Revealing Hidden Genuine Tripartite Nonlocality

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Nonlocal correlations arising from measurements on tripartite entangled states can be classified into two groups, one genuinely 3−way nonlocal and other local with respect to some bipartition. Still, whether a genuinely tripartite entangled quantum state can exhibit genuine 3−way nonlocality, remains a challenging problem so far as measurement context is concerned. Here we introduce a novel approach in this regard. We consider three tripartite quantum states none of which is genuinely 3−way nonlocal in a specific Bell scenario (three parties, two measurements per party, two outcomes per measurement), but they can exhibit genuine 3−way nonlocality when the initial states are subjected to stochastic local operations and classical communication (SLOCC). So, genuine 3−way nonlocality is a resource, which can be revealed by using a sequence of measurements.

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I. INTRODUCTION

The seminal work of J. S. Bell refuted EPR argument [1] claiming incompliances of Quantum theory. He in particular showed that there exist some correlations generated by measurements on a quantum system shared between distant parties that cannot be explained by any local hidden variable (LHV) theory [2]. Such type of correlations, referred to as nonlocal correlations, are witnessed via violation of a Bell inequality [3]. Apart from its importance as a foundational concept, nonlocality has also been used in various information-theoretic tasks [4–10]. For generation of nonlocal correlations, the quantum particles shared between distant parties must be entangled. However, the converse implication is not obvious. To be specific, though nonlocality can be considered as a generic notion for pure states [11, 12], no such definite conclusion can be drawn for mixed states, as initially shown by Werner who presented a class of bipartite entangled states admitting a LHV model in the particular case of projective measurement [13]. This model was later extended for general (positive-operator-valued-measurement, POVM) measurements [14] (see also [15]). Such states are referred to as local entangled states [16]. In this context, another important topic was discussed by Popescu[17] and Gisin [18] who showed that some local entangled states, unable to produce nonlocal correlations under projective measurements, when subjected to suitable sequential measurements, can exhibit nonlocal behavior (violates the Bell-CHSH inequality [19]). This process of revelation(or activation) of nonlocality of any state is referred to as its hidden nonlocality. In recent times it is shown that hidden nonlocality can be extracted even from those entangled states that admit a LHV model for POVMs [20]. There exist some other related works in the literature showing revelation of nonlocality of local entangled states by performing joint measurements on several copies of the state [21–27], or by placing many copies of the state in a quantum network [28–32]. All of these works simply point out the fact that context of measurement is important to reveal nonlocality of quantum states and ongoing research activities in this direction imply that it is still a challenging field of research.

Though questions related to revelation of hidden nonlocality of local entangled states, have been extensively discussed for bipartite states, the relation between entanglement and hidden nonlocality for multipartite systems is almost unexplored so far. For multipartite scenario, one should intuitively expect some more interesting and novel phenomena, due to the complex structure of multipartite entanglement. In particular, there is a hierarchy of different notions of entanglement in tripartite systems, the strongest of them being genuine tripartite entanglement (GTE) [33]. Analogous to entanglement in tripartite scenario, notion of genuine tripartite nonlocality (GTNL), discussed in [34–36], represents the strongest form of nonlocality for tripartite systems.

Now one may be interested to analyze whether hidden GTNL can be revealed under sequential measurements. In this context, Caban et al. [37] gave an example of a class of tripartite pure states ρ such that it does not violate the Svetlichny inequality [34] whereas $\rho \bigotimes \rho$ can violate it and hence can exhibit Svetlichny's notion of GTNL. They however referred this phenomenon as activation of violation of Svetlichny inequality. Recently a weaker (than Svetlichny's notion of GTNL) definition of GTNL has been introduced in [35, 36], known as genuine 3−way NS nonlocality (NS_2 nonlocality), which is better motivated both physically and from information theoretic view point. In this paper, we address the following question: consider some genuinely tripartite entangled states that do not exhibit NS_2 nonlocality individually in a spe-

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cific Bell scenario (three parties, two measurements per party, two outcomes per measurement) and also in hidden sense, i.e., even after being subjected to known useful local filters [38]. Is it then possible to find some sequential measurement protocol so that the final state resulting from the measurement protocol using these NS_2 local states, exhibits NS_2 nonlocality? We provide strong numerical evidence to this open problem. To be precise, we have framed a protocol based on sequential measurements which we refer to as sequential measurement protocol (SMP, see Fig.1). It involves three different tripartite quantum states. These three states, none of which was individually NS_2 nonlocal in the specific Bell scenario and not even after application of known useful local filters, when used in the SMP, generates a quantum state which is NS_2 nonlocal. However, as NS_2 nonlocality of the final state is revealed starting from NS_2 local initial states in the specific Bell scenario, so such revelation of hidden NS_2 nonlocality can be considered as revelation of weak hidden nonlocality. Moreover, the SMP can be used in principle even in the case when each of the states initially possessed by the parties has arbitrary amount of genuine entanglement.
Rest of our paper is organized as follows. In Sec. II, we give a brief introduction to some concepts and results which we will use in later sections. We introduce the sequential measurement protocol in Sec. III together with the states used in the protocol to exhibit hidden GTNL. In Sec. IV we discuss our observations in the context of revealing hidden GTNL. Finally we conclude in Sec.V discussing various aspects of our findings along with scope of future research works.

II. PRELIMINARIES

Before starting our discussion we provide all notions and facts necessary for further considerations.

A. Genuine tripartite nonlocality

To analyze the nature of correlations shared between three systems, different forms of nonlocality can be considered. The local tripartite correlations have the form:

$$P(a,b,c|x,y,z) = \sum_{\lambda} q_{\lambda} P_{\lambda}(a|x) P_{\lambda}(b|y) P_{\lambda}(c|z), \quad (1)$$

where a, b, c $\in \{0,1\}$ denote the outputs and x, y, z $\in \{0,1\}$ denote inputs of the parties Alice, Bob and Charlie respectively, $0 \leq q_{\lambda} \leq 1$ and $\sum_{\lambda} q_{\lambda} = 1$. $P_{\lambda}(a|x)$ is the conditional probability of getting outcome a when the measurement setting is x and λ is the hidden state; $P_{\lambda}(b|y)$ and $P_{\lambda}(c|z)$ are similarly defined. Tripartite correlations that cannot be written as in Eq.(1) are called nonlocal. Bell type inequalities based on local realism (Eq.(1)) fail to distinguish between bipartite and tripartite nonlocality [39–41]. In order to detect GTNL,

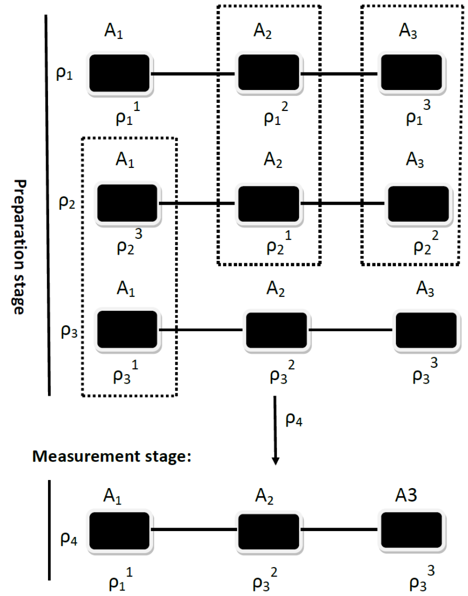

FIG. 1: *The figure depicts a particular sequential measurement protocol involving three parties. ρ_i^j denotes j^{th} particle of i^{th} state. Three states ρ_1 (Eq.(7)), ρ_2 (Eq.(8)) and ρ_3 (Eq.(9)) are distributed between the three parties A_1, A_2 and A_3 such that each of the three parties holds one particle from each of the three states. A_1 holds particles ρ_1^1, ρ_2^3 and ρ_3^1; A_2 holds ρ_1^2, ρ_2^1 and ρ_3^2 and A_3 holds particles ρ_1^3, ρ_2^2 and ρ_3^3. The sequential measurement protocol(SMP) is a particular example of WCCPI protocol. In the preparation stage, each of them performs Bell basis measurement on two of their respective three particles: A_1 performs Bell basis measurement on ρ_2^3 and ρ_3^1; A_2 performs Bell basis measurement on ρ_1^2, ρ_2^1 and A_3 performs Bell basis measurement on ρ_1^3 and ρ_2^2. Each bell-basis measurement is denoted by dotted box. Due to Bell basis measurements by each of the three parties and then communication of the results among themselves, resultant state ρ_4 (Eq.(10)) is generated at the end of the preparation stage. ρ_4 (Eq.(10)) is shared between A_1, A_2 and A_3. In the measurement stage, each of the parties A_1, A_2 and A_3 perform arbitrary projective measurements on their respective qubits of state ρ_4 (Eq.(10)). At the end of the measurement stage tripartite correlations will be generated in the SMP.*

Svetlichny introduced hybrid local-nonlocal form of correlations [34]:

$$P(abc|xyz) = \sum_{\lambda} q_{\lambda} P_{\lambda}(ab|xy) P_{\lambda}(c|z) +$$

$$\sum_{\mu} q_{\mu} P_{\mu}(ac|xz) P_{\mu}(b|y) + \sum_{\nu} q_{\nu} P_{\nu}(bc|yz) P_{\nu}(a|x); \quad (2)$$

where $0 \leq q_{\lambda}$, q_{μ}, q_{ν} ≤ 1 and $\sum_{\lambda} q_{\lambda} + \sum_{\mu} q_{\mu} + \sum_{\nu} q_{\nu} = 1$. This form of correlations are referred as Svetlichny local (S_2 local), otherwise Svetlichny nonlocal (S_2 nonlocal) [36]. Based on this, Svetlichny designed a tripartite Bell type inequality (known as Svetlichny inequality):

$$S \leq 4. \quad (3)$$

where $S = \langle x_0y_0z_0\rangle + \langle x_1y_0z_0\rangle - \langle x_0y_1z_0\rangle + \langle x_1y_1z_0\rangle$

$$+\langle x_0y_0z_1\rangle - \langle x_1y_0z_1\rangle + \langle x_0y_1z_1\rangle + \langle x_1y_1z_1\rangle.$$

Violation of this inequality guarantees S_2 nonlocality, sufficient to detect GTNL. While Svetlichny's notion of GTNL is often referred to in the literature, it has certain drawbacks. As has been pointed out in [35, 36, 42], Svetlichny's notion of GTNL is so general that correlations capable of two-way signaling are allowed among some parties. This may lead to grandfather-style paradoxes [36] and provide inconsistency in operational purposes [35, 43]. To remove this ambiguity, Bancal et al.[36], introduced genuine 3−way NS nonlocality (NS_2 nonlocality). Suppose $P(abc|xyz)$ be the tripartite correlation satisfying Eq.(2) with non-signalling criteria imposed on the bipartite correlations terms,

$$P_\lambda(a|x) = \sum_b P_\lambda(ab|xy) \quad \forall\, a,\, x,\, y, \tag{4}$$

$$P_\lambda(b|y) = \sum_a P_\lambda(ab|xy) \quad \forall\, b, x,\, y. \tag{5}$$

and similarly for $P_\mu(ac|xz)$ and $P_\nu(bc|yz)$. This form of correlations are called NS_2 local. Otherwise, they are NS_2 nonlocal. In analyzing the procedure of revelation of hidden GTNL, we have used the necessary and sufficient criteria for detecting GTNL provided by the whole set of 185 facet inequalities of the NS_2 local polytope in the presence of binary input and output (see Supplementary Material of [36]). Svetlichny inequality constitutes the 185th facet inequality. Throughout the paper we have used projective measurements to check nature of correlations generated by some tripartite quantum states.

B. Wirings And Classical Communication Prior To The Inputs(WCCPI Protocol)

This protocol may be considered as a set of allowed operations that cannot create nonlocality i.e., interpret nonlocality as a resource, analogous to entanglement which cannot be created by Local Operations and Classical communication(LOCC). This type of protocol was first used in [35] for framing multipartite nonlocality as a resource. The protocol introduced there involved single measurement. Later it was extended for sequential measurements in [44]. A *valid* WCCPI protocol for sequential measurements[44], characterizing basically correlation terms generated in any sequential scenario, mainly consists of two stages: *preparation stage* and *measurement stage*. In the preparation stage the parties are allowed to perform measurements on their respective physical systems and then communicate the corresponding outputs among each other. As the parties have not yet received any input for the final Bell test(going to take place in the measurement stage), classical communication is allowed in the preparation stage. However, this communication cannot be used to generate any sort of nonlocal correlations. The inputs of the parties for the final stage, i.e., the measurement stage depend on outputs that are obtained and communicated in the preparation stage. In the measurement stage no further communication is allowed between the parties. The permissible local operations of each party consist of processing the classical inputs and outputs and are referred to as *wirings*. The sequential correlations generated at the end of the measurement stage help in characterizing nonlocality as a resource. As already discussed before, nonlocality cannot be created by WCCPI. So GTNL cannot also be created by WCCPI protocol. In our present topic of discussion, we have introduced a measurement protocol which may be considered as a WCCPI protocol.

C. Genuine multipartite concurrence (C_{GM})

We briefly now describe C_{GM}, a measure of genuine multipartite entanglement. For pure n-partite states($|\psi\rangle$), this measure defined as [45] : $C_{GM}(|\psi\rangle) := \min_j \sqrt{2(1-\Pi_j(|\psi\rangle))}$ where $\Pi_j(|\psi\rangle)$ is the purity of j^{th} bipartition of $|\psi\rangle$. The expression of C_{GM} for X states are given in [46]. For tripartite X states,

$$C_{GM} = 2\max_i\{0, |\gamma_i| - w_i\} \tag{6}$$

with $w_i = \sum_{j\neq i}\sqrt{a_jb_j}$ where a_j, b_j and $\gamma_j(j = 1, 2, 3, 4)$ are the elements of the density matrix of tripartite X state:

$$\begin{bmatrix} a_1 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_1 \\ 0 & a_2 & 0 & 0 & 0 & 0 & \gamma_2 & 0 \\ 0 & 0 & a_3 & 0 & 0 & \gamma_3 & 0 & 0 \\ 0 & 0 & 0 & a_4 & \gamma_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & {\gamma_4}^* & b_4 & 0 & 0 & 0 \\ 0 & 0 & {\gamma_3}^* & 0 & & b_3 & 0 & 0 \\ 0 & {\gamma_2}^* & 0 & 0 & 0 & 0 & b_2 & 0 \\ {\gamma_1}^* & 0 & 0 & 0 & 0 & 0 & 0 & b_1 \end{bmatrix}$$

III. SEQUENTIAL MEASUREMENT PROTOCOL

Consider a measurement protocol connecting three distant observers $A_i(i = 1, 2, 3)$. n tripartite quantum states $\rho_j(j = 1, 2, ..., n)$ can be used in the protocol. Let each of n states $\rho_j(j = 1, 2, ..., n)$ fails to reveal GTNL in the specific Bell scenario. Each of these n states $\rho_j(j = 1, 2, ..., n)$ can be distributed between the three parties $A_i(i = 1, 2, 3)$ with some specification in distribution of qubits among the parties such that each of the three parties holds one particle from each of the n states. So each of the parties holds n qubits in his lab. This protocol is a particular example of WCCPI protocol. In the preparation stage, each party can perform some joint measurement on their respective $n-1$ particles and then communicate the results between themselves. At the end of measurements by all the three parties, ρ_{n+1}, a tripartite quantum state shared between A_1, A_2 and A_3, is generated. Clearly, as in any WCCPI protocol, the state ρ_{n+1} is output specific, i.e., depends on the output of the joint measurements performed by the parties in the preparation stage. In the measurement stage of the protocol, each of the three parties can now perform arbitrary projective measurements on their share of the physical system ρ_{n+1} but are not allowed to communicate among themselves thereby generating tripartite correlation terms whose nature can now be tested using some tripartite Bell inequality. We refer to this protocol of sequential measurements by the three parties sharing n states as Sequential Measurement Protocol (SMP). Now we have already discussed before that GTNL cannot be created by WCCPI protocol. Hence generation of GTNL by SMP, starting from three local initial states, guarantee revelation of hidden GTNL by our SMP. Our SMP can be considered as a particular type of sequential measurement protocol via which hidden GTNL can be revealed, analogous to the sequential measurement protocol introduced by Popescu for revealing hidden bipartite nonlocality [17]. We provide an explicit example of revelation of hidden GTNL for $n=3$ by using our SMP(see Fig.1). Suppose the three initial states shared between the three parties be given by:

$$\rho_1 = p_1|\psi_f\rangle\langle\psi_f| + (1-p_1)|001\rangle\langle 001| \tag{7}$$

with $|\psi_f\rangle = \cos\theta_1|000\rangle + \sin\theta_1|111\rangle$, $0 \leq \theta_1 \leq \frac{\pi}{4}$ and $0 \leq p_1 \leq 1$;

$$\rho_2 = p_2|\psi_m\rangle\langle\psi_m| + (1-p_2)|010\rangle\langle 010| \tag{8}$$

with $|\psi_m\rangle = \frac{|000\rangle+|111\rangle}{\sqrt{2}}$ and $0 \leq p_2 \leq 1$;

$$\rho_3 = p_3|\psi_l\rangle\langle\psi_l| + (1-p_3)|100\rangle\langle 100| \tag{9}$$

with $|\psi_l\rangle = \sin\theta_3|000\rangle + \cos\theta_3|111\rangle$, $0 \leq \theta_3 \leq \frac{\pi}{4}$ and $0 \leq p_3 \leq 1$. The i-th particle of each of $\rho_1(\rho_1^i)$ (Eq.(7)) and $\rho_3(\rho_3^i)$ is with the party $A_i(i=1,2,3)$ whereas the three particles of ρ_2, i.e., ρ_2^1, ρ_2^2 and ρ_2^3 are with parties A_2, A_3 and A_1 respectively. Hence each of the three parties $A_i(i=1,2,3)$ has three particles. Now in the preparation stage of the SMP, each of the three parties $A_i(i=1,2,3)$ performs Bell basis measurements on two of the three particles that each of them holds: A_1 performs Bell basis measurement on 3^{rd} particle of $\rho_2(\rho_2^3)$ and 1^{st} particle of $\rho_3(\rho_3^1)$; A_2 performs Bell basis measurement on 2^{nd} particle of $\rho_1(\rho_1^2)$ and 1^{st} particle of $\rho_2(\rho_2^1)$; A_3 performs Bell basis measurement on 3^{rd} particle of $\rho_1(\rho_1^3)$ and 2^{nd} particle of $\rho_2(\rho_2^2)$. After all the three parties have performed Bell basis measurement on their respective particles, they communicate the results among themselves, as a result of which ρ_4 is generated at the end of the preparation stage. If the output of each of the measurement is $|\psi^{\pm}\rangle(\frac{|01\rangle\pm|10\rangle}{\sqrt{2}})$, the resultant state (correcting phase term) is given by:

$$\rho_4 = \frac{p_3|\phi\rangle\langle\phi| + (1-p_3)\sin^2\theta_1|100\rangle\langle 100|}{\sin^2\theta_1 + p_3\cos 2\theta_1\sin^2\theta_3} \tag{10}$$

where $|\phi\rangle = \cos\theta_1\sin\theta_3|000\rangle + \sin\theta_1\cos\theta_3|111\rangle$. Eq.(10) points out that ρ_4 is independent of p_1 and p_2. Clearly the final state ρ_4 is obtained from the initial states $\rho_i(i=1,2,3)$ by means of post-selecting on particular results ($|\psi^{\pm}\rangle$) of local measurements. So preparation stage of this protocol can be considered as a particular instance of Stochastic Local Operations And Classical Communication (SLOCC). After ρ_4 is generated and shared between the parties in the preparation stage, each of the three parties A_1, A_2 and A_3 performs projective measurement on the state ρ_4 in the measurement stage. Now if the correlations generated from ρ_4 exhibits GTNL under the context that the initial states $\rho_i(i=1,2,3)$ fail to reveal the same, then that guarantees generation of hidden GTNL in the SMP. However ρ_4 can be generated for some other specification of SMP protocol also, specially for some different arrangement of particles between the parties $A_i(1,2,3)$ and for different outputs of Bell measurements. Having designed the SMP, we are now going to present our results.

IV. REVELATION OF HIDDEN GENUINE TRIPARTITE NONLOCALITY

In this section we discuss in details our observations which guarantee that the SMP introduced in the last section helps in revealing hidden GTNL. For that we consider two different notions of hidden GTNL: hidden S_2 nonlocality and hidden NS_2 nonlocality. Firstly we consider the former notion.

A. Revelation of hidden Svetlichny nonlocality

Existence of hidden S_2 nonlocality will be guaranteed if we can transform S_2 local $\rho_i(i=1,2,3)$ to ρ_4, capable of violating Eq.(3). Below we will show that the final state ρ_4, resulting from the preparation stage of the SMP, exhibits S_2 nonlocality, though the initial states $\rho_i(i=1,2,3)$ are S_2 local. The maximum value of the Svetlichny operator(S) upto projective measurements, for state $\rho_i(i=1,2,3)$ is given by (see Appendix A) :

$$B_1 = \max[4\sqrt{2}p_1\sin 2\theta_1, 4|(1-p_1-p_1\cos 2\theta_1)|],$$

$$B_2 = \max[4\sqrt{2}p_2, 4(1-p_2)]$$

and

$$B_3 = \max[4\sqrt{2}p_3\sin 2\theta_3, 4|(1-p_3-p_3\cos 2\theta_3)|] \tag{11}$$

respectively whereas that for the final state ρ_4, it is given by:

$$B_4 = \max[\frac{2\sqrt{2}\,p_3\sin 2\theta_1\sin 2\theta_3}{\sin^2\theta_1 + p_3\cos 2\theta_1\sin^2\theta_3},$$

$$\frac{2|(1-2p_3\sin^2\theta_3-\cos\theta_1)|}{\sin^2\theta_1 + p_3\cos 2\theta_1\sin^2\theta_3}]. \quad (12)$$

Since both the initial ($\rho_i, i = 1,2,3$) and final states (ρ_4) belong to the class of tripartite X states, their amount of genuine entanglement can be measured by Eq.(6). For the initial states $\rho_i(i = 1,2,3)$, the amount of GTE are given by:

$$C_{GM}^{\rho_1} = p_1\sin 2\theta_1,$$

$$C_{GM}^{\rho_2} = p_2$$

and

$$C_{GM}^{\rho_3} = p_3\sin 2\theta_3 \quad (13)$$

whereas that for ρ_4 is given by:

$$C_{GM}^{\rho_4} = \frac{p_3\sin 2\theta_1\sin 2\theta_3}{2(\sin^2\theta_1 + p_3\cos 2\theta_1\sin^2\theta_3)}. \quad (14)$$

The initial states $\rho_i(i = 1,2,3)$ are genuinely entangled for any nonzero value of the state parameters (Eq.(13)). It is clear from the maximum value of Svetlichny operator(Eqs.(11), (12)) and the measure of entanglement (Eqs.(13), (14)) of both initial states and final state, that each of them is S_2 local for $C_{GM}^{\rho_i} \leq \frac{1}{\sqrt{2}}(i = 1,2,3,4)$. Thus existence of hidden S_2 nonlocality can be observed if for some fixed values of the parameters of the three initial S_2 local states ($C_{GM}^{\rho_i} \leq \frac{1}{\sqrt{2}}$), the final state can have $C_{GM}^{\rho_4} > \frac{1}{\sqrt{2}}$. Now for $\theta_1 = 0.1$, $p_2 \leq \frac{1}{\sqrt{2}}$, $\theta_3 = 0.144$ and $p_1\,,p_3 \in [0,1]$, each of the initial states is S_2 local ($C_{GM}^{\rho_i} \leq \frac{1}{\sqrt{2}}$) whereas the resultant state ρ_4 violates Svetlichny inequality ($C_{GM}^{\rho_4} > \frac{1}{\sqrt{2}}$) for $p_3 \geq 0.5055$. In this explicit example, initial genuinely tripartite entangled states do not violate Svetlichny inequality but when used in preparation stage of our SMP, they can generate a state which exhibits S_2 nonlocality. This guarantees existence of hidden S_2 nonlocality for $p_3 \in [0.5055, 1]$ (See Fig.2).

Now use of local filters is known to be a standard method to reveal hidden nonlocality. Interestingly, our SMP can reveal hidden S_2 nonlocality using some initial states which are even incapable of exhibiting hidden S_2 nonlocality(i.e., cannot reveal S_2 nonlocality after being subjected to known useful local filters [38]). We proceed forward with an example. Let known useful local filters be applied on each of the three initial states $\rho_i(i = 1,2,3)$ to reveal hidden S_2 nonlocality of the individual state. The maximum value of Svetlichny operator S (Eq.(3)), in terms of state parameters, for each of the three states $\rho_i(i = 1,2,3)$, after applying known useful local filters, are derived(see Appendix B). Maximum values of S, in turn, provide constraints on the state parameters such that each of initial states ρ_i, has no S_2 nonlocality even after being subjected to local filtering. For a particular instance, when $\theta_1 = 0.1$, ρ_1, after being filtered, remains still S_2 local for $p_1 \in [0, 0.5025]$. Similarly second state(ρ_2), after being subjected to filtering remains S_2 local for $p_2 \in [0, 0.6666]$, but the range of p_3 for which ρ_3 exhibits S_2 nonlocality remains unaltered both before and after filtering when $\theta_3 = 0.144$ (see Appendix B). Hence each of the initial states $\rho_i(i = 1,2,3)$, under some restricted range of state parameters, has no hidden S_2 nonlocality. Now if these initial states under the said restricted range are used in the initial stage(preparation stage) of our SMP then S_2 nonlocality will be revealed for $p_3 \in [0.5055, 1]$. However, this range of revelation of hidden S_2 nonlocality in our SMP remains the same when the states $\rho_i(i = 1,2,3)$ are used without being filtered. This example thus suffices to justify our claim that our SMP can reveal hidden S_2 nonlocality even from some initial states which have no hidden S_2 nonlocality. This in turn points out the utility of SMP over the standard procedure of using local filters for revelation of hidden S_2 nonlocality. In the context of our discussion, it should be pointed out that in [37], hidden S_2 nonlocality was observed. But our method and the results differ from that discussed in [37]. It was shown there that if the three parties share two identical copies of the genuinely entangled state κ such that each of κ does not violate Svetlichny inequality, then $\kappa \bigotimes \kappa$ can violate Svetlichny inequality, maximal amount of violation being 4.2418. Moreover in our SMP, there exist initial states $\rho_i(i = 1,2,3)$ which do not violate Svetlichny inequality whereas the final state ρ_4 generated from the initial stage(preparation stage) of our SMP(Fig.1) can violate Svetlichny inequality maximally. For instance, if we consider $\rho_i(i = 1,2,3)$ as the three initial states with $\theta_3 = \theta_1$ and $p_3 = 1$, then with S_2 local version of these initial states, i.e. under some restricted range of θ_1, p_1 and p_2(Eq.(11)): $0 < \sin 2\theta_1 \leq \frac{1}{\sqrt{2}}$, $0 < p_1 \leq 1$, and $0 < p_2 \leq \frac{1}{\sqrt{2}}$, maximally entangled state $|\psi_m\rangle$ is obtained. Even with arbitrarily lower values of θ_1, p_1 and p_2, i.e., with initial states having lower values of C_{GM}, $|\psi_m\rangle$ can be obtained and hence maximal violation of Svetlichny inequality can be observed. This in turn points out utility of our SMP to check the existence of hidden S_2 nonlocality from experimental perspectives.

B. Revelation of hidden genuine 3−way NS nonlocality

Initial states used so far were S_2 local. However some of them were genuinely 3−way NS nonlocal as they can violate one of the 185 facets (except Svetlichny inequality). So revelation of hidden S_2 nonlocality via violation of Svetlichny inequality does not guarantee existence of hidden NS_2 nonlocality. For that purpose, all the initial states must be NS_2 local and the final state(resulting from the preparation stage of the SMP) must violate atleast one of these 185 facets. We now proceed to present instances in support of our claim that hidden NS_2 nonlocality can be revealed by our SMP.

Consider three genuinely tripartite entangled

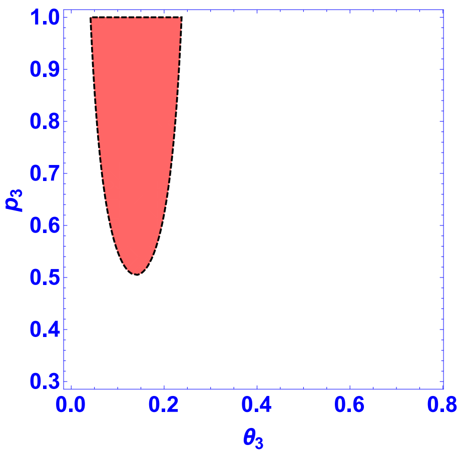

FIG. 2: *The region of revelation of hidden S_2 nonlocality for $\theta_1 = 0.1$ where the initial states $\rho_i(i = 1, 2, 3)$ are S_2 local. Similar type of regions of revelation of hidden S_2 nonlocality can be observed for different values of θ_1.*

states($\rho_i, i = 1, 2, 3$) satisfying all the 185 facets of NS_2 local polytope (for some restricted range of state parameters). Precisely, each of the three states are NS_2 local. If the final state ρ_4, resulting from the preparation stage of SMP, violates at least one of the 185 facets, then that implies revelation of hidden NS_2 nonlocality. For instance, consider ρ_1 (Eq.(7)) with $\theta_1 = 0.1$, $p1 < 0.509$, ρ_2 (Eq.(8)) with $p_2 < 0.6672$ and ρ_3 (Eq.(9)) with $\theta_3 = 0.3$, $p_3 < 0.9198$, then these initial states satisfy all the 185 facet inequalities. The final state ρ_4(Eq.(10))where $\theta_1 = 0.1$, $\theta_3 = 0.3$, violates some of the facet inequalities over varying range of the state parameter p_3. For $p_3 \geq 0.105$, 16th facet inequality (same numbering as in [36] has been used for convenience) is violated. This implies existence of hidden NS_2 nonlocality in the range $p_3 \in [0.105, 0.9198]$. These ranges of p_1, p_2, p_3 are found by numerical optimization using Mathematica software [47](see Appendix A). There exist many other specific NS_2 local initial states belonging to the three families of tripartite mixed states(Eqs.(7), (8), (9)) for which the state generated by the preparation stage of our SMP(Fig.1) can reveal hidden NS_2 nonlocality. We have thus succeeded to show the existence of hidden NS_2 nonlocality by our SMP. Some numerical observations are enlisted in Table(I). These observations justify our claim that arbitrarily small amount of GTE suffices to reveal hidden NS_2 nonlocality. Analogous to our approach in the case of S_2 nonlocality, here we consider three initial states, none of which is NS_2 nonlocal even after being subjected to filtering. Then these states when used in our SMP generate NS_2 nonlocal correlations. We provide with an explicit illustration in support of our claim. Let known useful filters be applied on each of the three initial states $\rho_i(i = 1, 2, 3)$. Fixing the state parameter of ρ_1 to be $\theta_1 = 0.1$, we apply known useful filters over it. After being filtered, it remains NS_2 local for $p_1 \in [0, 0.5025]$. Similarly second state(ρ_2), after filtration remains NS_2 local for $p_2 \in [0, 0.6666]$. However, for $\theta_3 = 0.3$, the range of p_3 for which ρ_3 exhibits NS_2 nonlocality does not change after applying filtering operation (see Appendix B). So for each of the three initial states $\rho_i(i = 1, 2, 3)$, after being subjected to local filtering, there exist some range of state parameters for which NS_2 nonlocality cannot be revealed. If these initial states under the said restricted range are used in our SMP then NS_2 nonlocality will be revealed for $p_3 \in [0.105, 0.9198]$. However, analogous to revelation of hidden S_2 nonlocality, this range of revelation of hidden NS_2 nonlocality in our SMP remains the same when the states $\rho_i(i = 1, 2, 3)$ are used without being subjected to filtration. Thus our SMP turns out to be more efficient compared to the standard procedure of using local filters for revelation of hidden NS_2 nonlocality.

ρ_1	ρ_2	ρ_3	Revelation Range
$p_1<0.509$	$p_2<0.6672$	$\theta_3=0.1,\ p_3<0.9901$	$p_3\in[0.504, 0.9901]$
$p_1<0.509$	$p_2<0.6672$	$\theta_3=0.5,\ p_3<0.8135$	$p_3\in[0.0425, 0.8135]$
$p_1<0.509$	$p_2<0.6672$	$\theta_3=0.7,\ p_3<0.7072$	$p_3\in[0.0243, 0.7072]$
$p_1<0.509$	$p_2<0.6672$	$\theta_3=0.785,\ p_3<0.6677$	$p_3\in[0.0202, 0.6677]$

TABLE I: The range of revelation of hidden genuine 3−way NS nonlocality for state parameter p_3 is given in the table for different fixed values of the state parameters of the NS_2 local initial $\rho_i(i = 1, 2, 3)$. These values were found by numerical optimization(by Mathematica Software). Here we consider a fixed value of state parameter(θ_1) of ρ_1, θ_1=0.1. Clearly range of revelation varies with variation of θ_1.

V. DISCUSSION

From our discussion so far we conclude that genuine 3−way NS nonlocality is some kind of resource, which can be revealed by a sequence of measurements. Usually it is believed that standard Bell scenario(i.e., in each run of the experiment, non-sequential local measurements are performed on a single copy of an entangled state) is suitable for a quantum state to exhibit genuine 3−way NS nonlocality. Our present work, however can be considered as an approach deviated from this usual belief. We have shown that three tripartite quantum states, unable to reveal genuine 3−way NS nonlocality in the standard Bell scenario, when used in our Sequential Measurement Protocol (SMP) can generate a state which is genuinely

3−way NS nonlocal. This implies that hidden genuine 3−way NS nonlocality can be revealed. Even our SMP emerges to be more efficient to reveal hidden genuine 3−way NS nonlocality compared to the standard procedure of using known useful local filters.

Besides, the preparation stage of our SMP protocol can also be interpreted as an entanglement swapping protocol. Consequently via this protocol we can give an affirmative answer for tripartite system to a query posed by Sen et al. [28]: consider some local entangled states, is it possible to find some entanglement swapping process, so that the swapped states, resulting from it, are capable of showing nonlocal behavior?

There are a number of possible generalizations of the above results. One may explore whether for any genuinely tripartite mixed entangled state, existence of at least one suitable SMP is guaranteed under which revelation of hidden GTNL is possible. Also, it will be interesting to generalize our SMP so as to demonstrate n partite hidden genuine nonlocality. Moreover, till now there exist various experimental works demonstrating tripartite nonlocality [48–50] and also hidden bipartite nonlocality [51]. In that context, one may expect to use our protocol for experimental verification of hidden GTNL. Besides, as GTNL implies GTE, our SMP can be used in the laboratory to detect GTE of the initial states in a device independent way [52, 53].

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VI. APPENDIX

A. Checking facets of NS_2 local polytope

Derivation of $B_i(i = 1, 2, 3, 4)$*:* In order to obtain the maximum value B_1 (Eq.(11) of main text) of the Svetlichny operator S (Eq.(3)) upto projective measurements we follow the method used in [42]. We consider the following measurements: $x_0 = \vec{x}.\vec{\sigma_1}$ or $x_1 = \vec{\acute{x}}.\vec{\sigma_1}$ on 1^{st} qubit, $y_0 = \vec{y}.\vec{\sigma_2}$ or $y_1 = \vec{\acute{y}}.\vec{\sigma_2}$ on 2^{nd} qubit, and $z_0 = \vec{z}.\vec{\sigma_3}$ or $z_1 = \vec{\acute{z}}.\vec{\sigma_3}$ on 3^{rd} qubit, where $\vec{x}, \vec{\acute{x}}, \vec{y}, \vec{\acute{y}}$ and $\vec{z}, \vec{\acute{z}}$ are unit vectors and σ_i are the spin projection operators that can be written in terms of the Pauli matrices. Representing the unit vectors in spherical coordinates, we have, $\vec{x} = (\sin\theta a_0 \cos\phi a_0, \sin\theta a_0 \sin\phi a_0, \cos\theta a_0)$, $\vec{y} = (\sin\alpha b_0 \cos\beta b_0, \sin\alpha b_0 \sin\beta b_0, \cos\alpha b_0)$ and $\vec{z} = (\sin\zeta c_0 \cos\eta c_0, \sin\zeta c_0 \sin\eta c_0, \cos\zeta c_0)$ and similarly, we define, $\vec{\acute{x}}, \vec{\acute{y}}$ and $\vec{\acute{z}}$ by replacing 0 in the indices by 1. Then the value of the operator S (Eq.(3)) with respect to the state ρ_1 (Eq.(7)) gives:

$$S(\rho_1) = \cos(\alpha b_0)(-1 + p_1 + p_1\cos(2\theta_1))(\cos(\zeta c_0)(\cos(\theta a_0) - \cos(\theta a_1)) + \cos(\zeta c_0)(\cos\theta a_0) + \cos(\theta a_1)) + \cos(\alpha b_1)(-1 + p_1 + p_1\cos(2\theta_1))(\cos(\zeta c_0)(\cos(\theta a_0) - \cos(\theta a_1)) - \cos(\zeta c_1)(\cos(\theta a_0) + \cos(\theta a_1))) + p_1\sin(2\theta_1)(\cos(\beta b_0 + \eta c_0 + \phi a_0)\sin(\alpha b_0)\sin(\zeta c_0)\sin(\theta a_0) + \cos(\beta b_1 + \eta c_0 + \phi a_0)\sin(\alpha b_1)\sin(\zeta c_0)\sin(\theta a_0) + \cos(\beta b_0 + \eta c_1 + \phi a_0)\sin(\alpha b_0)\sin(\zeta c_1)\sin(\theta a_0) - \cos(\beta b_1 + \eta c_1 + \phi a_0)\sin(\alpha b_1)\sin(\zeta c_1)\sin(\theta a_0) + \cos(\beta b_0 + \eta c_0 + \phi a_1)\sin(\alpha b_0)\sin(\zeta c_0)\sin(\theta a_1) - \cos(\beta b_1 + \eta c_0 + \phi a_1)\sin(\alpha b_1)\sin(\zeta c_0)\sin(\theta a_1) - \cos(\beta b_0 + \eta c_1 + \phi a_1)\sin(\alpha b_0)\sin(\zeta c_1)\sin(\theta a_1) - \cos(\beta b_1 + \eta c_1 + \phi a_1)\sin(\alpha b_1)\sin(\zeta c_1)\sin(\theta a_1)). \tag{15}$$

Hence in order to get maximum value of $S(\rho_1)$, we have to perform a maximization over 12 measurement angles. We first find the global maximum of $S(\rho_1)$ with respect to θa_0 and θa_1. We begin with by finding all critical points of $S(\rho_1)$ inside the region $R = [0, 2\pi] \times [0, 2\pi]$ which are namely $(0, 0)$, $(\frac{\pi}{2}, -\frac{\pi}{2})$,$(-\frac{\pi}{2}, \frac{\pi}{2})$, $(\frac{\pi}{2}, \frac{\pi}{2})$ and $(-\frac{\pi}{2}, -\frac{\pi}{2})$. Among all these critical points, $(\frac{\pi}{2}, \frac{\pi}{2})$ gives the global maximum of $S(\rho_1)$ if $\sqrt{2}p_1\sin(2\theta_1) > |(1 - p_1 - p_1\cos(2\theta_1))|$. Thus, we have:

$$S(\rho_1) \leq p_1\sin(2\theta_1)(\cos(\beta b_0 + \eta c_0 + \phi a_0)\sin(\alpha b_0)\sin(\zeta c_0)\sin(\theta a_0) + \cos(\beta b_1 + \eta c_0 + \phi a_0)\sin(\alpha b_1)\sin(\zeta c_0)\sin(\theta a_0) + \cos(\beta b_0 + \eta c_1 + \phi a_0)\sin(\alpha b_0)\sin(\zeta c_1)\sin(\theta a_0) - \cos(\beta b_1 + \eta c_1 + \phi a_0)\sin(\alpha b_1]\sin(\zeta c_1)\sin(\theta a_0) + \cos(\beta b_0 + \eta c_0 + \phi a_1)\sin(\alpha b_0)\sin(\zeta c_0)\sin(\theta a_1) - \cos(\beta b_1 + \eta c_0 + \phi a_1)\sin(\alpha b_1)\sin(\zeta c_0)\sin(\theta a_1) - \cos(\beta b_0 + \eta c_1 + \phi a_1)\sin(\alpha b_0)\sin(\zeta c_1)\sin(\theta a_1) - \cos(\beta b_1 + \eta c_1 + \phi a_1)\sin(\alpha b_1)\sin(\zeta c_1)\sin(\theta a_1)). \tag{16}$$

Now we carry out the same procedure over the following pair of variables $(\zeta c_0, \zeta c_1)$ and $(\alpha b_0, \alpha b_1)$, one by one. Similar to the previous case, critical point $(\frac{\pi}{2}, \frac{\pi}{2})$ gives the maximum value for both of these pair of variables. So, the last inequality in Eq.(16) takes the form

$$S(\rho_1) \leq p_1\sin(2\theta_1)(\cos\eta c_0(\cos(\beta b_0 + \phi a_0) + \cos(\beta b_1 + \phi a_0) + \cos(\beta b_0 + \phi a_1) - \cos(\beta b_1 + \phi a_1)) - \sin\eta c_0(\sin(\beta b_0 + \phi a_0) + \sin(\beta b_1 + \phi a_0) + \sin(\beta b_0 + \phi a_1) - \sin(\beta b_1 + \phi a_1)) + \cos\eta c_1(\cos(\beta b_0 + \phi a_0) - \cos(\beta b_1 + \phi a_0) - \cos(\beta b_0 + \phi a_1) - \cos(\beta b_1 + \phi a_1)) + \sin\eta c_1(-\sin(\beta b_0 + \phi a_0) + \sin(\beta b_1 + \phi a_0) + \sin(\beta b_0 + \phi a_1) + \sin(\beta b_1 + \phi a_1))). \tag{17}$$

which when maximized with respect to ηc_0 and ηc_1 gives:

$$S(\rho_1) \leq 2p_1 \sin(2\theta_1)\sqrt{(\cos A_{00} + \cos A_{10} + \cos(A_{01}) - \cos A_{11})^2 + (\sin A_{00} + \sin A_{10} + \sin A_{01} - \sin A_{11})^2} \quad (18)$$

where $A_{ij} = \beta b_i + \phi a_j$, $(i, j \in \{0, 1\})$. The last inequality is obtained by using the inequality $x\cos\theta + y\sin\theta \leq \sqrt{x^2 + y^2}$. Maximum value of the expression in Eq.(18) remains unaltered by putting any value of βb_0 and ϕa_0. In particular if we take $\beta b_0 = 0$ and $\phi a_0 = 0$, then maximum value is obtained for $(\beta b_1, \phi a_1) = (\frac{\pi}{2}, \frac{\pi}{2})$ and is equal to $4\sqrt{2}p_1 \sin 2\theta_1$. Again if $\sqrt{2}p_1 \sin(2\theta_1) < |(1 - p_1 - p_1 \cos(2\theta_1))|$, the critical point $(0, 0)$ gives the maximum value of $S(\rho_1)$. Then Eq.(15) reduces to $S(\rho_1) \leq 2(-1 + p_1 + p_1 \cos(2\theta_1))(\cos(\alpha b_0)\cos(\zeta c_0) - \cos(\alpha b_1)\cos(\zeta c_1))$. Now the critical point $(0, 0)$ gives the maximum value when we maximize the last expression with respect to αb_0 and αb_1 . Then the last inequality becomes $S(\rho_1) \leq 2(-1 + p_1 + p_1 \cos(2\theta_1))(\cos(\zeta c_0) - \cos(\zeta c_1))$. Again we maximize it with respect to ζc_0 and ζc_1. Critical point $(0, \pi)$ or $(\pi, 0)$ gives the maximum value depending on whether $p_1(1 + \cos(2\theta_1)) > 1$ or $p_1(1 + \cos(2\theta_1)) < 1$. Hence in any case $S(\rho_1) \leq 4|1 - p_1 - p_1 \cos(2\theta_1)|$. So $S(\rho_1) \leq \max[4\sqrt{2}p_1 \sin 2\theta_1, 4|1 - p_1 - p_1 \cos(2\theta_1)|]$ as stated in Eq.(11) of main text. Similarly one can obtain B_i $(i = 2, 3, 4)$. From these values of $B_i(\mathrm{i} = 1,\ 2,\ 3)$, one can get the range of p_i for which the corresponding initial state ρ_i satisfy Svetlichny inequality. For the final state ρ_4, we obtain the range of violation of Svetlichny inequality, by following the above analytical method. We now proceed to search for the maximum expectation value of operator$(NS_i(\rho_j), j = 1, 2, 3, 4)$ corresponding to the remaining $i^{th}, (i = 1, 2, ..., 184)$ facet inequality.

Checking the remaining 184 *facets:* The above method of maximization is applied for most of the remaining 184 facet inequalities excluding a few for which the upper bound of violation$(B_i(i = 1, 2, 3))$ is measurement specific, i.e. varies not only with the state parameters but also with variation of parameters characterizing the measurement settings. In order to find the range of p_i for those inequalities, we have performed numerical optimization by using Mathematica software [47]. We now give an example of such a facet inequality for which the analytical method of maximization does not hold good due to dependence of the upper bound of expectation value of the corresponding operator over measurement settings apart from state parameters. 3^{rd} facet(say), is given by : $NS_3 =$

$$-\langle x_0\rangle - \langle x_1\rangle - \langle x_0y_0\rangle - 2\langle y_1\rangle - \langle z_0\rangle + \langle x_1y_0\rangle - \langle x_0z_0\rangle$$

$$+\langle y_0z_0\rangle + \langle x_1y_0z_0\rangle - \langle x_0y_1z_0\rangle + \langle x_1y_1z_0\rangle - \langle z_1\rangle +$$

$$\langle x_1z_1\rangle - \langle y_0z_1\rangle - \langle x_0y_0z_1\rangle + \langle x_0y_1z_1\rangle + \langle x_1y_1z_1\rangle \leq 4. \quad (19)$$

The value of the operator NS_3 given by the 3^{rd} facet with respect to the state ρ_1(Eq.(7) of main text) under the projective measurement gives:

$$NS_3(\rho_1) = \cos\zeta c_1((1-p_1)(1+\cos\alpha b_0+\cos\theta a_0\cos\alpha b_0) - p_1\cos^2\theta_1 - p_1\cos\alpha b_0\cos^2\theta_1) - (1-p_1)\cos\theta a_0(1+\cos\alpha b_0)$$
$$-p_1\cos^2\theta_1\cos\theta a_0(1+\cos\alpha b_0+\cos\alpha b_0\cos\zeta c_1) + (\cos\alpha b_0\cos\theta a_1 - \cos\zeta c_1\cos\theta a_1 - 1)(1-p_1) - p_1\cos^2\theta_1\cos\theta a_1(1-\cos\alpha b_0$$
$$-\cos\zeta c_1) + \cos\zeta c_0(1-p_1-p_1\cos 2\theta_1+\cos\theta a_0(1-2p_1)+\cos\alpha b_0(-1+2p_1+(-1+p_1+p_1\cos 2\theta_1)\cos\theta a_1))$$
$$-\frac{1}{2}\cos\alpha b_1(\cos\zeta c_0(-2+p_1+p_1\cos 2\theta_1)(\cos\theta a_0-\cos\theta a_1) - 2p_1\cos^2\theta_1(-2+\cos\zeta c_0(\cos\theta a_1-\cos\theta a_0)+\cos\zeta c_1(\cos\theta a_1$$
$$+\cos\theta a_0)) + 2(2-3p_1+p_1\cos 2\theta_1 - \frac{1}{2}\cos\zeta c_1(-2+p_1+p_1\cos 2\theta_1)(\cos\theta a_1+\cos\theta a_0)) + (p_1\cos\zeta c_1\sin^2\theta_1$$
$$+p_1\cos\theta a_0\sin^2\theta_1)(1-\cos\alpha b_0) + p_1\cos\alpha b_0\cos\zeta c_1\cos\theta a_0\sin^2\theta_1 + p_1\cos\theta a_1\sin^2\theta_1(1+\cos\alpha b_0+\cos\zeta c_1) -$$
$$p_1\cos(\beta b_1+\eta c_0+\phi a_0)\sin\alpha b_1\sin\zeta c_0\sin 2\theta_1\sin\theta a_0 - p_1\cos(\beta b_0+\eta c_1+\phi a_0)\sin\alpha b_0\sin\zeta c_1\sin 2\theta_1\sin\theta a_0 +$$
$$p_1\cos(\beta b_1+\eta c_1+\phi a_0)\sin\alpha b_1\sin\zeta c_1\sin 2\theta_1\sin\theta a_0 + p_1\cos(\beta b_0+\eta c_0+\phi a_1)\sin\alpha b_0\sin\zeta c_0\sin 2\theta_1\sin\theta a_1$$
$$+p_1\cos(\beta b_1+\eta c_0+\phi a_1)\sin\alpha b_1\sin\zeta c_0\sin 2\theta_1\sin\theta a_1 + p_1\cos(\beta b_1+\eta c_1+\phi a_1)\sin\alpha b_1\sin\zeta c_1\sin 2\theta_1\sin\theta a_1) \quad (20)$$

Now to find the upper bound of $NS_3(\rho_1)$ in terms of state parameters, we need to maximize $NS_3(\rho_1)$ over all the variables parameterizing measurement settings.

However, for almost each of those variables there is no fixed critical point for which $NS_3(\rho_1)$ gives maximum value, it varies with the variation of state parameters. Hence, the analytical method that was followed for $S(\rho_1)$ cannot be applied. In order to overcome this difficulty, we apply numerical optimization by using Mathematica Software [47]. We consider a particular example. Let $\theta_1 = 0.1$. The measurement settings parameters vary with the other state parameter p_1, i.e., the maxima of $NS_3(\rho_1)$ with respect to any measurement parameter varies with state parameter p_1. So we maximize $NS_3(\rho_1)$ over all measurement parameters by using Mathematica Software. After maximizing numerically, it is observed that under the restriction $0 \leq p_1 \leq 0.509$, the maximum value of $NS_3(\rho_1)$ never exceeds 4. Hence the initial state ρ_1 with $\theta_1 = 0.1$ satisfies 3−rd facet when $0 \leq p_1 \leq 0.509$. We have applied this numerical method for all the facets for which the upper bound of violation depends over measurement settings apart from state parameters. In totality, i.e. considering all facets(some by analytical method and others by numerical method), it is checked that ρ_1 with $\theta_1 = 0.1$ satisfy all of the 185 facets when $0 \leq p_1 \leq 0.509$. Similar method is applied to find the range of p_1 for which ρ_1 satisfy all of 185 facets for different fixed values of θ_1. Just as for the initial state ρ_1, we have followed similar trend of analysis for the other two initial states ρ_2, ρ_3 and also for the resultant state ρ_4.

B. Local filtering and hidden Genuine Tripartite nonlocality

Here we will discuss the effect of using local filtering on the initial states $\rho_i (i = 1, 2, 3)$. Any local filtering transforms a tripartite state ρ in

$$\acute{\rho} = \frac{(F_1 \bigotimes F_2 \bigotimes F_3)\rho(F_1^\dagger \bigotimes F_2^\dagger \bigotimes F_3^\dagger)}{\mathrm{tr}((F_1 \bigotimes F_2 \bigotimes F_3)\rho(F_1^\dagger \bigotimes F_2^\dagger \bigotimes F_3^\dagger))} \tag{21}$$

where $F_j^\dagger F_j \leq I_2$ $(j = 1, 2, 3)$. It is shown in [38] for qubit case, the most general filters are of the form $F_j =$

$$\begin{pmatrix} \epsilon_j & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_j & -e^{i\phi_j \sin\theta_j} \\ e^{i\phi_j \sin\theta_j} & \cos\theta_j \end{pmatrix}$$

where ϵ_j, θ_j, ϕ_j are real parameters. It is argued in [38] that theoretically there is no reason to exclude the unitary matrix in F_j (which corresponds to a local unitary before the filter), yet in standard form of local filters, the contribution from the unitary matrix is ignored. In [38] it is also argued that all the known useful filters are diagonal. Especially for the qubit case, it seems that only the diagonal filters are relevant. Since we are dealing with qubit case only, we take the filters of the form $F_j =$

$$\begin{pmatrix} \epsilon_j & 0 \\ 0 & 1 \end{pmatrix}.$$

Here ϵ_j's$(j = 1, 2, 3)$ are filtering parameters and $0 \leq \epsilon_j \leq 1$. Now the application of local filtering on the state ρ_1(Eq.(7) in the main text) results in

$$\acute{\rho_1} = \frac{p_1|\phi_1\rangle\langle\phi_1| + (1-p_1)\epsilon_2^2\epsilon_3^2|100\rangle\langle 100|}{(1-p_1)\epsilon_2^2\epsilon_3^2 + p_1\epsilon_1^2\epsilon_2^2\epsilon_3^2\cos^2\theta_1 + p_1\sin^2\theta_1}. \tag{22}$$

where $|\phi_1\rangle = \epsilon_1\epsilon_2\epsilon_3\cos\theta_1|000\rangle + \sin\theta_1|111\rangle$.
To obtain the maximum value of the Svetlichny operator S with respect to projective measurements, for the state $\acute{\rho_1}$, we apply the same method as we used in the last section for the derivation of B_1. The maximum value is given by

$$\max[\frac{4\sqrt{2}p_1\epsilon_1\epsilon_2\epsilon_3\sin 2\theta_1}{(1-p_1)\epsilon_2^2\epsilon_3^2 + p_1\epsilon_1^2\epsilon_2^2\epsilon_3^2\cos^2\theta_1 + p_1\sin^2\theta_1},$$

$$\frac{4((1-p_1)\epsilon_2^2\epsilon_3^2 - p_1\epsilon_1^2\epsilon_2^2\epsilon_3^2\cos^2\theta_1 + p_1\sin^2\theta_1)}{(1-p_1)\epsilon_2^2\epsilon_3^2 + p_1\epsilon_1^2\epsilon_2^2\epsilon_3^2\cos^2\theta_1 + p_1\sin^2\theta_1}]. \tag{23}$$

Clearly, $\frac{4((1-p_1)\epsilon_2^2\epsilon_3^2 - p_1\epsilon_1^2\epsilon_2^2\epsilon_3^2\cos^2\theta_1 + p_1\sin^2\theta_1)}{(1-p_1)\epsilon_2^2\epsilon_3^2 + p_1\epsilon_1^2\epsilon_2^2\epsilon_3^2\cos^2\theta_1 + p_1\sin^2\theta_1} \leq 4$ for any value of $0 \leq p_1 \leq 1$ and $0 \leq \epsilon_j \leq 1$. So the filtered state $\acute{\rho_1}$ remains S_2 local if $\frac{4\sqrt{2}p_1\epsilon_1\epsilon_2\epsilon_3\sin 2\theta_1}{(1-p_1)\epsilon_2^2\epsilon_3^2 + p_1\epsilon_1^2\epsilon_2^2\epsilon_3^2\cos^2\theta_1 + p_1\sin^2\theta_1} \leq 4$. After maximizing the left hand side of the last inequality with respect to ϵ_j $(j = 1, 2, 3)$, we have

$$p_1 \leq \frac{2}{3 + \cos 2\theta_1} \tag{24}$$

From Eq.(24) one can get the range of p_1 for each non-zero value of θ_1 such that the filtered state $\acute{\rho_1}$ remains S_2 local, i.e. the initial state ρ_1 has no hidden S_2 nonlocality. Similarly the range of p_1 for each non-zero value of θ_1 for which the filtered state $\acute{\rho_1}$ satisfies remaining facet inequalities are obtained. For most of the facet inequalities, the analytical method(as followed in the previous section) is applicable excepting a few where the upper bound of the expectation value of the operator $NS_i(\rho_1)$ corresponding to the $i^{th}(i = 1, ..., 184)$ facet depends not only on the state parameters but also on the variables parameterizing measurement settings. For those few facets we have done numerical optimization by Mathematica software(as already discussed in the previous section). For instance, we consider 3^{rd} facet inequality. Let us fix the state parameter θ_1: $\theta_1 = 0.1$. For this fixed value of θ_1, numerical maximization of $NS_3(\rho_1)$ over all the measurement settings shows that under the restriction $\epsilon_j(j = 1, 2, 3) \in [0, 1]$ and $p_1 \in [0, 0.515]$, state $\acute{\rho_1}$ satisfies 3^{rd} facet inequality. After checking all of $NS_i(\rho_1), i = 1, ..., 185$, we arrive at the conclusion that for $\theta_1 = 0.1$ and $p_1 \in [0, 0.5025]$, state ρ_1 does not reveal any GTNL after the application of known useful local filters. We have applied the same procedure over other fixed values of θ_1. For other two initial states ρ_2 and ρ_3, we have made analysis in similar manner so as to obtain the range of p_2 and p_3(for a fixed value of θ_3) of ρ_2(Eq.(8) of the main text) and ρ_3(Eq.(9) of the main text) respectively for which they still do not reveal any hidden GTNL after the application of local filters.