

## $R$ -Parity violation in $B \rightarrow \pi^+ \pi^-$ decay

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### Abstract

We consider the impact of  $R$ -parity violating supersymmetry in the nonleptonic decay  $B \rightarrow \pi^+ \pi^-$ . This is one of the rare instances where new physics contributes to both  $B^0 - \bar{B}^0$  mixing and  $B \rightarrow \pi\pi$  decay. We do a numerical analysis to capture the interplay between these two effects and place constraints on the relevant parameter space.

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## I Introduction

In the standard model (SM), baryon and lepton numbers ( $B$  and  $L$ , respectively) are automatically conserved, while in supersymmetry (SUSY) neither gauge symmetry nor any such fundamental principles tells us as to why these discrete symmetries should be exactly respected. This prompts us to define in supersymmetric theories a discrete quantum number called  $R$ -parity as  $R = (-1)^{(3B+L+2S)}$  [1, 2], where  $S$  is the spin of the particle. In fact, it is not fair to be totally dictated by theoretical prejudice and *ab initio* abandon the  $R$ -parity violating (RPV) terms. A rather open-minded approach would be to keep the RPV interactions in the theory and constrain them from observations/ non-observations at different experiments. There is a rich phenomenological consequence of  $R$ -parity violation. When  $R$ -parity is violated, the lightest supersymmetric particle is no longer stable and superparticles need not be produced in pairs. As a result, one needs to reformulate the SUSY search strategies. A complete set of RPV interactions introduces 48 new parameters into the theory, and so it is important to constrain them from existing data. Upper bounds on RPV couplings emerge from proton stability,  $n-\bar{n}$  oscillation, neutrino masses and mixings, charged current universality, atomic parity violation,  $Z$  pole observables, meson decays and mixings, etc., a thorough account of which has been reviewed in [3].

During the last few years many new data on branching ratios (BRs) and CP asymmetries in different  $B$  decay channels are coming from the  $B$  factories. It is therefore worthwhile to examine the consequences of these data for RPV models. Let us first discuss why  $B$  decays in RPV scenario are interesting. In the  $R$ -parity conserving limit, leading SUSY contributions to  $B$  decays would come from penguins and/or boxes. As has been noted by several authors, e.g. [4], such SUSY loops are either suppressed, or at best comparable, with respect to the SM penguins on account of the heaviness of SUSY spectrum. *On the contrary, RPV might trigger such decays at tree level with suitable couplings turned on, and there lies its importance.* Moreover, such RPV couplings are complex, and their phases play their part in inducing new contribution to CP violation. As an example, processes which are purely penguin driven in the SM (e.g.  $b \rightarrow s\bar{d}d$  or  $b \rightarrow s\bar{s}s$ ), may not receive any appreciable corrections from SUSY loops, but tree level RPV contributions to them might be significant. Even otherwise,

in processes like  $b \rightarrow c\bar{c}s$  which is dominated by the SM tree diagram, a RPV tree contribution might have an effect in the extraction of the angle  $\beta$  of the unitarity triangle. All these effects in the context of  $B$  decaying into  $\pi K$ ,  $\phi K$ ,  $J/\psi K_S$  final states have been studied by many authors [5, 6].

*In this paper we shall restrict ourselves to  $B \rightarrow \pi^+\pi^-$  decay, and comment on other  $\pi\pi$  modes ( $\pi^0\pi^0$ ,  $\pi^0\pi^\pm$ ). The dominant SM contribution to  $B \rightarrow \pi^+\pi^-$  decay comes from a tree graph which has a suppression from  $V_{ub}$ . The RPV amplitude is driven by the product  $\lambda'_{i11}\lambda'^*_{i13}$ . For  $i = 2$  and  $3$ , the existing bound on this combination is rather modest ( $|\lambda'_{i11}\lambda'^*_{i13}| < 3.6 \times 10^{-3}$ ) [7]. The propagator of the RPV tree diagram is a slepton whose mass can be in the 100 GeV range. Thus the RPV amplitude can be comparable in size with the SM amplitude. It may be recalled at this stage that the presence of two interfering amplitudes of comparable magnitude is essential for a large direct CP violating asymmetry.*

Additionally, the above product coupling contributes to the  $B^0 - \bar{B}^0$  mixing through box graphs [7], and that makes the situation more tricky. In fact, this is one of the very few situations where a new physics operator can contribute to *both* mixing and decay with amplitudes comparable, or possibly even larger, than the SM contributions. The angle  $\beta$  of the unitarity triangle is determined from the  $B^0 - \bar{B}^0$  mixing phase  $\phi_M$ . The SM box diagram yields  $\phi_M$  to be  $2\beta$ . Now that the  $\lambda'_{i11}\lambda'^*_{i13}$  combination is turned on, there is new contribution *not only to decay into  $\pi\pi$  final states but also to  $B^0 - \bar{B}^0$  mixing at the same time. The twin rôle of these specific product couplings for the  $\pi\pi$  final states thus adds a new twist and brings in significant calculational intricacies.*

Now once the mixing phase gets contaminated, the immediate concern is what happens then to  $B$  decays into, for example,  $J/\psi K_S$  and  $\phi K_S$  final states? Supposing that only the above couplings, namely the  $\lambda'_{i11}\lambda'^*_{i13}$  combinations, are operative, there will not be any new contribution to the decay diagrams for the above final states, but the corresponding mixing induced asymmetries will get modified through new contribution to  $\phi_M$ . The CP asymmetries in the above channels are proportional to  $\sin 2\beta$  in the SM. Now both the asymmetries will measure not the angle  $\beta$  (defined as argument of  $V_{td}$ ) but some other  $\beta_{eff}$  carrying traces of new physics.

Now, as mentioned before, the  $\pi\pi$  final states from  $B$  decays require special attention as the given choice of RPV product couplings inducts new effects *simultaneously into mixing and decay*. Using the available data set on BRs and CP asymmetries (to be reviewed in the next section), we obtain useful constraints on the RPV product couplings. This is true even considering the fact that the present experimental errors are still quite large. For example, *we obtain a new bound  $|\lambda'_{i11}\lambda'^*_{i13}| \leq 0.0022$  for a slepton mass of around 100 GeV, which is a marginal improvement over its existing bound of 0.0036.* However, our bound is more general since we take into account the possibility of destructive interference between the SM and the RPV box amplitudes where as the previous bound was derived by saturating the experimental number by RPV box only. In addition, interesting correlations among  $\gamma$  (another angle of the unitarity triangle), the weak phase of the RPV product coupling, and the strong phase difference between the SM and RPV amplitudes emerge. It is to be noted that our *primary aim* is to put an upper bound on new physics parameters (RPV couplings) using the standard technique of accomodating as much of new physics as possible in the space between experimental data and SM predictions in the  $B \rightarrow \pi^+\pi^-$  channel. In the process, we observe a simultaneous enhancement of the BR in  $\pi^0\pi^0$  final state which is a big bonus in view of recent data showing large BR in this channel for which the SM prediction is too low.

## II Review of data and the relevant formulae

Data is available on all  $\pi\pi$  modes. Let us first look at the BRs (multiplied by  $10^6$ ) as they stand in ICHEP 2004 [8]:

$$\begin{aligned}
 Br(B \rightarrow \pi^+\pi^-) &= 4.6 \pm 0.4; \\
 Br(B \rightarrow \pi^0\pi^0) &= 1.51 \pm 0.28; \\
 Br(B^\pm \rightarrow \pi^\pm\pi^0) &= 5.5 \pm 0.6.
 \end{aligned}
 \tag{1}$$

We use  $B$  to indicate a flavor-untagged  $B^0$  or  $\overline{B^0}$ . Defining

$$\lambda = \exp(-i2\beta) \langle \pi^+ \pi^- | \mathcal{H} | \overline{B^0} \rangle / \langle \pi^+ \pi^- | \mathcal{H} | B^0 \rangle, \quad (2)$$

and the direct and mixing induced CP asymmetries as

$$a_{CP}^d = (1 - |\lambda|^2)/(1 + |\lambda|^2), \quad a_{CP}^m = 2\text{Im}\lambda/(1 + |\lambda|^2), \quad (3)$$

we write their present experimental numbers [8] as

$$\begin{aligned} S_{\pi\pi} &= -a_{CP}^m = -0.74 \pm 0.16, \\ C_{\pi\pi} &= a_{CP}^d = -0.46 \pm 0.13, \\ A_{CP}(\pi^+\pi^0) &= 0.01 \pm 0.07, \\ A_{CP}(\pi^0\pi^0) &= -0.28 \pm 0.39. \end{aligned} \quad (4)$$

The quantities  $a_{CP}^d$  and  $a_{CP}^m$  are obtained from the time-dependent measurement on  $B$ :

$$\begin{aligned} a_{\pi^+\pi^-}^{dm}(t) &= \frac{\Gamma(B^0(t) \rightarrow \pi^+\pi^-) - \Gamma(\overline{B^0}(t) \rightarrow \pi^+\pi^-)}{\Gamma(B^0(t) \rightarrow \pi^+\pi^-) + \Gamma(\overline{B^0}(t) \rightarrow \pi^+\pi^-)} \\ &= a_{CP}^d \cos \Delta m t + a_{CP}^m \sin \Delta m t, \end{aligned} \quad (5)$$

where  $\Delta m$  is the mass difference between the two mass eigenstates.

To motivate our further discussions, let us assume temporarily that only two interfering amplitudes contribute to the  $\overline{B^0} \rightarrow \pi^+\pi^-$  and denote them by

$$a_1 \exp(i\phi_1) \exp(i\delta_1) \quad \text{and} \quad a_2 \exp(i\phi_2) \exp(i\delta_2),$$

where  $\phi_i$ 's and  $\delta_i$ 's ( $i = 1, 2$ ) are the weak and the strong phases, respectively. We also use the notation

$$\Delta\delta = \delta_2 - \delta_1; \quad \Delta\phi = \phi_2 - \phi_1. \quad (6)$$

We, however, wish to emphasize that for the actual numerical analyses, we will take into account not only the SM tree diagram but also the SM penguin amplitudes as well<sup>1</sup>, in addition to new physics contributions.

The observables  $a_{CP}^d$  and  $a_{CP}^m$  can be expressed in terms of the above parameters. One obtains

$$a_{CP}^d = \frac{1 - |\lambda_{\pi\pi}|^2}{1 + |\lambda_{\pi\pi}|^2} = \frac{2a_1a_2 \sin \Delta\phi \sin \Delta\delta}{a_1^2 + a_2^2 + 2a_1a_2 \cos \Delta\phi \cos \Delta\delta}, \quad (7)$$

$$\text{and} \quad a_{CP}^m = \frac{2 \text{Im}\lambda_{\pi\pi}}{1 + |\lambda_{\pi\pi}|^2}, \quad \text{where} \quad (8)$$

$$\lambda_{\pi\pi} = e^{-i\phi_M} \frac{\langle \pi^+\pi^- | \mathcal{H}_{eff} | \overline{B_q} \rangle}{\langle \pi^+\pi^- | \mathcal{H}_{eff} | B_q \rangle} = \eta_{CP} e^{i(-\phi_M + 2\phi_1)} \frac{(a_1^2 + a_2^2 e^{2i\Delta\phi} + 2a_1a_2 e^{i\Delta\phi} \cos \Delta\delta)}{(a_1^2 + a_2^2 + 2a_1a_2 \cos(\Delta\delta - \Delta\phi))}. \quad (9)$$

Here  $\phi_M$  is the phase of the  $B^0 - \overline{B^0}$  mixing amplitude (this may include phases from the CKM elements as well as phases from new physics), and  $\eta_{CP}$  is the CP eigenvalue (+1) for the final state  $\pi^+\pi^-$ .

For the sake of completeness we also include the expressions for the  $\text{BR}(B \rightarrow \pi^+\pi^-)$ :

$$\begin{aligned} \text{BR}(B^0 \rightarrow \pi^+\pi^-) &\sim a_1^2 + a_2^2 + 2a_1a_2 \cos(\Delta\delta - \Delta\phi), \\ \text{BR}(\overline{B^0} \rightarrow \pi^+\pi^-) &\sim a_1^2 + a_2^2 + 2a_1a_2 \cos(\Delta\delta + \Delta\phi), \end{aligned} \quad (10)$$

where the phase space factors have been suppressed. When one averages over these two terms, one obtains the expression in the denominator of Eq. (7).

<sup>1</sup>The SM amplitudes in the naive factorization approach are given in Eq. (A1) of Appendix-A in Ref. [9].

In the SM  $a_1$  and  $a_2$  are identified with the tree and the top-mediated strong penguin amplitudes, respectively, so that  $\phi_1 = -\gamma$ ,  $\phi_2 = \beta$  and  $\phi_M = 2\beta$ . One expects  $a_2$  to be suppressed with respect to  $a_1$  due to the standard loop suppression factors. This is an approximate representation of the standard form of the amplitudes found in the literature [10], where both charm- and up-quark mediated strong penguins, as well as the electroweak penguins, are taken into account. One usually decomposes the top-penguin into terms proportional to  $V_{ub}V_{ud}^*$  and  $V_{cb}V_{cd}^*$  using the unitarity relationship. The term proportional to  $V_{ub}V_{ud}^*$  is dumped with the tree amplitude, since all of them carry the same weak phase, and this combination is usually called  $T_c$ . The terms proportional to  $V_{cb}V_{cd}^*$  does not have any weak phase in the leading order of the Wolfenstein representation; this group is called  $P_c$ . The electroweak penguins are expected to be much smaller than both  $T_c$  and  $P_c$ , and they can be neglected, or treated separately, depending upon the precision required. In the approximation that the penguins are much smaller than the tree amplitude, and among the penguins, the top-quark mediated one is the dominant, we can use the numbers quoted in the literature for  $T_c$  and  $P_c$  for  $a_1$  and  $a_2$ , respectively. As a standard prediction in all theoretical models, one finds  $|P_c/T_c| \sim 0.25 - 0.35$  [11]. Thus the observable  $a_{CP}^d$  appears to be small in the SM. Moreover, as we shall see below, the measured BR for  $\pi^+\pi^-$  mode turns out to be a bit smaller than the SM prediction, the degree of discrepancy depending upon the method of calculation.

Thus, the explanation of the BR( $B \rightarrow \pi^+\pi^-$ ) data needs a destructive interference between the two amplitudes leading to large  $|\cos \Delta\delta|$ . On the other hand, large asymmetry as experimentally measured requires large  $|\sin \Delta\delta|$ . These twin requirements constrain the parameter space.

### III The $B^0 - \overline{B^0}$ mixing and $B \rightarrow \pi^+\pi^-$ decay in $R$ -parity violating supersymmetry

It is well known that in order to avoid rapid proton decay one cannot have both lepton number and baryon number violating RPV model, and we shall work with a lepton number violating one. This leads to slepton/sneutrino mediated  $B$  decays, and slepton/sneutrino/squark mediated  $B^0 - \overline{B^0}$  mixing. We start with the superpotential

$$\mathcal{W}_{\lambda'} = \lambda'_{ijk} L_i Q_j D_k^c, \quad (11)$$

where  $i, j, k = 1, 2, 3$  are quark and lepton generation indices;  $L$  and  $Q$  are the SU(2)-doublet lepton and quark superfields and  $D^c$  is the SU(2)-singlet down-type quark superfield, respectively. For the process  $B \rightarrow \pi^+\pi^-$ , the relevant four-Fermi operator is of the form

$$\mathcal{H}_{\lambda'} = \frac{\lambda'_{i11} \lambda'_{i13}{}^*}{2m_{e_L}^2} (\overline{u} \gamma^\mu P_L u)_8 (\overline{d} \gamma_\mu P_R b)_8 + \text{h.c.} \quad (12)$$

where  $P_R(P_L) = (1 + (-)\gamma_5)/2$ , and the subscript 8 denotes a color octet combination. In the above formula  $i$  is the generation index of the slepton. The current bound on  $\lambda'_{111}$  is too restrictive ( $|\lambda'_{111}| < 3.5 \times 10^{-4}$  [3]), which rules out the possibility that this coupling plays any significant role in  $B$  decays. For  $i = 2$  or 3, the bound on the product  $\lambda'_{i11} \lambda'_{i13}{}^*$  is rather modest ( $|\lambda'_{i11} \lambda'_{i13}{}^*| < 3.6 \times 10^{-3}$ ) [7]. Following the standard practice we shall assume that the RPV couplings are hierarchical *i.e.*, only one combination of the couplings is numerically significant. For simplicity we choose to ignore the RPV penguin contributions, which are expected to be small even compared to the SM penguin amplitudes; this follows from the smallness of the relevant RPV couplings compared to the SM gauge couplings. The bounds on the RPV couplings that we eventually derive are insensitive to the inclusion of RPV penguins.

There is a much stronger bound on the product couplings of the type  $\lambda'_{i13} \lambda'_{i31} \leq 8 \cdot 10^{-8}$  from tree-level  $B^0 - \overline{B^0}$  mixing (see [3]); however, we consider only one type of product coupling to be nonzero (namely,  $\lambda'_{i11} \lambda'_{i13}{}^*$ ) in our analysis.

Our discussion of  $B^0 - \overline{B^0}$  mixing in the framework of an  $L$ -violating RPV model follows that of [12]. The off-diagonal element in the  $2 \times 2$  effective Hamiltonian causes the  $B^0 - \overline{B^0}$  mixing. The mass difference between

the two mass eigenstates  $\Delta M$  is given by (following the convention of [13])

$$\Delta M = 2|M_{12}|, \quad (13)$$

with the valid approximation  $|M_{12}| \gg |\Gamma_{12}|$ . Let the SM amplitude be

$$|M_{12}^{SM}| \exp(-2i\theta_{SM}) \quad (14)$$

where  $\theta_{SM} = \beta$  for the  $B^0 - \overline{B^0}$  system. We follow the  $(\alpha, \beta, \gamma)$  convention for the unitarity triangle [13].

If we have  $n$  number of new physics (NP) amplitudes with weak phases  $\theta_n$ , one can write

$$M_{12} = |M_{12}^{SM}| \exp(-2i\theta_{SM}) + \sum_{i=1}^n |M_{12}^i| \exp(-2i\theta_i). \quad (15)$$

This immediately gives the effective mixing phase  $\theta_{eff}$  as

$$\theta_{eff} = \frac{1}{2} \arctan \frac{|M_{12}^{SM}| \sin(2\theta_{SM}) + \sum_i |M_{12}^i| \sin(2\theta_i)}{|M_{12}^{SM}| \cos(2\theta_{SM}) + \sum_i |M_{12}^i| \cos(2\theta_i)}, \quad (16)$$

and the mass difference between mass eigenstates as

$$\Delta M = 2 \left[ |M_{12}^{SM}|^2 + \sum_i |M_{12}^i|^2 + 2|M_{12}^{SM}| \sum_i |M_{12}^i| \cos 2(\theta_{SM} - \theta_i) + 2 \sum_i \sum_{j>i} |M_{12}^j| |M_{12}^i| \cos 2(\theta_j - \theta_i) \right]^{1/2}. \quad (17)$$

These are going to be our basic formulae. The only task is to find  $M_{12}^i$  and  $\theta_i$ .

The SM mixing amplitude, dominated by the top-quark loop, is

$$\begin{aligned} M_{12}^{SM} &\equiv \frac{\langle \overline{B^0} | H_{eff}^{SM} | B^0 \rangle}{2m_B} \\ &= \frac{G_F^2}{6\pi^2} (V_{td} V_{tb}^*)^2 \eta_B m_B f_B^2 B_B m_W^2 S_0(x_t). \end{aligned} \quad (18)$$

where

$$S_0(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x^3 \ln x}{2(1-x)^3}. \quad (19)$$

It is easy to check from the RPV superpotential that there are two different kind of boxes that contribute to  $B^0 - \overline{B^0}$  mixing: first, the one where one has two sfermions flowing inside the loop, alongwith two SM fermions [14], and secondly, the one where one slepton, one  $W$  (or charged Higgs or Goldstone) and two up quarks complete the loop [7]. It is obvious that the first amplitude is proportional to the product of four  $\lambda'$  type couplings, and the second to the product of two  $\lambda'$  type couplings times  $G_F$ . We call them L4 and L2 boxes, respectively. The detailed calculations including the QCD corrections at the next-to-leading order (NLO), which we follow in our present analysis, may be found in [12].

In the presence of RPV, the  $\Delta B = 2$  effective Hamiltonian can be written as

$$\mathcal{H}_{eff}^{\Delta B=2} = \sum_{i=1}^5 c_i(\mu) O_i(\mu) + \sum_{i=1}^3 \tilde{c}_i(\mu) \tilde{O}_i(\mu) + \text{h.c.} \quad (20)$$

where  $\mu$  is the regularization scale, and

$$\begin{aligned} O_1 &= (\overline{b}\gamma^\mu P_L d)_1 (\overline{b}\gamma_\mu P_L d)_1, \\ O_2 &= (\overline{b}P_R d)_1 (\overline{b}P_R d)_1, \\ O_3 &= (\overline{b}P_R d)_8 (\overline{b}P_R d)_8, \\ O_4 &= (\overline{b}P_L d)_1 (\overline{b}P_R d)_1, \\ O_5 &= (\overline{b}P_L d)_8 (\overline{b}P_R d)_8, \end{aligned} \quad (21)$$

where  $P_{R(L)} = (1 + (-)\gamma_5)/2$ . The subscripts 1 and 8 indicate whether the currents are in colour-singlet or in colour-octet combination. The  $\tilde{O}_i$ s are obtained from corresponding  $O_i$ s by replacing  $L \leftrightarrow R$ . The Wilson coefficients  $c_i$  and  $\tilde{c}_i$  at  $q^2 = m_W^2$  and at the low energy scale have been taken from [15].

Now we come to the decay amplitude. The matrix element of the RPV operator for  $B \rightarrow \pi^+\pi^-$  is given, using conventional factorization [9], by

$$\langle \pi^+\pi^- | \mathcal{H}_{\lambda'} | \overline{B^0} \rangle = -\frac{1}{4} \frac{\lambda'_{i11} \lambda'_{i13}^*}{m_{\tilde{e}_{L_i}}^2} \frac{m_\pi^2}{(m_d + m_u)(m_b - m_u)} f_\pi F_0^{B \rightarrow \pi}(m_\pi^2)(m_B^2 - m_\pi^2) \quad (22)$$

where  $f_\pi$  is the pion decay constant and  $F_0$  is the Bauer-Stech-Wirbel (BSW) form factor. We use conventional factorization [9] for simplicity.

## IV Numerical results

We now turn to the numerical results. The calculational intricacies leading to our results are two-fold. First, we must bring in the RPV box diagram for reasons discussed above. The dependence on the weak phase associated with the complex RPV product couplings  $\lambda'_{i11} \lambda'_{i13}^*$  is therefore rather involved and intriguing. Second, although for illustration we have often discussed in the text the interference of SM tree and RPV tree amplitudes only, leading to CP asymmetries in the  $B \rightarrow \pi\pi$  channel, for actual numerical details we have included the SM penguin as the third interfering amplitude, and its effect is numerically non-negligible. Although the introduction of RPV introduces a few more parameters (see below), we now have sufficient number of constraints. As a consequence, the new results, in particular the constraints on the magnitude of  $\sin \Delta\delta$  and the product coupling  $\lambda'_{i11} \lambda'_{i13}^*$  have a very restrictive pattern.

The RPV model introduces four extra parameters compared to the SM: (a) the left slepton and sneutrino masses, which are equal up to the SU(2) breaking D-terms, (b) the magnitude of the product  $\lambda'_{i11} \lambda'_{i13}^*$  (which according to our convention can be either positive or negative), (c) the phase of this product, hereafter called  $\phi$  or the weak RPV phase, which can have any value between 0 and  $\pi$  to maintain consistency with the sign convention in (b), and (d) the strong phase between the SM tree and the RPV amplitudes varying between 0 and  $2\pi$ . We fix the slepton mass at 100 GeV which is consistent with the current bound coming from direct searches. The squarks are taken to be degenerate with the sleptons. Even though that is unrealistic from the Tevatron data, the numbers change only marginally. The magnitude of the product coupling and the remaining two parameters are randomly varied within their allowed ranges or bounds.

In order to carry out the numerical analysis we need some more inputs like quark masses, form factors and the relevant CKM elements. We use  $m_u = 4.2$  MeV,  $m_d = 7.6$  MeV,  $m_b = 4.88$  GeV, pion decay constant  $f_\pi = 132$  MeV, and the decay form factor in the BSW model  $F_0^{B \rightarrow \pi}(m_\pi^2) = 0.39$ . The relevant Wilson coefficients for  $b \rightarrow d$  transition in the LO scheme are taken from [9]. We expect the strong phase difference between the SM tree and the SM penguin amplitudes to be small, but do not impose it as a constraint. The CKM parameters whose values are not precisely known have been varied randomly within the range allowed by the CKM fit [16]. In particular  $V_{td}$  is allowed to lie in the range between 0.0030 and 0.0096. Arguably such ranges may change in the presence of RPV, since the  $B^0 - \overline{B^0}$  mixing amplitude and the resulting mass difference of  $B$  meson mass eigenstates ( $\Delta m_{B^0}$ ), an important ingredient of the CKM fit, are affected for reasons discussed above. In order to compensate for the restricted inputs we have not constrained the weak phase  $\gamma$  within the SM range, but varied it randomly in the entire range of 0 to  $\pi^2$ . The other important input parameter  $\sin(2\beta)$  has been varied in the range allowed by the CKM fit:  $0.725 \pm 0.033$  at 68% C.L. [16]. The justification for using such limits is that the usual CKM fit without the direct measurement of CP asymmetries yields  $\sin(2\beta)$  very close to this range. We have checked that none of our results, apart from the allowed range of RPV weak phase, depends

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<sup>2</sup>The angle  $\gamma$  is expected to lie in the first quadrant from the CKM fit when  $B^0 - \overline{B^0}$  mixing data, *i.e.*,  $\Delta m_{B^0}$ , is taken into account. Once we entertain the possibility of new physics in  $B^0 - \overline{B^0}$  mixing, that constraint is no longer applicable, and there is a second possible solution, with  $\gamma$  in the second quadrant [16].

sensitively on the choice of the angle  $\beta$ , and thus this analysis holds for some other slightly different CKM fits too.

The origin of the strong phase can at present only be modeled. In naive factorization, admittedly, there cannot be any strong phase between two tree amplitudes. But in pQCD, dynamical enhancement of annihilation and exchange topologies play a significant part in generating a significant strong phase difference. Our approach has been the following: (i) we have used factorization model as a simplistic approach to calculate amplitudes, *but* (ii) we have treated the strong phase difference between the dominant interfering amplitudes as a *phenomenological parameter*.

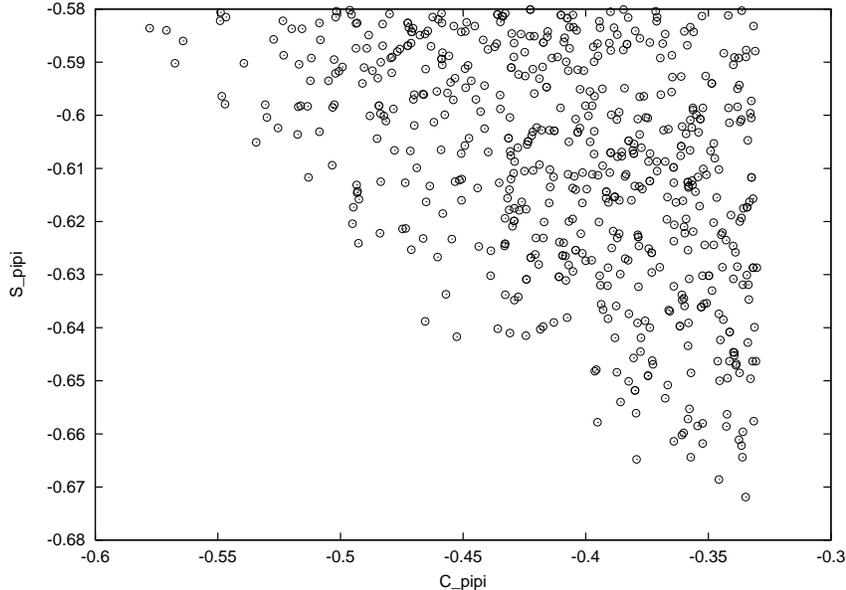


Figure 1: Correlation of  $S_{\pi\pi}$  and  $C_{\pi\pi}$ .

As stated above the effective Hamiltonian in Eq. (13) also leads to a pair of new box amplitudes for  $B^0 - \overline{B^0}$  mixing, *viz.*, L2 and L4. The first kind has two  $\lambda'$  vertices, two SU(2) gauge couplings, and two up quarks, one slepton and one  $W$  inside the box. We have neglected, for simplicity, the imaginary part which arises in this diagram from cutting the up quark lines. The second type has four  $\lambda'$  vertices. Neglecting the SM box completely, and taking the product coupling to be real, the authors in [7] found the conservative bound  $|\lambda'_{i11}\lambda'_{i13}| \leq 3.6 \times 10^{-3}$ . We, on the other hand, take into account the SM box and the possible phase of the RPV product coupling which is randomly varied over the range already given. This, as discussed above, modifies the phase  $\phi_M$  from its SM value of  $2\beta$  to  $2\beta_{eff}$ . We now impose the constraint that  $\sin(2\beta_{eff})$  should satisfy the observed CP-asymmetry in the  $B \rightarrow J/\psi K_S$  channel (*i.e.*,  $\beta_{eff}$ , which is a combination of  $\beta$ , RPV weak phase  $\phi$ , and the box amplitudes, should satisfy  $0.692 \leq \sin(2\beta_{eff}) \leq 0.758$ ).

We next list all the constraints imposed in our study of the allowed space of the RPV parameters: (i)  $\Delta m_{B^0}$ , (ii) CP asymmetry from the decay  $B \rightarrow J/\psi K_S$ , (iii)  $\text{BR}(B \rightarrow \pi^+\pi^-)$ , and (iv) the asymmetries  $C_{\pi\pi}$  and  $S_{\pi\pi}$ . In addition, we also impose the model independent constraint  $S_{\pi\pi}^2 + C_{\pi\pi}^2 < 1$ . All the experimental numbers are taken at  $1\sigma$ .

The random variation of the parameters subject to the constraints as discussed above leads to the scatter plots displayed in Figures 1 and 2. The following salient features are to be noted.

1. From Figure 1, one can see that though  $C_{\pi\pi}$  can be accommodated over its entire range,  $S_{\pi\pi}$  has a rather narrow allowed range:  $-0.67 < S_{\pi\pi} < -0.58$ . The correlation between  $C_{\pi\pi}$  and  $S_{\pi\pi}$  is also to be noted.
2. The unitarity triangle angle  $\gamma$  lies in the second quadrant:  $112^\circ < \gamma < 146^\circ$ . This range changes a bit if we change the allowed range of  $\sin(2\beta_{eff})$ . However, in no case it goes to the first quadrant, as happens in the

pure SM case. This is understandable: one needs to have destructive interference between the SM tree and the SM penguin amplitudes to lower the BR. If one tries to do this entirely with the RPV amplitude one ends up with unacceptable values of  $S_{\pi\pi}$  and  $C_{\pi\pi}$ . The above range of  $\gamma$  should *not* be interpreted as in conflict with the standard CKM fit results. The reason is that once we have new physics in  $B^0 - \overline{B}^0$  mixing, the  $V_{td}$  and  $\sin(2\beta)$  constraints no longer apply. Also note that the  $B \rightarrow \pi K$  analysis predicts  $\gamma$  in the second quadrant [10].

3. The RPV coupling has a new upper bound for 100 GeV sfermions (see Figure 2),

$$|\lambda'_{i11}\lambda'_{i13}| \leq 2.2 \times 10^{-3}. \quad (23)$$

The RPV weak phase lies in the third quadrant if this coupling is taken to be positive. This bound is more or less stable against the variation of  $\beta$ .

4. The strong phase difference between the SM tree and the SM penguin amplitudes can be kept small, as dictated by different theoretical models<sup>3</sup>. On the other hand, the strong phase difference between the SM tree and the RPV amplitudes, treated as a phenomenological parameter, has a very restrictive range:  $-40^\circ < \delta < 40^\circ$ . The reason for this restriction is that the CP asymmetries are mainly controlled by the SM tree and the RPV amplitudes.

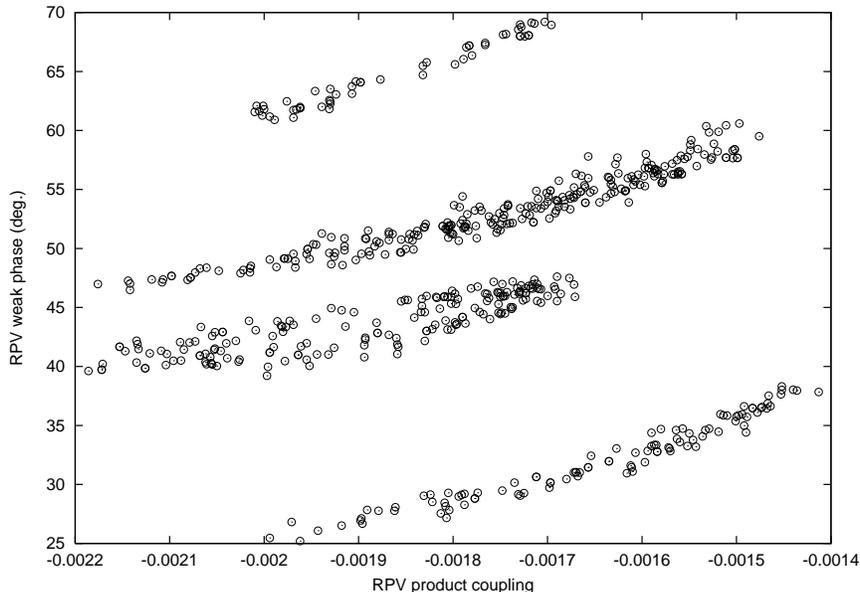


Figure 2: Allowed parameter space for the magnitude and the phase of the product coupling  $\lambda'_{i11}\lambda'_{i13}$ .

A comment on the robustness of our bound on the product coupling is now in order. If we vary the experimental data by  $\pm 2\sigma$  and allow for the uncertainties of the involved parameters, the constraint in Eq. (23) relaxes by about 20%.

So far we have focussed our attention on the quark level process  $b \rightarrow u\overline{u}d$  and studied its impact in  $B \rightarrow \pi^+\pi^-$  decay. It is now time to wonder what would be the impact of the SU(2) conjugate quark level process  $b \rightarrow d\overline{d}d$ ? Both operators contributes to  $B \rightarrow \pi^0\pi^0$  and to understand the nature of the SM and RPV contributions to this process it is important to recall that the quark composition of  $\pi^0$  is the antisymmetric combination  $(u\overline{u} - d\overline{d})/\sqrt{2}$ . In the SM, while  $b \rightarrow u\overline{u}d$  corresponds to a colour suppressed tree diagram,  $b \rightarrow d\overline{d}d$  can proceed only through penguin graphs. It turns out that a part of the  $b \rightarrow d\overline{d}d$  RPV amplitude is almost exactly (upto the SU(2) D-term splitting between sleptons and sneutrinos) cancelled by the corresponding  $b \rightarrow u\overline{u}d$

<sup>3</sup>Our analysis has been carried out in the context of naive factorization [17, 9] model. The strong phase is small here, since that comes only from the imaginary part of the respective Wilson coefficients. It is also small in QCD factorization model. In pQCD [18] it is not so small since the annihilation topologies are taken into account. However, one should note that the pQCD analysis uses  $T_c$  and  $P_c$ , which are *not exactly* identical to our tree and penguin amplitudes, respectively.

amplitude. However, another part remains, and that can significantly enhance the  $B \rightarrow \pi^0\pi^0$  branching ratio. The  $B^+ \rightarrow \pi^+\pi^0$  data is satisfied by the allowed RPV parameter space.

The RPV scenario that we have considered in this paper can be directly tested at colliders. If RPV indeed contributes to  $B$  decays as discussed in this paper, the associated light sleptons/sneutrinos, with masses in the range 100 – 300 GeV, mediating such decays are very likely to be produced at the Tevatron and, most certainly, at the LHC. Thus resonant production of sleptons/sneutrinos [19] will provide a useful cross-check of this scenario. The  $\lambda'_{i11}$  couplings (in particular,  $\lambda'_{211}$ ) give rise to a distinct collider signature in the form of like-sign dilepton signals. Such final states have low SM and  $R$ -parity conserving supersymmetry background. The dominant production mechanism is a  $\lambda'$  induced resonant charged slepton production at tree level at hadron colliders. This is followed by a  $R$ -parity conserving gauge decay of the charged slepton into a neutralino and a charged lepton. The neutralino can then decay via the crossed process to give rise to a second charged lepton, which due to the majorana nature of the neutralino can have the same charge as the hard lepton produced in the slepton decay. It is gratifying to note that the study of Ref. [19] shows that for a value of  $\lambda'_{211} = 0.05$ , which is perfectly compatible with our bound on the product  $\lambda'_{211}\lambda'_{213}$ , a smuon mass of about 300 GeV would be visible above the backgrounds with  $2 \text{ fb}^{-1}$  integrated luminosity at the Tevatron Run II, while for the same coupling a resonant smuon can be observed with a mass of 750 GeV at LHC with  $10 \text{ fb}^{-1}$  integrated luminosity. It is, therefore, reasonable to expect smuon signals already at the upgraded Tevatron.

## V Conclusions

1. RPV contribution to the decay  $B \rightarrow \pi^+\pi^-$  mode is interesting because the same new physics amplitude affects both  $B^0 - \overline{B}^0$  mixing and the decay. Their interplay leads to important constraints. The constraints are expected to be tighter as more data accumulate.
2. As we have shown, the upper bound on the magnitude of the RPV product coupling has been improved over the existing results. The RPV weak phase also gets restricted.
3. A bonus of RPV is that one can enhance the  $B \rightarrow \pi^0\pi^0$  branching ratio to a significant level, which is a much desired result in view of what one finds from the current experiments.

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