

Spectroscopic factors for alpha decay in the $N_p N_n$ scheme

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Abstract

Lifetime values for alpha decay in even-even nuclei with $Z = 84 - 98$ and $N = 128 - 152$ have been calculated in the supersymmetric fission model. The interaction between the alpha particle and the daughter nucleus has been formed in the double folding approach using a density dependent NN interaction. The densities have been obtained using the Relativistic Mean Field formalism. The spectroscopic factors for the decays have been deduced and are shown to vary smoothly as a function of effective numbers of valence nucleons, N_p and N_n chosen with a suitable core. The implication of such a smooth behaviour has been discussed.

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Simplified parametrization of various nuclear quantities may be obtained if the quantities are plotted as a function of $N_p N_n$, the product of effective number of valance particles (or holes)[1]. Various quantities such as deformation and B(E2) values[2, 3, 4], rotational moments of inertia in low spin states in the rare earth

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region[5], ground band energy systematics [6], core cluster decomposition in the rare earth region[7] and properties of excited states [8, 9] have been found to follow certain simple trends when expressed as a function of simple product of N_p and N_n or certain simple functions of the above two numbers. Essentially this simple functions are seen to represent $n - p$ interaction and bear smooth relationships with the observables.

It has often been pointed out that in view of the change in magic number and shell structure in various mass regions, the conventional counting of valence protons and neutrons may be inadequate. Extraction of effective number of valence particles in the $N_p N_n$ scheme may significantly improve the predictive capability of the scheme as well as point to the emergence of new shell structure in different mass regions[1, 10]. For example, Zhao *et al.*[10] have used the concept of effective valence number of protons and neutrons to study the breakdown of the shell gap at $Z=64$ in even-even, odd and odd-odd nuclei. Wolf and Casten[11] also have studied the shell gap at $Z = 64$ for $N_p N_n$ scheme.

Alpha decay in the heavy actinide nuclei presents a unique opportunity of testing various nuclear structure theories. It is known to take place through tunnelling of the potential barrier by the alpha particle. The spectroscopic factor in α -decay was introduced to incorporate the preformation probability. It contains the nuclear structure effects, and may be thought as the overlap between the actual ground state configuration of the parent and the configuration described by one α -particle coupled to the ground state of the daughter. Thus the tunnelling probability of the alpha particle is a quantity which is expected to be crucially dependent on the $n - p$ interaction and the quantity spectroscopic factor may show a certain correlation when expressed as a function of N_p and N_n .

In the present work, we have followed the microscopic superasymmetric fission model. The interaction potential between the alpha particle and the daughter nucleus has been obtained in the double folding model by folding the densities of the alpha particle and the daughter nucleus using some suitable interaction. Usually the densities are obtained from phenomenological description. However, microscopic densities obtained from mean field approaches may be expected to provide a better description of the densities and hence that of the process of alpha decay.

Relativistic Mean Field (RMF) approach is now a standard tool in low energy nuclear structure. It has been able to explain different features of stable and exotic nuclei like ground state binding energy, deformation, radius, excited states, spin-orbit splitting, neutron halo, etc. For a detailed discussion, readers are referred to the refs. [12, 13, 14]. Later related developments in studies of ground state structure include incorporating the effect of the resonant continuum in drip line

nuclei in both mean field [15] and Hartree Bogoliubov approach[16], study of new Lagrangian densities with coupling between mesons[17], etc. It is well known that in nuclei far away from the stability valley, the single particle level structure undergoes certain changes in which the spin-orbit splitting plays an important role. Being based on the Dirac Lagrangian density, RMF is particularly suited to investigate these nuclei because it naturally incorporates the spin degrees of freedom.

There exist different variations of the Lagrangian density as well as a number of different parameterizations in RMF. Recently, a new Lagrangian density has been proposed[17] which involves self-coupling of the vector-isoscalar meson as well as coupling between the vector-isoscalar meson and the vector-isovector meson. The corresponding parameter set is called FSU Gold[17]. This Lagrangian density was earlier employed to obtain the proton nucleus interaction to successfully calculate the half life for proton radioactivity[18]. We have also applied it to study alpha radioactivity in superheavy nuclei with $A > 282$ [19] and cluster radioactivity[20] in the same approach followed in the present work. This density seems very appropriate for a large mass region *viz.* medium mass to superheavy nuclei. In this work also, we have employed FSU Gold.

In the conventional RMF+BCS approach for even-even nuclei, the Euler-Lagrange equations are solved under the assumptions of classical meson fields, time reversal symmetry, no-sea contribution, etc. Pairing is introduced under the BCS approximation. Since accuracy of the nuclear density is very important in our calculation, we have solved the equations in co-ordinate space. The strength of the zero range pairing force is taken as 300 MeV-fm for both protons and neutrons.

The microscopic density dependent M3Y interaction (DDM3Y) is obtained from a finite range nucleon interaction by introducing a density dependent factor. This class of interactions has been employed widely in the study of nucleon-nucleus as well as nucleus-nucleus scattering, calculation of proton radioactivity, etc. In this work, we have employed the exponential density dependent interaction DDM3Y1

$$v(r, \rho_1, \rho_2, E) = C(1 + \alpha \exp(-\beta(\rho_1 + \rho_2)))(1 - 0.002E)u^{M3Y}(r) \quad (1)$$

used in Ref. [21] to study alpha-nucleus scattering. It uses the direct M3Y potential $u^{M3Y}(r)$ based on the G -matrix elements of the Reid[22] NN potential. The weak energy dependence was introduced[23] to reproduce the empirical energy dependence of the optical potential. The parameters used are the standard values *viz.* $C = 0.2845$, $\alpha = 3.6391$ and $\beta = 2.9605\text{fm}^2$. Here ρ_1 and ρ_2 are the densities of the α -particle and the daughter nucleus and E is the energy per nucleon of the α -particle in MeV. This interaction has been double folded with the theoretical densities of alpha

particle and the daughter nucleus in their ground states using the code DFPOT[24]. The assault frequency has been calculated from the decay energy following Gambhir *et al.*[25].

We have studied the alpha decays between the ground states of even-even nuclei for parents with $Z = 84 - 98$ and $N = 128 - 152$. The spectroscopic factor has been calculated as the ratio of the calculated half life to the experimentally observed value. Obviously it is expected to be less than unity. The results for the spectroscopic factors are presented in Table 1. Theoretical calculations exist for spectroscopic factors for some of these nuclei. Our results are comparable to those values. For example, our calculated value for S of ^{212}Po is 0.019 as compared to theoretical value 0.025 deduced in [26]. A value of 0.031 was obtained by Mohr[27] in a double folding model calculation using density from experimentally known charge distribution.

The spectroscopic factors for different chains of isotopes generally follow a trend though there are exceptions. However, we find that expressed in terms of the Casten factor $P = N_p N_n / (N_p + N_n)$, the points, with some exceptions, show a smooth trend when the N_p and N_n values are, or in other words, the core is, chosen suitably. As earlier pointed out, this method has already been followed [1, 10]. However, the implications of such choices of N_p and N_n values are usually not always obvious. We will invoke a number different of subshells in various nuclei that are designed to produce a smooth trend and then give microscopic justifications for these choices later in the paper.

For Po($Z = 84$) and Rn($Z=86$) isotopes, the masses are very close to the doubly magic core $Z = 82, N = 126$. Thus N_p and N_n are chosen simply as $N_p = Z - 82$ and $N_n = N - 126$, respectively. However, the values for Ra($Z = 88$) isotopes fall on the smooth curve only if we choose $N_p = 4$. This can be justified on the assumption of $Z = 92$ as a semiclosed shell as explained later. Similarly, for Th($Z = 90$) nuclei, the effective value needed to be chosen is also $N_p = 4$. U($Z = 92$) nuclei present a somewhat different scenario. For the two lightest U nuclei studied, i.e. $^{224,226}\text{U}$, we choose $N_p = 2$ while for all the heavier isotopes, $N_p = 4$ is taken as the effective valence proton number. The valence neutron number is taken to be $N_n = N - 126$ in all the above nuclei. For Cm($Z = 96$) and Cf($Z = 98$) isotopes, we choose $N_p = 2$ and 4, respectively. For Cm isotopes, the valence neutron number is taken to be $N_n = N - 138$ while For Cf isotopes, it is taken as $N_n = 152 - N$. However Pu ($Z = 94$) nuclei could not be fitted in the scheme. Interestingly, this is analogous to the case of some nuclei with $N=90-92$ which could not be fitted to the $N_p N_n$ scheme for B(E2) values[10] and for the ratio between the excitation energies of the first 4^+ and 2^+ states[8]. The resulting plot for spectroscopic factors as a function of $P(\geq 1.0)$ is shown in Figure 1. The smooth line is drawn only for the purpose of

guiding the eye. The results for Pu have also been plotted in Figure 1. As results for spectroscopic factor of Pu did not fall on the smooth line in Figure 1, it is difficult to assign N_p and N_n values in the present procedure. We have assigned $N_p=2$ and $N_n = N - 126$ for all the Pu nuclei.

It will be interesting to trace the origin of the effective number of valence particles in different isotopes. As already explained, for Po and Ra nuclei, the number of valence particles is simply calculated as those outside the $^{208}_{82}\text{Pb}$ core. Our calculation suggests that the next proton single particle level is $\pi 1h_{9/2}$. This fact is also experimentally verified by the ground state spin-parity of the odd proton nuclei with $Z = 83 - 91$. Particularly, the ground states of the odd mass nuclei $^{187-215}\text{Bi}(Z = 83)$, $^{197-217}\text{At}(Z = 85)$, $^{201-219}\text{Fr}(Z = 87)$, $^{207-219}\text{Ac}(Z = 89)$, $^{213-221}\text{Pa}(Z = 91)$ all have spin-parity $9/2^-$, whenever known. Theoretically, we find an interesting situation. The level just below the 82 shell gap, *i.e.* $\pi 3s_{1/2}$ and the three levels above it, *i.e.* $\pi 1h_{9/2}$, $\pi 2f_{7/2}$ and $\pi 1i_{13/2}$ are shown in Figure 2 for nuclei with $N - Z = 44$. It can be easily seen that the $\pi 1h_{9/2}$ state is going down sharply with increasing Z , thus opening up another gap at $Z=92$. Earlier mean field calculations of Rutz *et al.*[28] also suggested a shell gap at $Z = 92$ in the vicinity of nuclei with $N = 126$. Hence, in Ra nuclei protons are hole-like and N_p is taken as 4.

In nuclei with higher Z , there is a possibility of another gap opening up near $Z = 94$. Nuclei with $Z = 84 - 88$ that we have studied have small deformation in their ground state. This is evident from the measured $B(E2)$ values. However, near $Z = 92$, nuclei have large deformation. There is a possibility of new gaps opening up in this region for deformed nuclei. We should mention here that deformed shell gaps are not usually employed in the $N_p N_n$ scheme. In an earlier calculation with RMF, Long *et al.*[29] have shown that in nuclei with deformation $\beta \sim 0.3$ and with Z and N near 96 and 144, respectively, gaps of the order of 1.5-2 MeV appear at $Z = 94, 96$. There is a similar example in the neutron sector. Rare-earth nuclei such as Hf, Lu, etc are known to exhibit a deformed shell gap at $N = 94$. This possibly is the reason that Cm and Cf isotopes have $N_p = 2$ and 4, respectively while Th with $Z = 90$ has 4 proton holes. However, our calculation does not include the effects of deformation and it is difficult to be absolutely certain of this explanation. This fact also may explain why $^{224,226}\text{U}$ have $N_p = 2$ but fails for the heavier U nuclei where it was necessary to assign $N_p = 4$.

A similar situation also exists in the neutron sector. In nuclei with neutron number $N > 126$, the first level beyond $N = 126$ is calculated to be the $\nu 1i_{11/2}$ state. In lighter nuclei this state is close to the other states in the same shell. However, as neutron number increases, it comes down in energy rapidly opening a

gap between it and the other states in the same shell. For example, in ^{234}Cm , this gap is more than 3.8 MeV, a value more than double of that observed in ^{218}Po . Thus in Cm, $N_n = 138$ may act a subshell closure and the number of valence nucleons come out as $N = 138$. This cannot however explain why for Cf isotopes, it is necessary to have the number of valence neutrons at $N_n = N - 152$. This may possibly be another effect of deformation.

We expect that the observed smooth behaviour of the spectroscopic factor as a function of the parameter P should be reflected in some other properties also if the parameter P basically is a measure of proton-neutron interaction. With this in mind, we have chosen to study the ground state binding energy of the nuclei in this region. Our method is as follows. It is well known that correlations beyond mean field results are due principally to residual two body interaction. The residual interaction between similar nucleons is described by the zero range pairing force in the present calculation. However, the residual $n - p$ interaction has not been considered in the present calculation. So for a chain of isotopes, the difference between the calculated and the experimental binding energies may be a measure of the strength of $n - p$ interaction in a particular nucleus. However, not all differences can be ascribed to this effect. We have chosen one nucleus for each Z where, our scheme suggests that the number of valence neutrons is zero *i.e.* there is no valence $n - p$ pair. We expect the effect of $n - p$ interaction to be small in these nuclei and the difference between the calculated and experimental binding energies in these nuclei to be due to all the other effects combined. The difference in the change in the binding energy from the isotope with $N_n = 0$ for a particular Z between theory and experiment is taken as a measure of the contribution of $N_p N_n$ interaction and expressed as $\Delta_{\nu\pi}$. This difference has been plotted as function of P in Figure 3 in a scatter diagram. One can see a similar trend as observed in the case of spectroscopic factors particularly in the horizontal trend for low P values and the sharply upsloping values beyond $P = 3$. The results for Pu are somewhat ambiguous for two reasons. Firstly, as has already been pointed out, the N_p and N_n values are somewhat arbitrary. Secondly, the experimental binding energy value for Pu with $N_n = 0$ *i.e.* $N = 126$ is not known and we had to use an extrapolated value. In this case also, Pu nuclei do not conform to the general trend. However, the Cm isotopes also do not follow it. We have checked that the $B(E2)$ values for $2^+ \rightarrow 0^+$ also show a similar type of behaviour, indicating that the quadrupole deformation starts to increase beyond $P = 3$, consistent with the fact that the proton neutron interaction is the major cause of deformation.

To summarize, microscopic superasymmetric fission model has been employed to calculate the lifetime of alpha decay in even-even nuclei with $Z = 84 - 98$ and

$N = 128 - 152$. The interaction between the alpha particle and the daughter nucleus has been calculated in the double folding model using a density dependent NN interaction. The densities have been obtained using the Relativistic Mean Field formalism with the Lagrangian density FSU Gold. The spectroscopic factors for the decays are shown to vary smoothly as a function of effective numbers of valence nucleons, N_p and N_n . However, for a useful description, it was often necessary either to change the usual spherical subshell definitions or to utilize deformed subshells in counting N_p and N_n . The proton level $\pi h_{9/2}$ and the neutron level $\nu i_{11/2}$ occur just after the closed shells at $Z = 82$ and $N = 126$, respectively and act sometimes as subshells. Binding energy systematics also follow a similar smooth curve indicating the role of the residual $n - p$ interaction in spectroscopic factor. In view of the observed smooth behaviour, in future it will be interesting to investigate the origin of the observed effective valence nucleon numbers in more detail, particularly around $Z = 94$ and $N = 152$.

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Table 1: Logarithm of experimental half life values and spectroscopic factors (S) of alpha decay obtained in the present calculation.

Parent	$\log T_{ex}(s)$	S	Parent	$\log T_{ex}(s)$	S
^{212}Po	-6.524	0.019	^{232}U	9.337	0.149
^{214}Po	-3.784	0.034	^{234}U	12.889	0.142
^{216}Po	-0.839	0.046	^{236}U	14.868	0.166
^{218}Po	2.269	0.055	^{238}U	17.149	0.226
^{216}Rn	-4.347	0.059	^{228}Pu	0.041	0.015
^{218}Rn	-1.456	0.070	^{230}Pu	2.084	0.079
^{220}Rn	1.745	0.084	^{232}Pu	3.997	0.075
^{222}Rn	5.519	0.095	^{234}Pu	5.723	0.094
^{220}Ra	-1.745	0.063	^{236}Pu	7.955	0.089
^{222}Ra	1.580	0.070	^{238}Pu	9.442	0.088
^{224}Ra	5.500	0.092	^{240}Pu	11.316	0.135
^{226}Ra	10.703	0.125	^{242}Pu	13.073	0.115
^{218}Th	-6.931	0.039	^{244}Pu	15.402	0.102
^{220}Th	-5.013	0.068	^{240}Cm	6.370	0.055
^{222}Th	-2.552	0.049	^{242}Cm	7.148	0.066
^{224}Th	0.021	0.074	^{244}Cm	8.756	0.062
^{226}Th	3.263	0.101	^{246}Cm	11.176	0.066
^{228}Th	7.780	0.128	^{248}Cm	13.078	0.072
^{230}Th	12.376	0.162	^{242}Cf	2.443	0.056
^{232}Th	17.646	0.225	^{244}Cf	3.066	0.065
^{224}U	-3.046	0.030	^{246}Cf	5.109	0.048
^{226}U	-0.456	0.055	^{248}Cf	7.460	0.043
^{230}U	6.255	0.127	^{250}Cf	8.615	0.041

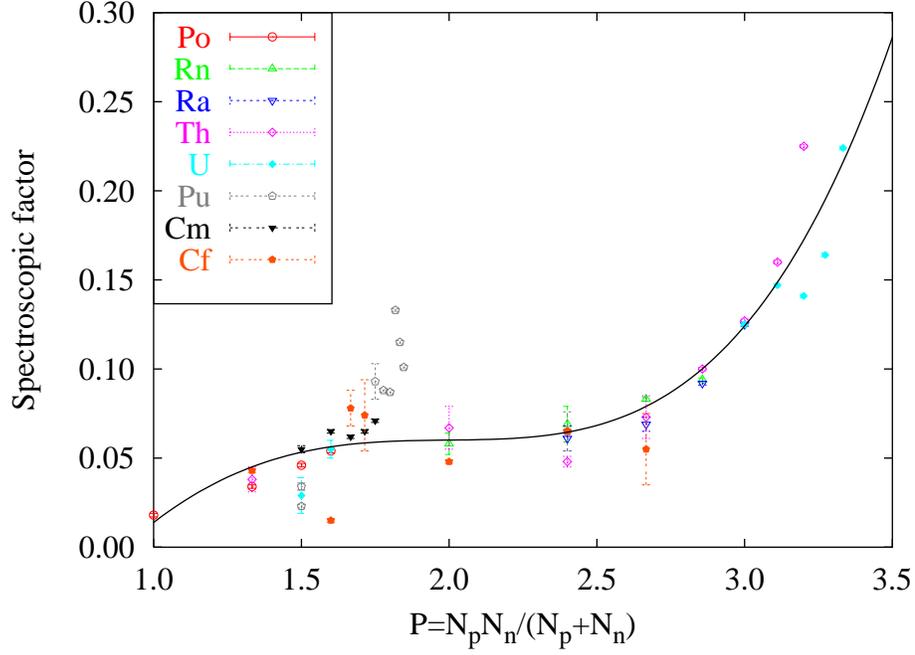


Figure 1: Spectroscopic factors S as a function of $P = N_p N_n / (N_p + N_n)$ for even-even nuclei between $Z=84$ and 98 . The N_p and N_n values for different isotopes are explained in the text.

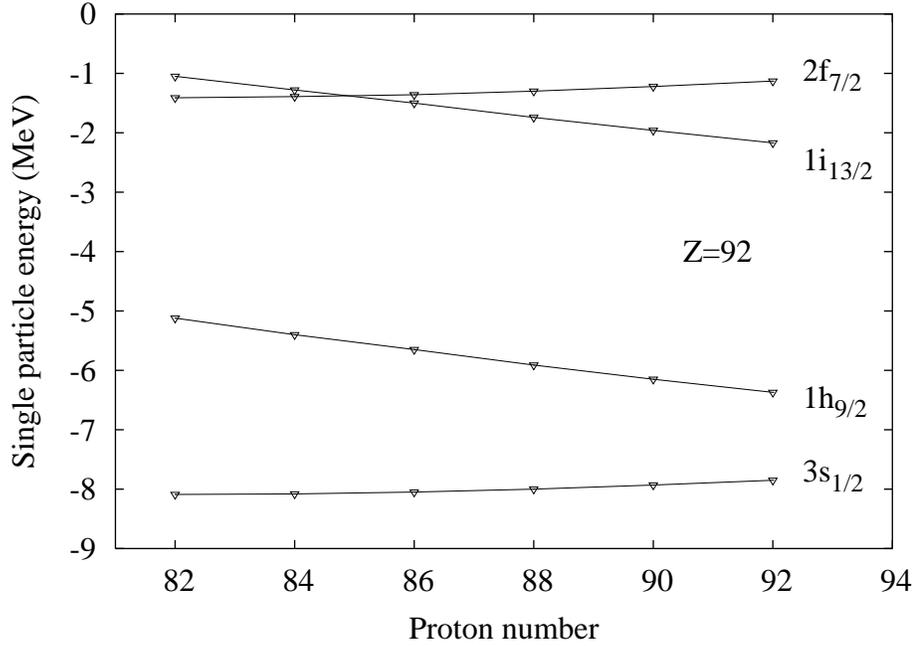


Figure 2: Calculated single particle proton levels near the proton shell closure $Z=82$ for nuclei with $N - Z = 44$.

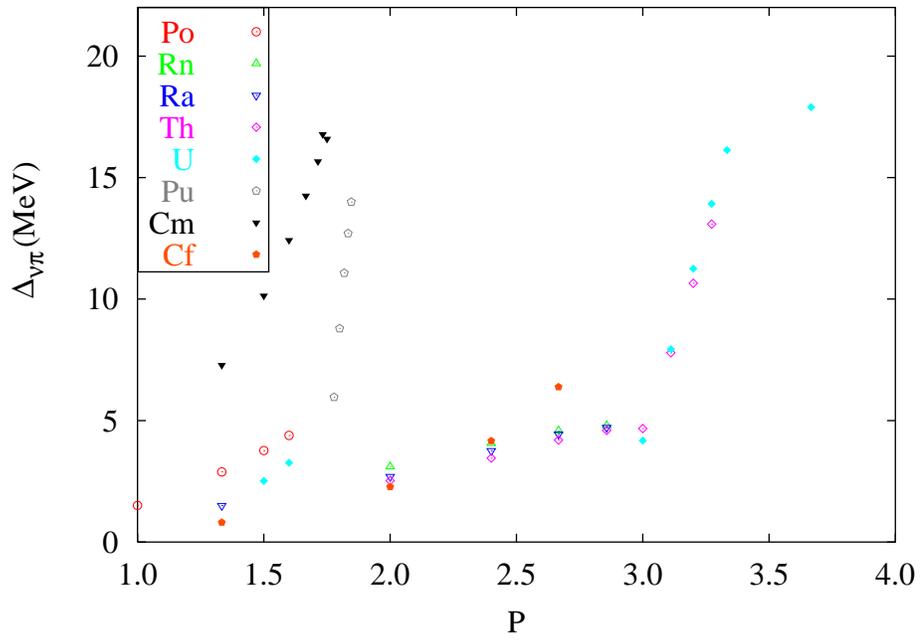


Figure 3: The quantity $\Delta_{\nu\pi}$ plotted as a function of P for even-even nuclei between $Z=84$ and 98 . See text for details.