 2}$ denotes the Svetlichny operator with the measurement observables on each side being incompatible qubit measurements. Note that quantum violation of tripartite steering inequality (22) by $2 \times 2 \times 2$ systems implies quantum violation of the Svetlichny biseparability inequality (23) by that $2 \times 2 \times 2$ systems. Because, for both of these two inequalities the upper bounds are the same. Hence the claim. \square

We have illustrated the above results with the Svetlichny family in Fig. 3.

V. DETECTION OF TRIPARTITE STEERING WITH MERMIN FAMILY

The Mermin family of tripartite correlations is defined as

$$P_{MF}^V(abc|A_x B_y C_z) = \frac{1 + (-1)^{a \oplus b \oplus c \oplus x \oplus y \oplus z} \delta_{x \oplus y \oplus 1, z} V}{8}, \quad (24)$$

where $0 < V \leq 1$, which can be obtained from the noisy three-qubit GHZ state for the measurements that give rise to the GHZ paradox; for instance, $A_0 = \sigma_x$, $A_1 = \sigma_y$, $B_0 = \sigma_x$, $B_1 = \sigma_y$, $C_0 = \sigma_x$ and $C_1 = -\sigma_y$. The Mermin family is Bell nonlocal for $V > \frac{1}{2}$ as it violates the MI given in Eq. (2). This implies that it certifies tripartite entanglement for $V > \frac{1}{2}$. In that range, the Mermin family is not genuinely nonlocal since it has a NLHV model as in Eq. (3) [33]. However, it certifies genuine tripartite entanglement for $V > \frac{1}{\sqrt{2}}$ in a fully device-independent way since it violates the Mermin inequality more than $2\sqrt{2}$ [34]. We will study tripartite steering of the Mermin family in our 1SDI and 2SDI scenarios.

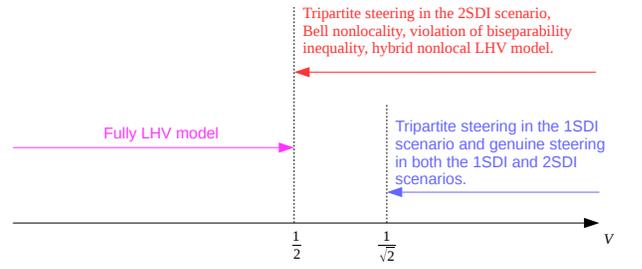


FIG. 4. Regions of the parameter V in which the Mermin family detects Bell-nonlocality, genuine steering and tripartite steering, has fully LHV model and hybrid nonlocal LHV model, and violates the biseparability inequality.

A. 1SDI scenario

We consider a tripartite 1SDI steering scenario where Charlie performs two dichotomic black-box measurements to prepare conditional two-qubit states on Alice and Bob's side on which Alice and Bob perform pair of incompatible qubit measurements that demonstrate EPR steering without Bell nonlocality. Now we present a Lemma which is useful to find out in which ranges the Mermin family detects genuine tripartite steering and tripartite steering in the context of the above 1SDI scenario.

Lemma 2. *In our 1SDI scenario mentioned above the Mermin family has a steering LHS-LHV model as in Eq. (14) in the range $0 < V \leq \frac{1}{\sqrt{2}}$ and has a fully LHS-LHV model iff $0 < V \leq \frac{1}{2\sqrt{2}}$.*

Proof. See Appendix C for the proof. \square

The above Lemma implies the following two propositions.

Proposition 5. *The Mermin family detects genuine tripartite steering iff $V > \frac{1}{\sqrt{2}}$ in the context of 1SDI scenario.*

Proof. Since the Mermin family violates the Mermin inequality more than $2\sqrt{2}$ for $V > \frac{1}{\sqrt{2}}$, it certifies genuine tripartite entanglement in the fully device independent scenario in that range [34]. Hence, the Mermin family certifies genuine tripartite entanglement in our 1SDI scenario as well for $V > \frac{1}{\sqrt{2}}$. This implies that, for $V > \frac{1}{\sqrt{2}}$, it does not have a steering LHS-LHV model as in Eq. (14) in our 1SDI scenario. On the other hand, following Lemma 2 we can state that the Mermin family has a steering LHS-LHV model as in Eq. (14) in the range $0 < V \leq \frac{1}{\sqrt{2}}$. The Mermin family, therefore, detects genuine tripartite steering iff $V > \frac{1}{\sqrt{2}}$. \square

Proposition 6. *The Mermin family detects tripartite steering iff $V > \frac{1}{\sqrt{2}}$ in the context of our 1SDI scenario.*

Proof. Mermin family detects entanglement between Charlie and Alice-Bob for $V > \frac{1}{\sqrt{2}}$ as it violates the Mermin inequality more than $2\sqrt{2}$ in this range [34]. Moreover, the steering

LHS-LHV model given in the proof of Lemma 2 for the Mermin family implies that, for $V \leq \frac{1}{\sqrt{2}}$, it can be reproduced by a $2 \times 2 \times d$ biseparable state of the form given by Eq. (21). Therefore, the Mermin family detects entanglement between Charlie and Alice-Bob iff $V > \frac{1}{\sqrt{2}}$. On the other hand, in the context of our 1SDI scenario, the Mermin family does not have a fully LHS-LHV model as in Eq. (13) for $V > \frac{1}{2\sqrt{2}}$ following Lemma 2. Combining these two facts we can state that the Mermin family detects entanglement between Charlie and Alice-Bob and does not have a fully LHS-LHV model as in Eq. (13) in the range $V > \frac{1}{\sqrt{2}}$ in our 1SDI scenario. Hence, in the context of our 1SDI scenario, the Mermin family detects tripartite steering iff $V > \frac{1}{\sqrt{2}}$. \square

From the Propositions 5 and 6, we observe the following two salient features: 1) in our 1SDI scenario, the Mermin family does not detect tripartite steering in the range $\frac{1}{2\sqrt{2}} < V \leq \frac{1}{\sqrt{2}}$ despite it does not have a fully LHS-LHV model in this range and 2) the ranges in which the Mermin family detects tripartite steering and genuine tripartite steering in our 1SDI scenario are the same.

B. 2SDI scenario

We will now study tripartite steering of the Mermin family in our 2SDI scenario where Bob and Charlie perform two dichotomic black-box measurements to prepare conditional single qubit states on Alice's side on which Alice performs two mutually unbiased qubit measurements. We are interested to find out in which ranges the Mermin family detects genuine tripartite steering and tripartite steering in the context of this 2SDI scenario.

Proposition 7. *The Mermin family detects genuine tripartite steering in our 2SDI scenario iff $V > \frac{1}{\sqrt{2}}$.*

Proof. Note that the Mermin family can be reproduced by a $2 \times 2 \times d$ dimensional biseparable state of the form given in Eq. (21) for $V \leq \frac{1}{\sqrt{2}}$. This implies that it does not detect genuine tripartite entanglement in the range $V \leq \frac{1}{\sqrt{2}}$ in our 2SDI scenario. On the other hand, the Mermin family detects genuine tripartite entanglement for $V > \frac{1}{\sqrt{2}}$ in the fully device independent scenario as the Mermin family violates the Mermin inequality more than $2\sqrt{2}$ in that range [34]. Hence, the Mermin family detects genuine tripartite entanglement in our 2SDI scenario as well for $V > \frac{1}{\sqrt{2}}$. The Mermin family, therefore, detects genuine tripartite steering in our 2SDI scenario iff $V > \frac{1}{\sqrt{2}}$. \square

Proposition 8. *The Mermin family detects tripartite steering in our 2SDI scenario for $V > \frac{1}{2}$.*

Proof. In Ref. [17], it has been shown that the violation of the following inequality (Eq. (21) in [17] with $N = 3$ and $T = 1$),

$$\langle M \rangle_{2 \times 2 \times 2}^{\text{LHS}} \leq 2, \quad (25)$$

detects tripartite steering in our 2SDI scenario. Here, M is the Mermin operator given in the Mermin inequality (2), $2 \times 2 \times 2$ indicates that Alice performs qubit measurements while Bob and Charlie perform black-box measurements. Note that the Mermin family violates the above steering inequality for $V > \frac{1}{2}$. Thus, the Mermin family detects tripartite steering for $V > \frac{1}{2}$ in the 2SDI scenario. \square

From aforementioned Propositions we observe the following two salient features: 1) the ranges in which the Mermin family detects tripartite steering and genuine tripartite steering in our 2SDI scenario are different and 2) the Mermin family detects more tripartite entangled states to be tripartite steerable in the 2SDI scenario than in the case of 1SDI scenario. Now we want to state the following important observation.

Observation 2. *Quantum violation of the tripartite steering inequality (25) by $2 \times 2 \times 2$ systems certifies genuine entanglement in that $2 \times 2 \times 2$ systems, even if genuine nonlocality or genuine steering is not detected.*

Proof. In Ref. [37], it was shown that the Mermin inequality detect genuine entanglement of three-qubit systems in the scenario where all three parties perform two mutually unbiased qubit measurements. This implies that the violation of the inequality (25) implies the presence of genuine entanglement if all three parties performs qubit measurements in mutually unbiased bases. Similar to the derivation of Svetlichny biseparability inequality presented in Appendix B, one can obtain following the Mermin biseparability inequality:

$$\langle M \rangle_{2 \times 2 \times 2}^{\text{Bi-sep}} \leq 2, \quad (26)$$

whose violation detects genuine tripartite entanglement of $2 \times 2 \times 2$ systems. Here, $\langle M \rangle_{2 \times 2 \times 2}$ denotes the Mermin operator with the measurement observables on each side being incompatible qubit measurements. Note that quantum violation of tripartite steering inequality (25) by $2 \times 2 \times 2$ systems implies quantum violation of the biseparability inequality (26) by that $2 \times 2 \times 2$ systems. Because, for both of these two inequalities the upper bounds are the same. Hence the claim. \square

We have illustrated the above results with the Mermin family in Fig. 4.

VI. CONCLUSION

In this work, we have studied tripartite EPR steering of quantum correlations arising from two local measurements on each side in the two types of partially device-independent scenarios: 1-sided device-independent scenario where one of the parties performs untrusted measurements while the other two parties perform trusted measurements and 2-sided device-independent scenario where one of the parties performs trusted measurements while the other two parties perform untrusted measurements.

We have studied tripartite steering and genuine tripartite steering in the 1-sided device-independent framework in the

following scenarios: one of the parties performs two dichotomic black-box measurements and the other two parties perform incompatible qubit measurements that demonstrate Bell nonlocality [16] in one of the types or perform incompatible measurements that demonstrate EPR steering without Bell nonlocality [17, 24] in the other type. In the context of these two scenarios, we have studied tripartite steering of two families of quantum correlations called Svetlichny family and Mermin family, respectively. We have shown that the ranges in which these families detect tripartite steering and genuine tripartite steering are the same.

On the other hand, in the 2-sided device-independent framework, the ranges in which the Svetlichny family and Mermin family detect tripartite steering and genuine tripartite steering are different. These studies reveal that tripartite steering in the 2-sided device-independent scenario is weaker than tripartite steering in the 1-sided device-independent scenario. That is the Svetlichny family and Mermin family in the 2-sided device-independent framework detect tripartite entanglement for a larger region than that in the 1-sided device-independent framework. Using biseparability inequality, it has been demonstrated that tripartite steering in the 2-sided device-independent framework implies the presence of genuine tripartite entanglement of $2 \times 2 \times 2$ quantum system, even if the correlation does not exhibit genuine nonlocality or genuine steering.

Similar to our tripartite 1-sided device-independent scenario considered in Section IV where the trusted parties Alice and Bob perform incompatible qubit measurements that demonstrate Bell-CHSH inequality violation, in Ref. [38], the authors considered a tripartite measurement scenario in which Alice and Bob perform incompatible qubit measurements that demonstrate maximal Bell-CHSH inequality violation. In the latter scenario, the authors studied an interesting feature of genuinely tripartite entangled states called “entangled entanglement” in which entanglement between measurement choices of Charlie and entanglement of the conditional states prepared on Alice and Bob’s side by these measurement choices occurs. On the other hand, in our work, we have studied steerability between measurement choices of Charlie and entanglement of the conditional states prepared on Alice and Bob’s side by these measurements on genuinely tripartite entangled states.

Note that in Ref. [19] the definition of genuine tripartite steering was presented and it was experimentally demonstrated in [20]. In their approach Alice, Bob and Charlie are all assumed to perform characterised measurements at some point (i. e., the trusted and untrusted parties are not fixed in their definition of genuine tripartite steering). On the other hand, trusted and untrusted parties are fixed (Alice, Bob are trusted and Charlie is untrusted in 1SDI scenario; Alice is trusted and Bob, Charlie are untrusted in 2SDI scenario) in the definitions of tripartite steering and genuine tripartite steering presented in this paper which is an advancement in the context of the notion of tripartite steering. In the steering scenarios considered in Ref. [21] noisy GHZ state demonstrates genuine tripartite steering in 1SDI scenario in a larger region compared to that in 2SDI scenario. On the other hand, the

two examples of quantum correlations presented in this study reveal that noisy GHZ state demonstrates tripartite steering (not genuine) in 2SDI scenario in a larger region compared to that in 1SDI scenario. One important point to be stressed here is that the procedures to detect genuine tripartite steering adopted in Refs. [21–23] are based on numerical calculations with the help of semidefinite program (SDP). But the advantage of our study is that the steering LHS-LHV model and the fully LHS-LHV model of the two families of correlations are derived analytically, not using SDP. The application of tripartite steering in the context of randomness certification has been studied in [22] using SDP. It is worth to be studied in future what advantage one can gain in the context of randomness certification in tripartite steering scenario considered by us in the present study.

In Ref. [24], the author proposed two inequalities for detecting genuine steering in the Svetlichny-type and Mermin-type one-sided device-independent scenarios. We have demonstrated that these inequalities do not detect genuine steering and they detect tripartite steering of $2 \times d \times d$ systems in the 2-sided device-independent framework. Further, the author argued that the violation of one of these inequalities imply genuine entanglement if one assumes only dimension of the trusted parties to be qubit dimension. However, the present study demonstrates that the violation of these inequalities do not detect genuine entanglement in this context, on the other hand, the violation of those inequalities may imply genuine entanglement in the scenario where the dimensions of all three parties are assumed to be qubit dimension.

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Appendix A: Proof for Lemma 1

We consider the following classical simulation scenario to demonstrate in which range the Svetlichny family has a steering LHS-LHV model as in Eq. (14) and a fully LHS-LHV model as in Eq. (13) in our 1SDI scenario considered in Section IV:

Scenario 1. *Charlie generates his outcomes by using classical variable λ which he shares with Alice-Bob. Alice and Bob share a two-qubit system for each value of λ and perform pair of incompatible qubit measurements that demonstrate Bell nonlocality of certain two-qubit states; for instance, the singlet state.*

For $0 < V \leq \frac{1}{\sqrt{2}}$, the Svetlichny family given by Eq.(20) can be written as

$$P_{SVF}^V(abc|A_x B_y C_z) = \sum_{\lambda=0}^3 r_\lambda P(ab|A_x B_y, \rho_{AB}^\lambda) P_\lambda(c|C_z) \quad (\text{A1})$$

where $r_0 = r_1 = r_2 = r_3 = \frac{1}{4}$, and $P_0(c|C_z) = P_D^{00}$, $P_1(c|C_z) = P_D^{01}$, $P_2(c|C_z) = P_D^{10}$, $P_3(c|C_z) = P_D^{11}$, here,

$$P_D^{\alpha\beta}(c|C_z) = \begin{cases} 1, & c = \alpha z \oplus \beta \\ 0, & \text{otherwise} \end{cases} \quad (\text{A2})$$

Here, $\alpha, \beta \in \{0, 1\}$. The four bipartite distributions $P(ab|A_x B_y, \rho_{AB}^\lambda)$ in Eq. (A1) are given as follows:

1. For $\lambda = 0$, it is given by,

$$P(ab|A_x B_y, \rho_{AB}^0) = \begin{array}{c} \begin{array}{cc} ab & \\ xy & \end{array} \\ \begin{array}{cc} 00 & 01 \\ 01 & 10 \\ 10 & 11 \\ 11 & \end{array} \end{array} \begin{pmatrix} \frac{1+\sqrt{2}V}{4} & \frac{1-\sqrt{2}V}{4} & \frac{1-\sqrt{2}V}{4} & \frac{1+\sqrt{2}V}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1-\sqrt{2}V}{4} & \frac{1+\sqrt{2}V}{4} & \frac{1+\sqrt{2}V}{4} & \frac{1-\sqrt{2}V}{4} \end{pmatrix}, \quad (\text{A3})$$

where each row and column corresponds to a fixed measurement (xy) and a fixed outcome (ab) respectively. Throughout the paper we will follow the same convention. Note that, each of the probability distributions must satisfy $0 \leq P(ab|A_x B_y, \rho_{AB}^0) \leq 1$, which implies that $0 < V \leq \frac{1}{\sqrt{2}}$.

This joint probability distribution at Alice and Bob's side can be reproduced by performing measurements of the observables corresponding to the operators $A_0 = \sigma_x$, $A_1 = \sigma_y$; and $B_0 = \frac{\sigma_x - \sigma_y}{\sqrt{2}}$, $B_1 = \frac{\sigma_x + \sigma_y}{\sqrt{2}}$ on the two-qubit state given by,

$$|\psi_0\rangle = \cos \theta |00\rangle + \frac{(1-i)\sin \theta}{\sqrt{2}} |11\rangle, \quad (\text{A4})$$

with $\sin 2\theta = \sqrt{2}V$; $0 \leq \theta \leq \frac{\pi}{4}$. $|0\rangle$ and $|1\rangle$ are the eigenstates of σ_z corresponding to the eigenvalues $+1$ and -1 respectively.

2. For $\lambda = 1$, it is given by,

$$P(ab|A_x B_y, \rho_{AB}^1) = \begin{pmatrix} \frac{1-\sqrt{2}V}{4} & \frac{1+\sqrt{2}V}{4} & \frac{1+\sqrt{2}V}{4} & \frac{1-\sqrt{2}V}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1+\sqrt{2}V}{4} & \frac{1-\sqrt{2}V}{4} & \frac{1-\sqrt{2}V}{4} & \frac{1+\sqrt{2}V}{4} \end{pmatrix}. \quad (\text{A5})$$

Note that, each of the probability distributions must satisfy $0 \leq P(ab|A_x B_y, \rho_{AB}^1) \leq 1$, which implies that $0 < V \leq \frac{1}{\sqrt{2}}$.

This joint probability distribution at Alice and Bob's side can be reproduced by performing measurements of the observables corresponding to the operators $A_0 = \sigma_x$, $A_1 = \sigma_y$; and $B_0 = \frac{\sigma_x - \sigma_y}{\sqrt{2}}$, $B_1 = \frac{\sigma_x + \sigma_y}{\sqrt{2}}$ on the two-qubit state given by,

$$|\psi_1\rangle = \cos \theta |00\rangle - \frac{(1-i)\sin \theta}{\sqrt{2}} |11\rangle, \quad (\text{A6})$$

with $\sin 2\theta = \sqrt{2}V$; $0 \leq \theta \leq \frac{\pi}{4}$.

3. For $\lambda = 2$, it is given by,

$$P(ab|A_x B_y, \rho_{AB}^2) = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1+\sqrt{2}V}{4} & \frac{1-\sqrt{2}V}{4} & \frac{1-\sqrt{2}V}{4} & \frac{1+\sqrt{2}V}{4} \\ \frac{1+\sqrt{2}V}{4} & \frac{1-\sqrt{2}V}{4} & \frac{1-\sqrt{2}V}{4} & \frac{1+\sqrt{2}V}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}. \quad (\text{A7})$$

Note that, each of the probability distributions must satisfy $0 \leq P(ab|A_x B_y, \rho_{AB}^2) \leq 1$, which implies that $0 < V \leq \frac{1}{\sqrt{2}}$.

This joint probability distribution at Alice and Bob's side can be reproduced by performing measurements of the observables corresponding to the operators $A_0 = \sigma_x$, $A_1 = \sigma_y$; and $B_0 = \frac{\sigma_x - \sigma_y}{\sqrt{2}}$, $B_1 = \frac{\sigma_x + \sigma_y}{\sqrt{2}}$ on the two-qubit state given by,

$$|\psi_2\rangle = \cos \theta |00\rangle + \frac{(1+i)\sin \theta}{\sqrt{2}} |11\rangle, \quad (\text{A8})$$

with $\sin 2\theta = \sqrt{2}V$; $0 \leq \theta \leq \frac{\pi}{4}$.

4. For $\lambda = 3$, it is given by,

$$P(ab|A_x B_y, \rho_{AB}^3) = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1-\sqrt{2}V}{4} & \frac{1+\sqrt{2}V}{4} & \frac{1+\sqrt{2}V}{4} & \frac{1-\sqrt{2}V}{4} \\ \frac{1-\sqrt{2}V}{4} & \frac{1+\sqrt{2}V}{4} & \frac{1+\sqrt{2}V}{4} & \frac{1-\sqrt{2}V}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}. \quad (\text{A9})$$

Note that, each of the probability distributions must satisfy $0 \leq P(ab|A_x B_y, \rho_{AB}^3) \leq 1$, which implies that $0 < V \leq \frac{1}{\sqrt{2}}$.

This joint probability distribution at Alice and Bob's side can be reproduced by performing measurements of the observables corresponding to the operators $A_0 = \sigma_x$, $A_1 = \sigma_y$; and $B_0 = \frac{\sigma_x - \sigma_y}{\sqrt{2}}$, $B_1 = \frac{\sigma_x + \sigma_y}{\sqrt{2}}$ on the two-qubit state given by,

$$|\psi_3\rangle = \cos \theta |00\rangle - \frac{(1+i)\sin \theta}{\sqrt{2}} |11\rangle, \quad (\text{A10})$$

with $\sin 2\theta = \sqrt{2}V$; $0 \leq \theta \leq \frac{\pi}{4}$.

Here it can be easily checked that the aforementioned observables corresponding to the operators $A_0 = \sigma_x$, $A_1 = \sigma_y$; and $B_0 = \frac{\sigma_x - \sigma_y}{\sqrt{2}}$, $B_1 = \frac{\sigma_x + \sigma_y}{\sqrt{2}}$ used to reproduce the joint probability distributions at Alice and Bob's side can demonstrate nonlocality of the singlet state given by, $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. Hence, the Svetlichny family given by Eq.(20) has a steering LHS-LHV model as in Eq. (14) in the range $0 < V \leq \frac{1}{\sqrt{2}}$ in Scenario 1.

In the steering LHS-LHV model given for the Svetlichny family as in Eq.(A1), the bipartite distributions $P(ab|A_x B_y, \rho_{AB}^\lambda)$ belong to the BB84 family up to local reversible operations (LRO) ¹,

$$P_{BB84}(ab|A_x B_y) = \frac{1 + (-1)^{a \oplus b \oplus x \cdot y} \delta_{x,y} W}{4} \quad (\text{A11})$$

where $W = \sqrt{2}V$ is a real number such that $0 < W \leq 1$. In Ref. [32], it has been shown that the BB84 family certifies two-qubit entanglement iff $W > \frac{1}{2}$. This implies that for $W \leq \frac{1}{2}$, it can be reproduced by a two-qubit separable state. Therefore, the bipartite distributions $P(ab|A_x B_y, \rho_{AB}^\lambda)$ in Eq. (A1) has a LHS-LHS decomposition for $V \leq \frac{1}{2\sqrt{2}}$. This implies that the Svetlichny family can be reproduced by a fully LHS-LHV model,

$$P_{SvF}^V(abc|A_x B_y C_z) = \sum_{\lambda} q_{\lambda} P(a|A_x, \rho_A^\lambda) P(b|B_y, \rho_B^\lambda) P_{\lambda}(c|C_z), \quad (\text{A12})$$

for $V \leq \frac{1}{2\sqrt{2}}$ in Scenario 1. Here, $P(a|A_x, \rho_A^\lambda)$ and $P(b|B_y, \rho_B^\lambda)$ are the distributions arising from the local hidden states ρ_A^λ and ρ_B^λ which are in \mathbb{C}^2 , respectively.

In Ref. [17], it has been shown that violation of the following inequality (Eq. (22) in [17] with N (Number of parties) = 3 and T (Number of trusted parties) = 2):

$$\langle S \rangle_{2 \times 2 \times ?}^{\text{LHS}} \leq 2, \quad (\text{A13})$$

detects non-existence of fully LHS-LHV model in Scenario 1. Here, S is the Svetlichny operator given in the Svetlichny inequality (4), $2 \times 2 \times ?$ indicates that Alice and Bob perform qubit measurements while Charlie performs black-box measurements. Note that the Svetlichny family violates the above inequality for $V > \frac{1}{2\sqrt{2}}$. Thus, the Svetlichny family does not have fully LHS-LHV model in the region $V > \frac{1}{2\sqrt{2}}$ in Scenario 1. Hence, we can conclude that the Svetlichny family has fully LHS-LHV model iff $0 < V \leq \frac{1}{2\sqrt{2}}$ in Scenario 1.

Appendix B: Derivation of the Svetlichny biseparability inequality

Here we derive a biseparability inequality that detect genuine entanglement of three-qubit systems by using the

Svetlichny operator in the scenario where each party performs incompatible qubit measurements. In this scenario, the tripartite correlations that can be reproduced by a biseparable three-qubit state has the following nonseparable LHS-LHS (NSLHS) model:

$$\begin{aligned} P(abc|A_x B_y C_z) = & \sum_{\lambda} p_{\lambda} P(a|A_x, \rho_A^\lambda) P(bc|B_y C_z, \rho_{BC}^\lambda) \\ & + \sum_{\lambda} q_{\lambda} P(ac|A_x C_z, \rho_{AC}^\lambda) P(b|B_y, \rho_B^\lambda) \\ & + \sum_{\lambda} r_{\lambda} P(ab|A_x B_y, \rho_{AB}^\lambda) P(c|C_z, \rho_C^\lambda), \quad (\text{B1}) \end{aligned}$$

with $\sum_{\lambda} p_{\lambda} + \sum_{\lambda} q_{\lambda} + \sum_{\lambda} r_{\lambda} = 1$. Here, $P(a|A_x, \rho_A^\lambda)$, $P(b|B_y, \rho_B^\lambda)$ and $P(c|C_z, \rho_C^\lambda)$ are the distributions which can be reproduced by the qubit states ρ_A^λ , ρ_B^λ and ρ_C^λ , respectively, and $P_{\lambda}(bc|B_y C_z, \rho_{BC}^\lambda)$, $P_{\lambda}(ac|A_x C_z, \rho_{AC}^\lambda)$ and $P_{\lambda}(ab|A_x B_y, \rho_{AB}^\lambda)$ can be reproduced by the 2×2 states ρ_{BC}^λ , ρ_{AC}^λ and ρ_{AB}^λ , respectively. Note that in the model given by Eq. (B1), the bipartite distributions at each λ level may have nonseparability.

The Svetlichny operator can be rewritten as follows:

$$S = CHSH_{AB} C_1 + CHSH'_{AB} C_0. \quad (\text{B2})$$

Here, $CHSH_{AB} = A_0 B_0 + A_0 B_1 + A_1 B_0 - A_1 B_1$ is the canonical CHSH (Clauser-Horne-Shimony-Holt) operator [16] and $CHSH'_{AB} = -A_0 B_0 + A_0 B_1 + A_1 B_0 + A_1 B_1$ is one of its equivalents. Note that the expectation value of the Svetlichny operator for the correlation which has the nonseparable LHS-LHS model as given in Eq. (B1) have the following form:

$$\begin{aligned} & \sum_{\lambda} p_{\lambda} \langle A_1 \rangle_{\rho_A^\lambda} \langle CHSH_{BC} \rangle_{\rho_{BC}^\lambda} + \sum_{\lambda} p_{\lambda} \langle A_0 \rangle_{\rho_A^\lambda} \langle CHSH'_{BC} \rangle_{\rho_{BC}^\lambda} \\ & + \sum_{\lambda} q_{\lambda} \langle CHSH_{AC} \rangle_{\rho_{AC}^\lambda} \langle B_1 \rangle_{\rho_B^\lambda} + \sum_{\lambda} q_{\lambda} \langle CHSH'_{AC} \rangle_{\rho_{AC}^\lambda} \langle B_0 \rangle_{\rho_B^\lambda} \\ & + \sum_{\lambda} r_{\lambda} \langle CHSH_{AB} \rangle_{\rho_{AB}^\lambda} \langle C_1 \rangle_{\rho_C^\lambda} + \sum_{\lambda} r_{\lambda} \langle CHSH'_{AB} \rangle_{\rho_{AB}^\lambda} \langle C_0 \rangle_{\rho_C^\lambda}. \quad (\text{B3}) \end{aligned}$$

Let us now argue that the above quantity is upper bounded by $2\sqrt{2}$. Consider the first line of the decomposition given in Eq. (B3). Suppose Bob and Charlie's correlation at each λ level of this line detects nonseparability. Then $\pm \langle CHSH_{BC} \rangle_{\rho_{BC}^\lambda} \pm \langle CHSH'_{BC} \rangle_{\rho_{BC}^\lambda} \leq 2\sqrt{2}$. Suppose Bob and Charlie's correlation at each λ level has a LHS-LHS model. Then also $\pm \langle CHSH_{BC} \rangle_{\rho_{BC}^\lambda} \pm \langle CHSH'_{BC} \rangle_{\rho_{BC}^\lambda} \leq 2\sqrt{2}$. In a similar way, considering the second line of the decomposition given in Eq. (B3), one can show that $\pm \langle CHSH_{AC} \rangle_{\rho_{AC}^\lambda} \pm \langle CHSH'_{AC} \rangle_{\rho_{AC}^\lambda} \leq 2\sqrt{2}$; and considering the third line of the decomposition given in Eq. (B3), one can show that $\pm \langle CHSH_{AB} \rangle_{\rho_{AB}^\lambda} \pm \langle CHSH'_{AB} \rangle_{\rho_{AB}^\lambda} \leq 2\sqrt{2}$. Therefore, any convex combination of the three above mentioned expression should be upper bounded by $2\sqrt{2}$. Hence, we can conclude that in the Scenario where each party performs incompatible qubit measurements, the Svetlichny operator is upper bounded by $2\sqrt{2}$ if the

¹ LRO is designed [31] as follows: Alice may relabel her inputs: $x \rightarrow x \oplus 1$, and she may relabel her outputs (conditionally on the input): $a \rightarrow a \oplus \alpha x \oplus \beta$ ($\alpha, \beta \in \{0, 1\}$); Bob can perform similar operations.

distributions at Alice and Bob's side can demonstrate EPR steering without Bell nonlocality of the singlet state given by $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. Hence, the Mermin family given by Eq.(24) has a steering LHS-LHV model as in Eq.(14) in the range $0 < V \leq \frac{1}{\sqrt{2}}$ in Scenario 2.

In the steering LHS-LHV model given for the Mermin family as in Eq.(C1), the bipartite distributions $P(ab|A_x B_y, \rho_{AB}^\lambda)$ belong to the CHSH family up to local reversible operations [31],

$$P_{CHSH}(ab|A_x B_y) = \frac{2 + (-1)^{a \oplus b \oplus xy} \sqrt{2}W}{8}, \quad (C7)$$

where $W = \sqrt{2}V$ is a real number such that $0 < W \leq 1$ and $0 < V \leq \frac{1}{\sqrt{2}}$. In Ref. [32], it has been that the CHSH family certifies two-qubit entanglement iff $W > \frac{1}{2}$. This implies that for $W \leq \frac{1}{2}$, it can be reproduced by a two-qubit separable state. Therefore, the bipartite distributions $P(ab|A_x B_y, \rho_{AB}^\lambda)$ in Eq. (C1) has a LHS-LHS decomposition for $V \leq \frac{1}{2\sqrt{2}}$. This implies that the Mermin family can be reproduced by a fully LHS-LHV model,

$$P_{MF}^V(abc|A_x B_y C_z) = \sum_{\lambda} q_{\lambda} P(a|A_x, \rho_A^{\lambda}) P(b|B_y, \rho_B^{\lambda}) P_{\lambda}(c|C_z), \quad (C8)$$

for $V \leq \frac{1}{2\sqrt{2}}$ in Scenario 2. Here, $P(a|A_x, \rho_A^{\lambda})$ and $P(b|B_y, \rho_B^{\lambda})$ are the distributions arising from the local hidden states ρ_A^{λ} and ρ_B^{λ} which are in \mathbb{C}^2 , respectively.

In Ref. [17], it has been shown that violation of the following inequality (Eq. (21) in [17] with $N = 3$ and $T = 2$),

$$\langle M \rangle_{2 \times 2 \times 2} \stackrel{\text{LHS}}{\leq} \sqrt{2}, \quad (C9)$$

detects non-existence of fully LHS-LHV model in Scenario 2. Here M is the Mermin operator given in the Mermin inequality (2), $2 \times 2 \times 2$ indicates that Alice and Bob perform qubit measurements while Charlie performs black-box measurements. Note that the Mermin family violates the above inequality for $V > \frac{1}{2\sqrt{2}}$. Thus, the Mermin family does not have fully LHS-LHV model in the region $V > \frac{1}{2\sqrt{2}}$ in Scenario 2. Hence, we can conclude that the Mermin family has fully LHS-LHV model iff $0 < V \leq \frac{1}{2\sqrt{2}}$ in Scenario 2.

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