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A note on the neutrino mass implications of the K2K experiment

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Abstract

The K2K experiment has presented the first results on the observation of ν_μ. They show a depletion compared to the expectations and are consistent with neutrino oscillations with a mass-splitting in the range favoured by the Super-Kamiokande atmospheric neutrino measurements. Here we examine the extent by which the range of Δm^2 obtained from the K2K measurements can vary due to the uncertainties in the flux, cross-section, and detector efficiency.

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Short title: Neutrino mass implications of the K2K experiment

I Introduction

A non-zero neutrino mass has far-reaching implications in particle physics, astrophysics, and cosmology. Any evidence in its support must be probed from as many different angles as possible. Atmospheric neutrinos provide a strong case for non-degenerate neutrino masses and mixing. Super-Kamiokande (SK) [1], MACRO [2], and Soudan 2 [3] studying neutrinos produced in interactions of cosmic rays with atmospheric nuclei show a significant suppresion of the ratio of ν_μ to ν_e with respect to the expectation from standard hadronic shower models. These observations can be explained by the phenomenon of neutrino oscillations in which ν_μ

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changes to some other neutrino, *e.g.*, ν_τ. Such oscillations can occur if neutrinos have a non-degenerate mass spectrum and if the neutrino mass eigenstates are not the same as the flavour eigenstates. The atmospheric neutrino data favour a mass splitting $\Delta m^2 = |m_1^2 - m_2^2| \simeq 10^{-3}$ eV^2 and maximal mixing, $\theta = \pi/4$. Neutrino oscillations have also been advanced as a natural solution to the long-standing solar neutrino problem, especially when the data from the SK and SNO experiments are taken together.

K2K [4] is the first accelerator based long-baseline oscillation experiment. It is designed precisely to probe the mass and mixing parameters favoured by the atmospheric neutrino data. It uses neutrinos produced by the 12 GeV proton beam of the KEK accelerator laboratory which are subsequently detected by the SK detector at a distance of 250 km. The average neutrino energy is $\langle E \rangle = 1.3$ GeV and the intense beam is 98.0% pure ν_μ. K2K also uses a near detector at KEK, 300m downstream from the pion production target, to monitor and study the neutrino beam. The experiment started taking data from June, 1999. The primary oscillation search mode is $\nu_\mu \to \nu_x$ oscillation (the ν_μ disappearance mode).

The first results from K2K [5] have been released. With 3.85 $\times 10^{19}$ protons on target (p.o.t.), in the 22.5 kton fiducial volume of the SK detector they observe 44 fully contained events in place of the (no oscillation) expectation of $63.9^{+6.1}_{-6.6}$ events. This corresponds to a depletion ratio

$$d \equiv \frac{\text{No. of observed events}}{\text{No. of expected events}} = 0.689^{+0.079}_{-0.060}. \tag{1}$$

This depletion is consistent with neutrino oscillations with a mass splitting in the atmospheric range and maximal mixing[1]. In this note, basing ourselves on the available information, we wish to examine to what extent this conclusion could be uncertain. Our finding is that the result is robust.

When oscillations are operative, the survival probability of a neutrino with energy E produced a distance L from a detector is expressed (in the two-flavour approximation) as:

$$P_{\mu\mu} = P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{1.27\Delta m^2(\text{eV}^2) L(\text{km})}{E(\text{GeV})}, \tag{2}$$

For the purpose of the present work, the nature of the partner neutrino state to which the ν_μ oscillates is immaterial so long as it does not mimic a ν_μ at the SK detector.

II The K2K results

In the presence of neutrino oscillations, the event rate in the K2K experiment, r, can be expressed as

[1] Based on 2.6×10^{19} p.o.t. [6] the depletion ratio was $d = 0.670^{+0.086}_{-0.070}$.

$$r = \int dES(E)P_{\mu\mu}(E), \tag{3}$$

where $P_{\mu\mu}$ is given by (2).

The oscillation parameters θ and Δm^2 will be determined from the K2K data. The experiment is sensitive to $(\Delta m^2, \sin^2 2\theta)$ around $(10^{-3}\text{eV}^2, 1)$ favoured by the SuperKamiokande experiment.

$S(E)$ is the 'energy spectrum' of the neutrinos from KEK as detected at the Kamioka site. This spectrum is given by the convolution of the standard (no oscillation) neutrino energy spectrum ϕ at the SK (far) detector, the interaction cross-section σ, and the detection efficiency ϵ (all in differential form), integrated over the final state parameters, *i.e.*, symbolically, $S = \int \phi\sigma\epsilon$.

In the published literature there is information on ϕ but not yet on the product $\sigma.\epsilon$ relevant for the far detector specification. As direct and accurate reconstruction of the spectrum is not possible, some indirect, approximate reconstruction was suggested by the K2K collaboration.

We have two sets of information in hand.

(i) The neutrino spectrum is considered to be practically zero above 5 GeV [4].

(ii) According to the K2K Monte Carlo simulation, for an exposure of 3.85×10^{19} p.o.t., assuming maximal mixing, we have the set of predictions for the event rate (r^{MC}) for different Δm^2 shown in Table 1:

Δm^2 in eV^2	1×10^{-4}	3×10^{-3}	5×10^{-3}	7×10^{-3}
r^{MC}	63.9	41.5	27.4	23.1

Table 1: The values of r^{MC} for different Δm^2 determined from the K2K Monte Carlo calculations [5] for maximal mixing ($\sin^2 2\theta = 1$).

III Impact of different $S(E)$

In the absence of direct information, a form for the spectrum was suggested [7] to be $S(E) \propto x^\alpha(1-x)^\beta$ with $x = E/(5\text{ GeV})$. It fits the expected behaviour of $S(E)$ at $x = 0$ and 1 and has a particularly simple form. The overall normalization and the parameters α and β were obtained by using a best fit to the data in Table 1. It was shown using this functional form of $S(E)$ that the preliminary data from K2K are consistent with the Δm^2 favoured by the SK atmospheric neutrino experiment.

Here, we ask the question: How sensitively does this conclusion depend on the form of $S(E)$? To this end, we consider several different functional forms for the spectrum $S(E)$. For each, the best-fit values of the parameters are determined by minimizing

$$\chi^2 = \sum_{i=2}^{4} \frac{(r_i - r_i^{MC})^2}{\sigma_i^2}, \tag{4}$$

where r_i is calculated using the chosen form of $S(E)$ and r_i^{MC} is taken from the K2K Monte Carlo results of Table 1 and, as in [7],

$$\sigma_i^2 = 44 + 0.01 r_i^2. \tag{5}$$

r^{MC} corresponding to $i = 1$, *i.e.*, $\Delta m^2 = 1.0 \times 10^{-4}$ eV^2, is used to determine the overall normalization constant N. In this way, in all cases the no oscillation limit is correctly reproduced. The functional forms that we have examined and the best-fit values of the parameters are listed in Table 2. These functions are plotted in Fig. 1.

Case	functional form	Normalization N	best fit values		
			α	β	χ^2
a	$Nx^\alpha(1-x)^\beta$	1370.7	1.14	2.78	0.044
b	$Nx^\alpha(1-x^2)^\beta$	518.3	0.85	3.81	0.063
c	$Nx^\alpha(1-x^3)^\beta$	307.7	0.60	5.50	0.068
d	$Nx^\alpha(1-x^4)^\beta$	215.7	0.42	7.48	0.061

Table 2: The different parametrizations of $S(E)$, the best-fit values of the parameters, and the values of χ^2.

All the above functions have a common nature in that they vanish at $x = 0$ and 1 and rise to a maximum value at some intermediate energy. The actual neutrino spectrum as a function of the energy is expected to have rising and falling regions at the two ends and an intermediate region where its variation is rather slow. To mimic this feature, we have also considered two further cases where we have chosen the functional forms of Cases a and b of Table 2 but 'chopped' the function at the values of E where it achieves half the maximum value. Between these points, E_1 and E_2, the function is assumed to be a constant. The points $x_i = E_i/(5$ GeV$)$, (i =1,2) and the best-fit values of the parameters in these cases are presented in Table 3 and the functions exhibited in Fig. 1.

Case	functional form	Normalization N	Half-Maxima		best fit values		
			x_1	x_2	α	β	χ^2
e	$Nx^\alpha(1-x)^\beta$	13136.3	0.128	0.529	2.00	4.61	0.065
f	$Nx^\alpha(1-x^2)^\beta$	3392.6	0.156	0.544	1.72	6.58	0.047

Table 3: Parametrizations of $S(E)$ with a constant region between x_1 and x_2 (see text). The best-fit values of the parameters and the values of χ^2 are shown.

We expect that the six rather different functional forms for $S(E)$ that we have chosen adequately capture the possible uncertainties in the neutrino spectrum, cross-section, and detection efficiency. We make two remarks. Firstly, the fits are all of very low χ^2. This is due to the comparatively large σ_i^2 from (5) associated with each datum point. Secondly, we find that over a rather broad region in the (α, β) parameter space around the best-fit points, χ^2 varies relatively little. In particular, the best-fit point we find in Case a is somewhat different from the one in [7] even though the functional forms are the same. For the latter we find a comparable but higher χ^2.

After having obtained the six different functional forms of $S(E)$ which best fit the Monte Carlo results of Table 1 we use them in (3) to obtain the predicted value for the number of events for the K2K experiment with 3.85×10^{19} p.o.t. These results are shown in Fig. 2 as a function of Δm^2 for the maximal mixing case ($\sin^2 2\theta = 1$). The two horizontal lines are the 1σ range of the experimental observation. The intercepts of the curves with this experimental range determine the allowed values of Δm^2 for the various cases. It is seen from Fig. 2 that the differences between the alternative cases (a - f) are comparatively marginal. There is a small spread in the lowest admissible values of Δm^2 in the different cases. But as the data improve even this will be removed. Since we have chosen $S(E)$ with a wide range of forms, it would not be unfair to conclude that the range of Δm^2 which is favoured by the K2K data can be predicted in a robust fashion.

Motivated by the best-fit to the atmospheric neutrino data, in this work we have used maximal mixing, *i.e.*, $\sin^2 2\theta = 1.0$, in (2). The SK data restrict the mixing angle rather tightly and at 90% C.L. one can allow $\sin^2 2\theta \gtrsim 0.9$. How much would our conclusions be affected if the lower limit of $\sin^2 2\theta$ is used in the analysis? We find, for example, for an $S(E)$ of the form of Case a (see Table 2) the best fit point moves to $(\alpha, \beta) = (2.39,\ 6.10)$[2]. The event rate as a function of Δm^2 for this case is also shown in Fig. 2. Notice that a small region around 2×10^{-2} eV^2 is allowed for this case. A large improvement in the K2K systematic and statistical uncertainties will be able to exclude this option.

IV Conclusion

The first results from the K2K long baseline experiment show a depletion in the number of events compared to the expectation. This is consistent with oscillations of the ν_μ to a neutrino of a different flavour with mass and mixing in the range favoured by the atmospheric neutrino results. In this note, we have shown that this conclusion is robust and is not altered even if the uncertainty in the initial neutrino spectrum, interaction cross-section, and detection efficiency are widely varied.

Note added: After the submission of the paper for publication, K2K announced results [8] based on 4.8×10^{19} p.o.t. They observe 56 events with a no-oscillation expectation of $80.1^{+6.2}_{-5.4}$. This corresponds to the depletion ratio (see Eq. (1)) $d = 0.699^{+0.051}_{-0.050}$, consistent with the earlier result. We find using our analysis that for the different parametrizations of $S(E)$ this corresponds to allowed ranges for Δm^2 shown in Table 4. It is gratifying that the best fit point found by the K2K group for maximal mixing ($\Delta m^2 = 2.8 \times 10^{-3}$ eV^2 [8]) lies within the allowed range for all parametrizations considered.

[2]This fit has $\chi^2 = 9.74 \times 10^{-3}$.

Case	Δm^2 Limits (10^{-3} eV^2)	
	Lower	Upper
a	1.57	3.61
b	1.60	3.72
c	1.52 22.48	3.40 25.06
d	1.47 21.77	3.73 26.38

Table 4: The ranges of Δm^2 allowed at 1σ for maximal mixing by the latest K2K results[8] for the alternative parametrizations of $S(E)$ (see Table 2). Notice that in cases (c) and (d) there are two allowed ranges.

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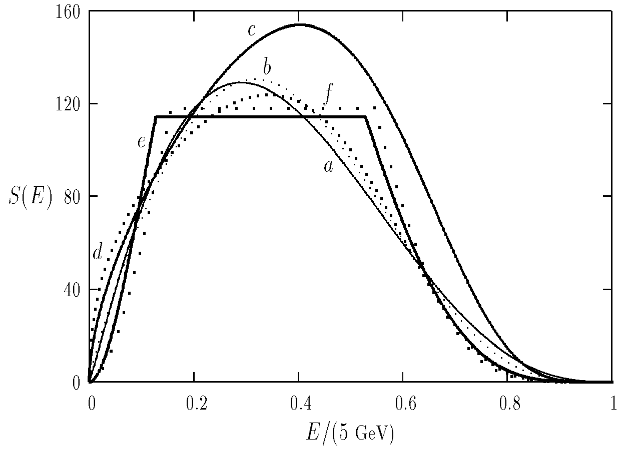

Figure 1: The spectrum $S(E)$ as a function of E. The cases to which the different curves correspond are indicated (see Tables 2 and 3).

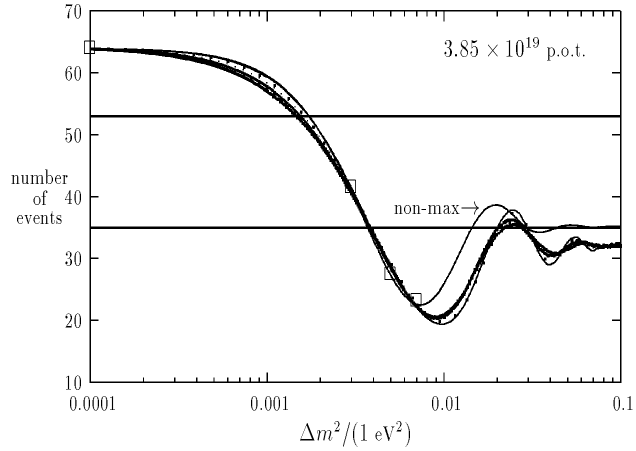

Figure 2: The number of events in the K2K experiment for an exposure of 3.85×10^{19} p.o.t. as a function of Δm^2 (at $\sin^2 2\theta = 1$). The region between the two horizontal lines is the current K2K measured range within 1 standard deviation. The boxes are the values from the K2K MC simulation (Table 1). The six curves obtained from the different forms for $S(E)$ considered in this work are shown (same conventions as in Fig. 1). The curve marked 'non-max' corresponds to the case of non-maximal mixing ($\sin^2 2\theta = 0.9$) discussed in the text.